S.H.M

☐ **Periodic Motion:**

Motions which repeat themselves after a regular interval of time are called periodic motion. The time interval after which the motion is repeated is called time period.

Ex: Earth's motion, vibrations of stretched string etc.

☐ **Oscillation:**

If a body in a periodic motion moves back & forth about a fixed position, the motion of the body is called oscillation or vibration.

Ex: vibration of tunning fork, Motion of a spring.

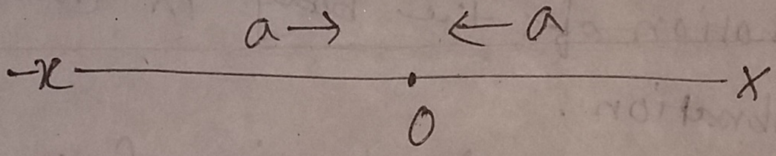
☐ **Simple harmonic Oscillation:**

The type of oscillation in which the body oscillates on a straight line, the acceleration of the body is always directed towards the mean position on the line and its magnitude is proportional to the displacement of the body from this point is called the simple harmonic oscillation or motion.

Taking the mean position as origin and the line of motion as the X-axis, we can define the equation of a simple harmonic ~~oscillation~~ ~~as motion~~. motion as,

$$a = -cx.$$

Where c is a positive constant. If x is positive a is negative and if x is negative, a is positive.



Example: Motion of a simple pendulum oscillating with very small amplitude. vibration of arms of tunning fork, vibration of a spring.

Characteristics of Simple harmonic oscillation:

1. The motion is periodic.
2. It is oscillatory.
3. It is linear motion.
4. Acceleration is proportional to the displacement.
5. Acceleration is opposite to displacement.

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Differential Equation of a simple Harmonic Oscillator / Motion :

A system executing simple harmonic motion is called a simple harmonic Oscillator. In simple harmonic motion, the force acting on the system at any instant, is directly proportional to the displacement from a fixed point in its path and the direction of this force is towards that fixed point. Thus, the system executes the motion under a linear restoring force. If the displacement of the system from a fixed point is x, the linear restoring force is $-Kx$, where K is a constant which is called the force constant. Thus no other force except the linear restoring force acts on a simple harmonic oscillator. As a result, the oscillation executes vibrations of constant amplitude and with a constant frequency. These Oscillations are called the free oscillations.

Let, a particle of mass m be executing simple harmonic oscillations. The acceleration of the particle at displacement x from a fixed point will be $\frac{d^2x}{dt^2}$. For a particle, restoring force $\propto$ displacement.

dis $\rightarrow x$

$v = \frac{dx}{dt}$

$a = \frac{d^2x}{dt^2}$

$$F \propto -x$$

$$ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

[where k is a constant which is known as force const.]

Acceleration of the particle,

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

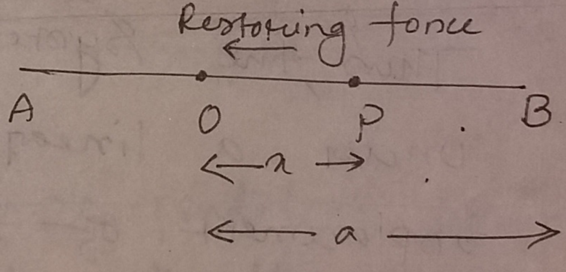

fig: Direction of force

Let, $\frac{k}{m} = \omega^2$

then,

Acceleration of the particle $\frac{d^2x}{dt^2} = -\omega^2 x$.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{—— (1)}$$

Eq^n (1) is known as differential equation of simple harmonic oscillator.

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Sol^n of differential equation of simple harmonic motion :

Let ~~the solution~~ us solve the differential equation

$$\text{Let } \omega^2 = K/m \qquad \text{—— (2)}$$

eq^n (1) becomes $\frac{d^2x}{dt^2} + \omega^2 x = 0$ —— (3)

eq^n (3) is a 2nd order differential equation.

Let us solve eq^n (3). Let the solution be of the form:

$$x = e^{qt} \qquad \text{—— (4)}$$

$$\left.\begin{aligned} \frac{dx}{dt} &= qe^{qt} \\ \text{and } \frac{d^2x}{dt^2} &= q^2 e^{qt} \end{aligned}\right\} \qquad \text{—— (5)}$$

using (5), ~~in~~ (4) in eq^n (3) $\Rightarrow$

$$q^2 e^{qt} + \omega^2 e^{qt} = 0$$

$$\text{or, } (q^2 + \omega^2) e^{qt} = 0$$

in this equation since $e^{qt} \neq 0$

$$\therefore (q^2 + \omega^2) = 0$$

$$\text{or, } q^2 = -\omega^2$$

$$\text{or, } q = \pm i\omega \qquad [\because i^2 = -1]$$

$$\therefore x = a e^{i\omega t} + b e^{-i\omega t} \qquad \left[\begin{aligned} e^{i\theta} &= \cos\theta + i\sin\theta \\ e^{-i\theta} &= \cos\theta - i\sin\theta \end{aligned}\right]$$

$$= a.(\cos\omega t + i\sin\omega t) + b(\cos\omega t - i\sin\omega t)$$

$$= (a+b)\cos\omega t + i(a-b)\sin\omega t$$

$$= c\cos\omega t + d\sin\omega t \quad \text{——— (6)}$$

Where, $c = (a+b)$ and $d = i(a-b)$

Now if $c = A\cos\delta$ and $d = -A\sin\delta$.

Putting this in (6) $\Rightarrow$

$$x = A\cos\delta\cos\omega t - A\sin\delta\sin\omega t$$

$$x = A\cos(\omega t + \delta) \quad \text{——— (7)}$$

eq^n (7) is the solution of the equation of a simple harmonic oscillator.

$$\left.\begin{aligned} x &= A\cos(\omega t + \delta) \\ x &= A\sin(\omega t + \delta) \end{aligned}\right\} \text{Sol}^n$$