Conservation of Linear momentum

Linear momentum of a Particle:

The momentum of a single particle is a vector $\vec{P}$ defined as the product of its mass m and its velocity $\vec{v}$. That is

$$\vec{P} = m\vec{v}$$

Expressed in modern terminology Newton's second Law reads

"The rate of change of momentum of a body is proportional to the resultant force acting on the body and is in the direction of that force.

In symbolic form. This becomes

$$\vec{F} = \frac{d\vec{P}}{dt}$$

So, $\vec{F} = m\vec{a}$

if m is constant then

$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v})$$

$$= m\left(\frac{d\vec{v}}{dt}\right)$$

$$= m\vec{a}.$$

The relations $\vec{F} = m\vec{a}$ and $F = \frac{d\vec{p}}{dt}$ for single particles are completely equivalent in classical mechanics.

if the momentum is a single particle is defined not as $m_0 v$ but as,

$$\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - v^2/c^2}}$$

here, we got a new definition of mass,

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Here, $\vec{v}$ is the speed of the particle

c is the speed of light.

m_0 is the "rest mass" (when $v = 0$)

Conservation of Linear Momentum:

Suppose that the sum of the external forces acting on a system is zero.

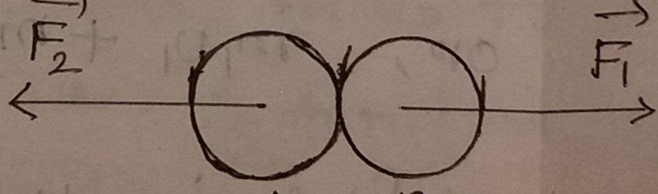
$$\frac{d\vec{P}}{dt} = 0 \quad \text{or,} \quad \vec{P} = \text{const.}$$

When the resultant external force acting on a system is zero the total vector momentum of the system remains constant.

This simple but quite general result is called the Principle of conservation of linear momentum.

We know from third Law of motion

$$\vec{F_1} = -\vec{F_2} \quad \text{--- (1)}$$



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We also know from 2nd Law of motion,

$$\vec{F} = \frac{d\vec{P}}{dt}$$

Now, put the value of $\vec{F}$ into eq^n (1)

$\therefore \vec{F_1} = -\vec{F_2}$

or, $\frac{d\vec{P_1}}{dt} = -\frac{d\vec{P_2}}{dt}$

or, $\frac{d\vec{P_1}}{dt} + \frac{d\vec{P_2}}{dt} = 0$

or, $\frac{d}{dt}(\vec{P_1} + \vec{P_2}) = 0$

or, $\frac{d}{dt}(\vec{P}) = 0$

or, $\vec{P} = \text{Const.}$

$\therefore \vec{P_i} = \vec{P_f}$

$\therefore m_{1i}v_{1i} + m_{2i}v_{2i} = m_{1f}v_{1f} + m_{2f}v_{2f}$.

or, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Which is known as conservation of linear momentum.

Problem:

1. Two bodies of masses 40 kg and 60 kg are moving opposite to each other with velocities $10 ms^{-1}$ and $5 ms^{-1}$ respectively and at a particular time they collide. after collision the bodies are combined to form a single body. with what velocity the combined body will move?

2. A neutron having mass of 1.67×10^{-27} kg and moving with velocity $10^8 ms^{-1}$ collides with deuteron at rest and sticks to it. Find the speed of the combination (Mass of deuteron is 3.34×10^{-27} kg)

3. A body of mass 5 kg at rest explodes into three pieces. Two pieces, each of mass 1 kg fly off perpendicular to each other with a speed of 100 m/s. Calculate the the velocity of the third. Piece Also Calculate the ratio of the kinetic energy of third piece and one of the small pieces.

Here, $m_3 v_3 = m_1 v_1 \cos 45^\circ + m_2 v_2 \cos 45^\circ$

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Center of Mass Motion :

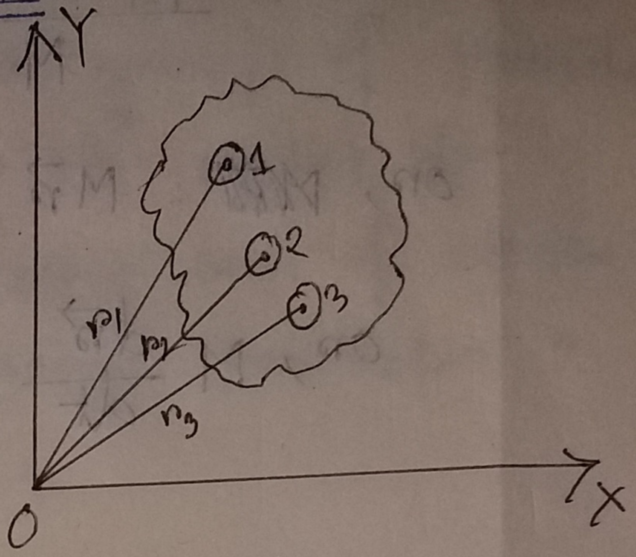

fig:

Considering the motion of a group of particles whose masses are $m_1, m_2 \cdots\cdots m_n$ and whose total mass is M.

and the total Mass remains constant with time.

So, we shall consider here M is not a constant

$$M = m_1 + m_2 + m_3 + \cdots\cdots + m_n$$

We have, for our fixed system of particles,

$$\vec{r} = \frac{r_1 m_1 + r_2 m_2 + \cdots\cdots + r_n m_n}{(m_1 + m_2 + \cdots\cdots + m_n)}$$

$$= \frac{\sum_{i=1}^{n} r_i m_i}{\sum^{n} M}$$

$$\vec{r} = \frac{\sum_{i=1}^{n} r_i m_i}{M}$$

or, ~~M~~ $M\vec{r} = \sum_{i=1}^{n} r_i m_i$

or, $M\frac{d\vec{r}}{dt} = \frac{d}{dt}\sum_{i=1} r_i m_i$

$$= m_1\frac{d\vec{r_1}}{dt} + m_2\frac{d\vec{r_2}}{dt} + \cdots + m_n\frac{d\vec{r_n}}{dt}$$

(here r is the position vector identifying the center of Mass in a Particular reference frame)

or, $M\frac{d\bar{v}}{dt} = \frac{d}{dt}\sum m_1\vec{v_1}$

or, $M\frac{d\bar{v}}{dt} = \frac{m_1 d\vec{v_1}}{dt} + m_2\frac{d\vec{v_2}}{dt} + \cdots + \frac{m_n d\vec{v_n}}{dt}$

We know, $\vec{F} = m\vec{a}$

$$M\vec{a} = m_1\vec{a_1} + m_2\vec{a_2} + \cdots + m_n\vec{a_n}$$

$$\vec{F} = \vec{F_1} + \vec{F_2} + \cdots + \vec{F_n}$$

$$\vec{F}_{total} = \vec{F}_{intertal} + \vec{F}_{external}$$

(Proved)

Question:

"The total mass of the group of particles times the acceleration of its center of mass is equal to the vector sum of all the forces acting on the group particles."