Assignment
EM-I (CSE)
Premier University, Chittagong
Topic: Function, Limit, Continuity and Differentiability

Define following terms with examples:

i) Domain and Range of a function
ii) Limit, continuity and Differentiability of a function at a point
iii) Successive Differentiation

1. a) Draw the graph of the function and state the domain and range $f(x)=(x-2)^2+3$.

   b) Find the domain of the function $f(x)=\sqrt{25-x^2}$.

2. A function f(x) as $f(x)=\dfrac{x^2+1}{x^2-1}$. Does the limit exist at x=-1?

3. Evaluate the limits: *i)* $\lim\limits_{x\to 0}\dfrac{5\sin^2 x}{1-\cos x}$, *ii)* $\lim\limits_{x\to 0} x\sin\left(\dfrac{a}{x}\right)$, *iii)* $\lim\limits_{x\to 2^+}\dfrac{x+1}{3x+6}$.

4. A function $f(x)$ is defined as follows:

   i) $f(x)=\begin{cases}3+2x & ;-\dfrac{3}{2}\le x<0\\ 3-2x & ;0\le x<\dfrac{3}{2}\end{cases}$, ii) $f(x)=\begin{cases}x\sin\dfrac{1}{x} & ;x\neq 0\\ 0 & ;x=0\end{cases}$

   Check the continuity and differentiability at $x=0$.

5. a) Find $\dfrac{dy}{dx}$ for $y=(1+x^2)^{\sin x}$.

   b) If $y=e^{\sin^{-1}x}$ and $z=e^{-\cos^{-1}x}$, the show that $\dfrac{dy}{dz}$ is independent of x.

   c) Find $\dfrac{dy}{dx}$ for $3x^4-5x^3y^2+2\sin y=0$.

6. If $x=\sin t,\ y=\sin ct$, show that $(1-x^2)y_2-xy_1-c^2y=0.$

7. If $y=\cos(m\sin^{-1}x)$, then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2-m^2)y_n=0.$

**You are requested to submit your assignment by 18/06/2022. Be careful about plagiarism. It is unethical.

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