Two Dimensional Motoin:

The Projectile motoin:

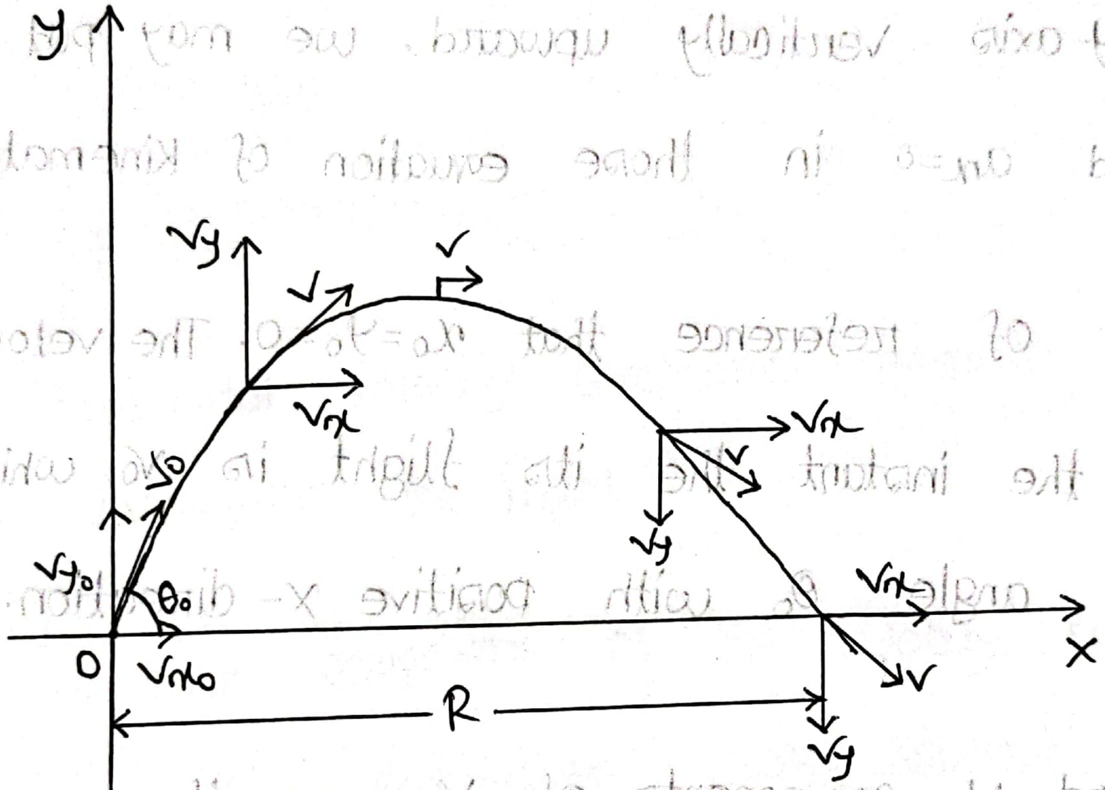

Fig. The Trajectory of a projectile is parabola where R is a horizontal range.

The motoin of a projectile is one of constant accelaration g, directed and thus should be described the eqn of Kinematics

There is no horizontal component of accelaration if we choose a reference frame with the positive y-axis vertically upward, we may put $a_y = -g$ and $a_x = 0$ in those equation of Kinematics.

The origin of reference that $x_0 = y_0 = 0$. The velocity at $t = 0$, the instant the its flight is v_0, which makes an angle θ_0 with positive x-direction.

The x and y components of v_0 are then,

$$V_{x_0} = V_0 \cos\theta_0$$

$$\text{and } V_{y_0} = V_0 \sin\theta_0$$

Because there is no horizontal component of accelaration the horizontal component velocity will be constant.

if $a_x = 0$

$$V_{x0} = V_{x_0} \cos\theta_0$$

So that,

$$V_x = V_0 \cos\theta_0 \quad (1)$$

The horizontal velocity component retains its initial value throughout the flight.

The vertical component of the velocity with change with time in accordance with vertical motoin with constant downward accelaration.

then, $a_y = -g$ and $V_{y_0} = V_0 \sin\theta_0$

So that, $V_y = V_0 \sin\theta_0 - gt$

The magnitude of the resultant at any instant is-

$$V = \sqrt{V_x^2 + V_y^2} \longrightarrow ②$$

The angle θ is represented by velocity vector,

$$\tan\theta = \frac{V_y}{V_x}$$

The velocity vector is tangent to the path of the particle at every point as shown in fig.

The x coordinate of the particles position at any instant of time,

$$x_0 = 0$$

$$\text{and } V_{x_0} = V_0 \cos\theta_0$$

$$x = (V_0 \cos\theta_0)t \longrightarrow ③$$

$$\boxed{S = Vt}$$

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The y co-ordinate obtained from with $y_0 = 0$, $a_y = -g$ and $a_{y_0} = V_0 \sin\theta_0$ is-

$$y = (V_0 \sin\theta_0)t - \frac{1}{2}gt^2 \quad \text{--- } ④$$

eqn ③ and ④ gives us x and y as function of common parameter t, the time in flight. By combining and eliminating t from them, we obtain-

$$y = (\tan\theta_0)x - \frac{g}{2(V_0 \cos\theta)^2}x^2 \quad \text{--- } ⑤$$

which relates y to x and is the eqn of the trajectory of the projectile sine $V_0 \cdot \theta_0$ and g are constant, this eqn has the forms

$$y = bx - ax^2$$

which is an eqn of parabola, hence the trajectory of a projectile is a parabola.

Time to reach maximum height:

Vertical Component velocity,

$$v_y = v_0 \sin\theta_0 - gt.$$

at maximum height final velocity

$$v_y = 0$$

$$\Rightarrow 0 = v_0 \sin\theta_0 - gt$$

$$\therefore t = \frac{v_0 \sin\theta_0}{g} \qquad (6)$$

Maximum Height:

Let the maximum height be H we know from equation of motion,

$$v^2 = v_0^2 + 2as$$

For vertical component of a projectile we can write,

$$v_y^2 = v_{y_0}^2 + 2a_y y$$

$a_y = -g$, $v_{y_0} = v_0 \sin\theta_0$

$v_y = 0$, $y = H$

So, $0^2 = (v_0 \sin\theta_0)^2 + 2(-g) \cdot H$

$\Rightarrow +2gH = v_0^2 \sin^2\theta_0$

$\Rightarrow H = \dfrac{v_0^2 \sin^2\theta_0}{2g}$

$$* \quad H = \frac{v_0^2 \sin^2\theta_0}{2g}$$

☐ Time of Flight :

At time $t = t$, Vertical

Displacement,

$$y = (v_0 \sin\theta_0)t - \frac{1}{2} g t^2$$

Let time of flight be T

In time T the projectile returns to the ground

i.e $y = 0$.

$$\therefore 0 = V_0 \sin\theta_0 \times T - \frac{1}{2} g T^2$$

$$\text{or, } \frac{gT}{2} = V_0 \sin\theta_0$$

$$\therefore T = \frac{2V_0 \sin\theta_0}{g}$$

$$* \; T = \frac{2V_0 \sin\theta_0}{g} ;$$

* T = 2 × Time to reach maximum height

Range:

The distance travelled along the horizontal direction in the time of flight T is called the range.

Let Range = R.

$$R = (V_0 \cos\theta_0) \times T$$

$$= V_0 \cos\theta_0 \times \frac{2V_0 \sin\theta_0}{g}$$

$$= \frac{v_0^2 \cdot 2\sin\theta_0 \cos\theta_0}{g}$$

$$\therefore R = \frac{v_0^2 \sin 2\theta}{g}$$

☐ Maximum Range:

$$\text{Range, } R = \frac{v_0^2 \sin 2\theta_0}{g}, \quad R = \frac{v_0^2 \sin 2\theta}{g}$$

R will be maximum,

when, $\sin 2\theta_0 = 1$

Or, $\sin 2\theta_0 = \sin 90^\circ$

Or, $2\theta_0 = 90^\circ$

$\therefore \theta_0 = 45^\circ$

$$R_{max} = \frac{v_0^2 \times 1}{g}$$

$$\therefore R_{max} = \frac{v_0^2}{g}$$

Mathemetical Problem:

A soccer player kicks a ball at an angle of 37° from the horizontal with an initial speed of $30\, ms^{-1}$ assuming that the balls moves in a vertical way.

a) Find the time at which ball reaches the highest point of its Trajectory.

Soln:

Here, $v_0 = 30\ ms^{-1}$

$\theta_0 = 37^\circ$

$g = 9.80\ ms^{-2}$

$$\text{So, } t = \frac{v_0 \sin\theta_0}{g}$$

$$= \frac{30 \times \sin 37^\circ}{9.80}$$

$$= 1.84\,s$$

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b) How hight does the ball go?

Soln:

Here, $v_0 = 30\ ms^{-1}$

$\theta_0 = 37^\circ$

$$So,\ H = \frac{v_0^2 \sin^2\theta_0}{2g}$$

$$= \frac{(30)^2 \times (\sin 37^\circ)^2}{2\times 9.80}$$

$$= 16.64\ m$$

c) What is Horizontal Range of the ball and how long is it in the air?

Soln:

$$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{30^2 \times \sin(2\times 37^\circ)}{9.80}$$

$$= 88.27\ m.$$

$$T = 2\times t = 2\times 1.84$$

$$= 3.68\ s$$

d) What is the velocity of the ball as it strikes the ground?

Soln:

$$V_{x_0} = V_0 \cos\theta_0$$

$$= 30 \times \cos 37^\circ$$

$$= 23.95\ ms^{-1}$$

$$V_{y_0} = V_0 \sin\theta_0 - gt$$

$$= 30 \times \sin 37^\circ - 9.8 \times 1.84$$

$$= 18.05\ \cancel{ms}^{-1} - 18.032 = 0.018\ ms^{-1}$$

$$\therefore V = \sqrt{V_{x_0}^2 + V_{y_0}^2}$$

$$= \sqrt{(23.95)^2 + \cancel{(18.05)^2}\ (0.018)^2}$$

$$= \cancel{30}\ 23.95\ ms^{-1}$$

An Aeroplane travels 130 miles straight angle 22.50 east due to north how far north the plane travel?

Solnº: An Aeroplane travels, Here,

$R = 130$ miles

$= 130 \times 1609$ m

$= 209170$ m

$dy = R\sin\theta$

$= 209170 \times \sin 22.5^\circ$

$\cancel{= 193247.8818 \text{ m}}$

$= 80045.8$ m

$dx = R\cos\theta$

$= 209170 \times \cos 22.5^\circ$

$= 193247.8$ m

The Speed of an automobile travelling due east uniformly reduced from 21 ms^{-1} to 14 ms^{-1} in distance of 81m.

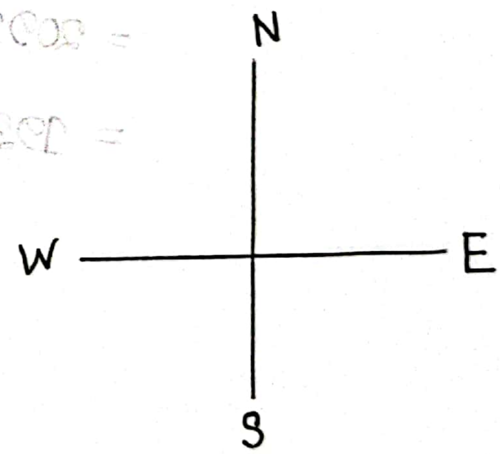
a) What is magnitude and direction of the constant accelaration.

Solⁿ: Here, v is final velocity and u is initial velocity, s is distance.

$v = 14\ ms^{-1}$

$u = 21\ ms^{-1}$

$s = 81\ m$



$\Rightarrow v^2 = u^2 + 2as$

$\Rightarrow 2as = v^2 - u^2$

$\Rightarrow a = \frac{v^2 - u^2}{2s} = \frac{14^2 - 21^2}{2 \times 81}$

$\therefore a = -1.51\ ms^{-2}$

direction - west

b) how much time has elapsed during this de-acceleration?

Solⁿ:

$$a = \frac{v-u}{t}$$

$$\Rightarrow t = \frac{v-u}{a} = \frac{19-21}{-1.51}$$

$$\cancel{= \frac{30-45}{-1.51}} = 4.63\,s$$

c) If one assumed that the car continues to de-accelaration at the same range, how much time would elapsed in bringing it to rest from 21 ms^{-1}.

Solⁿ: now, $v=0$

$$\therefore t = \frac{0-21}{-1.51} = 13.91 \approx 14\,s$$

d) What total distance is required to bring the car to rest from 21 ms^{-1}?

Soln:

$$S = \frac{v^2 - u^2}{2a}$$

$$= \frac{0 - 21^2}{-(2 \times 1.51)} = 146.02 \text{ m.}$$

Two Dimensional motion:

A bomber is flying at a constant horizontal velocity of 820 miles/hr at an elevation of 52,000 ft toward a point directly above its target. A what angle of sight ϕ should a bomb be released to strike the target?

$1m = 3.28 ft.$

$1 mile = 1609 m.$

Soln:

$V_i = 820$ miles/hr

$= \frac{820 \times 1609}{60 \times 60}$

$= 366.49 \ ms^{-1}$

Problem:

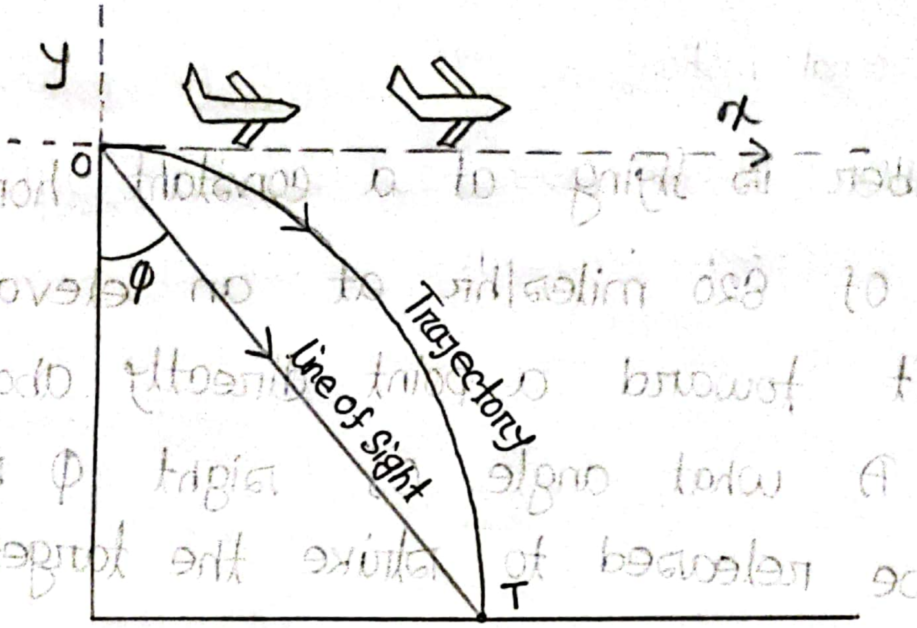


Fig. A bomb is released from an airplane with horizontal velocity V_0

We choose a reference frame fixed with respect to the earth, its origin O being the bomb release point. The motion of the bomb at the instant of release is the same as that of the bomber. Hence the initial projectile velocity V_0 is horizontal and its magnitude is 820 miles/hr. or 366.49 m/s.

Note that,

→ The time of fall of the bomb doesn't depend on the speed of the plane for a horizontal projection.

The horizontal distance traveled by the bomb in this time is given by

$$x = (v_0 \cos\theta_0)t \quad \text{——— ②}$$

Here,

$$x = (366.49) \times 57 \text{ m}$$

$$= 20889.9 \text{ m}$$

So the angle of sight should be,

$$\phi = \tan^{-1}\frac{x}{|y|} = \tan^{-1}\frac{20889.9}{|-15853.60|}$$

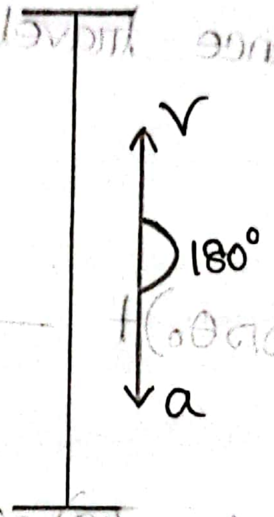
$$= 53^\circ$$

$$\therefore \phi = 53^\circ$$

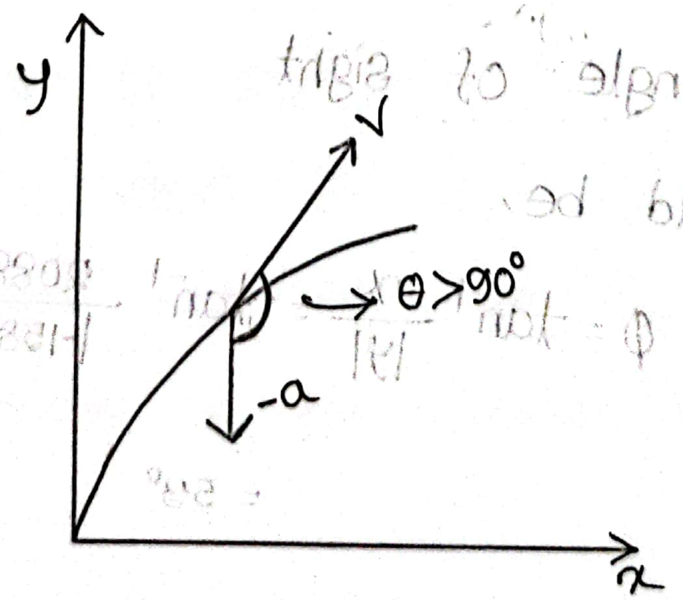
Circular Motion:

* Show the relation between velocity and accelaration for various motion by graphical represantation.

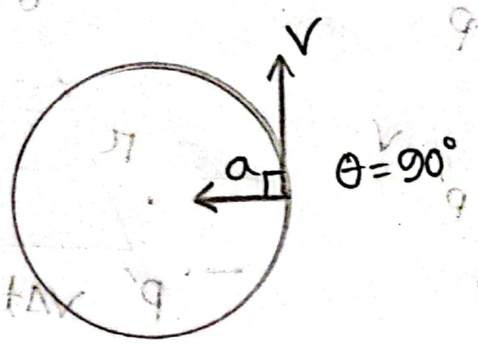
i) Ball trown Up:



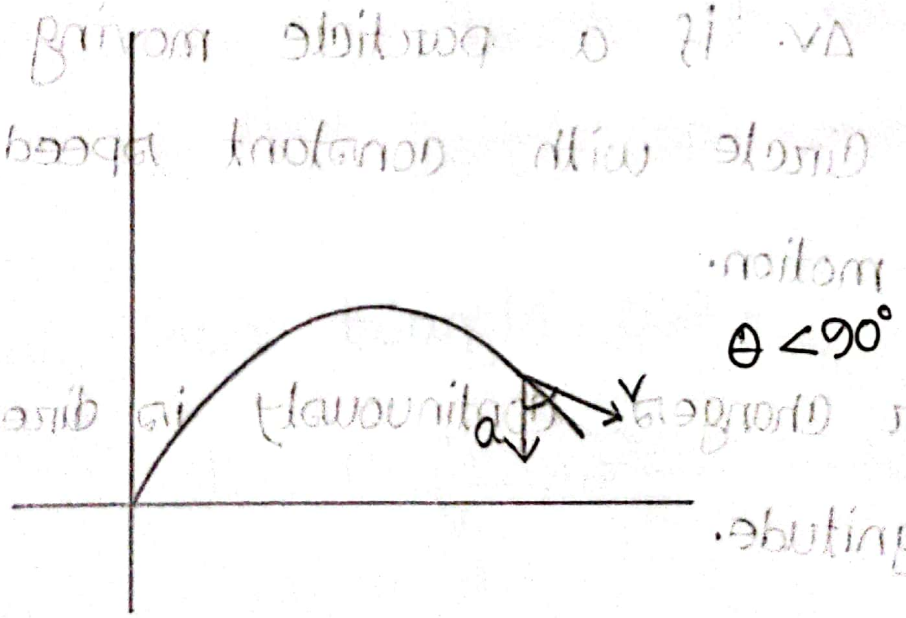
ii) Rise of Projectile:



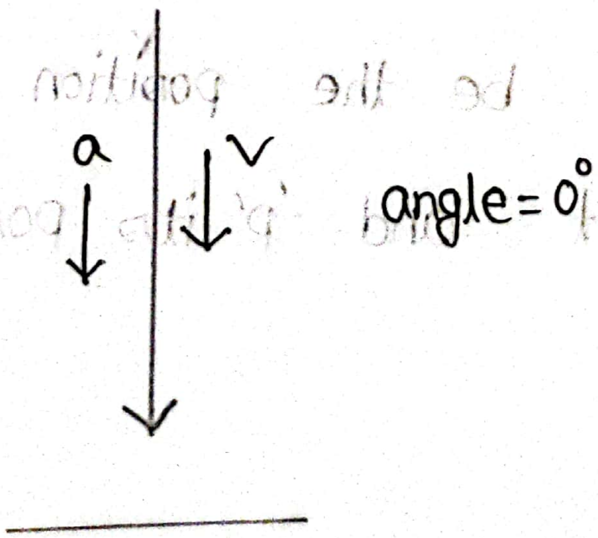
iii) Uniform Circular Motion:



iv) Fall of Projectile:



v) Ball thrown Down:



Uniform Circular Motion:

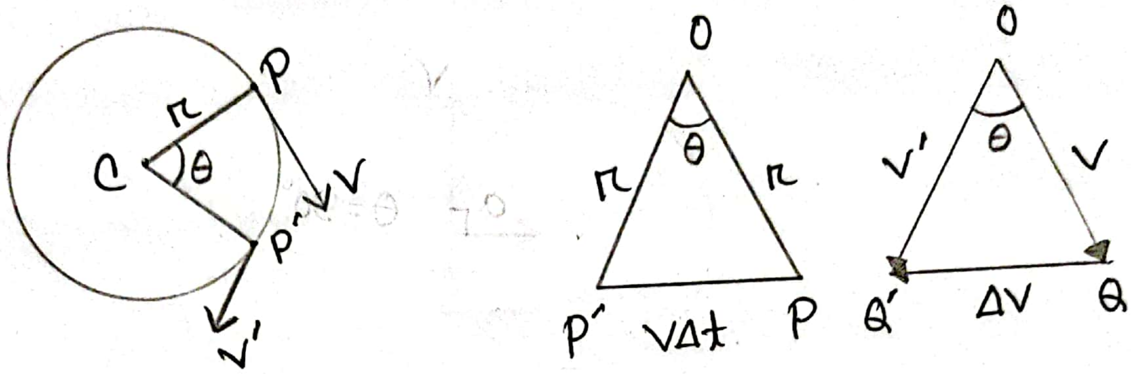


fig (a)

Fig: The Velocity at P and P' is shown and their change is Δv. if a particle moving in a ~~circular~~ circle with constant speed called Uniform motion.

The velocity vector changes continuously is direction but not in magnitude.

Let P be the position of the particle at time t and P' its position at time t+Δt.

The velocity at P is v, a vector tangent to the curve at P.

The velocity at p′ is v′, a vector tangent to the curve at p′.

vector v and v′ are equal in magnitude the speed being constant but their directions are different. The length of part traversed during Δt is the length PP′, which is equal to $v\Delta t$.

Now from triangle OQQ′ -

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r} \quad \text{—— (i) (approximately)}$$

or, $$\frac{\Delta v}{v\Delta t} = \frac{v}{r}$$

or, $$\frac{\Delta v}{\Delta t} = \frac{v^2}{r} \quad \text{—— (ii)}$$

and in the limit when $\Delta t \to 0$, this expression become exact,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \quad \text{--- (3)}$$

$\therefore$ from (ii) and (3) we get -

$$a = \frac{v^2}{r}$$

$$* \quad a = \frac{v^2}{r};$$

Centripetal Force, $F = ma$

$$= m \cdot \frac{v^2}{r}$$

$$= \frac{mv^2}{r}$$

$$* \quad F = \frac{mv^2}{r};$$

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Problem:

1. The moon revolves about the earth making a complete revolution in 27.3 days.

Assume that the orbit is circular and has a radius of 239,000 miles.

What is the magnitude of the accelaration of the moon towards the earth?

Sol^n: Here, $r = 239.000$ ~~mi~~ miles $\boxed{1 \text{ mile} = 1609 \text{ m}}$

$= 384551000$ m.

$T = 27.3$ days $\quad 1 \text{ day} = 24 \times 60 \times 60 = 86400$ s

$= 27.3 \times 86400$

$= 2.36 \times 10^6$ s

again, $S = vt$

$\therefore v = \frac{S}{T} = \frac{2.41 \times 10^8}{2.36 \times 10^6}$

$= 102.11$ m/s

$S = 2\pi r$

$= 2 \times \pi \times 3.85 \times 10^8$

$= 2.41 \times 10^9$ m

$\therefore a = \frac{v^2}{r}$

$= \frac{(102.11)^2}{384551000}$

$= 2.71 \times 10^{-5}$ m/s^2

2. Calculate the speed of an artificial earth satelite, assumming that it is travelling at an altitude h of 120 Km above the surface of the earth where, $g = 9.8\ m/s^2$.

The Radius of earth R is 6400 Km.

Soln:

Here, $h = 120\ Km$, $R+h = 6520\ Km$

$R = 6400\ Km$ $= 6520000\ m$.

$g = 9.8\ m/s^2 \longrightarrow a$

So, $g = \frac{v^2}{R+h}$

or, $g = \frac{v^2}{(R+h)}$

or, $v = \sqrt{g(R+h)}$

$= 7993.5\ ms^{-1}$