Optimizer 4.0 White Paper

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Abstract

Optimizer 4.0 is the latest update on the list of models for optimizing volume distribution. It allows more efficient and flexible optimization on more product combinations.

1 The Program

Let U be the set of products to optimize distribution. Let element u of U denote a specific product-combination, for example bulk-mesh-chicken or bag-pellet-cow. For now on, we use product/product-combination interchangeably. Define F_u as the set of factories that supply product u and C_u as the set of customers that purchase product u. Naturally, $\bigwedge_{u\in U} F_u \subseteq F$ and $\bigwedge_{u\in U} C_u \subseteq C$, for F and C the set of all factories and customers respectively. Note that the set U is constant on both the inbound section and the output section of the program

1.1 Objective Vector

The objective vector concatenates two smaller vectors, the inbound and outbound logistic cost vectors.

$$[\text{Objective Vector}] = [\text{Inbound Cost Vector}] \parallel [\text{Outbound Cost Vector}] \tag{1}$$

Subsequently,

$$[\text{Inbound Cost Vector}] = [\text{Inbound cost of all } f \in F_u \ \forall u \in U] \tag{2}$$

$$[\text{Outbound Cost Vector}] = [\text{Outbound cost of all } g \in (F \times C)_u \ \forall u \in U] \tag{3}$$

1.2 Constraints Matrix

Each of the constraints matrices are also divided into two parts corresponding to the inbound and outbound section of the objective vector. We use **IB** to refer to the inbound part and **OB** to refer to the outbound part. The subscript u in $\mathbf{OB_u}$ refers to row relating to product $u \in U$.

1.2.1 Demand Matrix

This matrix set the necessary constraints of satisfying all the demand of the customers for all products.

$$\mathbf{OB_u} = [\ [g \in F_u \times c]\ \forall c \in C_u] \tag{4}$$

where the inside hard-bracket denotes an Iverson braket of the form

$$[P] = \begin{cases} 1 & \text{if P is True;} \\ 0 & \text{otherwise.} \end{cases}$$

Vertically concatenate the $\mathbf{OB_u}$ for all $u \in U$, we get

$$\mathbf{OB} = \bigoplus_{u\in U} \Big(\mathop{\Big\|}_{|F_u|} I_{|C_u|} \Big) \tag{5}$$

Visually, this matrix is made up of block diagonal of multiple identiy matrix concatenate together.
This matrix has $\sum\limits_{u \in U} |C_u|$ rows and $\sum\limits_{u \in U} |F_u \times C_u|$ columns.
Since demand is independent of inbound logistics, the inbound part of the demand matrix is all zeros with $\sum\limits_{u \in U} |C_u|$ rows and $\sum\limits_{u \in U} |F_u|$ columns.

1.2.2 Product Capacity Matrix

This matrix defines capacity constraints on each product-combination. However practical settings, constraints on the joint-capacity of multiple related products are used more often. Nonetheless, we can recycle these twos for later constraints.

$$\mathbf{OB_u} = [\, [g \in f \times C_u] \, \forall f \in F_u] \tag{6}$$

Vertically concatenate the $\mathbf{OB_u}$ for all $u \in U$, we get

$$\mathbf{OB} = \bigoplus_{u \in U} (\bigoplus_{|F_u|} (\mathop{\Big\|}_{|C_u|} [1])) \tag{7}$$

Visually, this matrix is made up of block diagonal regular blocks diagonal 1, 1, 1, ..., each with length $|C_u|$ for each $u \in U$.
This matrix has $\sum\limits_{u \in U} |F_u|$ rows and $\sum\limits_{u \in U} |F_u \times C_u|$ columns.
Since product capacity is independent of inbound logistics, the inbound part of the product capacity matrix is all zeros with $\sum\limits_{u \in U} |F_u|$ rows and $\sum\limits_{u \in U} |F_u|$ columns.

1.2.3 Supply Matrix

The supply matrix dictates that there must be as much inbound volume as outbound volume (accounted for some loss factor).

The outbound section is set up exactly as that of the product capacity matrix.
Then the inbound part is square diagonal with dimension $\sum\limits_{u \in U} |F_u|$ rows and $\sum\limits_{u \in U} |F_u|$ columns.

$$\mathbf{IB} = -\mathrm{diag}\big\{\text{efficiency of } f \in F_u \; \forall u \in U\big\} \tag{8}$$

1.2.4 Joint Capacity Matrix

The joint capacity matrix accounts for situations where there a constrains on the combine production of a subset of product. Suppose we have a subset of products U', and we want that the total production. For example, we can have a cap on the production of bulk product in a factory, which would include all the associated sub-type. However, we can also have constraints on the global level between different factories. This arises when the pre-mix of one factory gets produced in another factory.

$$\sum_{u \in U' \subseteq U} V_{f_u} \leq K_{U'} \tag{9}$$

Mathematically, we can define a vector space of dimension $\sum_{u \in U} |F_u|$. The bases of this vector space are the rows of the outbound part of the product capacity matrix. This means that the possible constraints are linear combinations of the rows defined above. Suppose all coefficients are 0s and 1s, we have upto $2^{2^{\sum_{u \in U} |F_u|}}$ different joint capacity matrices.
Practically, it is advised to set up a hierarchy of product. This way, choosing a product from the top of the hierarchy will automatically choose the all the sub-type beneath it.
Let n be the number of joint constraints selected. Then, the outbound part of the joint-capacity matrix will have n rows, and $\sum\limits_{u \in U} |F_u \times C_u|$ columns. The inbound part will have n rows and $\sum\limits_{u \in U} |F_u|$ columns.

1.3 Joint Supply Matrix

Similarly to the joint supply matrix above, we can create a matrix referring to the joint combination of the supply. For example, suppose factory A has three products but only requires one inbound source. Then we have one element for the inbound cost, but three different groups of outbound cost. This is when we use the joint supply matrix.