Equilibrium Forces

Equilibrium of Forces:

If the resultant of all the forces acting on a body is zero, then the phenomenon is called equilibrium of force. The forces, which establish equilibrium, are called equilibrium of forces. The resultant of the components of the equilibrium forces is also zero. If forces $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$ ...... act on a body, according to the condition of equilibrium,

$$\sum \vec{F} = \vec{F_1} + \vec{F_2} + \cdots\cdots = 0$$

Also, $\sum F_x = 0$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

Again, $\sum \vec{F} = \sum m\vec{a} = 0$

$$\therefore \vec{a} = 0 \quad [\because m \neq 0]$$

i.e a body will remain in equilibrium when its acceleration is zero.

If a number of equilibrium forces act on a body at rest or in uniform motion, its state of rest or uniform motion will not change.

Force is a vector quantity.

So, rules of vector addition is applicable for the forces.

1. Equilibrium of two forces:
2. Equilibrium of three forces (non parallel)
3. Equilibrium of three parallel forces.

1. Equilibrium of two forces:

Two forces acting at a point maintain equilibrium, if they are of equal magnitude and directed oppositely in fig. Two forces $\vec{P}$ and $\vec{Q}$ are acting at a point or on a body along OA and OB respectively.

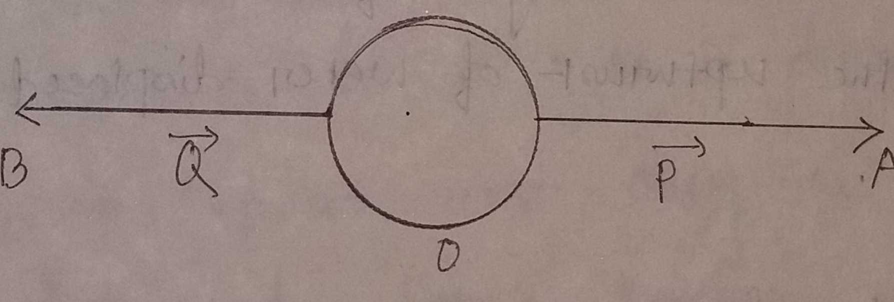

fig:

Here, $\vec{P} = -\vec{Q}$

or, $\vec{P} + \vec{Q} = 0$

Hence, the forces create equilibrium on the body. Condition of equilibrium for two forces :

1. The magnitude of two forces must be equal.

2. Two forces must be directed oppositely.

3. Two forces must act along the same line.

Example :

(a) Net force on floating crok is zero because the weight of crok is balanced by the upthrust of water displaced by the crok.

(b) Net force on a car moving on a rough road with constant speed is zero. Because, there is no acceleration.

2 Equilibrium of three non Parallel forces:

Conditions of equilibrium of three non parallel forces:

(a) Forces must be co-planer

(b) Forces will be act at the same point from different directions.

(c) Any one force will be equal to the resultant of the other two forces in magnitude but opposite in direction.

(d) The forces must obey the triangle law and Lami's theorem.

Triangle law of forces:

If there three non parallel forces acting at a point is directed in same order along three sides of triangle, then the forces establishes equilibrium.

Proof:

Let, $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are co-planer forces acting at a point O along three different directions in plane as shown in fig-1(a)

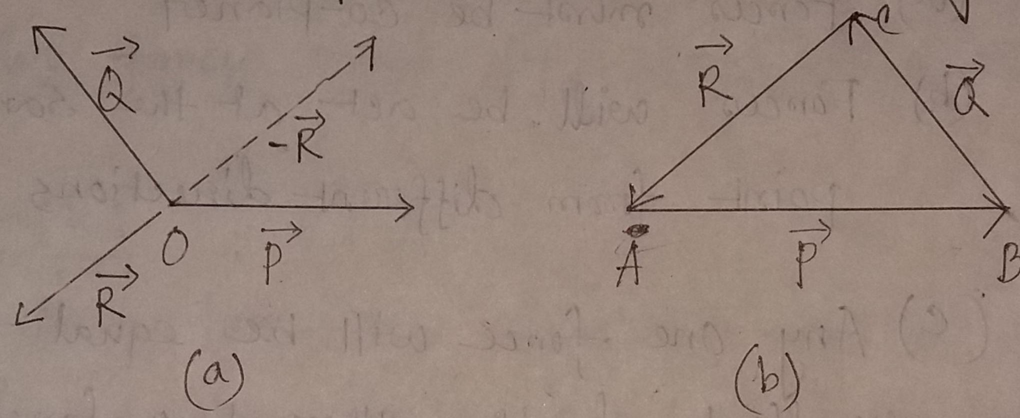

fig-1

Let us construct triangle ABC as shown in fig {1(b)} and the forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are acting along three sides AB, BC and CA respectively.

i.e. $\vec{AB} = \vec{P}$

$\vec{BC} = \vec{Q}$

$\vec{CA} = \vec{R}$

According to the triangle law of addition of vectors,

$$\vec{P} + \vec{Q} = \vec{AC} = -\vec{R}$$

or, $$\vec{P} + \vec{Q} + \vec{R} = 0$$

Thus, the forces establishe equilibrium and the forces are equilibrium forces.

Lami's theorem :

"If three non-parallel forces maintain equilibrium, then the magnitude of any one is proportional to the sine of the angle between the other two."

Let three forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ acting at a point O are in equilibrium. Angle between $\vec{Q}$ and $\vec{R} = \alpha$, angle between $\vec{R}$ and $\vec{P} = \beta$, angle between $\vec{P}$ and $\vec{Q} = \gamma$.

According to the Lami's theorem

$$\frac{\vec{P}}{\sin\alpha} = \frac{\vec{Q}}{\sin\beta} = \frac{\vec{R}}{\sin\gamma}$$

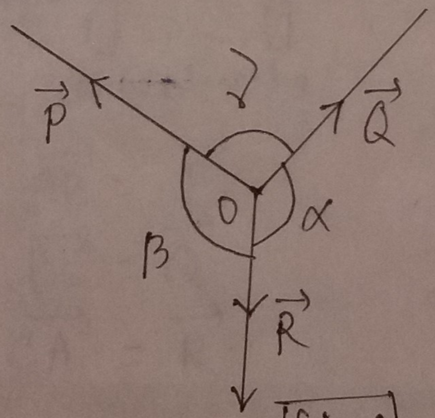

fig 2

Example :

Let two pins A and B are placed horizontally in the middle of the wall. (fig-3)

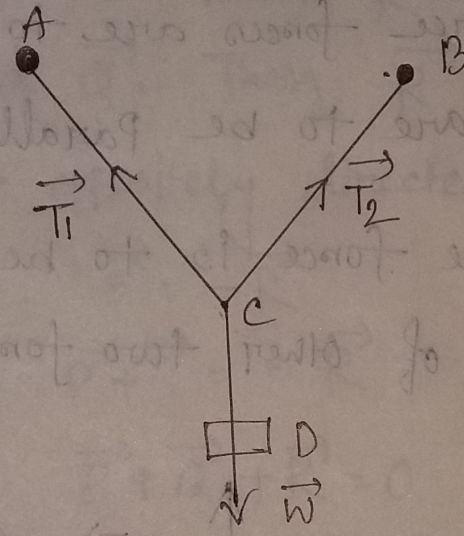

fig - 3

A long thread ACB is fastened with the pins. A body D is suspended from the point C. Let the weight of the body is $\vec{W}$. Along CA and CB part tension of the thread or upward forces be $\vec{T_1}$ and $\vec{T_2}$.

In this case, $\vec{T_1}$, $\vec{T_2}$ and $\vec{W}$ forces will establish equilibrium.

3 Equilibrium of three Parallel forces:

Parallel forces establish equilibrium, if they satisfy the following condition.

1. The three forces are to be co-planer.
2. Forces are to be Parallel to each Other.
3. Any One force is to be equal to the resultant of other two forces but directed oppositely.

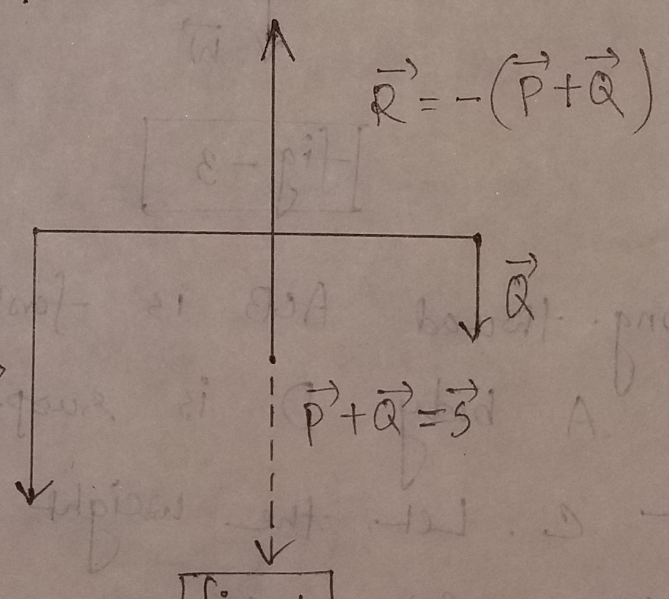

Fig-4

Let, three parallel forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are acting on a body and maintain equilibrium. (fig-4)

The resultant of $\vec{P}$ and $\vec{Q}$ is $\vec{S} = (\vec{P} + \vec{Q})$

According to the condition of equilibrium the vector $\vec{R}$ is the resultant of $\vec{P}$ and $\vec{Q}$. Then $\vec{R}$ and $\vec{S}$ will be equal but oppositely directed.

i.e. $\vec{S} = -\vec{R}$

Or, $\vec{P} + \vec{Q} = -\vec{R}$

Or, $\vec{P} + \vec{Q} + \vec{R} = 0$

Example:

1. While measuring weight of a body by balance, the weight of the body is placed on one pan. Weight on another pan and upward force on the middle stand of the balance remain in equilibrium. These three forces are parallel to each other.

2. The hawker on his shoulder carries weights at the two ends of a wooden rod. The weights at two ends of the rod and the upward force of his shoulder at the middle of the rod.

These three parallel forces maintain equilibrium.

Problem: Figure 1(a) shows a weight W hung by strings. Consider the knot at the junction of the three strings to be "the body". The body remains at rest under the action of the three forces shown in fig 1(b). Suppose we are given the magnitude of one of these forces. How can we find the magnitude of the other forces?

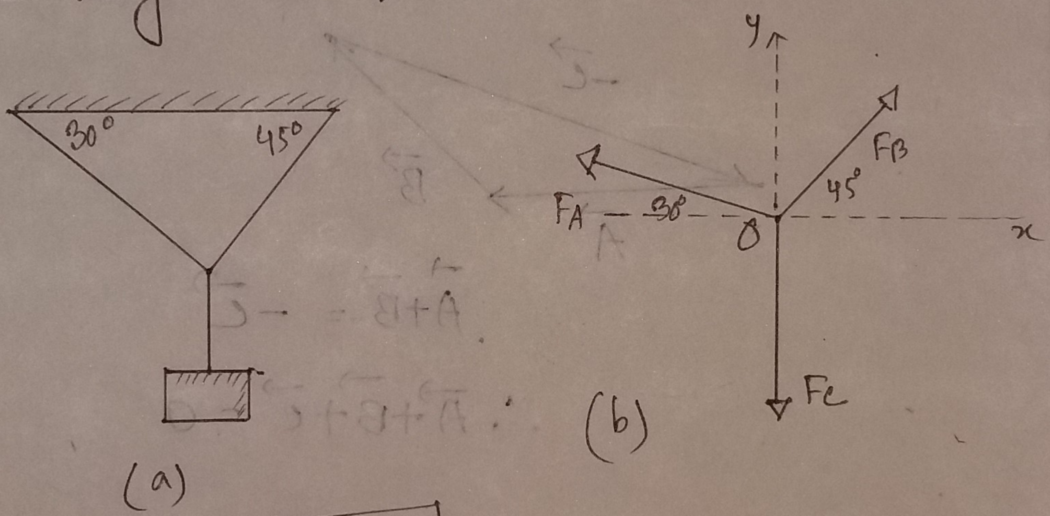

fig-1

If W = 150 N, then find $F_A = ?$

$F_B = ?$