Generally collisions are distinguished according to whether the kinetic energy of the system is conserved or not. If the kinetic energy of the system is conserved, it is called an *elastic collision*. If however, the kinetic energy is not conserved the collision is said to be *inelastic*. Collisions between atomic particles and nuclear particles are elastic. Collision between two large bodies is always inelastic.

Let a particle of mass m_1, moving with a velocity $\mathbf{v}_1$ along the X-axis, collide with another particle of mass m_2 at rest. After collision, the particle of mass m_1 moves with a velocity $\mathbf{v'}_1$ making an angle θ_1 with the X-axis and the particle of mass m_2 moves with a velocity $\mathbf{v}_2'$ making an angle θ_2 with the X-axis (Fig. 2·8).

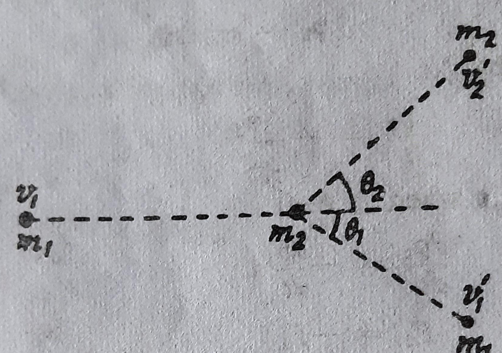

Fig. 2·8

The scalar components of momentum of the particles after collision are :

(*i*) $m_1 v_1' \cos \theta_1$ along positive X-axis.

(*ii*) $m_1 v_1' \sin \theta_1$ along negative Y-axis.

(*iii*) $m_2 v_2' \cos \theta_2$ along positive X-axis.

(*iv*) $m_2 v_2' \sin \theta_2$ along positive Y-axis.

Applying the principle of conservation of linear momentum, for X-components,

$$m_1 v_1 = m_1 v_1' \cos \theta_1 + m_2 v_2' \cos \theta_2 \qquad ...(i)$$

For Y-components

$$0 = m_2 v'_2 \sin \theta_2 - m_1 v_1' \sin \theta_1 \qquad ...(ii)$$

As the collision is elastic, the kinetic energy is also conserved.

$$\therefore \quad \tfrac{1}{2} m_1 v_1^2 = \tfrac{1}{2} m_1 (v_1')^2 + \tfrac{1}{2} m_2 (v_2')^2 \qquad ...(iii)$$

In actual experiments, θ_1 and θ_2 are observed and knowing the values of θ_1, θ_2, m_1 and v_1, the values of v_1' and v_2' are determined by using these equations.

Special case. When $\theta_1 = \theta_2 = 0$, the particle moves along the direction of $+X$-axis after collision. From equations (*i*), (*ii*) and (*iii*).

$$m_1 v_1 = m_1 v_1' + m_2 v_2' \qquad \ldots(iv)$$

$$\tfrac{1}{2}\, m_1 v_1^2 = \tfrac{1}{2}\, m_1(v_1') + {}^2\tfrac{1}{2}\, m_2(v_2')^2 \qquad \ldots(v)$$

From equation (*iv*)

$$m_1(v_1 - v_1') = m_2 v_2' \qquad \ldots(vi)$$

From equation (*v*)

$$m_1[v_1^2 - (v_1')^2] = m_2(v'_2)^2 \qquad \ldots(vii)$$

Dividing equation (*vii*) by (*vi*)

$$v_1 + v_1' = v_2' \qquad \ldots(viii)$$

Substituting this value of v_2' in equation (*vi*)

$$m_1(v_1 - v_1') = m_2(v_1 + v_1')$$

$$v_1' = \frac{(m_1 - m_2)v_1}{(m_1 + m_2)} \qquad \ldots(ix)$$

Subsuituting the value of v_1' in equation (*iii*)

$$v_2' = v_1 + \frac{(m_1 - m_2)\, v_1}{m_1 + m_2}$$

$$v_2' = \left(\frac{2m_1 v_1}{m_1 + m_2}\right) \qquad \ldots(x)$$

If $\qquad m_1 < m_2,$

from equations (*ix*) and (*x*)

$$v_1' = -v_1$$

$$v_2' = 0$$

It means that when a lighter particle moving with a certain velocity collides with a stationary heavier particle, it rebounds in the opposite direction with the same velocity.

Example. *When a hard rubber ball is dropped on a hard ground vertically, it rebounds practically with the same velocity upwards.*

General Case

Consider two spheres of masses m_1 and m_2 and moving with velocities v_1 and v_2 respectively. Let v_1' and v_2' be the respective velocities of the spheres after collision.

From the law of conservation of momentum

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \qquad \ldots(i)$$

When the collision is elastic,

Total initial kinetic energy = Total final kinetic energy

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_1 v_1'^2 + \tfrac{1}{2} m_2 v_2'^2 \qquad \ldots(ii)$$

From equation (*i*)

$$m_1(v_1 - v_1') = m_2(v_2' - v_2) \qquad \ldots(iii)$$

From equation (*ii*)

$$m_1(v_1^2-v_1'^2) = m_2(v_2'^2-v_2^2) \quad \text{...}(iv)$$

Dividing equation (*iv*) by (*iii*)

$$v_1+v_1' = v_2+v_2' \quad \text{...}(v)$$

or

$$v_1-v_2 = v_2'-v_1' \quad \text{...}(vi)$$

Equation (*vi*) shows that the relative velocity of approach before the collision is equal to the relative velocity of separation after the collision.

From equation (*vi*)

$$v_2' = v_1+v_1'-v_2 \quad \text{...}(vii)$$

Substituting this value of v_2' in equation (*iii*)

$$m_1(v_1-v_1') = m_2(v_1+v_1'-v_2-v_2)$$

$$v_1' = \left(\frac{m_1-m_2}{m_1+m_2}\right)v_1+\left(\frac{2m_2}{m_1+m_2}\right)v_2 \quad (viii)$$

Similarly, from equation (*vi*)

$$v_1' = v_2'+v_2-v_1 \quad \text{...}(ix)$$

Substituting this value of v_1' in equation (*iii*)

$$m_1(v_1-v_2'-v_2+v_1) = m_2(v_2'-v_2)$$

or

$$v_2' = \left(\frac{2m_1}{m_1+m_2}\right)v_1+\left(\frac{m_2-m_1}{m_1+m_2}\right)v_2 \quad \text{...}(x)$$

Special Case

(1) **When the two masses are equal**

$$m_1 = m_2$$

From equation (*viii*)

$$v_1' = v_2$$

From equation (*x*)

$$v_2' = v_1$$

Thus, during one dimensional elastic collision between two particles of equal masses, the particles simply exchange the velocities after the collision.

(2) **When the second particle is initially at rest**

$$v_2 = 0$$

From equation (*viii*)

$$v_1' = \left(\frac{m_1-m_2}{m_1+m_2}\right)v_1$$

From equation (*x*)

$$v_2' = \left(\frac{2m_1}{m_1+m_2}\right)v_1$$

Further, if $$m_1 = m_2$$

$$v_1' = 0$$

and $$v_2' = v_1$$

Thus, after collision the first particle comes to rest and the second particle moves with the initial velocity of the first particle.

(3) When the mass of the second particle initially at rest is too large as compared to the mass of the first particle

Here, $$m_2 >> m_1$$

From equation (*viii*)

$$v_1' \cong -v_1$$

From equation (*x*)

$$v_2' \cong 0$$

Thus, when a light particle collides with a heavy particle at rest, the velocity of the light particle is reversed and the heavy particle practically remains at rest.

Example. *When a hard rubber ball is dropped on the hard ground vertically, it rebounds practically with the same velocity upwards.*

Example 2·2. *A body of mass 5 kg at rest explodes into three pieces. Two pieces, each of mass 1 kg fly off perpendicular to each other with a speed of 100 m/s. Calculate the velocity of the third piece. Also calculate the ratio of the kinetic energy of third piece and one of the small pieces.*

Here $$m_1 = 1 \text{ kg}, \qquad m_2 = 1 \text{ kg},$$

$$m_3 = 3 \text{ kg}$$

$$v_1 = v_2 = 100 \text{ m/s}, \qquad v_3 = ?$$

Applying the law of conservation of momentum,

$$m_3 v_3 = m_1 v_1 \cos 45 + m_2 v_2 \cos 45$$

$$3 \times v_3 = 1 \times 100 \times \frac{1}{\sqrt{2}} + 1 \times 100 \times \frac{1}{\sqrt{2}}$$

$$v_3 = 47{\cdot}13 \text{ m/s}$$

$$\frac{E_3}{E_1} = \frac{\frac{1}{2} m_3 v_3^2}{\frac{1}{2} m_1 v_1^2} = \frac{3 \times (47{\cdot}13)^2}{1 \times (100)^2}$$

$$= 0{\cdot}6664$$

2·15. Impulse

The change in momentum of a body is called impulse.