

Regresión Lineal Logística

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Machine Learning vs Análisis Estadístico

Ejemplo pricing de una empresa: si quiero optimizar precios sera machine learning (no puedo interpretar coeficientes del modelo como en Analisis Estadístico). En machine learning hago pruebas de estabilidad (cambio en training) Si quieres algo que prediga directamente utilizas Machine Learning.

Si quieres entender porque ocurren las cosas y como se interpretan las cosas....Análisis estadístico. (como se fija y porque, por ejemplo, para explicar a un cliente)

Modelos machine learning y Analisis Estadísticos no son los mismo.

Tenemos:

- Modelo, saco coeficientes según que minimizo, no hay un modelo hay muchos modelos, hay diferentes coeficientes según si minimizo error, max beneficios, minimiza error cuadrado.... a partir de esa formula predecimos.
- Estimación
- Explotación

Analisis Estadístico

Regresión lineal: me interesan las variables que pueda controlar, sobre las que pueda hacer algun cambio para mejorar el resultado.

Hipótesis: creo que si bajo el precio subirán las ventas un 20%, hago un experimento y saco mi conclusion al 95%

Las variables tienen que ser independientes, la varianza tiene que ser igual para todas las observaciones, los errores tienen distribucion normal.

Si no se cumple lo anterior la interpretación nos dará datos pero no será completamente correcto. Pero a veces es mejor tener esta información a nada.

Estimación: MCO minimos cuadrados, el valor que tenía menos el valor que tiene por las variables predictivas. El error es la realidad menos la estimación. Lo elevo al cuadrado para que los negativos y positivos no se compensen. Busco el valor de "a"(coeficientes) que haga que eso sea más pequeño.(Los coeficientes solo son insesgados en MCO, el valor aprox el limite al valor real) Si utilizo valores absolutos me dará otros coeficientes.

RSA casos que no se pueden resolver. Buscar un password, hay tantas combinaciones que la tecnologia no lo permite.

gradient accent:busca optimos locales, iterando, tenga la forma que tenga la funcion, siempre te da algo.

Descomposición de la varianza:

Quiero calcular las variaciones de y (variable dependiente), tenemos la varianza de y, tengo la varianza del error. varianza y = varianza modelo + varianza error

varianza del modelo/varianza y = % coef denominacion que porcentaje de y está representado en el modelo

Hay que poner un criterio, entre que porcentaje tiene que estar. Hay cosas que serán totalmente subjetivos. Si todas las variables que tu quieras estudiar están en el modelo, para ti estará bien aunque salga error alto.

Ejemplo Regresion lineal: lm(cantidad~Precio,data=ventas)

Tengo que saber en que unidades estan las variables, miles.... Tiene que tener sentido el modelo, puede no ser lineal.

std.error si es igual que a, mal asunto es error es igual que el coeficiente. (cuando hago el lm)

t value es el coef entre estimate/15.778 o lo que es lo mismo a/std.error . El p valor es la probabilidad que sea mayor que el valor absoluto de t value.

Multiple R-squared:0.8233 el modelo ajusta el 0.82%, y está representado en el modelo este porcentaje (coef.determinacion)

Adjusted R-squared: Te penaliza por no tener algun parametro. Pero te da una ganancia. La gente se suele fijar en éste. N registros, K variables. Si aumento K le cociente es más grande

F-statistic: contraste de todo el modelo. el modelo sirve para algo?? compara modelos, mejora tener el modelo solo de la constante. p-value es menor de 0.01, aporta algo.

El contraste F te permite comparar un modelo con el siguiente, el primero tiene solo una variable, añado otra, entonces comparo los dos para ver si me sirve o no meter esa variable.

Práctica Regresión Lineal

Iniciamos librerias:

```
library(ggplot2)
library(effects)
library(plyr)
library(ROCR)

## Loading required package: gplots

##
## Attaching package: 'gplots'

## The following object is masked from 'package:stats':
##      lowess
```

Carga de datos:

```
creditos=read.csv("D:/master/data/Regresiones/creditos.csv",stringsAsFactors = FALSE)
```

Revisión de los datos:

- income es salario hora
- product contratados por el cliente
- education años educación, cuantos mas mas educación
- Balance saldo cuenta cliente

```
str(creditos)
```

```

## 'data.frame':   300 obs. of  10 variables:
## $ Income    : num  14.9 106 104.6 148.9 55.9 ...
## $ Rating    : int  283 483 514 681 357 569 259 512 266 491 ...
## $ Products  : int  2 3 4 3 2 4 2 2 5 3 ...
## $ Age       : int  34 82 71 36 68 77 37 87 66 41 ...
## $ Education : int  11 15 11 11 16 10 12 9 13 19 ...
## $ Gender    : chr  "Male" "Female" "Male" "Female" ...
## $ Mortgage  : chr  "No" "Yes" "No" "No" ...
## $ Married   : chr  "Yes" "Yes" "No" "No" ...
## $ Ethnicity : chr  "Caucasian" "Asian" "Asian" "Asian" ...
## $ Balance   : int  333 903 580 964 331 1151 203 872 279 1350 ...

```

```
head(creditos)
```

	Income	Rating	Products	Age	Education	Gender	Mortgage	Married	Ethnicity
## 1	14.891	283	2	34	11	Male	No	Yes	Caucasian
## 2	106.025	483	3	82	15	Female	Yes	Yes	Asian
## 3	104.593	514	4	71	11	Male	No	No	Asian
## 4	148.924	681	3	36	11	Female	No	No	Asian
## 5	55.882	357	2	68	16	Male	No	Yes	Caucasian
## 6	80.180	569	4	77	10	Male	No	No	Caucasian
## Balance									
## 1		333							
## 2		903							
## 3		580							
## 4		964							
## 5		331							
## 6		1151							

```
tail(creditos)
```

	Income	Rating	Products	Age	Education	Gender	Mortgage	Married	
## 295	42.915	205	4	42	13	Male	No	Yes	
## 296	27.272	149	5	67	10	Female	No	Yes	
## 297	65.896	370	1	49	17	Female	No	Yes	
## 298	55.054	321	3	74	17	Male	No	Yes	
## 299	20.791	204	1	70	18	Female	No	No	
## 300	24.919	372	3	76	11	Female	No	Yes	
## Ethnicity Balance									
## 295		Asian		0					
## 296		Caucasian		0					
## 297		Caucasian		293					
## 298		Asian		188					
## 299	African American			0					
## 300	African American			711					

```
summary(creditos)
```

	Income	Rating	Products	Age
## Min.	10.35	Min. : 93.0	Min. :1.000	Min. :24.00
## 1st Qu.	21.03	1st Qu.:235.0	1st Qu.:2.000	1st Qu.:41.00
## Median	33.12	Median :339.0	Median :3.000	Median :55.00

```

##  Mean    : 44.05   Mean    :348.1   Mean    :3.027   Mean    :54.98
##  3rd Qu.: 55.98   3rd Qu.:433.0   3rd Qu.:4.000   3rd Qu.:69.00
##  Max.   :186.63   Max.   :949.0   Max.   :8.000   Max.   :91.00
##  Education       Gender          Mortgage        Married
##  Min.   : 5.00   Length:300      Length:300      Length:300
##  1st Qu.:11.00   Class :character  Class :character  Class :character
##  Median :14.00   Mode  :character  Mode  :character  Mode  :character
##  Mean   :13.39
##  3rd Qu.:16.00
##  Max.   :20.00
##  Ethnicity       Balance
##  Length:300      Min.   : 0.00
##  Class :character 1st Qu.: 15.75
##  Mode  :character  Median : 433.50
##                      Mean   : 502.69
##                      3rd Qu.: 857.75
##                      Max.   :1809.00

```

Tratamiento de variables:

La regresion lineal y logistica trabaja con Factores El coef sobre la variable Educacion no es continua, aplico el mismo coeficiente a cada año que tengo de mas de educacion b1*x1, pero no es lo mismo 10 años que 11 que he terminado.(eso seria tratarlo como una variable continua)

```

creditos$Gender=as.factor(creditos$Gender)
creditos$Mortgage=as.factor(creditos$Mortgage)
creditos$Married=as.factor(creditos$Married)
creditos$Ethnicity=as.factor(creditos$Ethnicity)

summary(creditos)

```

	Income	Rating	Products	Age	
##	Min. : 10.35	Min. : 93.0	Min. :1.000	Min. :24.00	
##	1st Qu.: 21.03	1st Qu.:235.0	1st Qu.:2.000	1st Qu.:41.00	
##	Median : 33.12	Median :339.0	Median :3.000	Median :55.00	
##	Mean : 44.05	Mean :348.1	Mean :3.027	Mean :54.98	
##	3rd Qu.: 55.98	3rd Qu.:433.0	3rd Qu.:4.000	3rd Qu.:69.00	
##	Max. :186.63	Max. :949.0	Max. :8.000	Max. :91.00	
##	Education	Gender	Mortgage	Married	
##	Min. : 5.00	Male :132	No :268	No :117	Ethnicity
##	1st Qu.:11.00	Female:168	Yes: 32	Yes:183	African American: 78
##	Median :14.00				Asian : 81
##	Mean :13.39				Caucasian :141
##	3rd Qu.:16.00				
##	Max. :20.00				
##	Balance				
##	Min. : 0.00				
##	1st Qu.: 15.75				
##	Median : 433.50				
##	Mean : 502.69				
##	3rd Qu.: 857.75				
##	Max. :1809.00				

Test diferencia de medias Regresion lineal:

```
t.test(Income ~ Gender, data = creditos) #p-value=0.7345 t=0.3395, mean female=43,46, male=44,8
```

```
##  
## Welch Two Sample t-test  
##  
## data: Income by Gender  
## t = 0.3395, df = 284.51, p-value = 0.7345  
## alternative hypothesis: true difference in means is not equal to 0  
## 95 percent confidence interval:  
## -6.405656 9.075923  
## sample estimates:  
## mean in group Male mean in group Female  
## 44.80207 43.46693
```

Resultado: No hay evidencia significativa de que sean diferentes. No podemos rechazar la igualdad de las medias

Modelo Lineal En este caso, el R² es simplemente el cuadrado del coeficiente de correlación de Pearson, lo cual es sólo cierto para la regresión lineal simple

```
modeloT=lm(Income ~ Gender, data = creditos)  
summary(modeloT)
```

```
##  
## Call:  
## lm(formula = Income ~ Gender, data = creditos)  
##  
## Residuals:  
##     Min      1Q Median      3Q     Max  
## -34.45 -22.49 -11.33  11.99 143.17  
##  
## Coefficients:  
##             Estimate Std. Error t value Pr(>|t|)  
## (Intercept)  44.802     2.952 15.178 <2e-16 ***  
## GenderFemale -1.335     3.944 -0.338   0.735  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 33.91 on 298 degrees of freedom  
## Multiple R-squared:  0.0003843, Adjusted R-squared: -0.00297  
## F-statistic: 0.1146 on 1 and 298 DF,  p-value: 0.7352
```

Recta de regresión es $y = 44.802 - 1.335x$ ($x=1$ cuando es mujer, 0 cuando es hombre) std-error= 2.952
p-value=0.7352 cuando $x=1$, $y = 43,46$ Podría coger otro modelo e ir metiendo variables, el modelo perfecto es el que las variables fueran independientes si no lo son te generan ruido, cuando sube una baja la otra.
Resultado:

No aporta nada este modelo casi es el valor de la constante.

Regresión Lineal Individual

Mide correlación, no mide causa-efecto. Está relacionado, ejemplo, la edad influye en el impago, con la edad gana más dinero, lo que influye no es la edad es el ingreso... Si metes la variable ingreso la edad desaparece. Vamos viendo una a una las variables:

```
modeloInd1=lm(Income ~ Rating, data = creditos)# Rating es Puntuaje sobre la capacidad impago
#de 0-1000, cuanto mas grande mejor pagador.
summary(modeloInd1)#el rating explica el 60% de los ingresos o los ingresos explican el 60% del
```

```
##
## Call:
## lm(formula = Income ~ Rating, data = creditos)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -40.05 -15.74  -0.80  14.14  81.48
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -16.200514   3.139692  -5.16 4.52e-07 ***
## Rating       0.173088   0.008278   20.91 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 21.59 on 298 degrees of freedom
## Multiple R-squared:  0.5947, Adjusted R-squared:  0.5933
## F-statistic: 437.3 on 1 and 298 DF,  p-value: < 2.2e-16
```

```
#rating
```

```
modeloInd2=lm(Income ~ Products, data = creditos)#no influye
summary(modeloInd2)
```

```
##
## Call:
## lm(formula = Income ~ Products, data = creditos)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -36.86 -22.28 -10.90  12.55 143.32
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 41.8640    4.8091   8.705  <2e-16 ***
## Products     0.7237    1.4513   0.499    0.618
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 33.91 on 298 degrees of freedom
## Multiple R-squared:  0.0008337, Adjusted R-squared:  -0.002519
## F-statistic: 0.2487 on 1 and 298 DF,  p-value: 0.6184
```

```
modeloInd3=lm(Income ~ Age, data = creditos)
summary(modeloInd3)
```

```
##
```

```

## Call:
## lm(formula = Income ~ Age, data = creditos)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -37.58 -23.34 -10.35  10.45 145.97
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 30.7310   6.5135  4.718 3.67e-06 ***
## Age         0.2423   0.1131  2.143  0.0329 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 33.66 on 298 degrees of freedom
## Multiple R-squared:  0.01518, Adjusted R-squared:  0.01187
## F-statistic: 4.593 on 1 and 298 DF, p-value: 0.03291

```

```

modeloInd4=lm(Income ~ Education, data = creditos)#p-value 0.22 no es significativo
summary(modeloInd4)

```

```

##
## Call:
## lm(formula = Income ~ Education, data = creditos)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -38.65 -22.54 -11.81  12.12 143.05
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 54.5197   8.7430  6.236 1.54e-09 ***
## Education   -0.7814   0.6363 -1.228     0.22
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 33.83 on 298 degrees of freedom
## Multiple R-squared:  0.005035, Adjusted R-squared:  0.001696
## F-statistic: 1.508 on 1 and 298 DF, p-value: 0.2204

```

```

modeloInd5=lm(Income ~ Gender, data = creditos)
summary(modeloInd5)

```

```

##
## Call:
## lm(formula = Income ~ Gender, data = creditos)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -34.45 -22.49 -11.33  11.99 143.17
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
```

```

## (Intercept) 44.802      2.952   15.178 <2e-16 ***
## GenderFemale -1.335      3.944   -0.338    0.735
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 33.91 on 298 degrees of freedom
## Multiple R-squared: 0.0003843, Adjusted R-squared: -0.00297
## F-statistic: 0.1146 on 1 and 298 DF, p-value: 0.7352

modeloInd6=lm(Income ~ Mortgage, data = creditos)#no influye la hipoteca
summary(modeloInd6)

```

```

##
## Call:
## lm(formula = Income ~ Mortgage, data = creditos)
##
## Residuals:
##   Min     1Q Median     3Q    Max
## -38.64 -22.94 -10.58  12.06 143.20
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 43.432     2.069  20.991 <2e-16 ***
## MortgageYes  5.833     6.335   0.921    0.358
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 33.87 on 298 degrees of freedom
## Multiple R-squared: 0.002837, Adjusted R-squared: -0.0005095
## F-statistic: 0.8477 on 1 and 298 DF, p-value: 0.3579

```

```

modeloInd7=lm(Income ~ Married, data = creditos)
summary(modeloInd7)

```

```

##
## Call:
## lm(formula = Income ~ Married, data = creditos)
##
## Residuals:
##   Min     1Q Median     3Q    Max
## -34.76 -22.74 -10.78  11.93 141.53
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 42.404     3.134  13.533 <2e-16 ***
## MarriedYes    2.705     4.012   0.674    0.501
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 33.89 on 298 degrees of freedom
## Multiple R-squared: 0.001523, Adjusted R-squared: -0.001828
## F-statistic: 0.4545 on 1 and 298 DF, p-value: 0.5007

```

```

modeloInd8=lm(Income ~ Ethnicity, data = creditos)
summary(modeloInd8)

##
## Call:
## lm(formula = Income ~ Ethnicity, data = creditos)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -36.14 -22.87 -11.75  12.21 139.99
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 46.641     3.843   12.136 <2e-16 ***
## EthnicityAsian -3.557     5.384  -0.661   0.509
## EthnicityCaucasian -3.461     4.790  -0.723   0.471
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 33.94 on 297 degrees of freedom
## Multiple R-squared:  0.002059, Adjusted R-squared:  -0.004661
## F-statistic: 0.3064 on 2 and 297 DF, p-value: 0.7363

modeloInd9=lm(Income ~ Balance, data = creditos)#si influye, y explica el 0.1869 del income
summary(modeloInd9)

##
## Call:
## lm(formula = Income ~ Balance, data = creditos)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -57.246 -17.986 -5.543  9.792 119.167
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 28.295463  2.596872 10.896 < 2e-16 ***
## Balance      0.031349  0.003788  8.277 4.3e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 30.59 on 298 degrees of freedom
## Multiple R-squared:  0.1869, Adjusted R-squared:  0.1842
## F-statistic: 68.5 on 1 and 298 DF, p-value: 4.3e-15

```

Regresión Linial Múltiple

Introduzco todas las variables, ahora la hipoteca si que influye, porque las variables no son independientes a igualdad de todas las variables, los que tienen hipoteca tendrán menor rating....efectos conjuntos. Hemos capturado el 0.8965 del Income, todas las variables explican el 89,65 % de los ingresos. Multiple R-Squared. vamos a quitar el resto de las variables(menos rating, Balance y Mortgage), para ver las diferencias....

```

modeloMul1=lm(Income ~ ., data = creditos)
summary(modeloMul1)

##
## Call:
## lm(formula = Income ~ ., data = creditos)
##
## Residuals:
##    Min     1Q Median     3Q    Max 
## -19.484 -7.151 -2.572  5.736 33.034 
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -55.765554   4.614857 -12.084 <2e-16 ***
## Rating       0.414226   0.009515  43.533 <2e-16 ***
## Products     0.194289   0.480488   0.404   0.686  
## Age          -0.020811   0.038354  -0.543   0.588  
## Education    -0.114133   0.210614  -0.542   0.588  
## GenderFemale -1.042506   1.296913  -0.804   0.422  
## MortgageYes  39.923786   2.455449  16.259 <2e-16 ***
## MarriedYes   -1.028593   1.337745  -0.769   0.443  
## EthnicityAsian 1.740019   1.782636   0.976   0.330  
## EthnicityCaucasian 0.291304   1.571722   0.185   0.853  
## Balance      -0.091405   0.003220 -28.386 <2e-16 ***
## --- 
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 
##
## Residual standard error: 11.08 on 289 degrees of freedom
## Multiple R-squared:  0.8965, Adjusted R-squared:  0.893 
## F-statistic: 250.4 on 10 and 289 DF,  p-value: < 2.2e-16

```

Multiple R-squared:

Múltiple R-cuadrado se utiliza para evaluar como el modelo se ajusta a los datos. Te dice cuánto de la variación en la variable dependiente (la variable predicha) puede ser explicado por las variables independientes (las variables de predicción). Por ejemplo, un R valor de 0.75 -squared implica que el modelo puede explicar las tres cuartas partes de la variación en el resultado. Ahora, para entender la diferencia entre ellos, es importante saber que cada vez que se agrega una variable independiente al modelo , el valor R-cuadrado se incrementará . Porqué es eso ? Debido a que el modelo trata de capturar tanto la información como cualquier ruido en la nueva variable. No sabemos si el aumento en el valor R cuadrado es debido a la capacidad de predicción real de la nueva variable o debido a la casualidad. Ajustado R-cuadrado también proporciona la misma información que R cuadrado, pero se ajusta para el número de términos en el modelo. No aumenta monótonamente como R-cuadrado, pero sólo aumenta cuando la nueva variable en realidad tiene un efecto sobre el valor predicho. Se disminuye cuando la nueva variable no tiene ningún impacto real sobre el valor predicho.

R <- 0.8965 N <- 300 p <- 10

Radj <- 1-((1-R)*(N-1))/(N-p-1))

Comparación de modelos

Comparamos el Income con el Rating y el Income con el Rating y todas las demás. El Rating explicaba el 60% del Income. modeloInd1=lm(Income ~ Rating, data = creditos) modeloMul1=lm(Income ~ ., data = creditos)

```

anova(modeloInd1,modeloMul1)

## Analysis of Variance Table
##
## Model 1: Income ~ Rating
## Model 2: Income ~ Rating + Products + Age + Education + Gender + Mortgage +
##           Married + Ethnicity + Balance
##   Res.Df   RSS Df Sum of Sq    F    Pr(>F)
## 1     298 138964
## 2     289 35476  9    103488 93.673 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

#sale que tiene sentido

- Sum of squares
- df:degrees of freedom
- RSS: Sum squares
- F:F- ratio
- Pr(>F):p-value menor que 0.05

¿Cuales serian las variables que incluiriamos en el modelo?

```

modeloMul2=lm(Income ~Rating+Balance+Mortgage, data = creditos)
summary(modeloMul2)

```

```

##
## Call:
## lm(formula = Income ~ Rating + Balance + Mortgage, data = creditos)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -20.263  -7.425  -2.544   6.158  32.580
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -58.178638   2.153774 -27.01 <2e-16 ***
## Rating       0.412810   0.009254  44.61 <2e-16 ***
## Balance      -0.090950   0.003122 -29.13 <2e-16 ***
## MortgageYes  39.811486   2.408393  16.53 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11 on 296 degrees of freedom
## Multiple R-squared:  0.8955, Adjusted R-squared:  0.8944 
## F-statistic: 845.1 on 3 and 296 DF,  p-value: < 2.2e-16

```

Estas tres variables explican el 89.55% de la varianza del Income. 39.8 mas de income el que tiene hipoteca con respecto al que no. El dato que sale es con respecto a los que no tienen hipoteca.

```
anova(modeloInd1,modeloMul2)

## Analysis of Variance Table
##
## Model 1: Income ~ Rating
## Model 2: Income ~ Rating + Balance + Mortgage
##   Res.Df   RSS Df Sum of Sq    F    Pr(>F)
## 1     298 138964
## 2     296 35847  2     103117 425.74 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

La suma de cuadrados es casi igual que cuando comparamos anteriormente con todas las variables.

```
anova(modeloMul2,modeloMul1)
```

```
## Analysis of Variance Table
##
## Model 1: Income ~ Rating + Balance + Mortgage
## Model 2: Income ~ Rating + Products + Age + Education + Gender + Mortgage +
##           Married + Ethnicity + Balance
##   Res.Df   RSS Df Sum of Sq    F    Pr(>F)
## 1     296 35847
## 2     289 35476  7     370.7 0.4314 0.8822
```

Comparamos el modelo con las 3 variables con respecto al modelo que las incluía todas. No aporta ninguna información. No aporta nada el modelo Mul1 al modelo Mul2, si no fuera así hay una variable que se escapa.

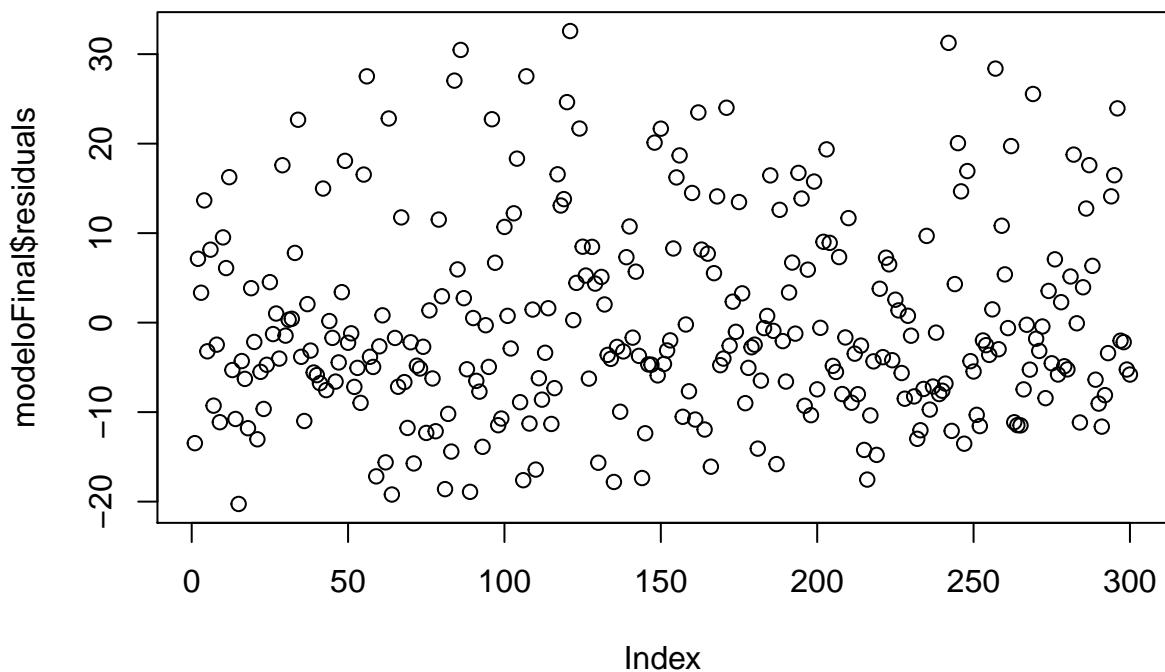
Análisis del Modelo

```
modeloFinal=lm(Income ~ Rating+Mortgage+Balance, data = creditos)
summary(modeloFinal)
```

```
##
## Call:
## lm(formula = Income ~ Rating + Mortgage + Balance, data = creditos)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -20.263  -7.425  -2.544   6.158  32.580
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -58.178638   2.153774 -27.01 <2e-16 ***
## Rating       0.412810   0.009254  44.61 <2e-16 ***
## MortgageYes  39.811486   2.408393  16.53 <2e-16 ***
## Balance      -0.090950   0.003122 -29.13 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

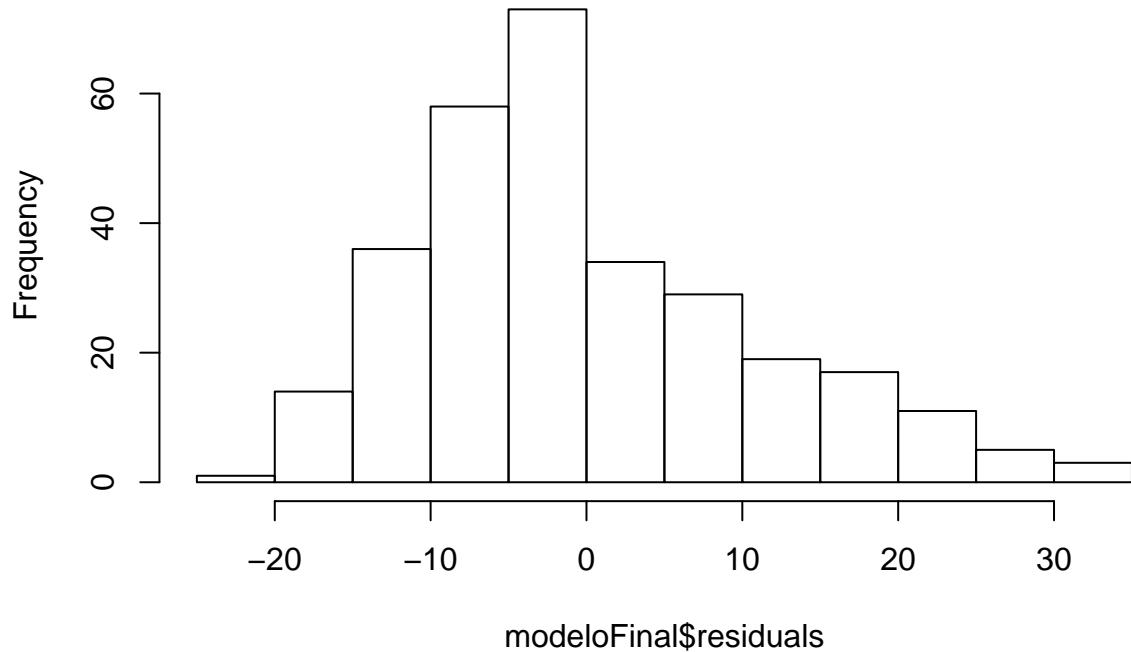
```
##  
## Residual standard error: 11 on 296 degrees of freedom  
## Multiple R-squared:  0.8955, Adjusted R-squared:  0.8944  
## F-statistic: 845.1 on 3 and 296 DF,  p-value: < 2.2e-16
```

```
plot(modeloFinal$residuals)
```



```
hist(modeloFinal$residuals)
```

Histogram of modeloFinal\$residuals



```
qqnorm(modeloFinal$residuals); qqline(modeloFinal$residuals, col=2)
```

Normal Q-Q Plot

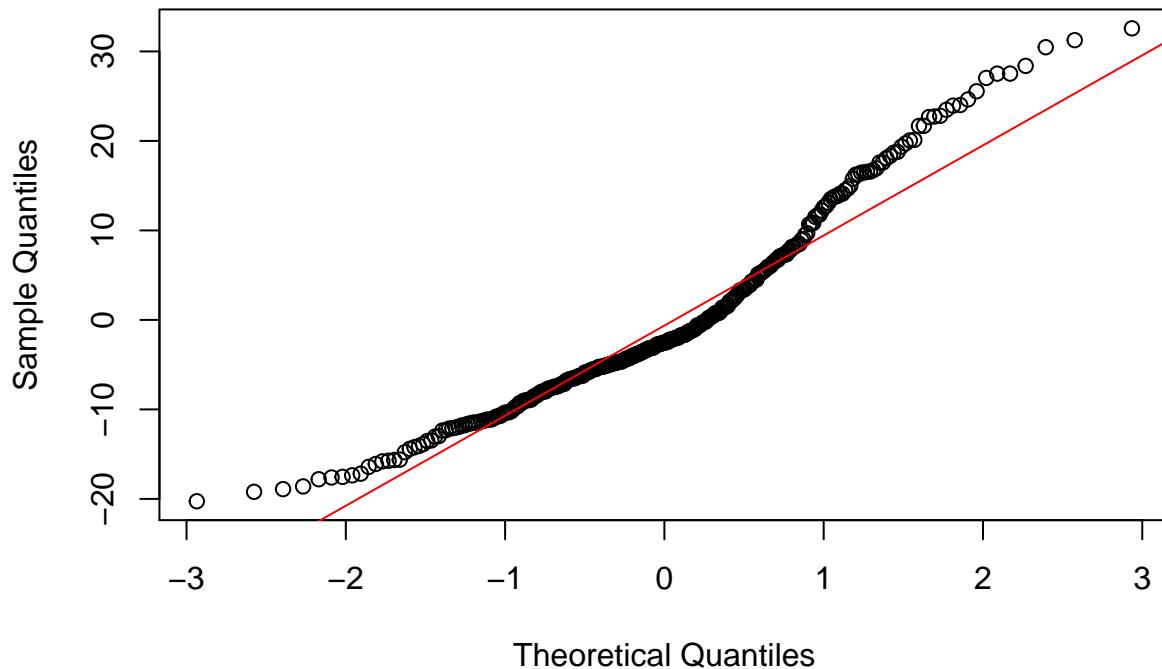


Gráfico de percentiles tienen que estar todos en la recta si es normal vemos que en las colas hay problemas. No vale la regresión lineal, no es insesgada, el valor de la población no se estima de manera exacta con el coeficiente.

```
confint(modeloFinal, level=0.95)
```

```
##                   2.5 %      97.5 %
## (Intercept) -62.41728868 -53.93998677
## Rating        0.39459856   0.43102128
## MortgageYes  35.07174280  44.55122908
## Balance       -0.09709433  -0.08480642
```

Indicamos al nivel que queremos los intervalos de confianza, entre que rangos van los coeficientes.

```
anova(modeloFinal, modeloMul1)
```

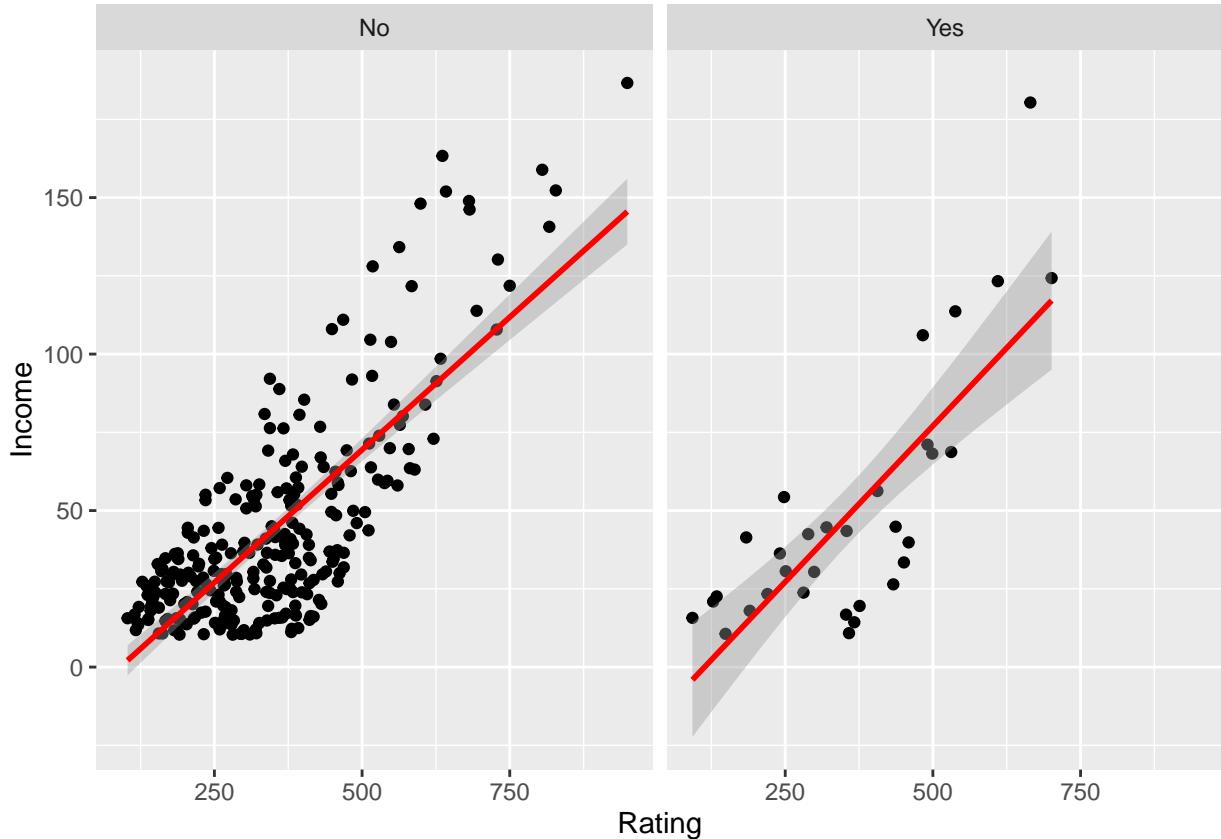
```
## Analysis of Variance Table
##
## Model 1: Income ~ Rating + Mortgage + Balance
## Model 2: Income ~ Rating + Products + Age + Education + Gender + Mortgage +
##           Married + Ethnicity + Balance
## Res.Df   RSS Df Sum of Sq    F Pr(>F)
## 1     296 35847
## 2     289 35476  7     370.7 0.4314 0.8822
```

Modelo Final comparado con modelo con todas las variables. El resultado es que el modelo con todas no aporta nada al modelo Final.

Los que tienen hipoteca no los estima tan bien, las rectas son muy buenas en los centros, em los bordes no, no valen para predecir.

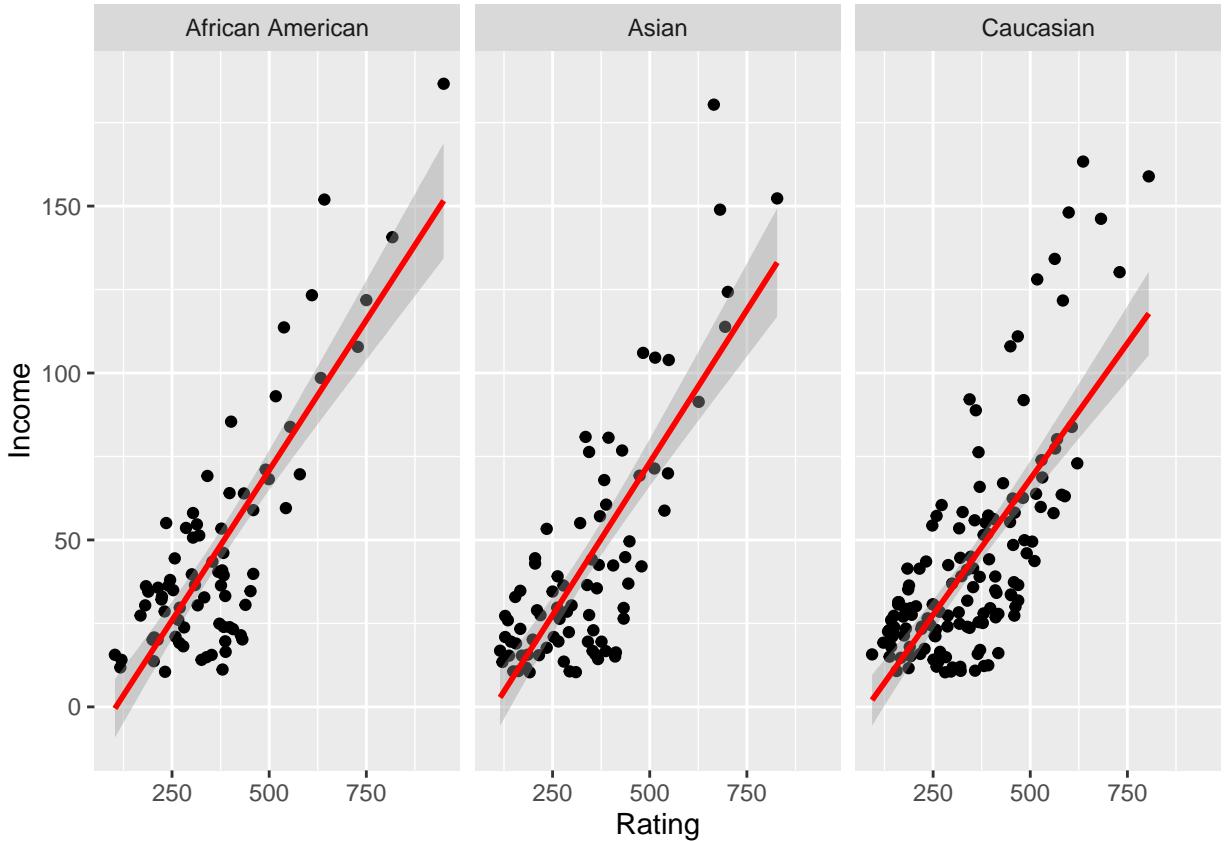
hay diferencias en las pendientes, por eso sale significativa mortgage

```
ggplot(creditos, aes(x = Rating, y = Income)) + geom_point() + facet_grid(~ Mortgage) +  
  geom_smooth(method = "lm", se=TRUE, color="red", formula = y ~ x)
```



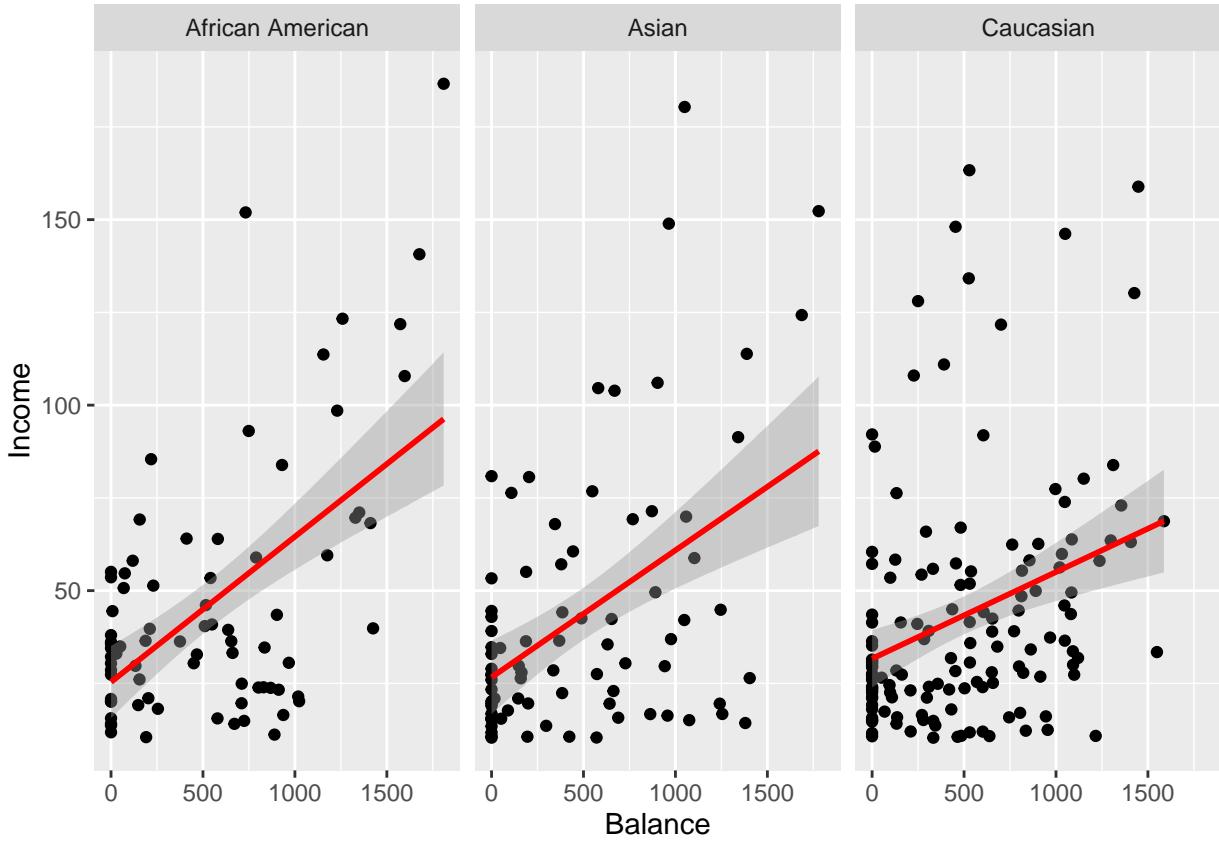
mas o menos todas se solapan por eso en el modelo no son significativas, no es conjunto aquí no tenemos en cuenta el resto de variables

```
ggplot(creditos, aes(x = Rating, y = Income)) + geom_point() + facet_grid(~ Ethnicity) +  
  geom_smooth(method = "lm", se=TRUE, color="red", formula = y ~ x)
```



```
ggplot(creditos, aes(x = Balance, y = Income)) + geom_point() + facet_grid(~ Mortgage) +
  geom_smooth(method = "lm", se=TRUE, color="red", formula = y ~ x)
```

```
ggplot(creditos, aes(x = Balance, y = Income)) + geom_point() + facet_grid(~ Ethnicity) +
  geom_smooth(method = "lm", se=TRUE, color="red", formula = y ~ x)
```



Análisis de interacciones

Ejemplo: Modelo ingresos dependen de $y=a+b\text{sexo}+c\text{estado civil}$, puedo inventarme una nueva variable $d\text{sexoEdad}$ (este efecto estoy diciendo que la edad afecta conjuntamente con el sexo)

```
modeloInter1=lm(Income ~ Balance+Rating*Mortgage+Balance:Mortgage, data = creditos)
#con el : añado la variable sola
summary(modeloInter1)
```

```
##
## Call:
## lm(formula = Income ~ Balance + Rating * Mortgage + Balance:Mortgage,
##      data = creditos)
##
## Residuals:
##    Min     1Q   Median     3Q    Max 
## -20.654 -7.174 -2.280  5.881 32.890 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -56.444729  2.259586 -24.980 < 2e-16 ***
## Balance      -0.090967  0.003376 -26.942 < 2e-16 ***
## Rating       0.407830  0.009885  41.258 < 2e-16 ***
## MortgageYes  23.681928  5.404132   4.382 1.64e-05 ***
```

```

## Rating:MortgageYes    0.050340   0.026038   1.933   0.0542 .
## Balance:MortgageYes -0.002179   0.008176  -0.266   0.7901
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.83 on 294 degrees of freedom
## Multiple R-squared:  0.8994, Adjusted R-squared:  0.8977
## F-statistic: 525.6 on 5 and 294 DF,  p-value: < 2.2e-16

```

Influyen en Income el Balance, el Rating y el Mortgage.

```

modeloInter2=lm(Income ~ Rating*Mortgage+Balance, data = creditos)
summary(modeloInter2)

```

```

##
## Call:
## lm(formula = Income ~ Rating * Mortgage + Balance, data = creditos)
##
## Residuals:
##      Min       1Q     Median      3Q      Max
## -20.693  -7.213  -2.370   5.676  32.913
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -56.612697   2.166470 -26.131 < 2e-16 ***
## Rating                   0.408803   0.009171  44.578 < 2e-16 ***
## MortgageYes               24.046930   5.219430   4.607 6.08e-06 ***
## Balance                  -0.091339   0.003070 -29.750 < 2e-16 ***
## Rating:MortgageYes      0.044344   0.013085   3.389 0.000797 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.81 on 295 degrees of freedom
## Multiple R-squared:  0.8994, Adjusted R-squared:  0.898
## F-statistic: 659.1 on 4 and 295 DF,  p-value: < 2.2e-16

```

Si que encuentra relación entre Rating y Mortgage para explicar la varianza de Income.

```

modeloInter3=lm(Income ~ Rating:Mortgage+Balance, data = creditos)
summary(modeloInter3)

```

```

##
## Call:
## lm(formula = Income ~ Rating:Mortgage + Balance, data = creditos)
##
## Residuals:
##      Min       1Q     Median      3Q      Max
## -21.240  -7.669  -1.837   7.255  35.209
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -52.879756   2.076767 -25.46  <2e-16 ***

```

```

## Balance          -0.088485   0.003108  -28.47   <2e-16 ***
## Rating:MortgageNo    0.395390   0.008988   43.99   <2e-16 ***
## Rating:MortgageYes   0.493467   0.012566   39.27   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.18 on 296 degrees of freedom
## Multiple R-squared:  0.8921, Adjusted R-squared:  0.891
## F-statistic:  816 on 3 and 296 DF,  p-value: < 2.2e-16

```

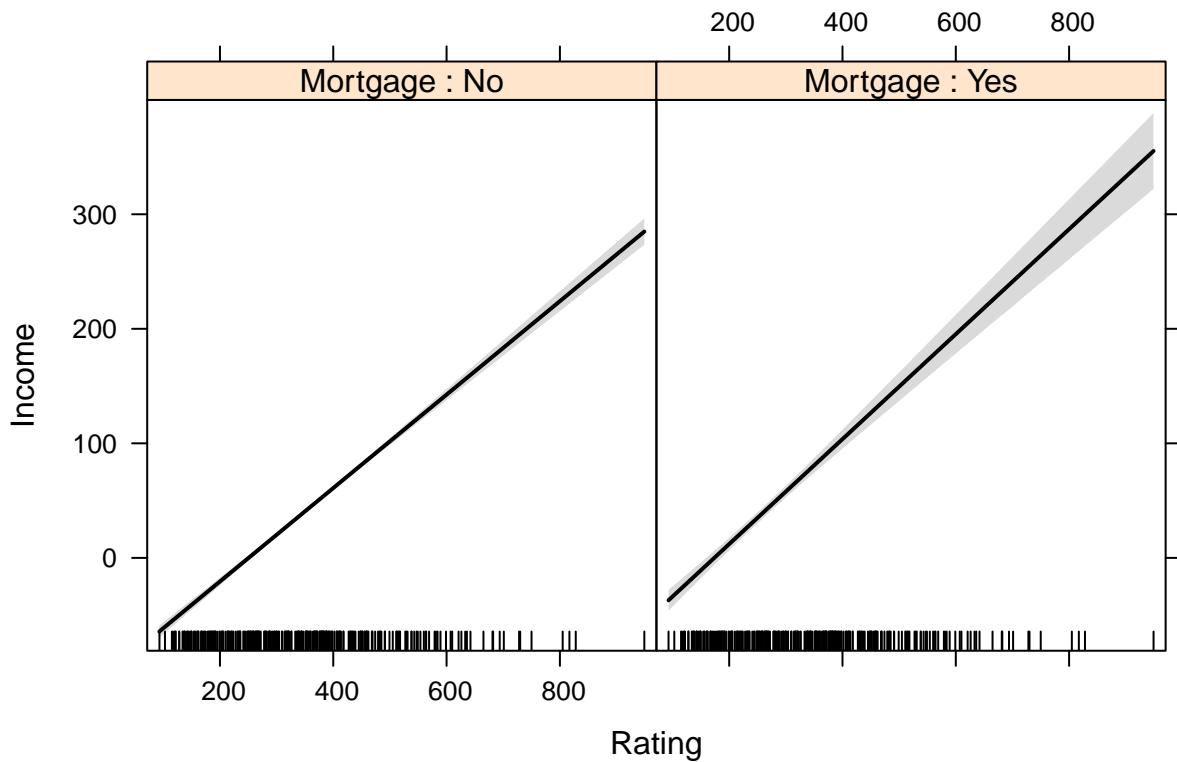
separa el efecto del rating en dos bloques si mortgage es no incrementa el rating 0.39

```

efecto1 <- effect("Rating*Mortgage", modeloInter1, xlevels = 10)
plot(efecto1)#la diferencia es que aqui le metes el modelo, con lo que le metes las

```

Rating*Mortgage effect plot



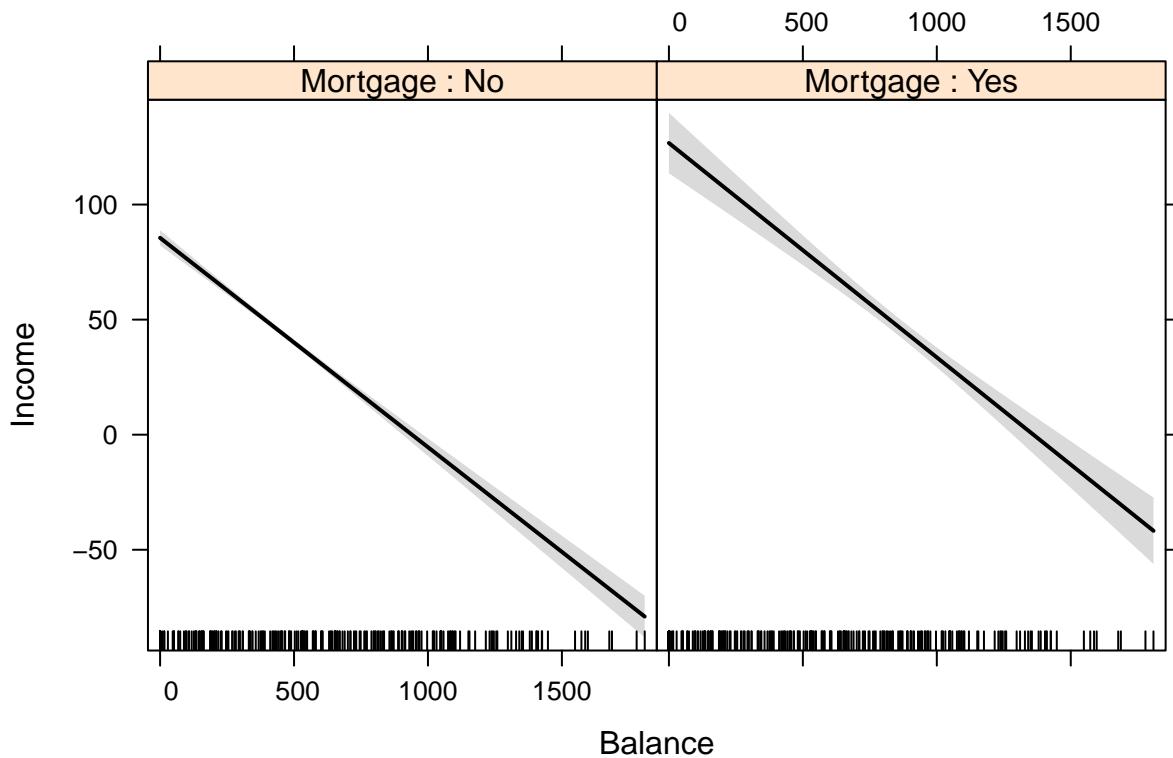
```
#relaciones con todas las variables
```

```

efecto2 <- effect("Balance*Mortgage", modeloInter1, xlevels = 10)
plot(efecto2)

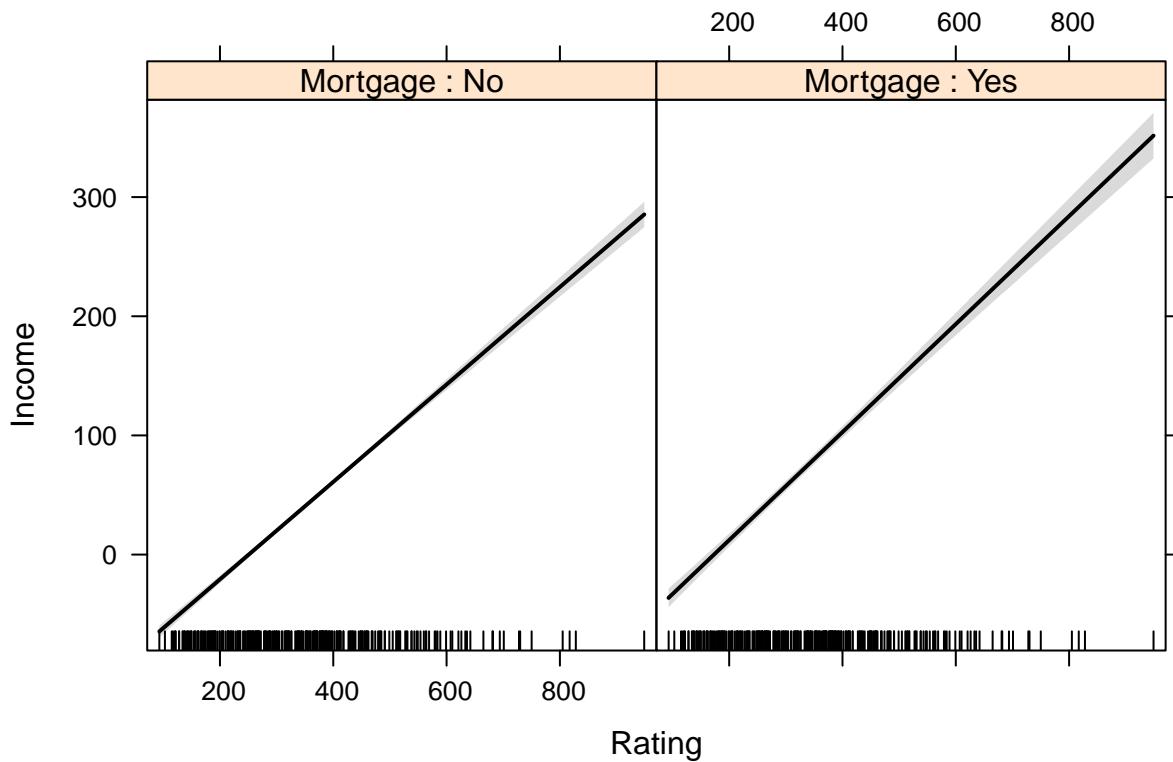
```

Balance*Mortgage effect plot



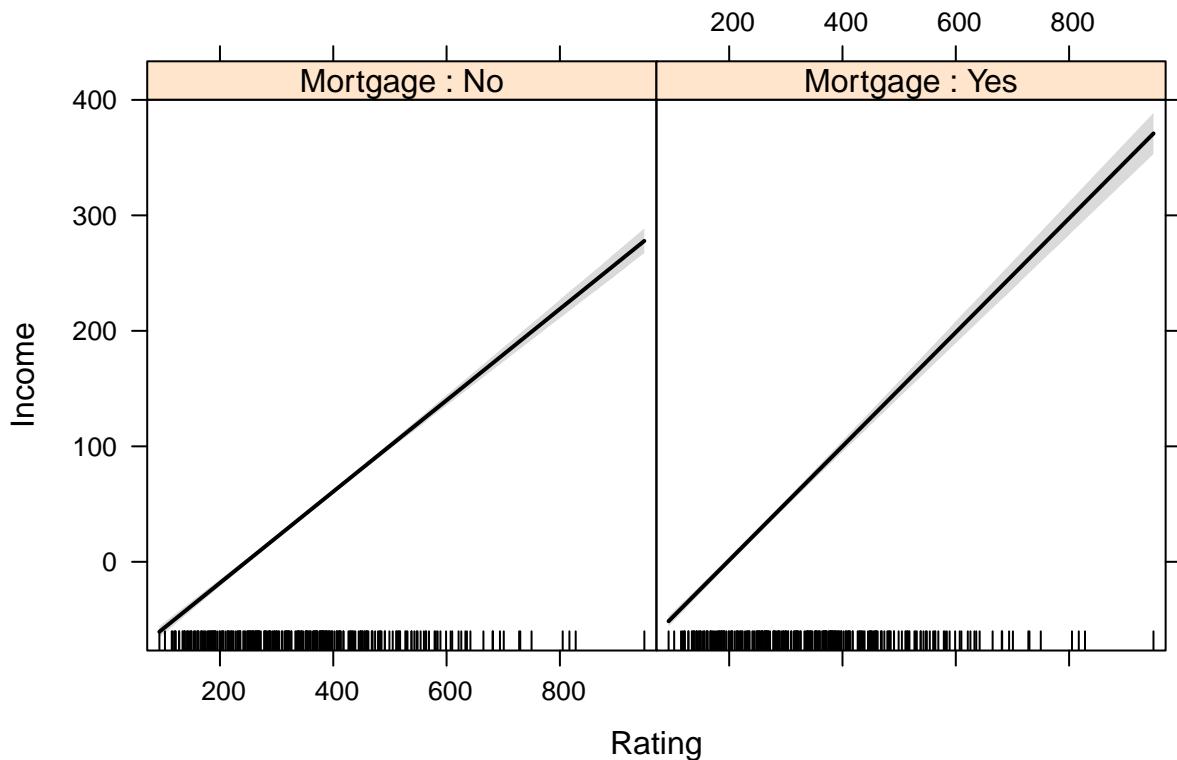
```
efecto3 <- effect("Rating*Mortgage", modeloInter2, xlevels = 10)
plot(efecto3)
```

Rating*Mortgage effect plot



```
efecto4 <- effect("Rating:Mortgage", modeloInter3, xlevels = 10)
plot(efecto4)
```

Rating*Mortgage effect plot

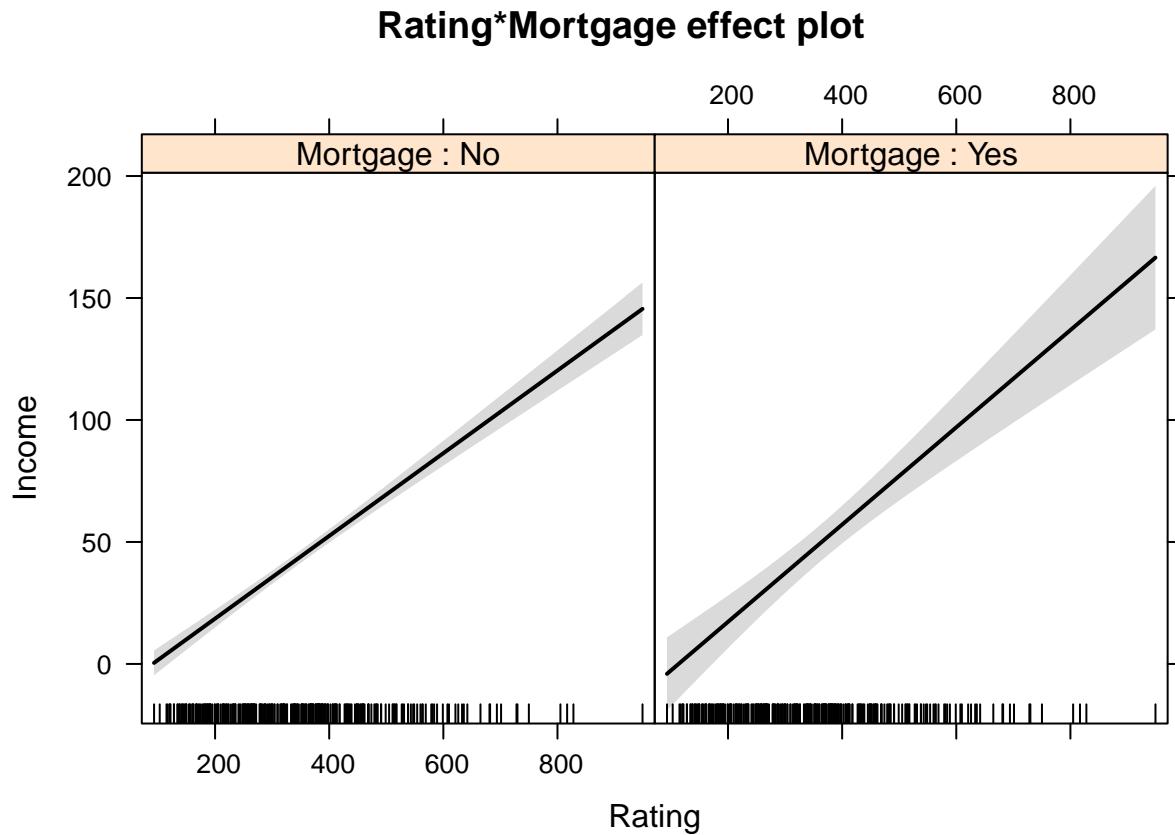


```
modeloInter5=lm(Income ~ Rating*Mortgage, data = creditos)
summary(modeloInter5)
```

```
##
## Call:
## lm(formula = Income ~ Rating * Mortgage, data = creditos)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -39.427 -15.551 -0.579  13.538  70.846 
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -15.322939   3.321568 -4.613 5.91e-06 ***
## Rating       0.169505   0.008795 19.274 < 2e-16 ***
## MortgageYes -7.283979  10.207199 -0.714   0.476    
## Rating:MortgageYes  0.029810   0.026109   1.142   0.254  
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 21.59 on 296 degrees of freedom
## Multiple R-squared:  0.5974, Adjusted R-squared:  0.5934 
## F-statistic: 146.4 on 3 and 296 DF,  p-value: < 2.2e-16
```

Aquí la hipoteca no representa nada en el Income

```
efecto5 <- effect("Rating*Mortgage", modeloInter5, xlevels = 10)
plot(efecto5)
```



Analisis de variable Balance

```
modeloBalance=lm(Balance ~ ., data = creditos)
summary(modeloBalance)

##
## Call:
## lm(formula = Balance ~ ., data = creditos)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -196.28  -83.07  -12.69   63.88  310.43 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -542.32300  42.50152 -12.760 <2e-16 ***
## Income       -8.05229   0.28367 -28.386 <2e-16 ***
## Rating        4.03048   0.06378  63.189 <2e-16 ***
## Products      5.86109   4.49787   1.303  0.1936    
## Age          -0.77749   0.35725  -2.176  0.0303 *  
## 
```

```

## Education          -1.41408   1.97605  -0.716   0.4748
## GenderFemale      -5.91052   12.18127  -0.485   0.6279
## MortgageYes       425.89676  19.73226  21.584   <2e-16 ***
## MarriedYes        -10.20768  12.55437  -0.813   0.4168
## EthnicityAsian    13.03881   16.74156   0.779   0.4367
## EthnicityCaucasian 2.56574   14.75205   0.174   0.8620
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 104 on 289 degrees of freedom
## Multiple R-squared:  0.9521, Adjusted R-squared:  0.9504
## F-statistic: 574.1 on 10 and 289 DF,  p-value: < 2.2e-16

```

Todas las variables explican el 95.21 del Balance. Income, Rating y Mortgage son representativas, Age un 0.0303 de p-valor.

Variables que incluiríamos en el modelo

```

modeloBalanceFin=lm(Balance ~ Income*Age+Rating+Mortgage, data = creditos)
summary(modeloBalanceFin)

```

```

##
## Call:
## lm(formula = Balance ~ Income * Age + Rating + Mortgage, data = creditos)
##
## Residuals:
##      Min      1Q      Median      3Q      Max
## -193.151  -83.464   -2.763   59.774  299.597
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -601.66420  36.21284 -16.615   <2e-16 ***
## Income       -6.94221   0.65555 -10.590   <2e-16 ***
## Age          0.16438   0.59065   0.278   0.7810
## Rating       4.03363   0.06235  64.694   <2e-16 ***
## MortgageYes  429.59524  19.40727  22.136   <2e-16 ***
## Income:Age   -0.01959   0.01032  -1.898   0.0587 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 103.1 on 294 degrees of freedom
## Multiple R-squared:  0.952, Adjusted R-squared:  0.9512
## F-statistic: 1167 on 5 and 294 DF,  p-value: < 2.2e-16

```

```
anova(modeloBalanceFin,modeloBalance)
```

```

## Analysis of Variance Table
##
## Model 1: Balance ~ Income * Age + Rating + Mortgage
## Model 2: Balance ~ Income + Rating + Products + Age + Education + Gender +
##           Mortgage + Married + Ethnicity

```

```

##   Res.Df      RSS Df Sum of Sq      F Pr(>F)
## 1    294 3127543
## 2    289 3125213  5     2329.9 0.0431 0.9989

```

p-valor de 0.9989 incluir el resto de variables no aporta nada al modelo anterior.

MODELOS LINEALES GENERALIZADOS: REGRESION LOGISTICA

Modelo que se utiliza cuando la variable "y" es dicotómica, si o no, true o false, bernoulli con probabilidad p. aplicaré la función sigmoidal, el resultado es la probabilidad de ocurrencia, para cada familia de parámetros iniciales la probabilidad de que ocurra mi output. En los modelos de clasificación todos cortan en el 0.5, acepto +0.5. que hago entre 0.5 y 1?? minimizas el capital ponderado por riesgo. Buscas un óptimo de beneficio y coste. el modelo ordena y yo elijo donde corto.

Máxima verosimilitud: para la regresión lineal si se cumple todo, MCO = max.verosimilitud insesgado: el método usado no se va a aproximar a la media poblacional. Máxima verosimilitud, me acepta un montón de modelos. Se minimizan los errores cuadráticos, hay un método iterativo de Nelder para encontrar los coeficientes mediante iteraciones.

Matriz de confusión: mezcla la realidad con el modelo, sacamos una serie de ratios para medir como de bueno es el modelo. True positive rate = TP/TP+FN (el porcentaje de los que hemos clasificado bien sobre el total de los que eran buenos)

Curva ROC representa la selección del modelo, cuanto más alta mejor el modelo, se calcula el área debajo de la curva. cuanto más cercano a 1 mejor, a 0.5 peor. Tengo que elegir el punto de corte. Mide la capacidad predictiva de un modelo, se usa más en machine learning que en análisis estadístico.

Comparativa de modelos:

- relativos: R^2 ajustado (en regresión lineal)
- absolutos: BIC bayesian mide la verosimilitud (regresión lineal, logística,) siempre se elige el más bajo. AIC akaike
- Comparativos: Contraste F para comparar varios modelos

Ejemplo regresión lineal generalizado:

Nº VECES ALQUILA BICIS EN MES, NO PUEDE SER NORMAL PORQUE NO PUEDE SER NEGATIVO...

Carga de Datos

campana de venta de depósitos, la "y" es si compra o no el depósito, el objetivo es quien va a contratar mi producto.

```
BANK=read.csv2("bank-full.csv")
```

datos extraídos de <https://archive.ics.uci.edu/ml/datasets/Bank+Marketing>

Revisión Básica Dataset

```
str(BANK)
```

```
## 'data.frame': 45211 obs. of 17 variables:
## $ age      : int 58 44 33 47 33 35 28 42 58 43 ...
## $ job      : Factor w/ 12 levels "admin.", "blue-collar", ... : 5 10 3 2 12 5 5 3 6 10 ...
## $ marital   : Factor w/ 3 levels "divorced", "married", ... : 2 3 2 2 3 2 3 1 2 3 ...
## $ education: Factor w/ 4 levels "primary", "secondary", ... : 3 2 2 4 4 3 3 3 1 2 ...
## $ default   : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 2 1 1 ...
## $ balance   : int 2143 29 2 1506 1 231 447 2 121 593 ...
## $ housing   : Factor w/ 2 levels "no", "yes": 2 2 2 2 1 2 2 2 2 ...
## $ loan      : Factor w/ 2 levels "no", "yes": 1 1 2 1 1 1 2 1 1 ...
## $ contact   : Factor w/ 3 levels "cellular", "telephone", ... : 3 3 3 3 3 3 3 3 3 ...
## $ day       : int 5 5 5 5 5 5 5 5 5 ...
## $ month     : Factor w/ 12 levels "apr", "aug", "dec", ... : 9 9 9 9 9 9 9 9 9 9 ...
## $ duration  : int 261 151 76 92 198 139 217 380 50 55 ...
## $ campaign  : int 1 1 1 1 1 1 1 1 1 ...
## $ pdays     : int -1 -1 -1 -1 -1 -1 -1 -1 -1 ...
## $ previous  : int 0 0 0 0 0 0 0 0 0 ...
## $ poutcome  : Factor w/ 4 levels "failure", "other", ... : 4 4 4 4 4 4 4 4 4 4 ...
## $ y         : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 1 1 1 ...
```

```
head(BANK)
```

```
##   age      job marital education default balance housing loan contact
## 1 58    management married tertiary      no    2143     yes  no unknown
## 2 44    technician single secondary      no      29     yes  no unknown
## 3 33 entrepreneur married secondary      no      2     yes  yes unknown
## 4 47 blue-collar married unknown      no    1506     yes  no unknown
## 5 33      unknown single unknown      no      1      no  no unknown
## 6 35    management married tertiary      no    231     yes  no unknown
##   day month duration campaign pdays previous poutcome y
## 1   5   may     261        1    -1      0 unknown no
## 2   5   may     151        1    -1      0 unknown no
## 3   5   may      76        1    -1      0 unknown no
## 4   5   may      92        1    -1      0 unknown no
## 5   5   may     198        1    -1      0 unknown no
## 6   5   may     139        1    -1      0 unknown no
```

```
summary(BANK)
```

```
##   age      job      marital      education
## Min.   :18.00  blue-collar:9732  divorced: 5207  primary   :6851
## 1st Qu.:33.00  management :9458   married  :27214  secondary :23202
## Median  :39.00  technician :7597   single   :12790  tertiary  :13301
## Mean    :40.94  admin.    :5171           unknown   :1857
## 3rd Qu.:48.00  services   :4154
## Max.   :95.00  retired   :2264
##          (Other)  :6835
##   default      balance      housing      loan      contact
## no :44396  Min.   :-8019  no :20081  no :37967  cellular :29285
## yes: 815   1st Qu.: 72   yes:25130  yes: 7244  telephone: 2906
```

```

##          Median : 448                      unknown  :13020
##          Mean   : 1362
##          3rd Qu.: 1428
##          Max.   :102127
##
##      day       month      duration      campaign
##  Min.   : 1.00  may     :13766  Min.   : 0.0  Min.   : 1.000
##  1st Qu.: 8.00  jul     : 6895  1st Qu.: 103.0  1st Qu.: 1.000
##  Median :16.00  aug     : 6247  Median : 180.0  Median : 2.000
##  Mean   :15.81  jun     : 5341  Mean   : 258.2  Mean   : 2.764
##  3rd Qu.:21.00  nov     : 3970  3rd Qu.: 319.0  3rd Qu.: 3.000
##  Max.   :31.00  apr     : 2932  Max.   :4918.0  Max.   :63.000
##           (Other): 6060
##      pdays      previous      poutcome      y
##  Min.   :-1.0   Min.   : 0.0000  failure: 4901  no  :39922
##  1st Qu.:-1.0   1st Qu.: 0.0000  other   : 1840  yes : 5289
##  Median :-1.0   Median : 0.0000  success: 1511
##  Mean   :40.2   Mean   : 0.5803  unknown:36959
##  3rd Qu.:-1.0   3rd Qu.: 0.0000
##  Max.   :871.0  Max.   :275.0000
##

```

Formateo de Variables

```

BANK$day=as.factor(BANK$day)
BANK$campaign=as.factor(BANK$campaign)
BANK$IND_PREVIO=as.factor(as.numeric(BANK$pdays!=1))

```

```
str(BANK)
```

```

## 'data.frame': 45211 obs. of 18 variables:
## $ age      : int 58 44 33 47 33 35 28 42 58 43 ...
## $ job      : Factor w/ 12 levels "admin.", "blue-collar", ... : 5 10 3 2 12 5 5 3 6 10 ...
## $ marital   : Factor w/ 3 levels "divorced", "married", ... : 2 3 2 2 3 2 3 1 2 3 ...
## $ education : Factor w/ 4 levels "primary", "secondary", ... : 3 2 2 4 4 3 3 3 1 2 ...
## $ default   : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 2 1 1 ...
## $ balance   : int 2143 29 2 1506 1 231 447 2 121 593 ...
## $ housing   : Factor w/ 2 levels "no", "yes": 2 2 2 2 1 2 2 2 2 2 ...
## $ loan      : Factor w/ 2 levels "no", "yes": 1 1 2 1 1 1 2 1 1 1 ...
## $ contact   : Factor w/ 3 levels "cellular", "telephone", ... : 3 3 3 3 3 3 3 3 3 3 ...
## $ day       : Factor w/ 31 levels "1", "2", "3", "4", ... : 5 5 5 5 5 5 5 5 5 5 ...
## $ month     : Factor w/ 12 levels "apr", "aug", "dec", ... : 9 9 9 9 9 9 9 9 9 9 ...
## $ duration  : int 261 151 76 92 198 139 217 380 50 55 ...
## $ campaign  : Factor w/ 48 levels "1", "2", "3", "4", ... : 1 1 1 1 1 1 1 1 1 1 ...
## $ pdays     : int -1 -1 -1 -1 -1 -1 -1 -1 -1 ...
## $ previous  : int 0 0 0 0 0 0 0 0 0 ...
## $ poutcome  : Factor w/ 4 levels "failure", "other", ... : 4 4 4 4 4 4 4 4 4 4 ...
## $ y         : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 1 1 1 ...
## $ IND_PREVIO: Factor w/ 2 levels "0", "1": 1 1 1 1 1 1 1 1 1 ...

```

```
head(BANK)
```

```
##   age          job marital education default balance housing loan contact
## 1 58    management married tertiary      no    2143     yes  no unknown
## 2 44    technician single secondary      no      29     yes  no unknown
## 3 33 entrepreneur married secondary      no       2     yes  yes unknown
## 4 47 blue-collar married unknown      no   1506     yes  no unknown
## 5 33        unknown single unknown      no       1     no  no unknown
## 6 35    management married tertiary      no    231     yes  no unknown
##   day month duration campaign pdays previous poutcome y IND_PREVIO
## 1   5   may     261         1    -1       0 unknown no      0
## 2   5   may     151         1    -1       0 unknown no      0
## 3   5   may      76         1    -1       0 unknown no      0
## 4   5   may     92         1    -1       0 unknown no      0
## 5   5   may    198         1    -1       0 unknown no      0
## 6   5   may    139         1    -1       0 unknown no      0
```

```
summary(BANK)
```

```
##   age          job marital education
## Min. :18.00  blue-collar:9732  divorced: 5207  primary  : 6851
## 1st Qu.:33.00 management :9458   married :27214  secondary:23202
## Median :39.00 technician :7597   single  :12790  tertiary :13301
## Mean   :40.94 admin.     :5171           unknown  : 1857
## 3rd Qu.:48.00 services   :4154
## Max.   :95.00 retired    :2264
##           (Other)   :6835
##   default      balance   housing   loan      contact
## no :44396   Min.   : -8019   no :20081   no :37967  cellular :29285
## yes: 815   1st Qu.:     72   yes:25130   yes: 7244  telephone: 2906
##           Median :    448
##           Mean   :  1362
##           3rd Qu.: 1428
##           Max.   :102127
##
##   day          month duration campaign
## 20   : 2752   may     :13766   Min.   : 0.0  1   :17544
## 18   : 2308   jul     : 6895   1st Qu.: 103.0 2   :12505
## 21   : 2026   aug     : 6247   Median : 180.0 3   : 5521
## 17   : 1939   jun     : 5341   Mean   : 258.2 4   : 3522
## 6    : 1932   nov     : 3970   3rd Qu.: 319.0 5   : 1764
## 5    : 1910   apr     : 2932   Max.   :4918.0 6   : 1291
## (Other):32344 (Other): 6060           (Other): 3064
##   pdays      previous   poutcome   y      IND_PREVIO
## Min.   : -1.0  Min.   : 0.0000  failure: 4901  no :39922  0:36954
## 1st Qu.: -1.0  1st Qu.: 0.0000  other   : 1840  yes: 5289   1: 8257
## Median : -1.0  Median : 0.0000  success: 1511
## Mean   : 40.2   Mean   : 0.5803  unknown:36959
## 3rd Qu.: -1.0  3rd Qu.: 0.0000
## Max.   :871.0   Max.   :275.0000
##
```

Modelo de Regresión Logística

```
model_logit=glm(y~., data=BANK,family=binomial(link="logit"))
summary(model_logit)

##
## Call:
## glm(formula = y ~ ., family = binomial(link = "logit"), data = BANK)
##
## Deviance Residuals:
##      Min        1Q     Median        3Q       Max 
## -5.9323   -0.3700   -0.2447   -0.1474    3.4526 
##
## Coefficients:
##                               Estimate Std. Error z value Pr(>|z|)    
## (Intercept)              -2.897e+00  9.902e-01 -2.925 0.003442 ** 
## age                      1.697e-04  2.217e-03  0.077 0.938994    
## jobblue-collar          -2.767e-01  7.324e-02 -3.778 0.000158 *** 
## jobentrepreneur         -3.337e-01  1.266e-01 -2.636 0.008398 ** 
## jobhousemaid            -4.948e-01  1.369e-01 -3.613 0.000303 *** 
## jobmanagement           -1.461e-01  7.378e-02 -1.980 0.047655 *  
## jobretired               2.327e-01  9.821e-02  2.370 0.017799 *  
## jobself-employed         -2.925e-01  1.127e-01 -2.596 0.009445 ** 
## jobservices              -1.998e-01  8.461e-02 -2.361 0.018201 *  
## jobstudent               3.794e-01  1.099e-01  3.454 0.000553 *** 
## jobtechnician             1.519e-01  6.947e-02 -2.186 0.028806 *  
## jobunemployed             1.795e-01  1.126e-01 -1.594 0.110947    
## jobunknownd              2.818e-01  2.358e-01 -1.195 0.232120    
## maritalmarried            1.767e-01  5.936e-02 -2.976 0.002919 ** 
## maritalsingle             7.966e-02  6.779e-02  1.175 0.239965    
## educationsecondary        1.860e-01  6.524e-02  2.851 0.004362 ** 
## educationtertiary         3.747e-01  7.587e-02  4.939 7.83e-07 *** 
## educationunknown          2.283e-01  1.051e-01  2.173 0.029766 *  
## defaultyes                1.014e-02  1.632e-01 -0.062 0.950448    
## balance                   1.154e-05  5.199e-06  2.219 0.026510 *  
## housingyes                6.434e-01  4.432e-02 -14.516 < 2e-16 *** 
## loanyes                   -4.191e-01  6.039e-02 -6.940 3.93e-12 *** 
## contacttelephone          -1.705e-01  7.570e-02 -2.253 0.024285 *  
## contactunknownd          -1.584e+00  7.430e-02 -21.318 < 2e-16 *** 
## day2                      -1.536e-01  1.870e-01 -0.821 0.411586    
## day3                      3.656e-02  1.887e-01  0.194 0.846354    
## day4                      2.827e-02  1.823e-01  0.155 0.876753    
## day5                      -1.918e-01  1.821e-01 -1.054 0.292036    
## day6                      -1.854e-01  1.861e-01 -0.996 0.319123    
## day7                      -3.237e-01  1.890e-01 -1.713 0.086768 .  
## day8                      4.270e-02  1.835e-01  0.233 0.816056    
## day9                      6.352e-02  1.888e-01  0.336 0.736564    
## day10                     5.136e-01  2.074e-01  2.477 0.013259 *  
## day11                     -1.374e-02  1.861e-01 -0.074 0.941157    
## day12                     3.184e-01  1.822e-01  1.748 0.080547 .  
## day13                     4.331e-01  1.832e-01  2.364 0.018100 *  
## day14                     1.642e-01  1.842e-01  0.892 0.372566    
## day15                     2.420e-01  1.823e-01  1.327 0.184476
```

## day16	6.602e-02	1.862e-01	0.355	0.722867
## day17	-5.570e-01	1.863e-01	-2.990	0.002788 **
## day18	-8.157e-02	1.815e-01	-0.450	0.653068
## day19	-5.871e-01	1.989e-01	-2.951	0.003169 **
## day20	-4.347e-01	1.852e-01	-2.348	0.018889 *
## day21	-4.649e-03	1.873e-01	-0.025	0.980196
## day22	1.710e-01	1.948e-01	0.878	0.380183
## day23	5.176e-01	2.021e-01	2.561	0.010446 *
## day24	2.563e-02	2.333e-01	0.110	0.912496
## day25	3.283e-01	1.983e-01	1.656	0.097783 .
## day26	2.638e-01	2.017e-01	1.308	0.190813
## day27	6.479e-01	1.950e-01	3.323	0.000891 ***
## day28	5.705e-02	1.970e-01	0.290	0.772112
## day29	-1.264e-01	1.990e-01	-0.636	0.525073
## day30	4.531e-01	1.830e-01	2.476	0.013296 *
## day31	7.429e-02	2.534e-01	0.293	0.769351
## monthaug	-7.381e-01	8.604e-02	-8.579	< 2e-16 ***
## monthdec	6.776e-01	1.804e-01	3.757	0.000172 ***
## monthfeb	-2.295e-01	9.722e-02	-2.361	0.018244 *
## monthjan	-1.294e+00	1.328e-01	-9.744	< 2e-16 ***
## monthjul	-9.496e-01	8.384e-02	-11.326	< 2e-16 ***
## monthjun	4.070e-01	9.782e-02	4.160	3.18e-05 ***
## monthmar	1.476e+00	1.244e-01	11.867	< 2e-16 ***
## monthmay	-5.895e-01	8.068e-02	-7.307	2.72e-13 ***
## monthnov	-6.728e-01	9.378e-02	-7.175	7.25e-13 ***
## monthoct	7.763e-01	1.118e-01	6.940	3.91e-12 ***
## monthsep	7.602e-01	1.229e-01	6.186	6.16e-10 ***
## duration	4.249e-03	6.532e-05	65.051	< 2e-16 ***
## campaign2	-3.464e-01	4.474e-02	-7.743	9.72e-15 ***
## campaign3	-2.707e-01	6.017e-02	-4.499	6.83e-06 ***
## campaign4	-4.846e-01	7.878e-02	-6.151	7.69e-10 ***
## campaign5	-5.887e-01	1.124e-01	-5.237	1.63e-07 ***
## campaign6	-6.608e-01	1.350e-01	-4.895	9.81e-07 ***
## campaign7	-7.511e-01	1.840e-01	-4.083	4.45e-05 ***
## campaign8	-6.394e-01	2.088e-01	-3.062	0.002197 **
## campaign9	-7.289e-01	2.782e-01	-2.620	0.008795 **
## campaign10	-6.882e-01	3.317e-01	-2.075	0.038030 *
## campaign11	-2.000e-01	3.102e-01	-0.645	0.519114
## campaign12	-1.579e+00	5.503e-01	-2.869	0.004117 **
## campaign13	-1.107e+00	4.865e-01	-2.275	0.022905 *
## campaign14	-7.318e-01	5.706e-01	-1.283	0.199652
## campaign15	-3.909e-01	5.529e-01	-0.707	0.479517
## campaign16	-1.590e+00	9.686e-01	-1.642	0.100589
## campaign17	-2.482e-01	5.282e-01	-0.470	0.638437
## campaign18	-1.251e+01	1.848e+02	-0.068	0.946028
## campaign19	-1.275e+01	2.045e+02	-0.062	0.950297
## campaign20	-1.126e+00	1.090e+00	-1.033	0.301830
## campaign21	-5.108e-01	1.040e+00	-0.491	0.623358
## campaign22	-1.235e+01	2.838e+02	-0.044	0.965291
## campaign23	-1.250e+01	2.902e+02	-0.043	0.965647
## campaign24	-1.361e+00	1.316e+00	-1.035	0.300902
## campaign25	-1.261e+01	2.822e+02	-0.045	0.964351
## campaign26	-1.181e+01	3.866e+02	-0.031	0.975638
## campaign27	-1.210e+01	4.443e+02	-0.027	0.978268

```

## campaign28      -1.262e+01  3.306e+02  -0.038  0.969549
## campaign29      1.733e-01  1.268e+00   0.137  0.891307
## campaign30      -1.195e+01  4.961e+02  -0.024  0.980774
## campaign31      -1.236e+01  3.969e+02  -0.031  0.975157
## campaign32      1.750e+00  1.092e+00   1.602  0.109187
## campaign33      -1.177e+01  5.844e+02  -0.020  0.983929
## campaign34      -1.245e+01  6.391e+02  -0.019  0.984461
## campaign35      -1.223e+01  7.156e+02  -0.017  0.986359
## campaign36      -1.217e+01  7.213e+02  -0.017  0.986538
## campaign37      -1.109e+01  1.027e+03  -0.011  0.991383
## campaign38      -1.161e+01  8.270e+02  -0.014  0.988803
## campaign39      -1.178e+01  1.455e+03  -0.008  0.993540
## campaign41      -1.119e+01  9.548e+02  -0.012  0.990649
## campaign43      -1.078e+01  8.198e+02  -0.013  0.989513
## campaign44      -1.122e+01  1.455e+03  -0.008  0.993848
## campaign46      -1.178e+01  1.455e+03  -0.008  0.993542
## campaign50      -1.117e+01  1.027e+03  -0.011  0.991318
## campaign51      -1.039e+01  1.455e+03  -0.007  0.994303
## campaign55      -1.480e+01  1.455e+03  -0.010  0.991886
## campaign58      -1.083e+01  1.455e+03  -0.007  0.994065
## campaign63      -1.025e+01  1.455e+03  -0.007  0.994383
## pdays          8.241e-05  3.072e-04   0.268  0.788535
## previous        1.164e-02  6.733e-03   1.728  0.083903 .
## poutcomeother   2.162e-01  9.060e-02   2.387  0.016995 *
## poutcomesuccess 2.220e+00  8.293e-02  26.771  < 2e-16 ***
## poutcomeunknown 4.351e-01  9.656e-01   0.451  0.652304
## IND_PREVIO1     4.669e-01  9.683e-01   0.482  0.629678
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 32631  on 45210  degrees of freedom
## Residual deviance: 21269  on 45092  degrees of freedom
## AIC: 21507
##
## Number of Fisher Scoring iterations: 14

```

Nos da un valor mucho mas pequeño de AIC 21507

```

model_logit1=glm(y~job+marital+education+default+balance+housing+loan+contact+month+poutcome, data=BANK
summary(model_logit1)

```

```

##
## Call:
## glm(formula = y ~ job + marital + education + default + balance +
##       housing + loan + contact + month + poutcome, family = binomial(link = "logit"),
##       data = BANK)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -2.3429  -0.4762  -0.3823  -0.2487   2.9771
## 
```

```

## Coefficients:
##                               Estimate Std. Error z value Pr(>|z|)
## (Intercept)           -1.268e+00 1.068e-01 -11.872 < 2e-16 ***
## jobblue-collar      -1.253e-01 6.490e-02  -1.931 0.053471 .
## jobentrepreneur     -1.967e-01 1.112e-01  -1.769 0.076953 .
## jobhousemaid        -2.849e-01 1.197e-01  -2.381 0.017270 *
## jobmanagement       -5.305e-02 6.566e-02  -0.808 0.419082
## jobretired          4.538e-01 7.798e-02   5.820 5.90e-09 ***
## jobself-employed    -9.552e-02 9.917e-02  -0.963 0.335483
## jobservices         -8.396e-02 7.450e-02  -1.127 0.259714
## jobstudent          3.309e-01 9.773e-02   3.386 0.000710 ***
## jobtechnician        -6.585e-02 6.188e-02  -1.064 0.287233
## jobunemployed       1.262e-01 9.770e-02   1.292 0.196280
## jobunknowm          -1.978e-01 2.077e-01  -0.952 0.341032
## maritalmarried     -2.106e-01 5.161e-02  -4.081 4.49e-05 ***
## maritalsingle        8.151e-02 5.556e-02   1.467 0.142387
## educationsecondary  1.508e-01 5.654e-02   2.667 0.007659 **
## educationtertiary   3.116e-01 6.575e-02   4.740 2.14e-06 ***
## educationunknown    1.990e-01 9.244e-02   2.153 0.031319 *
## defaultyes          -1.329e-01 1.470e-01  -0.904 0.366006
## balance              1.703e-05 4.463e-06   3.816 0.000136 ***
## housingyes          -5.398e-01 3.813e-02 -14.157 < 2e-16 ***
## loanyes              -3.969e-01 5.312e-02  -7.472 7.91e-14 ***
## contacttelephone    -2.883e-01 6.408e-02  -4.499 6.83e-06 ***
## contactunknown       -1.346e+00 6.339e-02 -21.228 < 2e-16 ***
## monthaug             -9.711e-01 6.848e-02 -14.181 < 2e-16 ***
## monthdec              5.651e-01 1.621e-01   3.485 0.000491 ***
## monthfeb             -4.419e-01 7.506e-02  -5.887 3.93e-09 ***
## monthjan             -1.071e+00 1.063e-01 -10.077 < 2e-16 ***
## monthjul             -7.875e-01 6.773e-02 -11.627 < 2e-16 ***
## monthjun              1.052e-01 8.094e-02   1.300 0.193738
## monthmar              1.063e+00 1.103e-01   9.639 < 2e-16 ***
## monthmay              -5.021e-01 6.345e-02  -7.913 2.52e-15 ***
## monthnov              -8.507e-01 7.457e-02 -11.409 < 2e-16 ***
## monthoct              6.755e-01 9.781e-02   6.907 4.96e-12 ***
## monthsep              6.544e-01 1.075e-01   6.089 1.13e-09 ***
## poutcomeother        2.516e-01 7.967e-02   3.159 0.001586 **
## poutcomesuccess      2.264e+00 7.346e-02  30.813 < 2e-16 ***
## poutcomeunknown      3.486e-02 5.156e-02   0.676 0.498980
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 32631  on 45210  degrees of freedom
## Residual deviance: 27282  on 45174  degrees of freedom
## AIC: 27356
##
## Number of Fisher Scoring iterations: 6

```

Regresion logistica, es un modelo lineal igual que antes el coef de casados es -2.1, si hacemos $e^{-0.21} = 0.81$, los casados tiene $1/0.81 = 1.23$ tienen 1.23 veces mas de contratar los casados. Casado es igual a divorciado, soltero no es igual a divorciado hay que tener cuidado con el caso base, normalmente se pone el que tenga más registros. Cuando hay variables categoricas jugamos a cambiar el orden. las variables se estan comparando

con una. Si cogemos como caso base el que tenga el coef mas pequeño conseguiremos que todos los coef sean positivos (ES LO MEJOR), se hace un relevel,ganas en interpretabilidad de los modelos. Intento no quitar variables porque luego puedo tener mas datos y a lo mejor tiene más importancia. Si no se como agruparlos los dejo separados. Aquí no hay R², maximizamos la verosimilitud, te compara el modelo Null. AIC cuanto más pequeño mejor.

```
model_probit1=glm(y~job+marital+education+default+balance+housing+loan+contact+month+poutcome, data=BANK)
summary(model_probit1)

##
## Call:
## glm(formula = y ~ job + marital + education + default + balance +
##       housing + loan + contact + month + poutcome, family = binomial(link = "probit"),
##       data = BANK)
##
## Deviance Residuals:
##      Min        1Q     Median        3Q       Max
## -2.3690   -0.4792   -0.3836   -0.2513    3.0684
##
## Coefficients:
##                               Estimate Std. Error z value Pr(>|z|)
## (Intercept)           -7.929e-01 5.657e-02 -14.017 < 2e-16 ***
## jobblue-collar      -5.775e-02 3.325e-02  -1.737 0.082405 .
## jobentrepreneur     -9.856e-02 5.625e-02  -1.752 0.079729 .
## jobhousemaid        -1.537e-01 6.107e-02  -2.517 0.011826 *
## jobmanagement        -3.585e-02 3.478e-02  -1.031 0.302638
## jobretired           2.496e-01 4.240e-02   5.887 3.92e-09 ***
## jobself-employed     -5.269e-02 5.190e-02  -1.015 0.310009
## jobservices          -4.880e-02 3.831e-02  -1.274 0.202662
## jobstudent            1.986e-01 5.490e-02   3.617 0.000298 ***
## jobtechnician         -3.917e-02 3.241e-02  -1.209 0.226850
## jobunemployed         6.648e-02 5.244e-02   1.268 0.204910
## jobunknowm           -1.213e-01 1.091e-01  -1.111 0.266409
## maritalmarried        -1.136e-01 2.693e-02  -4.220 2.44e-05 ***
## maritalsingle          4.783e-02 2.914e-02   1.641 0.100742
## educationsecondary    7.190e-02 2.869e-02   2.506 0.012215 *
## educationtertiary     1.613e-01 3.406e-02   4.735 2.19e-06 ***
## educationunknown       9.669e-02 4.843e-02   1.996 0.045885 *
## defaultyes            -6.124e-02 7.134e-02  -0.858 0.390676
## balance                1.047e-05 2.456e-06   4.265 2.00e-05 ***
## housingyes             -2.705e-01 1.984e-02  -13.636 < 2e-16 ***
## loanyes                -1.943e-01 2.618e-02  -7.422 1.15e-13 ***
## contacttelephone       -1.551e-01 3.413e-02  -4.545 5.50e-06 ***
## contactunknowm         -6.339e-01 3.094e-02  -20.486 < 2e-16 ***
## monthaug               -5.082e-01 3.736e-02  -13.601 < 2e-16 ***
## monthdec                3.585e-01 9.630e-02   3.723 0.000197 ***
## monthfeb               -2.301e-01 4.153e-02  -5.541 3.00e-08 ***
## monthjan                -5.711e-01 5.601e-02  -10.195 < 2e-16 ***
## monthjul                -4.115e-01 3.651e-02  -11.271 < 2e-16 ***
## monthjun                1.445e-02 4.374e-02   0.330 0.741171
## monthmar                6.623e-01 6.634e-02   9.984 < 2e-16 ***
## monthmay                -2.585e-01 3.454e-02  -7.484 7.23e-14 ***
## monthnov                -4.455e-01 3.991e-02  -11.162 < 2e-16 ***
## monthoct                4.316e-01 5.745e-02   7.512 5.82e-14 ***
```

```

## monthsep      4.147e-01  6.327e-02   6.553 5.62e-11 ***
## poutcomeother 1.377e-01  4.343e-02   3.171 0.001517 **
## poutcomesuccess 1.334e+00  4.225e-02  31.574 < 2e-16 ***
## poutcomeunknown 1.695e-02  2.756e-02   0.615 0.538435
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 32631  on 45210  degrees of freedom
## Residual deviance: 27309  on 45174  degrees of freedom
## AIC: 27383
##
## Number of Fisher Scoring iterations: 6

```

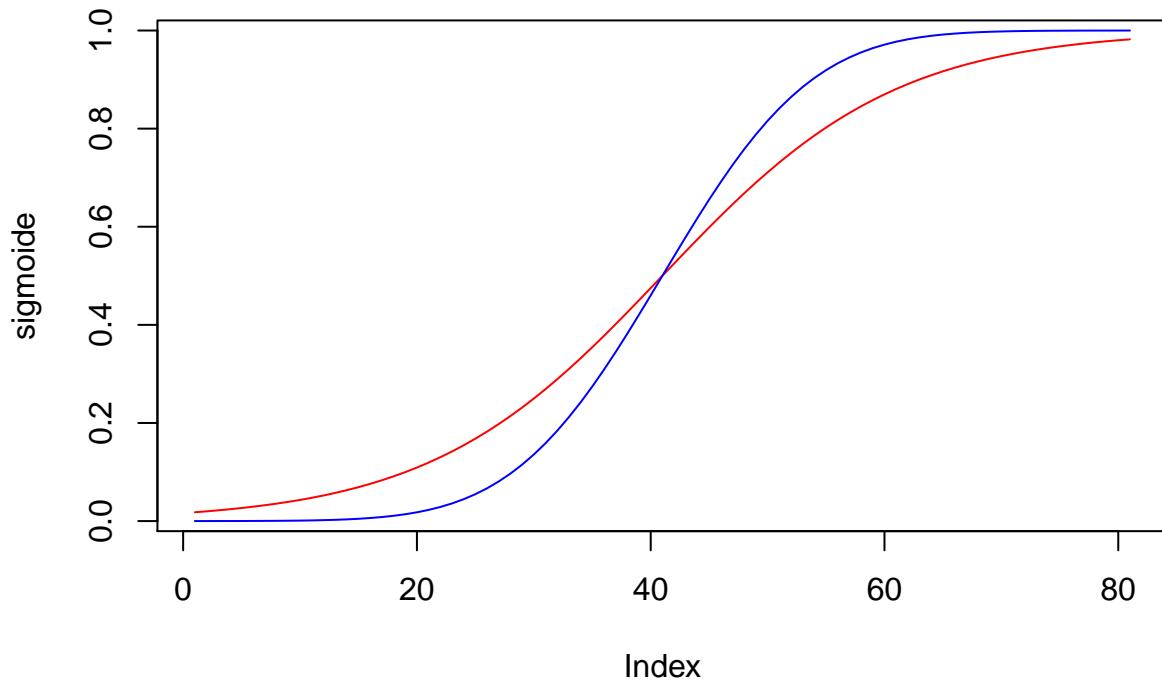
el efecto es un poco diferente, lo que si que es igual es el signo. Es un poco mas pequeño el AIC del logit

Diferencia entre el logit y el probit

```

X=seq(from=-4,to=4,by=0.1)
sigmoide=1/(1+exp(-X))
cumulative<-pnorm(X, 0, 1)
plot(sigmoide,type="l",col="red")
lines(cumulative,col="blue")

```



la roja es la sigmoide, es menos rigida. la azul se usa cuando quieres muy fiables y muy sensibles(medicina)

Evaluación del Modelo

Me quedo con el modelo logit1 probablemente tendría que quitar variables, como evaluo? tengo un monton de 0 y 1, mi modelo me devuelve probabilidades.

```
BANK$prediccion=predict(model_logit1,type="response")
head(BANK$prediccion)

## [1] 0.02831273 0.03067238 0.01374012 0.02339745 0.04753360 0.02743051
```

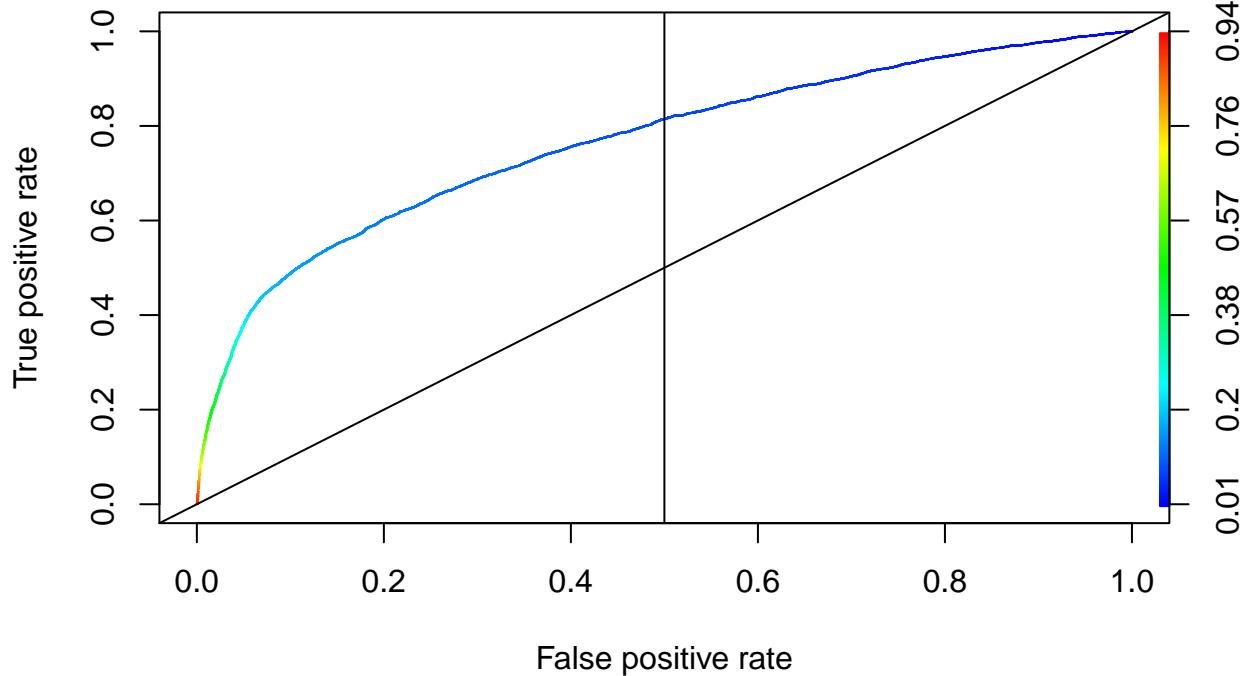
me puede dar el resultado despues de aplicar el sigmoide

```
Pred_auxiliar= prediction(BANK$prediccion, BANK$y, label.ordering = NULL)
auc.tmp = performance(Pred_auxiliar, "auc");
auc_model_logit1_train = as.numeric(auc.tmp@y.values)
auc_model_logit1_train #para medir la capacidad predictiva del modelo

## [1] 0.7641777
```

```
CURVA_ROC_model_logit1_train <- performance(Pred_auxiliar,"tpr","fpr")

plot(CURVA_ROC_model_logit1_train,colorize=TRUE)
abline(a=0,b=1)
abline(v=0.5)
```



cuando corte en la curva estoy capturando el 0.5 de los positivos y el 0.1 de los negativos

Capacidad del Modelo

```
mean(as.numeric(BANK$y)-1)
```

```
## [1] 0.1169848
```

Restamos 1, porque al pasar a numerico me pone 2 y 1, el 11% contratan de cada 100 que coja. “

```
aggregate(BANK$prediccion~BANK$y, FUN=mean)
```

```
##   BANK$y  BANK$prediccion
## 1     no      0.09752806
## 2    yes      0.26384662
```

comparo las predicciones con respecto al modelo, a los que si han contratado mi modelo le da una probabilidad de 26%, le está dando más probabilidad a los que han contratado.

Puesta en valor de un modelo: Fijación del Threshold

```
ALPHA=0.5  
Confusion=table(BANK$y ,BANK$prediccion>=ALPHA)  
Confusion
```

```
##  
##          FALSE   TRUE  
##    no    39398    524  
##    yes    4338    951
```

BANK predicciones son las probabilidades de que sean mayores que 0.5 BANK tiene 5289 positivos, y 39922 negativos. En alpha=0.5, hay 951 True positive and 524 false positive

```
Accuracy= (sum(BANK$y=="yes" & BANK$prediccion>=ALPHA)+sum(BANK$y=="no" & BANK$prediccion<ALPHA))/length(BANK$y)  
Accuracy
```

```
## [1] 0.8924598
```

TruePositive+True Negative/TOTALobs, es el porcentaje de aciertos, aciertas un 89%

Precisión

Precision is the number of True Positives divided by the number of True Positives and False Positives. Put another way, it is the number of positive predictions divided by the total number of positive class values predicted. It is also called the Positive Predictive Value (PPV).

```
Precision=sum(BANK$y=="yes" & BANK$prediccion>=ALPHA)/sum(BANK$y=="yes")  
Precision
```

```
## [1] 0.1798071
```

Recall is the number of True Positives divided by the number of True Positives and the number of False Negatives. Put another way it is the number of positive predictions divided by the number of positive class values in the test data. It is also called Sensitivity or the True Positive Rate. Recall 951/(951+524)

```
recall=sum(BANK$y=="yes" & BANK$prediccion>=ALPHA)/sum(BANK$prediccion>=ALPHA)  
recall
```

```
## [1] 0.6447458
```

La cobertura la han calculado como la precision:

```
Cobertura=sum(BANK$y=="yes" & BANK$prediccion>=ALPHA)/sum(BANK$y=="yes")  
Cobertura
```

```
## [1] 0.1798071
```

951/5289, Truepositive/suma todos los positivos BANK\$y

Modificación de ALPHA

Si bajamos alpha, la precision es peor, la cobertura es mayor. Tengo más errores en la tabla de confusión.

Criterio maximizar F1-Score

En estadística análisis de clasificación binaria , la F 1 puntuación (también F-Resultado o F-medida) es una medida de la exactitud de una prueba. Se considera tanto la precisión p y la retirada r de la prueba para calcular la puntuación: p es el número de resultados positivos correctos dividido por el número de todos los resultados positivos, y r es el número de resultados positivos correctos dividido por el número de positivos resultados que deberían haber sido devueltos. El F 1 puntuación puede ser interpretado como un promedio ponderado de la precisión y la recuperación , en donde un F 1 puntuación alcanza su mejor valor en 1 y lo peor a 0.

```
Precisionf1 <- Precision  
Precisionf1  
  
## [1] 0.1798071  
  
Recallf1 <- recall  
Recallf1  
  
## [1] 0.6447458  
  
F=2*((Precisionf1*Recallf1)/(Precisionf1+Recallf1))  
F  
  
## [1] 0.2811946
```

Índice Fowlkes-Malvas

Índice Fowlkes-Malvas [1] es una evaluación externa método que se utiliza para determinar la similitud entre dos agrupamientos (clusters obtenidos después de un algoritmo de agrupamiento). Esta medida de similitud podría ser o bien entre dos agrupamientos jerárquicos o de una agrupación, a la nomenclatura de referencia. Un valor más alto para el índice Fowlkes-Malvas indica una mayor similitud entre los clusters y las clasificaciones de referencia.

```
FM=sqrt(Precisionf1*Recallf1)  
FM  
  
## [1] 0.3404848
```

MODELOS LINIALES GENERALIZADOS: MODELO POISSON

Carga de Datos

```
BICIS=read.csv("hour.csv")
```

datos extraídos de <https://archive.ics.uci.edu/ml/datasets/Bike+Sharing+Dataset>

Revisión básica dataset

```
str(BICIS)
```

```
## 'data.frame': 17379 obs. of 17 variables:
## $ instant : int 1 2 3 4 5 6 7 8 9 10 ...
## $ dteday   : Factor w/ 731 levels "2011-01-01","2011-01-02",...: 1 1 1 1 1 1 1 1 1 1 ...
## $ season   : int 1 1 1 1 1 1 1 1 1 ...
## $ yr       : int 0 0 0 0 0 0 0 0 0 ...
## $ mnth     : int 1 1 1 1 1 1 1 1 1 ...
## $ hr       : int 0 1 2 3 4 5 6 7 8 9 ...
## $ holiday  : int 0 0 0 0 0 0 0 0 0 ...
## $ weekday  : int 6 6 6 6 6 6 6 6 6 ...
## $ workingday: int 0 0 0 0 0 0 0 0 0 ...
## $ weathersit: int 1 1 1 1 1 2 1 1 1 ...
## $ temp     : num 0.24 0.22 0.22 0.24 0.24 0.24 0.22 0.2 0.24 0.32 ...
## $ atemp    : num 0.288 0.273 0.273 0.288 0.288 ...
## $ hum      : num 0.81 0.8 0.8 0.75 0.75 0.75 0.8 0.86 0.75 0.76 ...
## $ windspeed: num 0 0 0 0 0 0.0896 0 0 0 ...
## $ casual   : int 3 8 5 3 0 0 2 1 1 8 ...
## $ registered: int 13 32 27 10 1 1 0 2 7 6 ...
## $ cnt      : int 16 40 32 13 1 1 2 3 8 14 ...
```

```
head(BICIS)
```

```
##   instant      dteday season yr mnth hr holiday weekday workingday
## 1        1 2011-01-01     1 0     1 0     0       6         0
## 2        2 2011-01-01     1 0     1 1     0       6         0
## 3        3 2011-01-01     1 0     1 2     0       6         0
## 4        4 2011-01-01     1 0     1 3     0       6         0
## 5        5 2011-01-01     1 0     1 4     0       6         0
## 6        6 2011-01-01     1 0     1 5     0       6         0
##   weathersit temp atemp hum windspeed casual registered cnt
## 1            1 0.24 0.2879 0.81 0.0000 3       13 16
## 2            1 0.22 0.2727 0.80 0.0000 8       32 40
## 3            1 0.22 0.2727 0.80 0.0000 5       27 32
## 4            1 0.24 0.2879 0.75 0.0000 3       10 13
## 5            1 0.24 0.2879 0.75 0.0000 0        1 1
## 6            2 0.24 0.2576 0.75 0.0896 0        1 1
```

```
summary(BICIS)
```

```
##   instant      dteday      season      yr
## Min.   : 1   2011-01-01: 24   Min.   :1.000   Min.   :0.0000
## 1st Qu.: 4346 2011-01-08: 24   1st Qu.:2.000   1st Qu.:0.0000
```

```

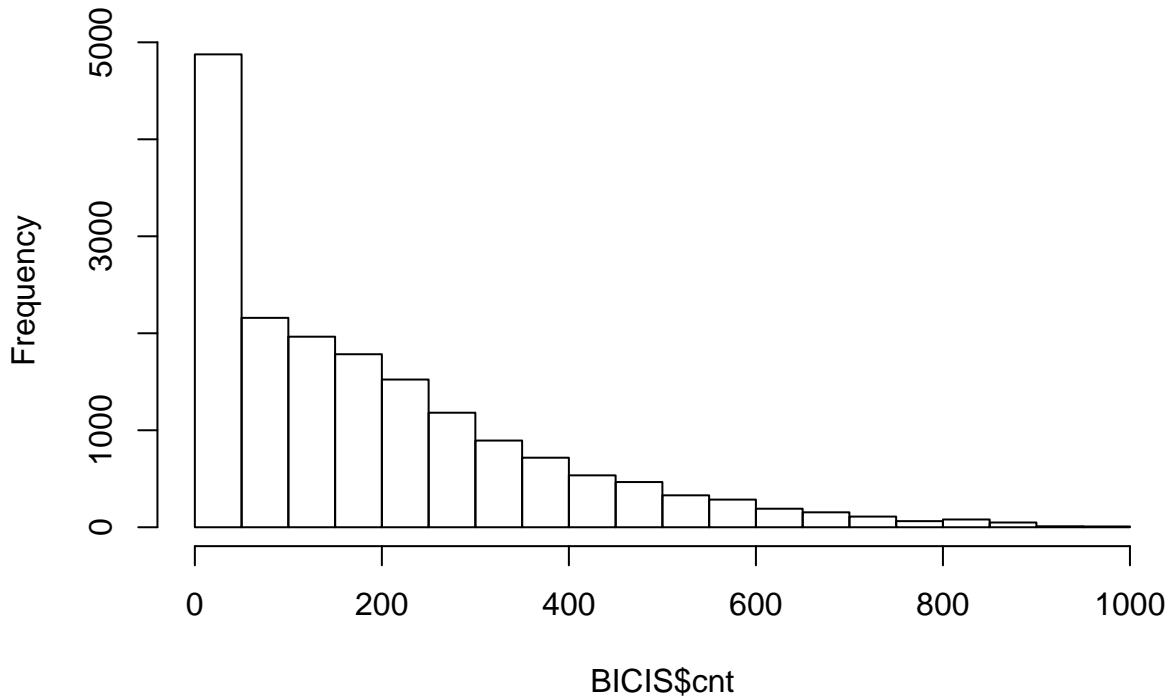
## Median : 8690 2011-01-09: 24 Median :3.000 Median :1.0000
## Mean   : 8690 2011-01-10: 24 Mean   :2.502 Mean   :0.5026
## 3rd Qu.:13034 2011-01-13: 24 3rd Qu.:3.000 3rd Qu.:1.0000
## Max.   :17379 2011-01-15: 24 Max.   :4.000 Max.   :1.0000
##          (Other) :17235
##      mnth        hr       holiday      weekday
## Min.   : 1.000  Min.   : 0.00  Min.   :0.00000  Min.   :0.000
## 1st Qu.: 4.000  1st Qu.: 6.00  1st Qu.:0.00000  1st Qu.:1.000
## Median : 7.000  Median :12.00  Median :0.00000  Median :3.000
## Mean   : 6.538  Mean   :11.55  Mean   :0.02877  Mean   :3.004
## 3rd Qu.:10.000 3rd Qu.:18.00  3rd Qu.:0.00000  3rd Qu.:5.000
## Max.   :12.000  Max.   :23.00  Max.   :1.00000  Max.   :6.000
##
##      workingday    weathersit      temp      atemp
## Min.   :0.0000  Min.   :1.000  Min.   :0.020  Min.   :0.0000
## 1st Qu.:0.0000  1st Qu.:1.000  1st Qu.:0.340  1st Qu.:0.3333
## Median :1.0000  Median :1.000  Median :0.500  Median :0.4848
## Mean   :0.6827  Mean   :1.425  Mean   :0.497  Mean   :0.4758
## 3rd Qu.:1.0000  3rd Qu.:2.000  3rd Qu.:0.660  3rd Qu.:0.6212
## Max.   :1.0000  Max.   :4.000  Max.   :1.000  Max.   :1.0000
##
##      hum      windspeed      casual      registered
## Min.   :0.0000  Min.   :0.0000  Min.   : 0.00  Min.   : 0.0
## 1st Qu.:0.4800  1st Qu.:0.1045  1st Qu.: 4.00  1st Qu.: 34.0
## Median :0.6300  Median :0.1940  Median :17.00  Median :115.0
## Mean   :0.6272  Mean   :0.1901  Mean   :35.68  Mean   :153.8
## 3rd Qu.:0.7800  3rd Qu.:0.2537  3rd Qu.:48.00  3rd Qu.:220.0
## Max.   :1.0000  Max.   :0.8507  Max.   :367.00  Max.   :886.0
##
##      cnt
## Min.   : 1.0
## 1st Qu.: 40.0
## Median :142.0
## Mean   :189.5
## 3rd Qu.:281.0
## Max.   :977.0
##

```

Modelos Regresión de Poisson

```
hist(BICIS$cnt)
```

Histogram of BICIS\$cnt



```
mean(BICIS$cnt)
```

```
## [1] 189.4631
```

```
sd(BICIS$cnt)
```

```
## [1] 181.3876
```

Modelo Poisson quitando las variables instant, dteday, casual y registered

```
model_poisson=glm(cnt~.-instant-dteday-casual-registered, family=poisson(link = "log"), data=BICIS) #a l
summary(model_poisson)
```

```
##
## Call:
## glm(formula = cnt ~ . - instant - dteday - casual - registered,
##       family = poisson(link = "log"), data = BICIS)
##
## Deviance Residuals:
##     Min      1Q      Median      3Q      Max
## -30.221   -8.748   -3.022    3.962   38.708
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
```

```

## (Intercept) 3.734e+00 3.953e-03 944.69 <2e-16 ***
## season      1.165e-01 1.098e-03 106.12 <2e-16 ***
## yr          4.393e-01 1.143e-03 384.37 <2e-16 ***
## mnth        7.090e-03 3.580e-04 19.80 <2e-16 ***
## hr          4.571e-02 9.071e-05 503.86 <2e-16 ***
## holiday     -1.319e-01 3.738e-03 -35.29 <2e-16 ***
## weekday     7.712e-03 2.796e-04 27.58 <2e-16 ***
## workingday   2.123e-02 1.235e-03 17.18 <2e-16 ***
## weathersit   -2.112e-02 1.051e-03 -20.10 <2e-16 ***
## temp         4.289e-02 1.802e-02 2.38 0.0173 *
## atemp        1.651e+00 2.042e-02 80.87 <2e-16 ***
## hum          -1.018e+00 3.557e-03 -286.19 <2e-16 ***
## windspeed    3.070e-01 4.786e-03 64.15 <2e-16 ***
##
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
## Null deviance: 2891591 on 17378 degrees of freedom
## Residual deviance: 1685957 on 17366 degrees of freedom
## AIC: 1796884
##
## Number of Fisher Scoring iterations: 5

```

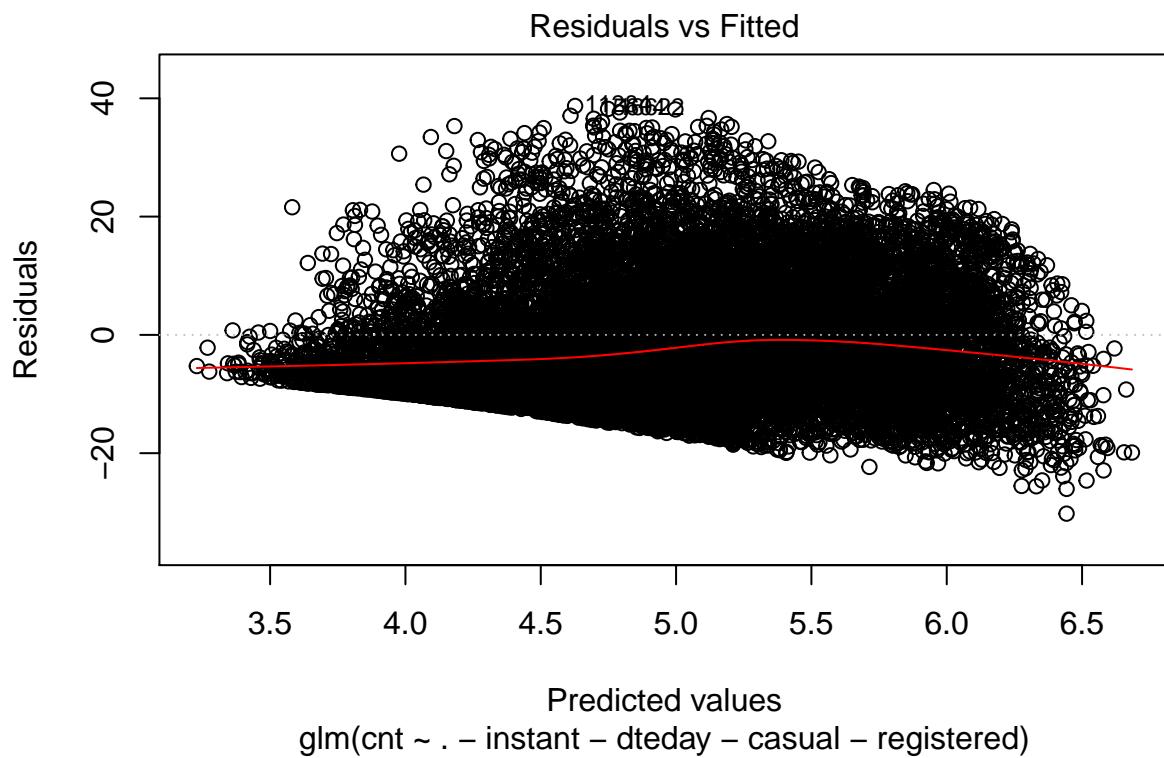
```
model_poisson
```

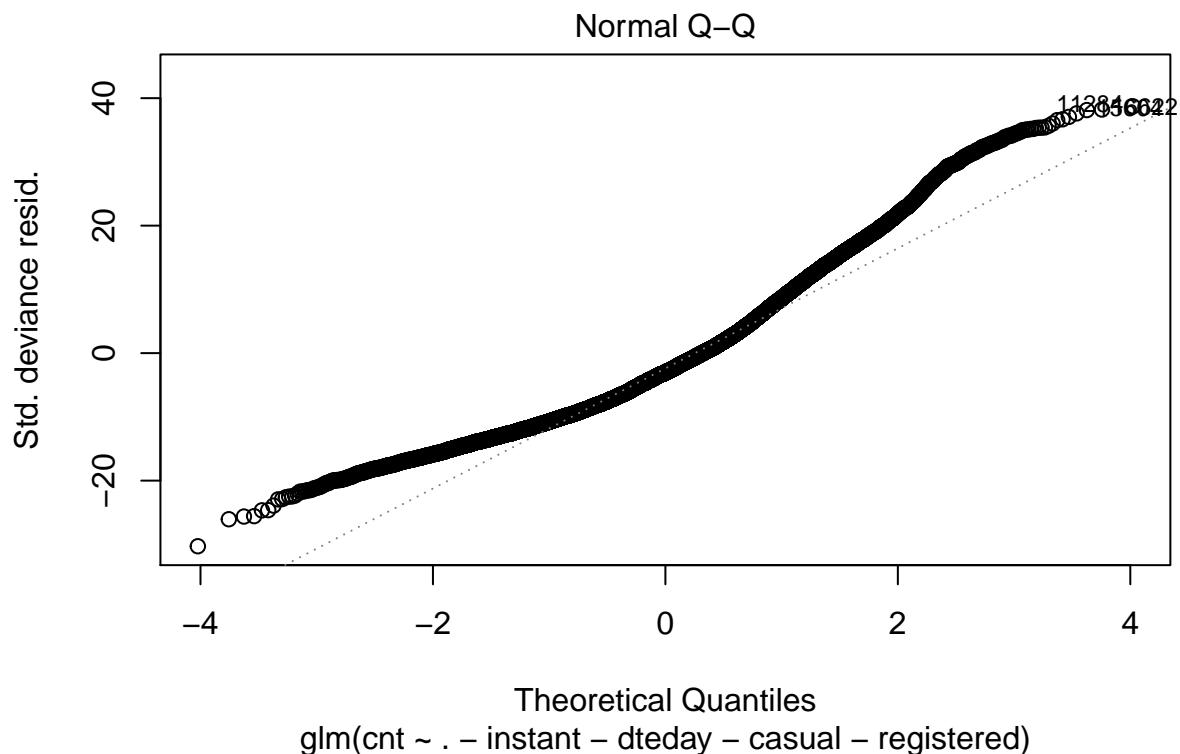
```

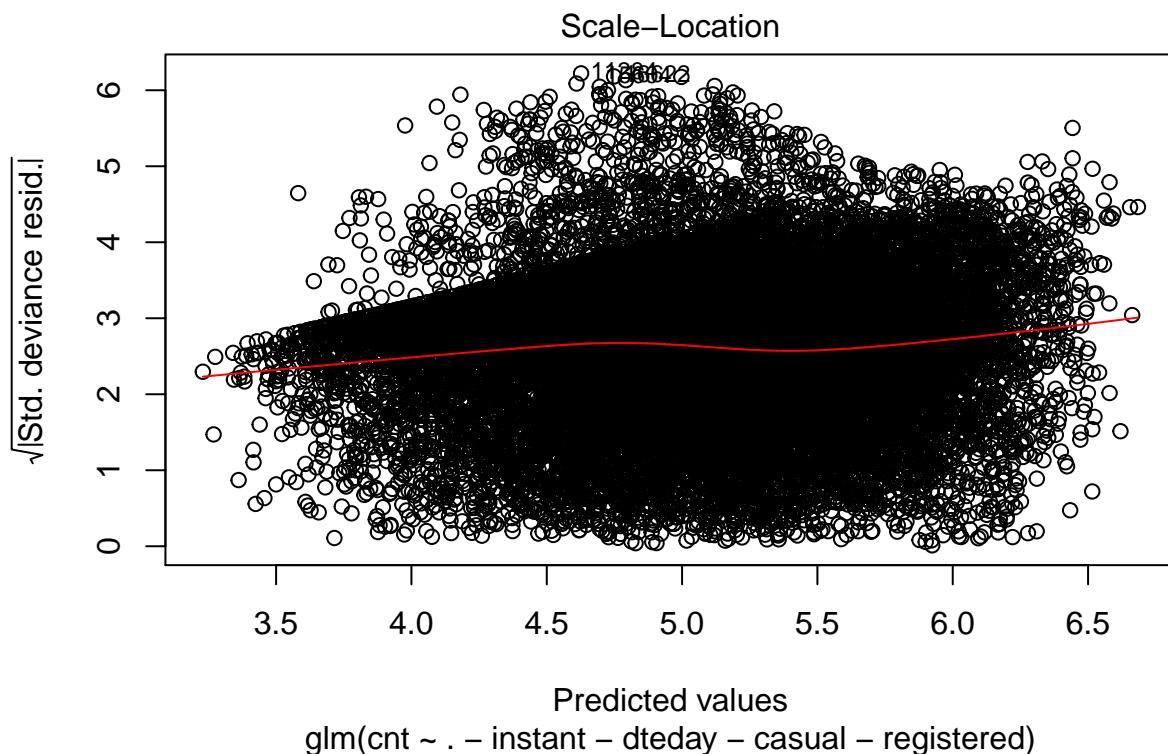
##
## Call: glm(formula = cnt ~ . - instant - dteday - casual - registered,
##           family = poisson(link = "log"), data = BICIS)
##
## Coefficients:
## (Intercept)      season          yr          mnth          hr
## 3.734459      0.116493      0.439322      0.007090      0.045705
## holiday       weekday       workingday  weathersit       temp
## -0.131925      0.007712      0.021229     -0.021123      0.042889
## atemp          hum          windspeed
## 1.651171     -1.018081      0.306994
##
## Degrees of Freedom: 17378 Total (i.e. Null); 17366 Residual
## Null Deviance: 2892000
## Residual Deviance: 1686000  AIC: 1797000

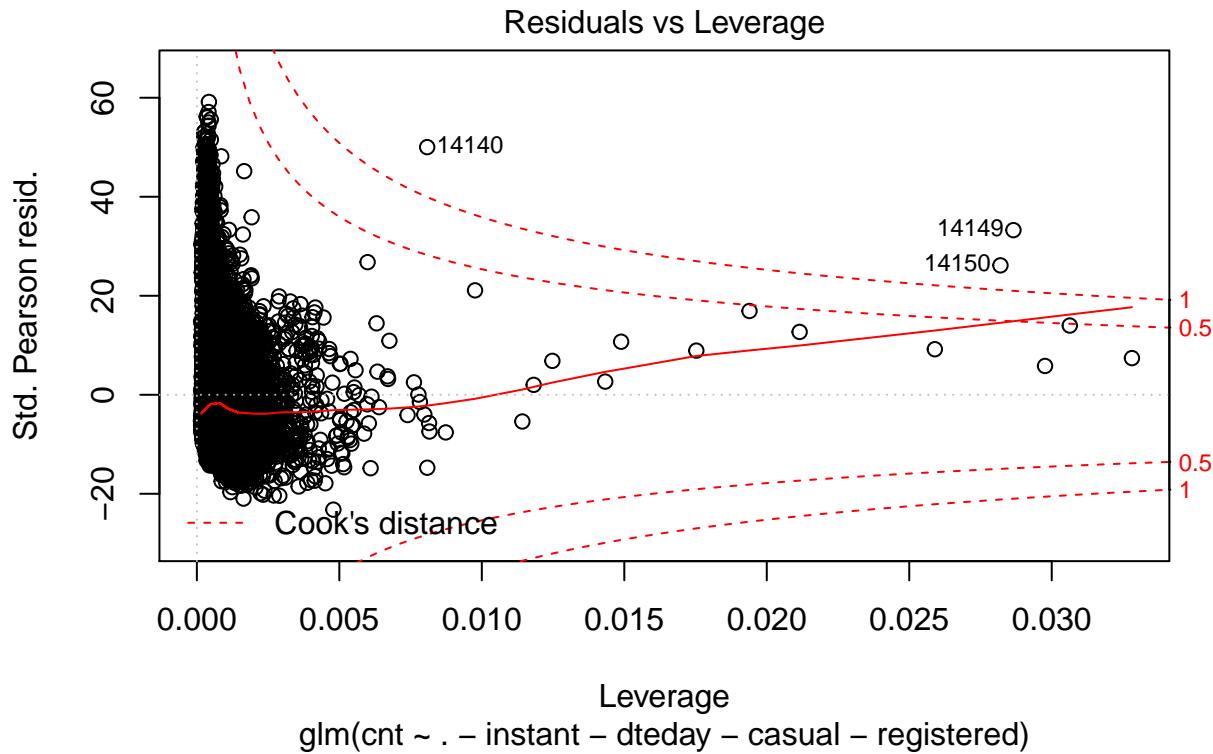
```

```
plot(model_poisson)
```









Todas las variables son significativas menos temp.

```
BICIS$prediccion=predict(model_poisson,type="response") #prediccion valores con el modelo de poisson
head(BICIS)
```

```
##   instant      dteday season yr mnth hr holiday weekday workingday
## 1       1 2011-01-01     1 0     1 0      0       6        0
## 2       2 2011-01-01     1 0     1 1      0       6        0
## 3       3 2011-01-01     1 0     1 2      0       6        0
## 4       4 2011-01-01     1 0     1 3      0       6        0
## 5       5 2011-01-01     1 0     1 4      0       6        0
## 6       6 2011-01-01     1 0     1 5      0       6        0
##   weathersit temp atemp hum windspeed casual registered cnt prediccion
## 1           1 0.24 0.2879 0.81 0.0000     3      13 16 34.61232
## 2           1 0.22 0.2727 0.80 0.0000     8      32 40 35.66395
## 3           1 0.22 0.2727 0.80 0.0000     5      27 32 37.33181
## 4           1 0.24 0.2879 0.75 0.0000     3      10 13 42.19960
## 5           1 0.24 0.2879 0.75 0.0000     0       1  1 44.17310
## 6           2 0.24 0.2576 0.75 0.0896     0       1  1 44.26412
```

```
SCE=sum((BICIS$cnt-BICIS$prediccion)^2) #suma cuadrado de los errores
```

```
STC=sum((BICIS$cnt-mean(BICIS$cnt))^2)
```

```
R2=1-(SCE/STC)
```

```
R2
```

```
## [1] 0.3793564
```

Interpretación del resultado: El 37.94% de la varianza de cnt está explicada por las variables de nuestro modelo, la varianza residual es de 62,06%.

R2 = Varianza Explicada / Total Varianza

- possibility 1 R2 <- cor(y,predict(mod))^2
- possibility 2 R2 <- 1 - (sum((y-predict(mod))^2)/sum((y-mean(y))^2))

Formateo variables

```
BICIS=read.csv("hour.csv")
```

```
BICIS$season=as.factor(BICIS$season)
BICIS$yr=as.factor(BICIS$yr)
BICIS$mnth=as.factor(BICIS$mnth)
BICIS$hr=as.factor(BICIS$hr)
BICIS$holiday=as.factor(BICIS$holiday)
BICIS$weekday=as.factor(BICIS$weekday)
BICIS$workingday=as.factor(BICIS$workingday)
BICIS$weathersit=as.factor(BICIS$weathersit)
```

```
model_poisson=glm(cnt~.-instant-dteday-casual-registered, family=poisson(link = "log"),data=BICIS)
summary(model_poisson)
```

```
##
## Call:
## glm(formula = cnt ~ . - instant - dteday - casual - registered,
##      family = poisson(link = "log"), data = BICIS)
##
## Deviance Residuals:
##       Min        1Q        Median         3Q        Max
## -24.9615   -3.7666   -0.8567    3.0347   22.3749
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) 2.917336  0.006790 429.640 < 2e-16 ***
## season2     0.274076  0.003680  74.487 < 2e-16 ***
## season3     0.267228  0.004211  63.457 < 2e-16 ***
## season4     0.457991  0.004081 112.212 < 2e-16 ***
## yr1         0.468566  0.001151 407.080 < 2e-16 ***
## mnth2       0.113516  0.003776  30.060 < 2e-16 ***
## mnth3       0.223665  0.003937  56.817 < 2e-16 ***
## mnth4       0.181289  0.005235  34.629 < 2e-16 ***
## mnth5       0.244655  0.005477  44.668 < 2e-16 ***
## mnth6       0.196362  0.005585  35.158 < 2e-16 ***
## mnth7       0.098808  0.006064  16.294 < 2e-16 ***
## mnth8       0.195102  0.005898  33.078 < 2e-16 ***
## mnth9       0.270869  0.005427  49.914 < 2e-16 ***
## mnth10      0.187711  0.005395  34.794 < 2e-16 ***
## mnth11      0.061117  0.005303  11.524 < 2e-16 ***
## mnth12      0.045360  0.004676   9.700 < 2e-16 ***
```

```

## hr1      -0.466697  0.008182 -57.038 < 2e-16 ***
## hr2      -0.839682  0.009313 -90.160 < 2e-16 ***
## hr3      -1.507858  0.012163 -123.967 < 2e-16 ***
## hr4      -2.110448  0.015858 -133.084 < 2e-16 ***
## hr5      -0.956562  0.009787 -97.738 < 2e-16 ***
## hr6       0.400501  0.006619  60.509 < 2e-16 ***
## hr7       1.422874  0.005666  251.117 < 2e-16 ***
## hr8       1.916567  0.005423  353.411 < 2e-16 ***
## hr9       1.391883  0.005648  246.429 < 2e-16 ***
## hr10      1.123194  0.005806  193.438 < 2e-16 ***
## hr11      1.269596  0.005717  222.071 < 2e-16 ***
## hr12      1.447484  0.005642  256.544 < 2e-16 ***
## hr13      1.427091  0.005663  251.991 < 2e-16 ***
## hr14      1.364773  0.005707  239.121 < 2e-16 ***
## hr15      1.405023  0.005693  246.794 < 2e-16 ***
## hr16      1.628120  0.005593  291.123 < 2e-16 ***
## hr17      2.036233  0.005445  373.972 < 2e-16 ***
## hr18      1.970299  0.005443  362.018 < 2e-16 ***
## hr19      1.674864  0.005518  303.540 < 2e-16 ***
## hr20      1.377556  0.005648  243.882 < 2e-16 ***
## hr21      1.121995  0.005800  193.434 < 2e-16 ***
## hr22      0.864956  0.006005  144.039 < 2e-16 ***
## hr23      0.483910  0.006419   75.382 < 2e-16 ***
## holiday1    -0.160979  0.003797 -42.398 < 2e-16 ***
## weekday1     0.051206  0.002167  23.626 < 2e-16 ***
## weekday2     0.060927  0.002103  28.971 < 2e-16 ***
## weekday3     0.066408  0.002103  31.582 < 2e-16 ***
## weekday4     0.067338  0.002089  32.228 < 2e-16 ***
## weekday5     0.093581  0.002089  44.798 < 2e-16 ***
## weekday6     0.079608  0.002091  38.063 < 2e-16 ***
## workingday1    NA        NA        NA        NA
## weathersit2   -0.064256  0.001422 -45.175 < 2e-16 ***
## weathersit3   -0.492963  0.002864 -172.124 < 2e-16 ***
## weathersit4   -0.469206  0.067067  -6.996 2.63e-12 ***
## temp          0.164396  0.019469   8.444 < 2e-16 ***
## atemp         0.946850  0.020326  46.584 < 2e-16 ***
## hum           -0.205716  0.004129 -49.824 < 2e-16 ***
## windspeed     -0.109958  0.004869 -22.581 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
## Null deviance: 2891591 on 17378 degrees of freedom
## Residual deviance: 572011 on 17326 degrees of freedom
## AIC: 683018
##
## Number of Fisher Scoring iterations: 5

model_poisson=glm(cnt~.-workingday-instant-dteday-casual-registered, family=poisson(link = "log"),data=
summary(model_poisson)
```

```

## 
## Call:
```

```

## glm(formula = cnt ~ . - workingday - instant - dteday - casual -
##      registered, family = poisson(link = "log"), data = BICIS)
##
## Deviance Residuals:
##      Min        1Q     Median        3Q       Max
## -24.9615   -3.7666   -0.8567    3.0347   22.3749
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 2.917336  0.006790 429.640 < 2e-16 ***
## season2     0.274076  0.003680  74.487 < 2e-16 ***
## season3     0.267228  0.004211  63.457 < 2e-16 ***
## season4     0.457991  0.004081 112.212 < 2e-16 ***
## yr1         0.468566  0.001151 407.080 < 2e-16 ***
## mnth2       0.113516  0.003776  30.060 < 2e-16 ***
## mnth3       0.223665  0.003937  56.817 < 2e-16 ***
## mnth4       0.181289  0.005235  34.629 < 2e-16 ***
## mnth5       0.244655  0.005477  44.668 < 2e-16 ***
## mnth6       0.196362  0.005585  35.158 < 2e-16 ***
## mnth7       0.098808  0.006064  16.294 < 2e-16 ***
## mnth8       0.195102  0.005898  33.078 < 2e-16 ***
## mnth9       0.270869  0.005427  49.914 < 2e-16 ***
## mnth10      0.187711  0.005395  34.794 < 2e-16 ***
## mnth11      0.061117  0.005303  11.524 < 2e-16 ***
## mnth12      0.045360  0.004676  9.700 < 2e-16 ***
## hr1        -0.466697  0.008182 -57.038 < 2e-16 ***
## hr2        -0.839682  0.009313 -90.160 < 2e-16 ***
## hr3        -1.507858  0.012163 -123.967 < 2e-16 ***
## hr4        -2.110448  0.015858 -133.084 < 2e-16 ***
## hr5        -0.956562  0.009787 -97.738 < 2e-16 ***
## hr6         0.400501  0.006619  60.509 < 2e-16 ***
## hr7         1.422874  0.005666 251.117 < 2e-16 ***
## hr8         1.916567  0.005423 353.411 < 2e-16 ***
## hr9         1.391883  0.005648 246.429 < 2e-16 ***
## hr10        1.123194  0.005806 193.438 < 2e-16 ***
## hr11        1.269596  0.005717 222.071 < 2e-16 ***
## hr12        1.447484  0.005642 256.544 < 2e-16 ***
## hr13        1.427091  0.005663 251.991 < 2e-16 ***
## hr14        1.364773  0.005707 239.121 < 2e-16 ***
## hr15        1.405023  0.005693 246.794 < 2e-16 ***
## hr16        1.628120  0.005593 291.123 < 2e-16 ***
## hr17        2.036233  0.005445 373.972 < 2e-16 ***
## hr18        1.970299  0.005443 362.018 < 2e-16 ***
## hr19        1.674864  0.005518 303.540 < 2e-16 ***
## hr20        1.377556  0.005648 243.882 < 2e-16 ***
## hr21        1.121995  0.005800 193.434 < 2e-16 ***
## hr22        0.864956  0.006005 144.039 < 2e-16 ***
## hr23        0.483910  0.006419  75.382 < 2e-16 ***
## holiday1   -0.160979  0.003797 -42.398 < 2e-16 ***
## weekday1    0.051206  0.002167  23.626 < 2e-16 ***
## weekday2    0.060927  0.002103  28.971 < 2e-16 ***
## weekday3    0.066408  0.002103  31.582 < 2e-16 ***
## weekday4    0.067338  0.002089  32.228 < 2e-16 ***
## weekday5    0.093581  0.002089  44.798 < 2e-16 ***

```

```

## weekday6      0.079608   0.002091   38.063 < 2e-16 ***
## weathersit2 -0.064256   0.001422  -45.175 < 2e-16 ***
## weathersit3 -0.492963   0.002864 -172.124 < 2e-16 ***
## weathersit4 -0.469206   0.067067  -6.996 2.63e-12 ***
## temp         0.164396   0.019469   8.444 < 2e-16 ***
## atemp        0.946850   0.020326   46.584 < 2e-16 ***
## hum          -0.205716   0.004129  -49.824 < 2e-16 ***
## windspeed    -0.109958   0.004869  -22.581 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
## Null deviance: 2891591  on 17378 degrees of freedom
## Residual deviance: 572011  on 17326 degrees of freedom
## AIC: 683018
##
## Number of Fisher Scoring iterations: 5

```

Quitando working day mi indice AIC es el mismo.

BICIS\$cnt

##	[1]	16	40	32	13	1	1	2	3	8	14	36	56	84	94	106	110	93
##	[18]	67	35	37	36	34	28	39	17	17	9	6	3	2	1	8	20	53
##	[35]	70	93	75	59	74	76	65	53	30	22	31	9	8	5	2	1	3
##	[52]	30	64	154	88	44	51	61	61	77	72	76	157	157	110	52	52	20
##	[69]	12	5	2	1	2	4	36	94	179	100	42	57	78	97	63	65	83
##	[86]	212	182	112	54	48	35	11	6	6	2	2	3	33	88	195	115	57
##	[103]	46	79	71	62	62	89	190	169	132	89	43	42	19	11	4	2	1
##	[120]	4	36	95	219	122	45	59	84	67	70	62	86	172	163	112	69	48
##	[137]	52	23	17	7	1	1	5	34	84	210	134	63	67	59	73	50	72
##	[154]	87	187	123	95	51	39	36	15	25	16	16	7	1	5	2	9	15
##	[171]	20	61	62	98	102	95	74	76	69	55	30	28	37	34	22	25	12
##	[188]	11	4	1	1	1	6	10	19	49	49	83	75	72	82	92	62	48
##	[205]	41	38	20	15	6	5	1	3	1	3	3	31	77	188	94	31	30
##	[222]	52	54	47	45	74	178	155	95	74	38	24	18	12	3	3	6	27
##	[239]	99	217	130	54	35	57	52	63	47	76	136	95	51	32	20	29	19
##	[256]	7	6	1	5	16	54	128	81	39	35	55	49	44	49	68	139	137
##	[273]	83	56	57	33	20	7	2	2	3	4	3	28	72	202	139	38	37
##	[290]	52	83	42	60	78	162	144	99	64	40	30	15	14	5	1	1	8
##	[307]	17	70	158	117	44	53	61	77	64	68	90	159	139	92	68	52	36
##	[324]	27	28	20	12	8	5	1	3	10	23	33	59	72	89	101	118	129
##	[341]	128	83	84	74	41	57	26	44	39	23	16	15	1	2	1	3	18
##	[358]	32	79	93	104	118	91	113	99	105	67	61	57	28	21	18	17	16
##	[375]	8	2	3	1	5	13	33	47	57	64	80	93	86	93	82	71	92
##	[392]	60	33	27	13	4	3	22	28	35	61	125	133	99	83	41	33	20
##	[409]	3	7	3	2	7	32	90	197	109	47	52	70	78	75	82	104	197
##	[426]	161	112	76	59	59	28	13	5	2	1	1	6	35	101	249	143	57
##	[443]	68	84	98	81	70	91	215	185	152	126	57	56	31	21	6	2	1
##	[460]	1	5	27	68	217	166	63	59	78	73	62	65	97	161	120	96	53
##	[477]	41	34	27	13	12	11	7	3	2	8	27	40	53	63	70	84	75
##	[494]	103	83	67	54	59	45	39	30	33	22	13	18	5	3	1	2	19

##	[511]	28	58	99	116	87	110	77	65	55	49	50	35	25	28	21	7	1
##	[528]	1	1	5	15	84	177	102	40	46	63	60	45	57	70	184	153	106
##	[545]	81	59	35	24	9	5	2	1	9	36	108	238	144	55	61	106	93
##	[562]	68	84	116	222	225	146	119	45	53	40	17	5	10	1	8	30	72
##	[579]	58	28	41	48	47	36	43	36	26	24	84	104	79	59	38	27	16
##	[596]	9	3	2	1	4	16	60	157	101	49	30	29	31	38	41	80	149
##	[613]	109	89	62	58	26	23	28	20	15	8	3	2	5	34	34	55	64
##	[630]	78	65	99	120	107	91	68	58	43	36	32	33	33	29	11	8	1
##	[647]	3	3	12	38	64	59	97	84	122	109	123	77	65	55	33	28	21
##	[664]	21	7	7	1	2	2	8	37	72	185	112	69	48	68	54	86	44
##	[681]	86	161	156	111	78	56	34	17	8	3	2	2	3	22	52	135	116
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##	[766]	220	127	51	64	86	82	91	90	99	205	155	103	71	43	46	31	39
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##	[800]	83	84	69	56	45	59	39	44	20	13	2	1	1	8	23	45	89
##	[817]	117	174	182	161	182	157	121	78	21	26	27	62	30	15	5	3	1
##	[834]	2	10	30	95	230	118	55	47	66	64	60	50	114	216	175	128	88
##	[851]	78	37	25	10	4	2	1	3	2	39	100	243	135	48	50	65	50
##	[868]	64	51	83	176	152	101	56	54	29	12	17	7	3	2	7	43	99
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##	[1055]	122	255	222	161	118	92	73	33	17	6	6	4	4	12	47	126	285
##	[1072]	179	88	90	101	123	81	106	146	274	222	175	139	124	76	44	32	8
##	[1089]	7	8	1	7	50	116	272	169	95	113	185	176	191	172	232	327	224
##	[1106]	162	124	72	107	77	29	31	17	7	3	3	6	22	45	55	106	123
##	[1123]	119	155	196	171	120	127	88	59	47	33	44	29	17	16	17	18	1
##	[1140]	2	2	24	55	104	161	182	198	212	168	147	146	101	42	42	56	47
##	[1157]	54	37	13	4	5	1	3	8	19	63	57	81	90	93	103	116	87
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##	[1191]	69	86	196	167	118	105	82	77	21	6	4	1	2	8	36	96	235
##	[1208]	139	51	69	67	87	78	72	94	222	209	141	109	91	61	39	11	7
##	[1225]	4	2	4	58	104	252	137	57	79	95	88	69	63	71	177	137	115
##	[1242]	89	83	55	50	9	10	3	2	6	11	35	73	114	49	59	115	83
##	[1259]	85	55	102	189	157	90	62	63	50	39	28	27	12	8	2	10	7
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## [2007] 81 106 104 86 87 136 299 294 190 132 83 55 35 11 10 4 4
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## [17375] 119 89 90 61 49

```

```
BICIS$prediccion
```

```
## NULL
```

```

BICIS$prediccion=predict(model_poisson,type="response")
SCE=sum((BICIS$cnt-BICIS$prediccion)^2)
STC=sum((BICIS$cnt-mean(BICIS$cnt))^2)
R2=1-sum((BICIS$cnt-BICIS$prediccion)^2)/sum((BICIS$cnt-mean(BICIS$cnt))^2)

```

```
R2
```

```
## [1] 0.7557846
```

COMO VALIDAS SI UN MODELO FUNCIONA:

Realidad: tengo un historico de variables, hago un modelo, lo estimo, tengo la predicción, para gente nueva tengo mi predicción. A la gente que tengo prediccion Si, le voy a hacer campaña. Como evaluo la capacidad de la campaña? Clientes que tenías SI y le has hecho accion, ves el % exito entre.. Clientes que tenias NO y no les has hecho acción, ves el % exito esto me da 4,7 Mi modelo tiene propensos SI y propensos NO, lo normal es hacer accion SI y accion NO. En este modelo no has incentivado al que no es propenso. Hay que hacer acción sobre el no propenso. El problema es el tamaño de no prepensos que incluyes.

Calculas propensos Si con accion si / propensos Si accion no + Prop Si acc No/prop No Acc No

Pasa un año, como entrenas el modelo dentro de un año cuando no funcione. Ya estan influenciado con la accion comercial puedo usar los que no he aplicado accion pero eran propensos.