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$$\delta H = 1 \times C_v dT + \frac{P \cdot dV}{J} \quad \dots(i)$$

But  $\delta H = C_p dT \quad \dots(ii)$

$$P \cdot dV = r \cdot dT$$

$$C_p dT = C_v dT + \frac{r \cdot dT}{J}$$

$$C_p - C_v = \frac{r}{J} \quad \dots(iii)$$

Here  $C_p$  and  $C_v$  represent the specific heats for 1 gram of a gas and  $r$  is the ordinary gas constant.

If  $C_p$  and  $C_v$  are the gram-molecular specific heats of gas, then

$$C_p - C_v = \frac{R}{J} \quad \dots(iv)$$

Here  $R$  is the universal gas constant.

### 6.15 Gas Equation During an Adiabatic Process

Consider 1 gram of the working substance (ideal gas) perfectly insulated from the surroundings. Let the external work done by the gas be  $\delta W$ .

Applying the first law of thermodynamics

$$\delta H = dU + \delta W$$

But  $\delta H = 0$

and  $\delta W = P \cdot dV$

where  $P$  is the pressure of the gas and  $dV$  is the change in volume.

$$0 = dU + \frac{P \cdot dV}{J} \quad \dots(i)$$

As the external work is done by the gas at the cost of its internal energy, there is fall in temperature by  $dT$ .

$$dU = 1 \times C_v \times dT$$

$$C_v \cdot dT + \frac{P \cdot dV}{J} = 0 \quad \dots(ii)$$

For an ideal gas

$$PV = rT \quad \dots(iii)$$

Differentiating,

$$P \cdot dV + V \cdot dP = r \cdot dT$$

Substituting the value of  $dT$  in equation (ii),

$$C_v \left[ \frac{P \cdot dV + V \cdot dP}{r} \right] + \frac{P \cdot dV}{J} = 0$$

$$C_v [P \cdot dV + V \cdot dP] + r \cdot \frac{P \cdot dV}{J} = 0$$

$$\text{But, } \frac{r}{J} = C_p - C_v$$

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$$\therefore C_v \cdot P \cdot dV + C_v \cdot V \cdot dP + C_p \cdot P \cdot dV - C_v \cdot P \cdot dV = 0$$

$$C_p \cdot P \cdot dV + C_v \cdot V \cdot dP = 0$$

Dividing by  $C_v \cdot PV$ ,

$$\frac{C_p}{C_v} \cdot \frac{dV}{V} + \frac{dP}{P} = 0$$

But

$$\frac{C_p}{C_v} = \gamma$$

$$\therefore \frac{dP}{P} + \gamma \frac{dV}{V} = 0$$

Integrating,  $\log P + \gamma \log V = \text{const.}$ 

$$\log PV^\gamma = \text{const.}$$

or

$$PV^\gamma = \text{const.} \quad \dots(iv)$$

This is the equation connecting pressure and volume during an adiabatic process.

$$\text{Taking } PV = rT$$

$$\text{or } P = \frac{rT}{V}$$

$$\left( \frac{rT}{V} \right) \cdot V^\gamma = \text{const.}$$

But  $r$  is const.

$$rTV^{\gamma-1} = \text{const.}$$

$$TV^{\gamma-1} = \text{const.}$$

...(v)

Also

$$V = \frac{rT}{P}$$

$$P \left[ \frac{rT}{P} \right]^\gamma = \text{const.}$$

$$\frac{rrT^\gamma}{Pr^{-1}} = \text{const.}$$

or

$$\frac{Pr^{-1}}{T^\gamma} = \text{const.}$$

...(vi)

or

Thus, during an adiabatic process

$$(i) \quad PV^\gamma = \text{const.}$$

$$(ii) \quad TV^{\gamma-1} = \text{const. and}$$

$$(iii) \quad \frac{Pr^{-1}}{T^\gamma} = \text{const.}$$

**Example 6.2.** A motor car tyre has a pressure of 2 atmospheres at the room temperature of 27°C. If the tyre suddenly bursts, find the resulting temperature.

Here,

$$P_1 = 2 \text{ atmospheres}$$

$$T_1 = 27 + 27$$

$$= 300 \text{ K}$$

$$P_2 = 1 \text{ atmosphere}$$

$$T_2 = ?$$

$$\gamma = 1.4$$

$$\frac{P_1^{\gamma-1}}{T_1^{\gamma}} = \frac{P_2^{\gamma-1}}{T_2^{\gamma}}$$

$$\left( \frac{P_2}{P_1} \right)^{\gamma-1} = \left( \frac{T_2}{T_1} \right)^{\gamma}$$

$$\left( \frac{1}{2} \right)^{0.4} = \left( \frac{T_2}{300} \right)^{1.4}$$

$$0.4 \log (0.5) = 1.4 [\log T_2 - \log 300]$$

$$-0.1204 = 1.4 \log T_2 - 3.4680$$

$$1.4 \log T_2 = 3.4680 - 0.1204$$

$$= 3.3476$$

$$\log T_2 = \frac{3.3476}{1.4}$$

$$= 2.3911$$

$$T_2 = 246.1 \text{ K}$$

$$= -26.9^\circ\text{C}$$

*Example 6.3.* A quantity of air at  $27^\circ\text{C}$  and atmospheric pressure is suddenly compressed to half its original volume. Find the final (i) pressure and (ii) temperature.

$$(i) P_1 = 1 \text{ atmosphere}; P_2 = ?, \gamma = 1.4$$

$$V_1 = V; \quad V_2 = \frac{V}{2}$$

During sudden compression, the process is adiabatic

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_2 = P_1 \left[ \frac{V_1}{V_2} \right]^\gamma$$

$$= 1[2]^{1.4}$$

$$= 2.636 \text{ atmospheres}$$

$$(ii) V_1 = V; \quad V_2 = \frac{V}{2}$$

$$T_1 = 300 \text{ K}; T_2 = ?$$

$$\gamma = 1.4$$

$$T_1 (V_1)^{\gamma-1} = T_2 (V_2)^{\gamma-1}$$

$$T_2 = T_1 [2]^{1.4-1}$$

$$= 300[2]^{0.4}$$

$$= 395.9 \text{ K}$$

$$= 122.9^\circ\text{C}$$

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**Example 6.4.** Air is compressed adiabatically to half its volume. Calculate the change in its temperature. (Delhi 1969)

Let the initial temperature be  $T_1$  K and the final temperature  $T_2$  K.

$$\begin{aligned}\text{Initial volume} &= V_1 \\ \text{Final volume} &= V_2 \\ &= \frac{V_1}{2}\end{aligned}$$

During an adiabatic process

$$\begin{aligned}T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ T_2 &= T_1 \left[ \frac{V_1}{V_2} \right]^{\gamma-1} \\ T_2 &= T_1 [2]^{\gamma-1}\end{aligned}$$

$$\begin{aligned}\text{But } \gamma \text{ for air} &= 1.40 \\ T_2 &= T_1 [2]^{1.40-1} \\ T_2 &= T_1 [2]^{0.40} \\ T_2 &= 1.319 T_1\end{aligned}$$

Change in temperature

$$\begin{aligned}&= T_2 - T_1 \\ &= 1.319 T_1 - T_1 \\ &= 0.319 T_1 \text{ K}\end{aligned}$$

**Example 6.5.** 1 gram molecule of a monoatomic ( $\gamma = 5/3$ ) perfect gas at  $27^\circ\text{C}$  is adiabatically compressed in a reversible process from an initial pressure of 1 atmosphere to a final pressure of 50 atmospheres. Calculate the resulting difference in temperature.

[Delhi (Hons.) 1973]

In a reversible adiabatic process

$$\begin{aligned}\frac{P_1^{\gamma-1}}{T_1^\gamma} &= \frac{P_2^{\gamma-1}}{T_2^\gamma} \\ \text{or } \left( \frac{P_2}{P_1} \right)^{\gamma-1} &= \left( \frac{T_2}{T_1} \right)^\gamma\end{aligned}$$

$$\begin{aligned}\text{Here, } P_2 &= 50, \\ P_1 &= 1, \\ T_1 &= 273 + 27 \\ &= 300 \text{ K} \\ T_2 &=? \\ \gamma &= \frac{5}{3}\end{aligned}$$

$$\therefore (50)^{2/3} = \left( \frac{T_2}{300} \right)^{5/3}$$

$$\text{or } \frac{2}{3} \log (50) = \frac{5}{3} [\log T_2 - \log 300]$$

$$\begin{aligned}T_2 &= 1,434 \text{ K} \\ &= 1,161^\circ\text{C}\end{aligned}$$

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**Example 6.6.** A quantity of dry air at  $27^\circ\text{C}$  is compressed (i) slowly and (ii) suddenly to  $1/3$  of its volume. Find the change in temperature in each case, assuming  $\gamma$  to be 1.4 for dry air.

[Agra 1969; Delhi 71, 75]

- (1) When the process is slow, the temperature of the system remains constant. Therefore, there is no change in temperature.
- (2) When the compression is sudden, the process is adiabatic.

Here

$$V_1 = V, \quad V_2 = \frac{V}{3}$$

$$T_1 = 300 \text{ K}, \quad T_2 = ?$$

$$\gamma = 1.4$$

$$T_2 (V_2)^{\gamma-1} = T_1 [V_1]^{\gamma-1}$$

$$T_2 = T_1 \left[ \frac{V_1}{V_2} \right]^{\gamma-1}$$

or

$$T_2 = 300 \left[ \frac{3V}{V} \right]^{1.4-1}$$

$$= 300 [3]^{1.4-1}$$

or

$$T_2 = 465.5 \text{ K}$$

$$= 192.5^\circ\text{C}$$

The temperature of air increases by

$$192.5 - 27 = 165.5^\circ\text{C} \text{ or } 165.5 \text{ K}$$

**Example 6.7.** A certain mass of gas at NTP is expanded to three times its volume under adiabatic conditions. Calculate the resulting temperature and pressure.  $\gamma$  for the gas is 1.40.

[Delhi (Hons.) 75]

- (1) Here,  $V_1 = V, \quad V_2 = 3V$
- $T_1 = 273 \text{ K} \quad T_2 = ?$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

or

$$T_2 = T_1 \left[ \frac{V_1}{V_2} \right]^{\gamma-1}$$

$$T_2 = 273 \left[ \frac{1}{3} \right]^{1.4-1}$$

$$T_2 = 176 \text{ K} = -97^\circ\text{C}$$

- (2) Here,  $V_1 = V, \quad V_2 = 3V$

$$P_1 = 1 \text{ atmosphere}, \quad P_2 = ?$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$P_2 = P_1 \left[ \frac{V_1}{V_2} \right]^{\gamma}$$

or

$$P_2 = 1 \left( \frac{1}{3} \right)^{1.4}$$

$$P_2 = 0.2148 \text{ atmosphere}$$

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## 6.16 Slopes of Adiabatics and Isothermals

In an isothermal process

$$PV = \text{const.}$$

Differentiating,

$$PdV + VdP = 0$$

or

$$\frac{dP}{dV} = -\frac{P}{V} \quad \dots(i)$$

In an adiabatic process

$$PV^\gamma = \text{const.}$$

Differentiating,

$$P\gamma V^{\gamma-1} dV + VdP = 0$$

$$\frac{dP}{dV} = -\frac{\gamma P}{V} \quad \dots(ii)$$

Therefore, the slope of an adiabatic is  $\gamma$  times the slope of the isothermal.

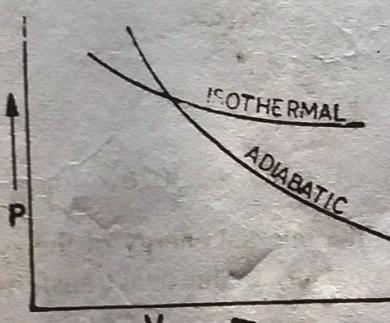


Fig. 6.5

Hence, the adiabatic curve is steeper than the isothermal curve (Fig. 6.5) at a point where the two curves intersect each other.

## 6.17 Work Done During an Isothermal Process

When a gas is allowed to expand isothermally, work is done by it.

Let the initial and final volumes be  $V_1$  and  $V_2$ , respectively. In Fig. 6.6, the area of the shaded strip represents the work done for a small change in volume  $dV$ . When the volume changes from  $V_1$  to  $V_2$ ,

$$\text{Work done} = \int_{V_1}^{V_2} P \cdot dV = \text{area } ABba \quad \dots(i)$$

Fig. 6.6 represents the indicator diagram. Considering one gram molecule of the gas

$$PV = RT$$

$$P = \frac{RT}{V}$$

or

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$$W = RT \int_{V_1}^{V_2} \frac{dV}{V}$$

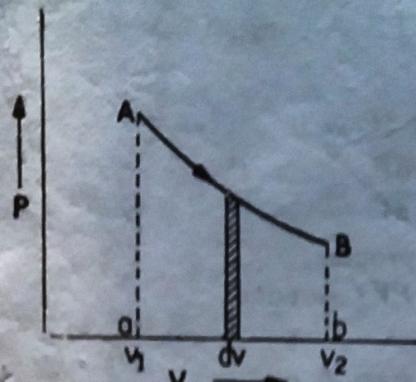


Fig. 6.6

$$= RT \log_e \frac{V_2}{V_1}$$

$$W = RT \times 2.3026 \log_{10} \frac{V_2}{V_1} \quad \dots (iii)$$

Also

$$P_1 V_1 = P_2 V_2$$

or

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$\therefore W = RT \times 2.3026 \times \log_{10} \frac{P_1}{P_2} \quad \dots (iv)$$

Here, the change in the internal energy of the system is zero (because the temperature remains constant). So the heat transferred is equal to the work done.

### 6.18 Work Done During an Adiabatic Process

During an adiabatic process, the gas expands from volume  $V_1$  to  $V_2$ . As shown by the indicator diagram (Fig. 6.7) the work done for an increase in

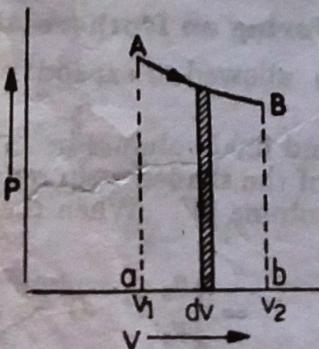


Fig. 6.7.

volume  $dV = P.dV$ . Work done when the gas expands from  $V_1$  to  $V_2$  is given by,

$$W = \int_{V_1}^{V_2} P dV = \text{Area } ABba$$

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During an adiabatic process,

$$PV^\gamma = \text{const} = K$$

or

$$P = \frac{K}{V^\gamma}$$

$$\begin{aligned} W &= K \int_{V_1}^{V_2} \frac{dV}{V^\gamma} \\ &= \frac{K}{1-\gamma} \left[ \frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}} \right] \end{aligned} \quad \dots(i)$$

Since A and B lie on the same adiabatic

$$P_1 V_1^\gamma = P_2 V_2^\gamma = K$$

$$W = \frac{1}{1-\gamma} \left[ \frac{K}{V_2^{\gamma-1}} - \frac{K}{V_1^{\gamma-1}} \right]$$

$$W = \frac{1}{1-\gamma} \left[ \frac{P_2 V_2^\gamma}{V_2^{\gamma-1}} - \frac{P_1 V_1^\gamma}{V_1^{\gamma-1}} \right]$$

$$= \frac{1}{1-\gamma} [P_2 V_2 - P_1 V_1] \quad \dots(ii)$$

Taking  $T_1$  and  $T_2$  as the temperatures at the points A and B respectively and considering one gram molecule of the gas

$$P_1 V_1 = RT_1$$

and

$$P_2 V_2 = RT_2$$

Substituting these values in equation (ii)

$$W = \frac{1}{1-\gamma} [RT_2 - RT_1] \quad \dots(iii)$$

Here, heat transferred is zero because the system is thermally insulated from the surroundings. The decrease in the internal energy of the system (due to fall in temperature) is equal to the work done by the system and vice versa.

**6.19. Relation Between Adiabatic and Isothermal Elasticities****1. Isothermal Elasticity**

During an isothermal process

$$PV = \text{const}$$

Differentiating,

$$PdV + VdP = 0$$

$$\text{or } \frac{V.dP}{-dV} = P \quad \dots(iv)$$

From the definition of elasticity of a gas

$$\begin{aligned} E_{iso} &= \frac{dP}{-dV/V} \\ &= \frac{V.dP}{-dV} \end{aligned} \quad \dots(v)$$