Eq^n of displacement of a particle executing simple harmonic motion:

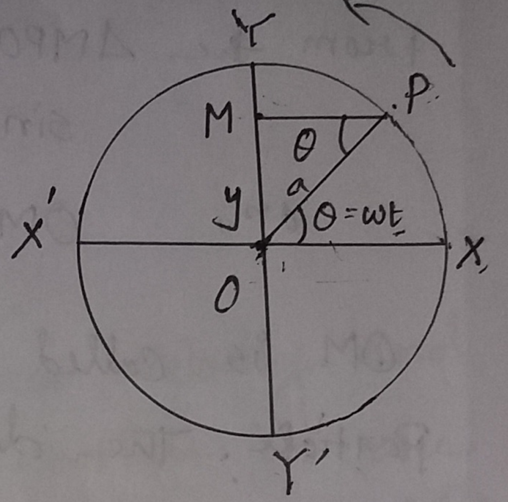


fig - 1

Let P be a particle moving on the circumference of a circle of radius a with a uniform velocity v [fig-1]. Let ω be the uniform angular velocity of the particle ($v = a\omega$).

The circle along which P moves is called the circle of reference. As the particle P moves round the circle continuously with uniform velocity, the foot of the perpendicular M vibrates along the diameter YY'. If the motion of P is uniform, then the motion of M is periodic. i.e it takes same time to vibrate once between the points Y and Y'. At any instant the distance of M from the center O of the circle is called the displacement. If the particle moved from X to P in time t, then,

$$\angle POX = \angle MPO = \theta = \omega t.$$

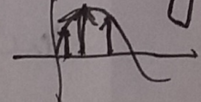
From the ΔMPO,

$$\sin\theta = \sin\omega t = \frac{OM}{a} \quad [OP = a]$$

or, $$OM = y = a\sin\omega t$$

OM is called displacement of the vibrating particle. The displacement of a vibrating particle at any instant can be defined as its distance from the mean position of rest.

The maximum displacement of a vibrating particle is called its amplitude.



$$\text{Displacement} = y = a\sin\omega t \quad \text{——— (1)}$$

The rate of change of displacement is called the velocity of the vibrating particle.

$$\text{Velocity} = \frac{dy}{dt} = a\omega\cos\omega t \quad \text{——— (2)}$$

The rate of change of velocity of a vibrating particle is called its acceleration

$\therefore$ Acceleration = Rate of change of velocity.

$$= \frac{d}{dt}\left(\frac{dy}{dt}\right)$$

$$= \frac{d^2y}{dt^2} = -a\omega^2 \sin\omega t$$

$$= -\omega^2 . a\sin\omega t$$

$$= -\omega^2 y \quad \text{———} \quad (3)$$

The change in the displacement, velocity and acceleration of a vibrating particle in one complete vibrations are given in these eq^ns.

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Graphical representation of SHM:

Displacement graph:

(i)

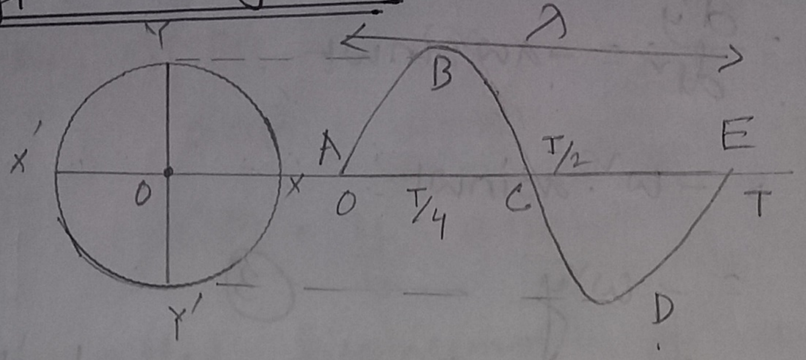

fig: displacement curve.

here, $y = a\sin\omega t$

$= a\sin 2\pi \frac{t}{T}$

The displacement graph is a sine curve represented by ABCDE. fig(1).

(ii) Velocity graph:

Here,

$v = \frac{dy}{dt}$

$= \pm a\omega\cos\omega t$

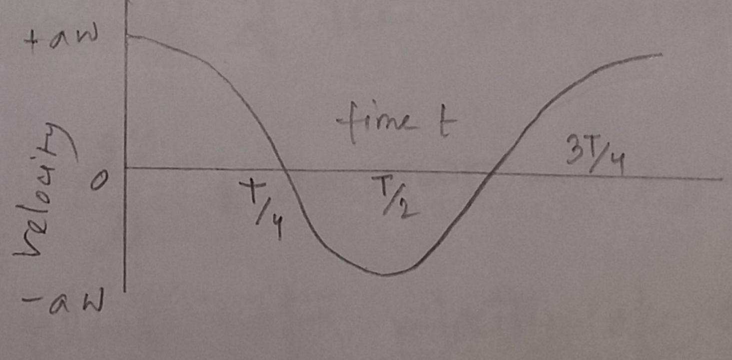

fig 2: velocity-time graph.

(iii) Acceleration graph :

Here,

$$a = \frac{d}{dt}(v)$$

$$= \frac{d}{dt}\left(\frac{dy}{dt}\right)$$

$$= \frac{d^2y}{dt^2}$$

$$= -\omega^2 a \sin\omega t.$$

$$\therefore a = -a\omega^2 \sin\omega t.$$

Which shown in graph.

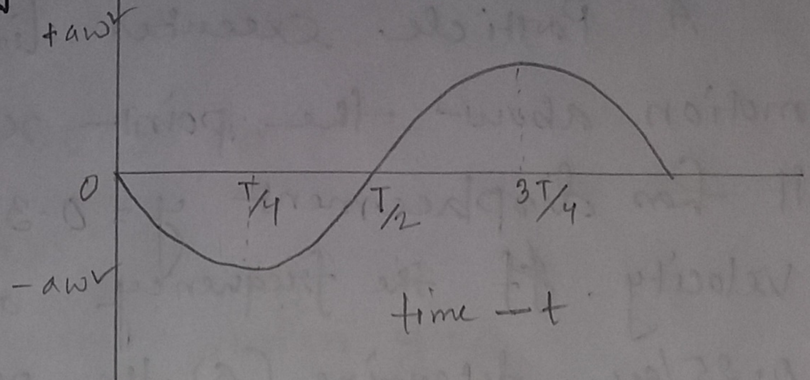

fig: Acceleration-time curve.

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Q. Show that for a particle executing S.H.M its velocity at any instant

$$v = \frac{dy}{dt} = \omega\sqrt{a^2 - y^2}$$

Soln: The displacement $y = a\sin\omega t$ —— (1)

velocity, $v = \frac{dy}{dt} = a\omega\cos\omega t$ —— (2)

From (1) ⇒ $\sin\omega t = \frac{y}{a}$

$\cos\omega t = \sqrt{1-\sin^2\omega t}$

$= \sqrt{1 - y^2/a^2}$

$[\sin^2\theta + \cos^2\theta = 1]$

$\cos\theta = \sqrt{1-\sin^2\theta}$

$\therefore \frac{dy}{dt} = a\omega\sqrt{1 - y^2/a^2}$

$\therefore v = \omega\sqrt{a^2 - y^2}$

(Proved)

Problem:

A Particle executes linear harmonic motion about the point $x = 0$, At $t = 0$, it has displacement $y = 0.37$ and zero velocity. If the frequency of the motion is $0.25/sec$. determine (a) the period

(b) the amplitude

(c) the maximum speed and

(d) the maximum acceleration.

Soln:

(a) $T = \frac{1}{n} = \frac{1}{0.25} = 4$ sec

(b) $y = a\sin(\omega t + \delta)$

at $t = 0$, $y = 0.37$ cm

$\therefore a\sin\delta = 0.37$ —— (I)

Again, $v = \frac{dy}{dt} = \omega a\cos(\omega t + \delta)$

at $t = 0$, $v = 0$

$\therefore \omega a\cos\delta = 0$

or, $a\cos\delta = 0$ —— (II)

From (i) and (ii)

$a^2 = (0.37)^2$

or, $a = \pm 0.37$ cm.

(c) $v = \omega\sqrt{a^2 - y^2}$

v is maximum, when $y = 0$

$\therefore v_{max} = \omega . a = a \times 2\pi n$

$= 0.37 \times 2 \times 3.14 \times 0.25$

$= 0.5809$ cm/sec

(d) $\text{acceleration}_{max} = -\omega^2 a$

$= -(2\pi n)^2 . a$

$= -(2 \times 3.14 \times 0.25)^2 \times 0.37$

$= -0.912013$ cm/sec^2.

Energy Conservation in Simple harmonic Motion:

For a Simple harmonic Oscillator in which no dissipative forces act the total mechanical energy, $E = T + V$ is conserved.

Here, $T \rightarrow$ is the kinetic energy.

and V is the potential energy.

For Simple harmonic motion we have the displacement:

$$x = A\cos(\omega t + \delta) \quad \text{——— } (1)$$

For conservative system the relation between the potential energy, V and the force, F is:

$$F = -dV/dx \quad \text{———}$$

$$dV = -F dx \quad \text{——— } (2)$$

For simple harmonic Oscillator,

$$F = -Kx \quad \text{——— } (3)$$

Using eq^n (3) in eq^n (2) we get,

$$dV = Kx\,dx$$

$$\int dV = K\int x\,dx$$

$$V = \frac{1}{2}Kx^2 \quad \text{——— } (4)$$

Using eq^n (1) in eq^n (4) we have,

$$V = \frac{1}{2}KA^2\cos^2(\omega t + \delta) \quad \text{——— } (5)$$

From eq^n 1, we can write,

$$v = dx/dt = -\omega A \sin(\omega t + \delta).$$

The Kinetic energy, T at any instant is $\frac{1}{2}mv^2$.

$$\therefore T = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \delta).$$

in simple harmonic motion, we know,

$\omega^2 = K/m$, then.

$$T = \frac{1}{2} \cancel{m} (K/\cancel{m}) A^2 \sin^2(\omega t + \delta).$$

$$= \frac{1}{2} K A^2 \sin^2(\omega t + \delta) \text{ ——— } (6)$$

From eq^n (5) and (6) we can write,

$$E = T + V$$

$$= \frac{1}{2} KA^2 [\sin^2(\omega t + \delta) + \cos^2(\omega t + \delta)]$$

$$= \frac{1}{2} KA^2 \text{ ——— } (7)$$

$$E \propto A^2$$

Thus the total energy of a particle executing S.H.M is proportional to the square of the amplitude of the motion.

fig add

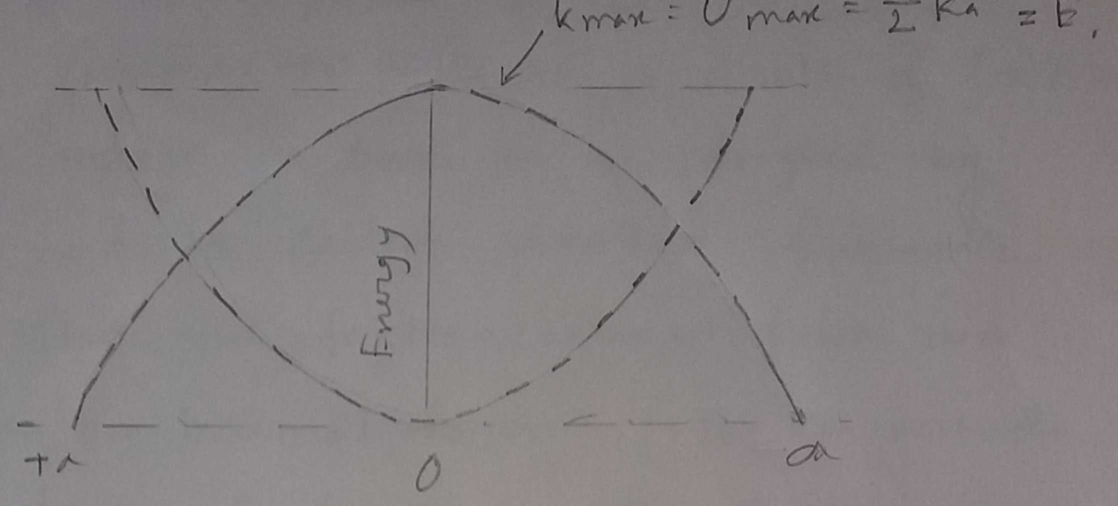

fig-1.

fig shows the kinetic, potential and total energies as a function of time.