CHANGE OF COORDINATE

Rotation of Axes

If the origin is shifted to a point (-3, 2), axes remaining parallel, the new coordinates of the point are (3, 1). Find the old coordinates.

Solution:

New origin $(-3, 2) \equiv (h, k)$

Let $(X, Y) \equiv (3, 1)$ be the new coordinates and (x, y) be the old coordinates

We have $x = X + h$ and $y = Y + k$

$x = 3 - 3 = 0$ and $Y = 1 + 2 = 3$

Ans: The old coordinates of point (3, 1) are (0, 3)

If the origin is shifted to a point (1, 2), axes remaining parallel, find the new coordinates of the point (2, -3)

Solution:

New origin $(1, 2) \equiv (h, k)$

Let $(x, y) \equiv (2, -3)$ be the old coordinates and (X, Y) be the new coordinates

We have $X = x - h$ and $Y = y - k$

$\therefore X = 2 - 1 = 1$ and $Y = -3 - 2 = -5$

Ans: The new coordinates of point (2, -3) are (1, -5)

The point (3, 8) becomes (-2, 1) after the shift of origin. Find the coordinates of the point, where the origin is shifted.

Solution:

Let the new origin be (h, k)

Let $(x, y) \equiv (3, 8)$ be the old coordinates and $(X, Y) \equiv (-2, 1)$ be the new coordinates

We have $h = x - X$ and $k = y - Y$

$\therefore h = 3 + 2 = 5$ and $Y = 8 - 1 = 7$

Ans: The coordinates of the point, where the origin is shifted are (5, 7)

When the origin is shifted to $(-1,2)$ by the translation of axes, find the transformex equation of $x^2+y^2+2x-4y+1=0$.

Solution :

Given equation is $x^2+y^2+2x-4y+1=0$

We take new origin $(h,k)=(-1,2)$, then

$x = X+h \Rightarrow x = X-1$

$y = Y+k \Rightarrow y = Y+2$

From the given equation, the transformed equation is

$$(X-1)^2+(Y+2)^2+2(X-1)-4(Y+2)+1=0$$

$$\Rightarrow (X^2+1-2X)+(Y^2+4+4Y)+2X-2-4Y-8+1=0$$

$$\Rightarrow X^2+Y^2-4=0$$

$\therefore$ The required transformed equation is

$$X^2+Y^2-4=0$$

$When\ the\ origin\ is\ shifted\ to\ the\ point\ (2, 3),$
$the\ transformed\ equation\ of\ a\ curve\ is$
$x^2+3xy-2y^2+17x-7y-11=0.$ $Find\ the$
$original\ equation\ of\ the\ curve.$

$Solution:$

$Given\ transformed\ equation\ is\ taken\ as$

$$X^2+3XY-2Y^2+17X-7Y-11=0$$

$We\ take\ origin\ (h,k)=(2,3),\ then$

$$X=x-h \Rightarrow X=x-2,$$

$$Y=y-k \Rightarrow Y=y-3$$

$From\ the\ given\ transformed\ equation,$
$original\ equation\ is$

$$(x-2)^2+3(x-2)\ (y-3)-2(y-3)^2+$$
$$17(x-2)-7(y-3)-11=0$$

$$\Rightarrow x^2+4-4x+3xy-9x-6y+18-2y^2-18$$
$$+12y+17x-34-7y+21-11=0$$

$$\Rightarrow x^2+3xy-2y^2+4x-y-20=0$$

$\therefore The\ required\ original\ equation\ is$

$$x^2+3xy-2y^2+4x-y-20=0$$

Find thetransformed equation of $x^2+2\sqrt{3xy}-y^2=2a^2$, *when the axes are rotated through an angle* $\frac{\pi}{6}$.

Solution :

The given original equation is

$$x^2+2\sqrt{3}\,xy-y^2=2a^2$$

Angle of rotation $\theta=\frac{\pi}{6}=30^\circ$, *then*

$$x=X\cos\theta-Y\sin\theta$$

$$\Rightarrow X\cos 30^\circ - Y\sin 30^\circ$$

$$=X\left(\frac{\sqrt{3}}{2}\right)-Y\left(\frac{1}{2}\right)\Rightarrow x=\frac{\sqrt{3}X-Y}{2}$$

$$y=Y\cos\theta+X\sin\theta\Rightarrow Y\cos 30^\circ+X\sin 30^\circ$$

$$=Y\left(\frac{\sqrt{3}}{2}\right)+X\left(\frac{1}{2}\right)\Rightarrow y=\frac{\sqrt{3}Y+X}{2}$$

From original equation, the transformed equation is

$$\left(\frac{\sqrt{3}X-Y}{2}\right)^2+2\sqrt{3}\left(\frac{\sqrt{3}X-Y}{2}\right)\left(\frac{\sqrt{3}Y+X}{2}\right)-\left(\frac{\sqrt{3}Y+X}{2}\right)^2=2a^2$$

$$\Rightarrow\frac{(\sqrt{3}X-Y)^2+2\sqrt{3}(\sqrt{3}X-Y)(\sqrt{3}Y+X)-(\sqrt{3}Y+X)^2}{4}=2a^2$$

$$\Rightarrow 3X^2+Y^2-2\sqrt{3}(3XY+\sqrt{3}X^2-\sqrt{3}Y^2-XY)-$$
$$3Y^2-X^2+2\sqrt{3}XY=4(2a^2)$$

$$\Rightarrow 3X^2+Y^2-2\sqrt{3}XY+6\sqrt{3}XY+6X^2-2\sqrt{3}XY-$$
$$3Y^2-X^2-2\sqrt{3}XY=8a^2$$

$$\Rightarrow 8X^2-8Y^2=8a^2$$

$$\Rightarrow 8(X^2-Y^2)=8a^2\Rightarrow X^2-Y^2=a^2$$

$\therefore$ *The required transformed equation is* $X^2-Y^2=a^2$

3. *Find the point to which the origin is to be shifted so as to remove the first degree terms from the equation* $4x^2+9y^2-8x+36y+4=0$

Solution :

The given equation is $4x^2+9y^2-8y+36y+4=0$

Comparing the equation with

$ax^2+2hxy+by^2+2gx+2fy+c=0$

Then, $a=4$, $b=9$, $c=4$, $h=0$, $g=-4$, $f=18$

$\therefore$ The required point $=$

$$\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$$

$$=\left(\frac{0(18)-9(-4)}{4(9)-(0)^2}, \frac{(-4)(0)-(4)(18)}{4(9)-(0)^2}\right)$$

$$=\left(\frac{36}{36}, \frac{-72}{36}\right)=(1, -2)$$

$\therefore$ Required point is $(1,-2)$