(ii) Longitudinal wave motion : In this type of wave motion, the particles of the medium oscillate to and fro about their mean or equlibrium position, along the direction of propagation of the wave motion itself. The wave motion, therefore, travels in the form of *compressions* (or *condensations*) and *rarefactions i.e.*, in the particles of the medium getting closer together and further apart alternately. This type of wave motion is possible in media possessing elasticity of volume, *i.e.*, in solids, liquids as well as gases. Waves produced in a spring or helix when one end of it is suddenly compressed or pulled out and then released or sound waves in air are examples of this type of wave motion (Fig. 4.2). As in the case of transverse wave motion, one compression and the adjoining rarefaction constitute a wave or pulse and a succession of them, a wavetrain

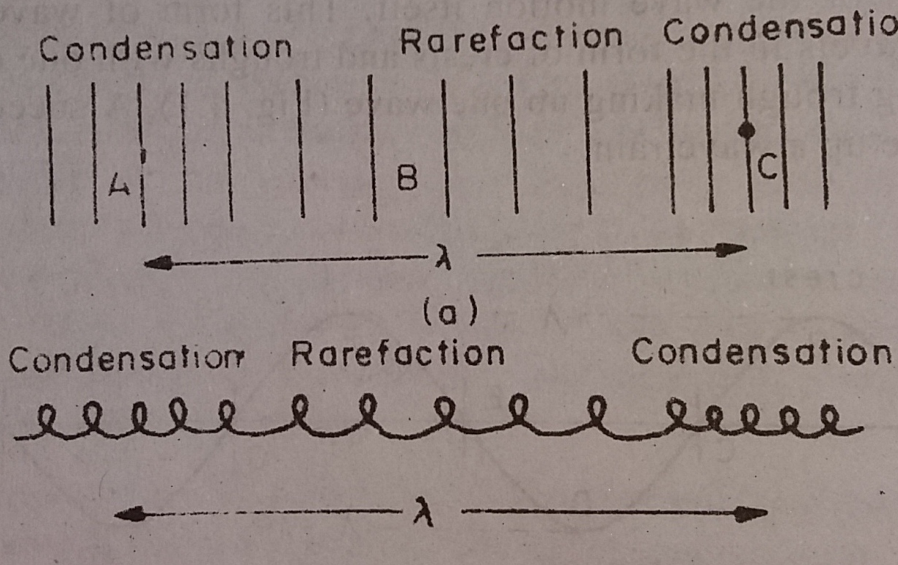

Fig. 4.2

Characteristics of wave motion

Before proceeding further, the important characteristics of wave motion, whether transverse or longitudinal, may be summarized below :

(i) Wave motion is a disturbance produced in a medium by the repeated periodic motion of the particles of the medium. It is only this disturbance which travels forward through the medium as the wave while the particles of the medium vibrate about their mean positions – they are not propagated through the medium.

(ii) The velocity with which the wave propagates through the medium and the velocity with which the particles of the medium vibrate about their mean positions are different. *While the velocity of the wave is constant, the velocity of the particles is different at different positions.* The velocity of the particle is maximum at the mean position and zero at the extreme position of the particle.

(iii) There is a phase difference between the particles of the medium. The particle ahead starts vibrating a little later than a particle just preceeding it.

4.3 Some definitions connected with wave motion

(i) Wavelength : Since a wave or a pulse is produced in the time taken by a particle of the medium to complete one full oscillation about its mean position, *wavelength may be defined as the distance travelled by the wave in the time in which the particle completes one vibration.* It may also be defined as *the distance between any two nearest particles of the medium which are in the same phase.*

(ii) Amplitude : *It is the maximum displacement of the particle from its mean position of rest.*

(iii) Time period (T) : *It is the time taken by a particle to complete one vibration.*

(iv) Frequency (n) : *The number of complete oscillations made by a particle of the medium i.e., the number of waves produced, in one second is called the frequency of the wave (or vibration).*

Suppose frequency = n

Time taken to complete n vibrations = 1 second

Time taken to complete 1 vibration = $\frac{1}{n}$ second

By definition time taken to complete one vibration is the time period (T).

$$\therefore \quad T = \frac{1}{n} \ ; \quad \text{or } nT = 1$$

Frequency x Time period = 1

(ix) Phase : The phase of a vibrating particle is defined as *the ratio of the displacement of the vibrating particle at any instant to the amplitude of the vibrating particle (y/a). It is also defined as the fraction of the time interval that has elapsed since the particle crossed the mean position of rest in the positive direction.* The phase is equal to the *angle* swept by the radius vector since the vibrating particle last crossed its mean position of rest.

(x) Wave front : According to origin of wave motion a vibrating particle placed at a point in a homogeneous medium, extending in all directions, communicates its motion to all its neighbouring particles. The neighbouring particles which have thus been disturbed then perform, in turn, the motion of the vibrating particle. Due to this periodic vibration of the particles a wave motion is produced which travel in every direction with equal velocity. The wave motion, therefore, reach all particles which are at equal distances from the point simultaneously. The position of all these particles can be represented by the surface of a sphere drawn with the position of the vibrating particle as the centre. With time the wave advances into spheres of gradually increasing radius. Such a sphere is known as a wavefront. A wavefront at any instant of time may, therefore, be defined as *the loci of all the neighbouring particles in the medium which are just being disturbed at that instant of time and are consequently in the same state of vibration.*

In a homogeneous medium, the wavefronts are always actually spherical. But if a wavefront is considered at a considerable distance from the source, then any small portion of the wavefront can be considered *plane*.

4.4 Expression for a plane progressive wave

A plane progressive wave is one which travels onward through the medium in a given direction without attenuation i.e., with its amplitude constant.

A progressive wave may be either transverse or longitudinal. In either case, there exists a regular phase difference between any two successive particles of the medium. A typical waveform is shown in Fig. 4.3. Let a wave originating at O, travel to the right along the x-axis. If we start counting the time at the moment when the particle at

Substituting this value of $\frac{d^2x}{dt^2}$ in eqn (1)

We get,

Left hand side $= -\omega^2 x + \omega^2 x = 0$

or, Left hand side = Right hand side.

Thus, we see that $x = A \sin(\omega t + \delta)$ satisfies eqn (1). Hence, it is a solution of differential eqn of simple harmonic oscillation.

Difference betn transverse and Longitudinal waves are following.

Transverse	Longitudinal
1. When the vibration of the particles of the medium are perpendicular to the direction of propagation of the wave, the wave is called a transverse wave.	1. When the vibration of the particles of medium are parallel to the direction of propagation of the wave, the wave is called Longitudinal wave.
2. The wave propagates producing crest and	2. The wave propagates producing compression and

Transverse	Longitudinal
and trough in the medium.	rarefaction in the medium.
3. One crest and trough constitute a wavelength.	3. One compression and rarefaction constitute a wavelength.
4. Polarization takes place.	4. Polarization does not take place.

Difference bet^n progressive and stationary waves are given below:

Progressive	Stationary
1. All particles in a progressive wave are in periodic motion.	1. All particles in a stationary wave, except those are at rest.
2. The amplitude of each particles is same but their displacement is not same at a definite instant of time.	2. The amplitude is not same though the time period of the particles are same.

Progressive	Stationary
3. The vibration of one particle is transffered to next particle, as a result the wave advances through the medium with a definite velocity.	3. The waves remain stationary in the ~~remain~~ medium and the compressions and rarefactions simply appear and disappear at some places of equal interval.
4. The phases of the particles of the medium transmitted from one particle to another.	4. All the particles betn two consecutive nodes are at same phase but the amplitudes are different.
5. The particles of the medium never come to rest.	5. In one complete cycle, All the particles of the medium come to rest twice.
6. In case of transverse progressive wave one crest and one trough constitute a wavelength and in case of Longitudinal progressive wave one compression and one rarefaction constitute a wavelength.	6. The distance betn three consecutive antinodes or three consecutive nodes is the wavelength of a stationary wave.

4.9 *Energy density and energy current (intensity) of a plane progressive wave*

It has already been mentioned that in a progressive wave motion, the energy derived from the source is passed on from particle to particle, so that there is a regular transmission of energy across every section of the medium. The term energy density of a plane progressive wave means the total (kinetic + potential) energy per unit volume of the medium through which the wave is passing.

In order to obtain an expression for energy density, let us start with the equation of a plane progressive wave, which is

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

where the symbols have their usual meanings.

Then the velocity of the particle,

$$U = \frac{dy}{dt} = \frac{2\pi v}{\lambda} a \cos \frac{2\pi}{\lambda}(vt - x) \tag{4.21}$$

and the acceleration of the particle,

$$\frac{dU}{dt} = \frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2} a \sin \frac{2\pi}{\lambda}(vt - x)$$

$$= -\frac{4\pi^2 v^2}{\lambda^2} \cdot y \tag{4.22}$$

kinetic energy per unit volume

Let us consider unit volume of the medium in the form of an extremely thin element of the medium parallel to the wavefront. Now density is mass per unit volume and since unit volume is being considered here,

Mass of the element = ρ, the density of the medium.

Again, *since the layer is very thin, the velocity of all the particles in it may be assumed to be the same. Thus, the kinetic energy per unit volume of the medium*

$$= \frac{1}{2}(\text{mass})(\text{velocity})^2$$

$$= \frac{1}{2} \cdot \rho \cdot U^2$$

$$= \frac{1}{2} \cdot \rho \cdot \left[\frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda} (vt - x) \right]^2$$

$$= \frac{2\pi^2 a^2 v^2}{\lambda^2} \cdot \rho \cdot \cos^2 \left[\frac{2\pi}{\lambda} (vt - x) \right] \quad (4.23)$$

potential energy per unit volume

Now, *the work done per unit volume for a small displacement dy of the layer*

= force x displacement.

force = mass x acceleration

$$= \rho \times \frac{d^2y}{dt^2}$$

$$= \rho \times \frac{4\pi^2 v^2}{\lambda^2} \cdot y$$

[the minus sign in the expression for d^2y/dt^2 which merely indicates the direction of the force has been ignored].

Hence work done per unit volume for a small displacement dy of the layer,

$$= \frac{4\pi^2 v^2 \rho}{\lambda^2} \, y \, dy$$

Then, the total work done when the layer is displaced from 0 to y,

$$= \int_0^Y \frac{4\pi^2 v^2 \rho}{\lambda^2} \, y.dy = \frac{4\pi^2 v^2 \rho}{\lambda^2} \int_0^Y y.dy$$

$$= \frac{4\pi^2 v^2 \rho}{\lambda^2} \cdot \frac{y^2}{2} = \frac{2\pi^2 v^2 \rho}{\lambda^2} \cdot y^2$$

$$= \frac{2\pi^2 v^2 \rho}{\lambda^2} \cdot a^2 \sin^2 \left[\frac{2\pi}{\lambda} (vt - x) \right]$$

Obviously, this work must be stored up in the medium in the form of potential energy.

Hence,

Potential energy (P.E.) per unit volume of the medium

$$= \frac{2\pi^2 v^2 \rho}{\lambda^2} a^2 \sin^2\left[\frac{2\pi}{\lambda}(vt - x)\right] \quad (4.24)$$

energy density

Thus, the total energy per unit volume of the medium or *the energy density of the plane progressive wave, E = K.E. + P.E.*

$$\text{or, } E = \frac{2\pi^2 a^2 v^2 \rho}{\lambda^2}\left[\cos^2\frac{2\pi}{\lambda}(vt - x) + \sin^2\frac{2\pi}{\lambda}(vt - x)\right]$$

$$= \frac{2\pi^2 v^2 . \rho}{\lambda^2} . a^2 \quad (4.25)$$

$$= 2\pi^2\left(\frac{v}{\lambda}\right)^2 . \rho . a^2$$

$$= 2\pi^2 n^2 a^2 \rho \quad (4.26)$$

where $n = v/\lambda$ is the frequency of the wave.

It is interesting to note that *although both kinetic and potential energies of the wave depend upon the values of x and t, its total energy or the energy density is quite independent of either.*

4.10 Energy current – intensity of a wave

In case the cross-section of the beam be unity, expressions (4.25) and (4.26) give the total energy of the beam or the wave *per unit length.*

If v is the velocity of the wave, then a new length v of the medium is set into motion every second; therefore, the energy transferred per second must be the energy contained in length v. This rate of flow of energy per unit area of cross-section of the wavefront along the direction of wave propagation is called the *energy current (C) or the energy flux of the wave* and is obviously equal to $E \times v$.