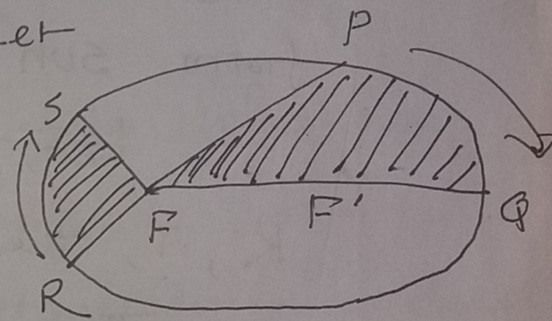
Gravity & Gravitation

Kepler's Law:

There are three Laws of planetary motion which are known as Kepler's Law.
The laws are stated below:

1. **First Law: (law of orbit):** The path of a planet is elliptical with the sun at a focus.

Explanation: Let F and F′ are two foci of an elliptical orbit PQRS. Let the sun is at the focus F. According to the first Law of Kepler's each planet rotates around the ellipse keeping sun at F.



2. **Second Law (law of area):** The line joining a planet to the Sun sweeps out equal area in equal time.

Explanation: Let, a planet take time, t in moving from position P to Q. If the planet travels in the same time from position R, to S. Then the area PFQ must be equal to the area of RFS.

Third Law (Law of time period):
The square of the time period of a planet is proportional to the cube of a semi major axis.

Explanation: The path of a planet is elliptical around the sun. So, the distance of a planet from sun is different in different time.

Let, the length of the semi major axis be R, T, the time period of the planet.

Then, according to third law $T^2 \propto R^3$.

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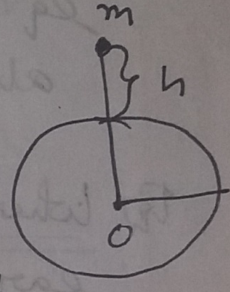
Q. Show that the value of g is maximum at the earth's surface and decreases above and below the earth's surface

Body above the surface of the earth:

Let, R be the radius and M the mass of the earth. Let a body is situated at a height h from the surface of the earth. So, the distance of the body from the center of the earth $= R + h$.



Then, the acceleration due to gravity g at the height h is,

$$g' = G \frac{M}{(R+h)^2} \quad \text{.....} (1)$$

On the surface of the earth,

$$g = G \frac{M}{R^2} \quad \text{———} (2)$$

Dividing eq^n (1) by eq^n (2), we get,

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2}$$

$$= \frac{1}{\left(1 + \frac{h}{R}\right)^2}$$

$$= \left(1 + \frac{h}{R}\right)^{-2}$$

if $h << R$, then

$$\frac{g'}{g} = 1 - \frac{2h}{R}$$

or, $g' = g\left(1 - \frac{2h}{R}\right)$ —— (3)

i.e., $g' < g$.

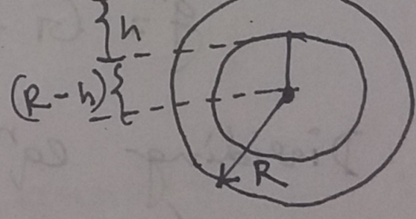
eqn (3) shows that with the increase of altitude h the value of g decreases.

(b) When body situated below the surface of the earth:

Let a body is situated at a depth h inside the earth and g' - the acceleration due to graving at that point. So, the distance of the body from the centre of the earth $= R - h$.

Volume of the Portion of the earth of radius.

$$R - h = \frac{4}{3}\pi(R-h)^3.$$



and Mass,

$$M' = \frac{4}{3}\pi(R-h)^3 \times \rho$$ [ρ = Average density of the earth].

$$\therefore g'' = \frac{GM'}{(R-h)^2}$$

$$= \frac{G \times \frac{4}{3}\pi (R-h)^3 \times \rho}{(R-h)^2}$$

or, $g'' = \frac{4}{3}\pi G\rho (R-h)$. —— (1)

Dividing eqn $g = G\frac{M}{R^2}$ by eqn (1)

we get,

$$\frac{g''}{g} = \frac{\frac{4}{3}\pi G\rho (R-h)}{\frac{GM}{R^2}}$$

$$= \frac{\frac{4}{3}\pi G\rho (R-h)}{\frac{G \times \frac{4}{3}\pi R^3 \times \rho}{R^2}}$$

$$= \frac{R-h}{R}$$

$$= 1 - \frac{h}{R}$$

or, $g' = g\left(1 - \frac{h}{R}\right)$ —— (2)

$\therefore g'' < g$

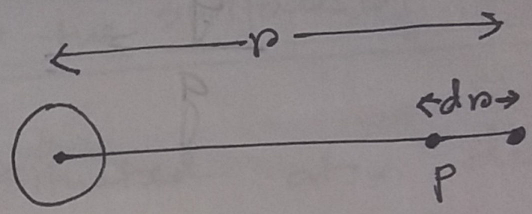
(showed)

Gravitational potential :

The gravitational potential at a point in the gravitational field is the amount of work done in moving a unit mass from infinity to that point. It is denoted by V.

Gravitational potential due to a point mass :

Let a body of mass M be placed at point A. Let P be a point at a distance r from A.



The gravitational potential at point P is to be determined. At point P the attractive force on unit mass is $F = G\frac{M \times 1}{r^2} = \frac{GM}{r^2}$ .... it acts along PA.

Let, potential at a point P be V.

The amount of work done i.e. potential in moving the unit mass by a small distance dr is,

$$dV = F dr \cos 0^\circ = F dr.$$

So, total work done in bringing the unit mass from infinity to point P is,

$$V = \int dV = \int_{r=\infty}^{r=r} \frac{GM}{r^2} \times dr .$$

or, $$V = GM \int_{r=\infty}^{r=r} \frac{1}{r^2} \, dr .$$

$$= GM \left[-\frac{1}{r} \right]_{\infty}^{r}$$

$$= GM \left[-\frac{1}{r} + \frac{1}{\infty} \right]$$

$$= GM \left[-\frac{1}{r} \right]$$

$$V = -\frac{GM}{r} .$$

The gravitational potential at infinity is maximum and is equal to zero. Potential is negative or less than zero towards the field.

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Escape velocity:

If we throw a body vertically upwards, it returns to the ground due to the earth's gravitational attraction. If the body is projected with such a velocity that it overcomes the gravitational attraction and never comes back to the earth, such a velocity is known as the escape velocity.

Def^n: The minimum velocity with which a body is projected as to enable it just to overcome the gravitational attraction and does not comeback to the earth is called escape velocity.

If the body is raised a distance dr against the force of attraction, the work done,

$$dW = F\,dr = \frac{GMm}{r^2}\,dr$$

where, $M \rightarrow$ mass of earth,
$m \rightarrow$ " " object
$r \rightarrow$ distance
$G \rightarrow$ Gravitational const.

So, total work done for taking the body out of the earth's attraction. i.e work done to throw the body to infinity froom the surface of the earth,

$$W = \int dW = \int_R^\infty \frac{GMm}{r^2}\, dr$$

or, $W = GMm \int_R^\infty \frac{1}{r^2} = GMm \left[-\frac{1}{r}\right]_R^\infty$

$\therefore\ W = \frac{GMm}{R}$

Let, the escape velocity be v_e. So, its initial kinetic energy $= \frac{1}{2} m v_e^2$. The body escapes the gravitational pull by spending this amount of energy.

$\therefore\ \frac{1}{2} m v_e^2 = \frac{GMm}{R}$

or, $v_e^2 = \frac{2GM}{R}$ ——— (1)

The acceleration due to gravity,

$$g = \frac{GM}{R^2} = \frac{GM}{R \times R}$$

or, $\frac{GM}{R} = gR$ ——— (2)

$\therefore\ v_e^2 = 2gR$

or, $v_e = \sqrt{2gR}$ ——— (3) which is the expression of escape velocity.

□ Artificial satellites:

It is found that if a body is raised to 930 km from the surface of the earth between 8.05 kms^{-1} to 11.1 kms^{-1}. Then it will be a satellite and will move round the earth as the moon does.

The satellites launched from the earth and put in stable orbit round the earth is called artificial satellites.

The scientists of soviet union launched the first artificial satellite (Sputnik 1) on 4th october, 1957.

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Problems: With what velocity should a body be projected vertically upwards from the surface of the earth so that it may just attain a height of R/2. Where R is the radius of the earth. (R = 6400 km).

Sol^n: Work done in moving the body from the surface of the earth to a height $R/2$ above the earth's surface

$$W = \int_R^{(R+R/2)=3R/2} \left(\frac{GMm}{x^2}\right) dx$$

$$= GMm \left[-\frac{1}{x}\right]_R^{3R/2}$$

$$= -GMm \left[\frac{1}{3R/2} - \frac{1}{R}\right]$$

$$\therefore W = \frac{GMm}{3R}$$

$$K.E = \frac{1}{2} m v^2$$

$$\text{or, } \frac{1}{2} m v^2 = \frac{GMm}{3R} \quad \text{—(1)}$$

At the surface of the earth

$$mg = \frac{GMm}{R^2} \quad \text{—(2)}$$

Deviding (1) ÷ (2)

$$\frac{v^2}{2g} = \frac{R}{3}$$

$$v = \sqrt{\frac{2gR}{3}}$$

$$= \sqrt{\frac{2 \times 9.8 \times 6.4 \times 10^6}{3}}$$

$$= 6.467 \text{ km/s}.$$

Q The mass and Satellite of Jupitar are 1.9×10^{27} kg and 7×10^7 m respectively. Calculate the escape velocity.

We know:

$$V_e = \sqrt{2gR}$$

$$= \sqrt{2 \times \frac{GM}{R^2} \times R}$$

$$= \sqrt{2 \frac{GM}{R}}$$

$$= \left[\frac{2 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{27}}{7 \times 10^7}\right]^{\frac{1}{2}}$$

$$= 6.03 \times 10^4 \text{ ms}^{-1}.$$

Here,

$G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}

M = Mass of jupiter $= 1.9 \times 10^{27}$ kg

R = radius of the earth $= 7 \times 10^7$ m.

At what height above the surface of the earth the value of acceleration due to gravity will be 1 % of it's value at the earth's surface : [$R = 6.4 \times 10^3$ km]

We know,

$$g' = g\left(\frac{R}{R+h}\right)^2$$

But $g' = \frac{1}{100} g$

$$\therefore \frac{g}{100} = g\left(\frac{R}{R+h}\right)^2$$

or, $\frac{1}{100} = \left(\frac{R}{R+h}\right)^2$

or, $\frac{1}{10} = \frac{R}{R+h}$

or, $R+h = 10R$

or, $h = 9R = 9 \times 6.4 \times 10^3$ km

$= 5.76 \times 10^4$ km.

Ans: 5.76×10^4 km