The resultant vibration of the particle will depend upon the value of α. fig - 1 represents the resultant vibration for values of a changing from 0 to 2π.

Special Cases:

(i) If $\alpha = 0$ or 2π; $\cos\alpha = 1$: $\sin\alpha = 0$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

or, $\frac{x}{a} - \frac{y}{b} = 0$

or, $y = \frac{b}{a} x$.

This represents the equation of a straight line BD (fig - 2) i.e, the particle vibrates simple harmonically along the line DB.

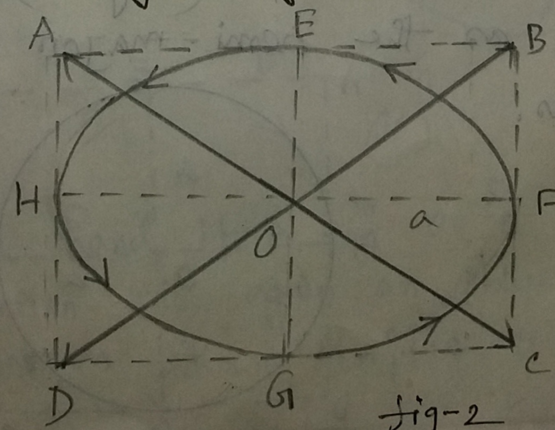


fig-2

(ii) If $\alpha = \pi$; $\sin\alpha = 0$

$\cos\alpha = -1$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

$$\left(\frac{x}{a} + \frac{y}{b}\right) = 0$$

$$y = -\frac{b}{a}x \quad \text{——} (6)$$

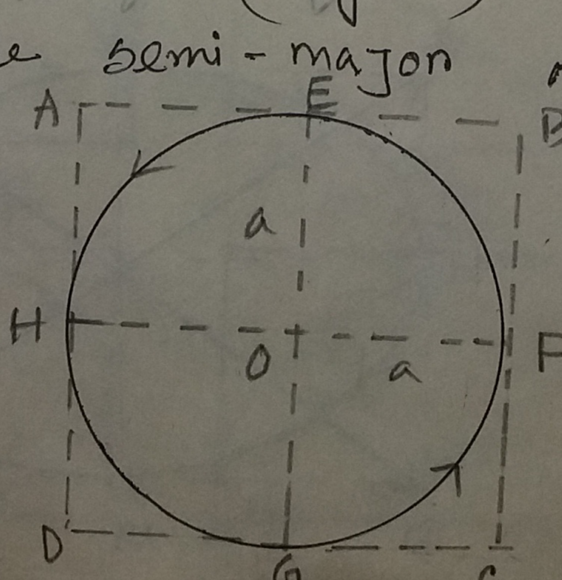
This represents the equation of a straight line AC. fig-(2)

(iii) If $\alpha = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

$\sin\alpha = 1$; $\cos\alpha = 0$

$$\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1$$

This represents the equation of an ellipse EHGF (fig-2) with a and b as the semi-major and semi-minor axes.



(fig-3)

(iv) If $a = \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

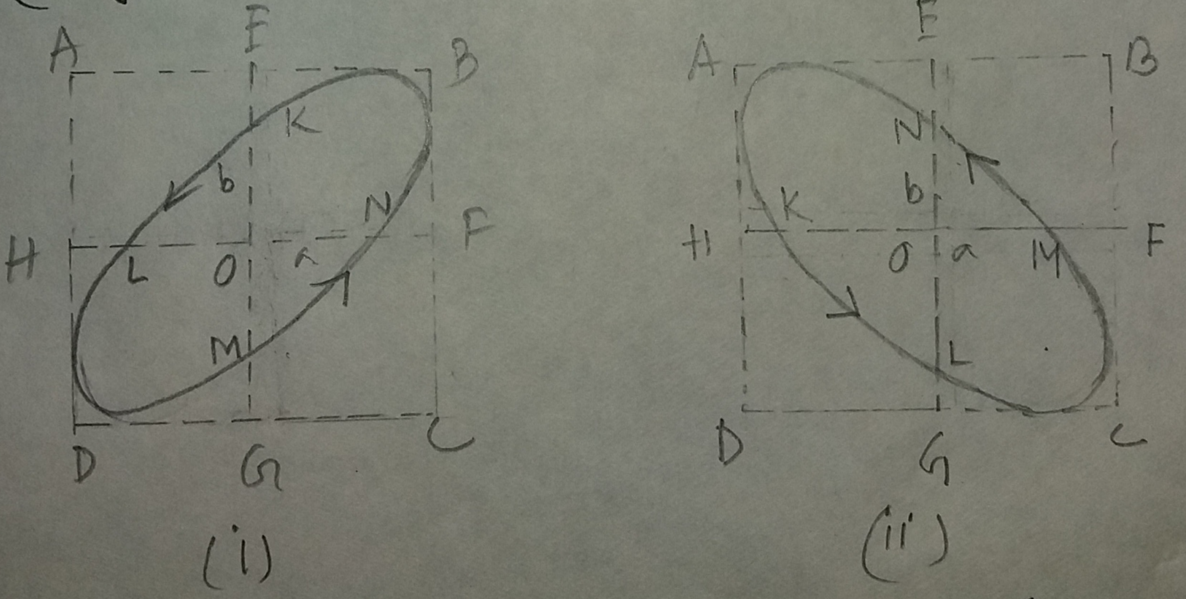
and $a = b$

then, $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$

or, $x^2 + y^2 = a^2$.

This represents the equation of a circle of radius a (fig - 3).

(v) $a = \frac{\pi}{4}$ or $\frac{7\pi}{4}$, the resultant vibrations is an oblique ellipse KLMN as shown in (fig - 4(i)).



on the other hand if $a = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$, the resultant vibration is again an oblique ellipse KLMN as shown in (fig 4-(ii))

The cycle of change is repeated after every time period.

CHAPTER IV

WAVE MOTION

Wave motion : types of waves-transverse and longitudinal wave motion-some definitions connected with wave motion-Expression for a plane progressive wave-Differential equation of wave motion-Particle velocity-Phase or wave velocity-Energy density of a plane progressive wave-Energy current-intensity of a wave.

4.1 Wave motion-types of waves

Wave motion can be thought of as the transport of energy and momentum from one point in space to another without the transport of matter. For example, water waves, sound waves, light waves and radio waves are all known to carry energy of one form or the other from one place to another. In the case of water and sound wave, although a medium is necessary, there is no bulk motion of the intervening medium. In the case of light and radio waves, no intervening medium is at all necessary. Thus wave motion may be divided into two broad categories.

(i) Mechanical wave motion : This sort of wave motion is possible only in media (solid, liquid or gas) which possess intertia as well as elasticity. Water waves and sound waves are examples of this type of wave motion and are, therefore, referred to as mechanical waves.

(ii) Electromagnetic (or non-mechanical) wave motion : No material medium is necessary for the propagation of this sort of wave motion. Light and radio waves which can travel through empty space, belong to this category and are, therefore, referred to as non-mechanical or electromagnetic waves.

As sound waves are mechanical waves, only mechanical type of wave motion – to be referred simply as wave motion from now on, will be discussed in this chapter.

Production and propagation of wave motion

The production and propagation of wave motion through a medium which possesses elasticity and inertia will now be investigated. No particle of an elastic medium can be disturbed without affecting its immediate neighbour and, tending to recover its original position; it first stores up potential energy *and then* converts it back into kinetic energy. The neighbouring *particle which* has thus been disturbed then performs a similar *motion, so that* each successive particle repeats, in turn, the movements *of its predecessor* a little later than it and then hands down the *same on to its* successor. This results in a transference of energy from *particle to* particle all along the line. One complete oscillation of a *particle of* the medium obviously produces one single wave or a pulse *and its* repeated periodic motion, a succession of waves or a wavetrain.

A wave motion may thus be defined as a disturbance or a condition that travels onwards through a medium due to the repeated periodic motion of its particles about their mean or equilibrium positions, each particle repeating the movements of its predecessor a little later than it and handing it on to its successor, so that there is a regular phase difference between one particle and the next.

It must be clearly understood that what is propagated in a wave motion is only a state of motion of the matter – not the matter itself. The wave motion is a form of dynamic condition, arising out of the vibration of the particles of the medium about their mean positions, that is propagated from one point to the other point in the medium. According to the laws of physics, any dynamic condition is related to momentum and energy. Thus, in conclusion, *it may be said that in wave motion momentum and energy are transferred or propagated*. It is not a case of propagation of matter as a whole.

The simplest type of periodic motion performed by a particle is, of course, the simple harmonic motion. The corresponding wave motion is, therefore, called a simple harmonic or sinusoidal wave motion. This is the most general type of wave motion and will be dealt with in the following discussion.

It may be emphasized again that, *but for the properties of elasticity and inertia, it would not have been possible for a wave motion to be produced in or propagated through a medium.* As will be seen later, these two properties in fact determine the velocity of propagation of the wave motion through the medium. A wave may travel through a medium over fairly large distances. In order that the wave may do so without any attenuation (*i.e.*, without any decrease in its amplitude), a third property is also necessary, *viz.*, that *the medium should offer the least frictional resistance* so as not to unduly damp the periodic motion of the particles.

A wave motion which progresses onwards through a medium, with energy transferred across every section of it, is called a *travelling or a progressive wave motion* to distinguish it from what is called a *standing or stationary wave motion* in which there is no onward movement of the wave motion through the medium and hence no transference of energy across any section of it.

4.2 Transverse and longitudinal wave motion

There are two distinct types of wave motion :

(i) transverse and (ii) longitudinal.

(i) *Transverse wave motion* : In transverse wave motion, the particles of the medium oscillate up and down about their mean or equilibrium position in a direction at right angles to the direction of propagation of the wave motion itself. This form of wave motion therefore travels in the form of crests and troughs with one crest and an adjoining trough making up one wave (Fig. 4.1). A succession of waves make up a wavetrain.

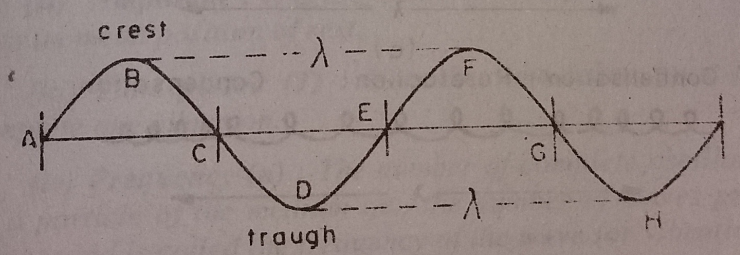


Fig. 4.1

Transverse wave motion is possible in media which possess *elasticity of shape or rigidity i.e.*, solids. Waves travelling along a stretched string is an example of transverse wave motion. Although water and liquids do not possess the property of rigidity, transverse wave motion is still possible in them because they possess another equally effective property of resisting any vertical displacement of their particles (or keeping their level). Gases, however, possess neither rigidity nor do they resist any vertical displacement of their particles (or keep their levels). *A transverse wave motion is, therefore, not possible in a gaseous medium.*

(ii) Longitudinal wave motion : In this type of wave motion, the particles of the medium oscillate to and fro about their mean or equlibrium position, along the direction of propagation of the wave motion itself. The wave motion, therefore, travels in the form of *compressions* (or *condensations*) and *rarefactions i.e.*, in the particles of the medium getting closer together and further apart alternately. This type of wave motion is possible in media possessing elasticity of volume, *i.e.*, in solids, liquids as well as gases. Waves produced in a spring or helix when one end of it is suddenly compressed or pulled out and then released or sound waves in air are examples of this type of wave motion (Fig. 4.2). As in the case of transverse wave motion, one compression and the adjoining rarefaction constitute a wave or pulse and a succession of them, a wavetrain

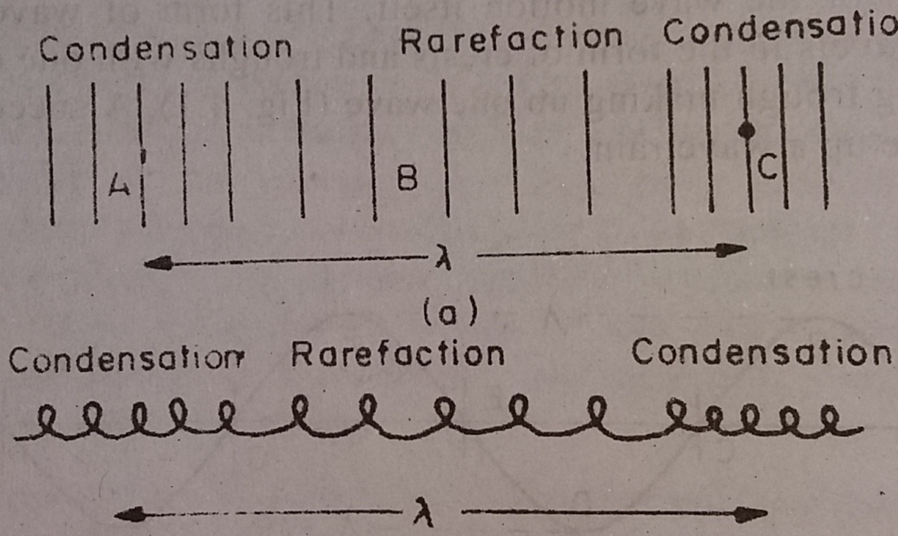


Fig. 4.2

Characteristics of wave motion

Before proceeding further, the important characteristics of wave motion, whether transverse or longitudinal, may be summarized below :

(i) Wave motion is a disturbance produced in a medium by the repeated periodic motion of the particles of the medium. It is only this disturbance which travels forward through the medium as the wave while the particles of the medium vibrate about their mean positions – they are not propagated through the medium.