8.4. When both the Source and the Observer are in Motion

When the source moves towards the observer and the observer moves away from the source. Suppose a source S is producing a sound of pitch n and wavelength λ. The velocity of sound is v (Fig. 8.5). The velocity of the source is a and the velocity of the observer is b.

Let the source move towards the observer with a velocity a and the observer move away from the source with a velocity b.

The apparent wavelength

$$\lambda' = \left(\frac{v-a}{n}\right).$$

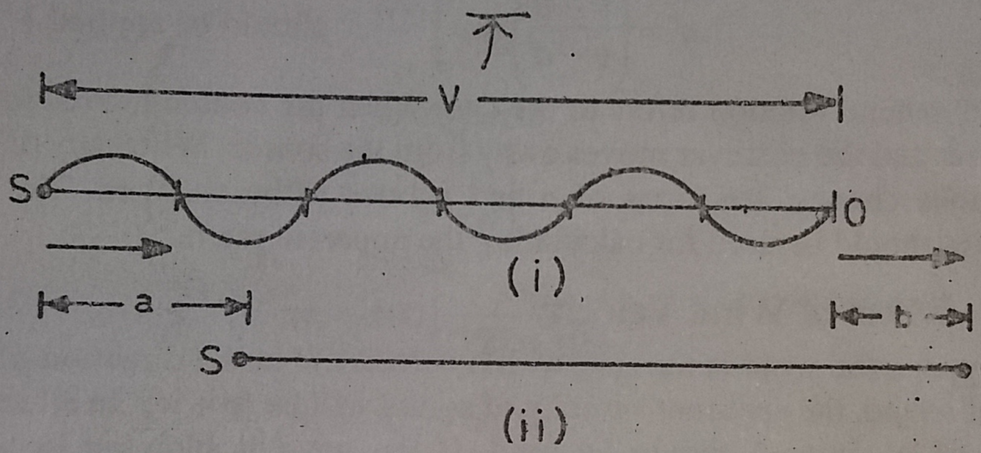

Fig. 8.5.

and
$$n' = \left(\frac{v-b}{\lambda'}\right)$$

$\therefore$
$$n' = \left(\frac{v-b}{v-a}\right)n \qquad \ldots (1)$$

Special cases

(a) When the source and the observer move towards each other

In equation (1), taking b to be negative,

$$n' = \left[\frac{v-(-b)}{v-a}\right]n$$

$$n' = \left(\frac{v+b}{v-a}\right)n \qquad \ldots (2)$$

(b) When the source and observer move away from each other.

In equation (1), taking a to be negative

$$n' = \left[\frac{v-b}{v-(-a)}\right]n$$

$$n' = \left(\frac{v-b}{v+a}\right)n \qquad \ldots (3)$$

(*c*) **Source moving away from the observer and the observer moving towards the source**

In equation (1), taking both a and b negative,

$$n' = \left[\frac{v-(-b)}{v-(-a)}\right]n$$

$$n' = \left(\frac{v+b}{v+a}\right)n \qquad \ldots (4)$$

[**Note.** While solving numerical problems, the general formula

$$n' = \left(\frac{v-b}{v-a}\right)n \qquad \text{should be applied.]}$$

The general relation refers to the case when the source moves towards the observer and the observer moves away from the source. When any of these two directions change, the signs of a and b have to be changed. The modified formula should be used for calculating the apparent pitch.

8.5. Effect of Wind Velocity

Suppose the wind is moving with a velocity w in the direction of propagation of sound, the apparent velocity of sound will be $(v+w)$. In all relations, in place of v, $(v+w)$ should be used. If the wind is blowing in a direction opposite to the direction of propagation of sound, the apparent velocity of sound will be $(v-w)$. In all relations, in place of v, $(v-w)$ should be used.

The general relation will be

$$n' = \left[\frac{(v+w)-b}{(v+w)-a}\right]n \qquad \ldots (5)$$

Here the wind direction is the same as the direction of propagation of sound.

When the direction of wind is opposite to the direction of propagation of sound,

$$n' = \left[\frac{(v-w)-b}{(v-w)-a}\right]n \qquad \ldots (6)$$

8.6. Doppler Effect in Light

There is a change in the frequency of light radiation when the source or the observer move with respect to one another. This phenomenon is known as Doppler effect. It is similar to the apparent change in the pitch of sound when either the source or the observer is in motion with respect to one another.

According to the theory of relativity, a material medium is not necessary for the propagation of light waves whereas for sound waves material medium is a necessity. In sound, the Doppler effect is asymmetric *i.e.*, the apparent frequen-

(*v*) When the source and the observer move away from each other and each is moving with a velocity v.

$$\nu' = \nu\left[1 - \frac{v}{c}\right]^2 \qquad \ldots (5)$$

The phenomenon of Doppler effect is of great importance in light. It shows that there is a shift in the frequency when the source of light moves with a velocity comparable to the velocity of light. In the field of astronomy, Doppler effect has been found to be very useful.

Applications

(1) **Velocity and rotation of the sun.** By the study of Doppler shift from the light received from the western and eastern edges of the sun, it has been found that the shift is due to a velocity of about 2 km/s. Moreover, no such shift has been observed from the light received from the north and the south edges. This shows that the sun rotates about the north-south axis.

(2) **Discovery of double stars.** By the constant observation of the sky, it has been found that some of the stars that appear to be single are actually double stars and are known as spectroscopic binaries. These stars revolve about each other. When one is approaching the earth, and the other is going away from the earth, there is a shift in their spectral lines and a single spectral line is split up into two lines whose separation depends upon time and the time period is equal to the time period of revolution of the stars. By this method a number of double-star systems have been found.

(3) **Red shift.** It has been observed that some distant nebulae are moving away with a velocity greater than 20×10^3 km/s and the important spectral lines appear to shift towards the red end of the spectrum by 200 Å. This gives the idea that the universe is expanding.

(4) **Saturn's rings.** The planet Saturn has been found to be surrounded by concentric rings. With the help of Doppler effect it has been found that these rings are not solids but consist of a number of 'satellites' moving around the Saturn in these orbits. If the rings were solids, the outer edge of the ring should have greater velocity than the inner edge. But according to the principle of a satellite

$$\frac{mV^2}{R} = \frac{GMm}{R^2} \quad \text{or} \quad V^2 = \frac{GM}{R}$$

i.e., the velocity of the satellite in the inner orbit is more than that in the outer orbit. This fact has been established by the Doppler shift. Thus, the rings of the Saturn are not solids but there are a large number of satellites moving in these orbits called the rings of Saturn.

Example 8.1. *A motor car sounding a horn at a frequency of 100* hertz *moves away from a stationary observer towards a rigid flat wall with a velocity* *of 36* km/hr. *How many beats per second will be heard by the observer? The velocity of sound in air at room temperature* = *350* m/s.

Here $V = 350$ m/s

$a = 36$ km/hr $= 10$ m/s

$n = 100$ hertz

Here the observer receives sound of apparent frequency n_1 from the moving source directly and also sound of apparent frequency n_2 from the sound waves reflected from the wall.

$$n_1 = n\left[\frac{V}{V+a}\right]$$

$$n_1 = 100\left[\frac{350}{350+10}\right]$$

$$n_1 = 97{\cdot}23 \text{ hertz}$$

$$n_2 = n\left[\frac{V}{V-a}\right]$$

$$n_2 = 100\left[\frac{350}{350-10}\right]$$

$$n_2 = 102{\cdot}9 \text{ hertz}$$

Number of beats/s

$$n_2 - n_1 = 102{\cdot}9 - 97{\cdot}23 = \mathbf{5{\cdot}67}.$$

Example 8.2 *The apparent frequency of the whistle of an engine changes in the ratio 6 : 5 as the engine passes a stationary observer. If the velocity of sound is 352* m/s, *calculate the velocity of the engine.* (Delhi, 1994)

(*i*) When the source moves towards the stationary observer,

$$n_1 = \left(\frac{v}{v-a}\right)n \qquad \ldots (1)$$

(*ii*) When the source moves away from the stationary observer,

$$n_2 = \left(\frac{v}{v+\alpha}\right)n \qquad \ldots (2)$$

Dividing (1) by (2)

$$\frac{n_1}{n_2} = \frac{v+a}{v-a}$$

Here $\frac{n_1}{n_2} = \frac{6}{5}$ (given)

$$v = 352 \text{ m/s}$$

$$a = ?$$

$$\therefore \quad \frac{6}{5} = \frac{352 + a}{352 - a} \quad \text{or} \quad a = \mathbf{32 \text{ m/s.}}$$

Hence the velocity of the engine is 32 m/s.

Example 8.3. *Two trains travelling in opposite directions at 100* km/hr *each, cross each other while one of them is whistling. If the frequency of the note is 800* Hz *find the apparent pitch as heard by an observer in the other train:*

(a) before the trains cross each other

(b) after the trains have crossed each other.

Velcocity of sound in air =340 m/s. [Delhi (Sub.), 1986]

Here, $V = 340 \text{ m/s.}$

$$a = 100 \text{ km/hr} = \frac{250}{9} \text{ m/s}$$

$$b = 100 \text{ km/hr} = \frac{250}{9} \text{ m/s}$$

$$n = 800 \text{ Hz}$$

(a) In the first case, the source and the observer are moving towards each other. Therefore,

$$n' = n\left[\frac{V + b}{V - a}\right]$$

$$n' = 800\left[\frac{340 + \left(\frac{250}{9}\right)}{340 - \left(\frac{250}{9}\right)}\right]$$

$$n' = 800\left(\frac{3310}{2810}\right)$$

$$n' = \mathbf{942{\cdot}3 \text{ Hz.}}$$

(b) In the second case, the source and the observer are moving away from each other.

$$n' = n\left[\frac{V - b}{V + a}\right]$$

$$n' = 800\left[\frac{340 + \left(\frac{250}{9}\right)}{340 - \left(\frac{250}{9}\right)}\right]$$

$$n' = 800\left(\frac{2810}{3310}\right)$$

$$n' = \mathbf{679{\cdot}2\ Hz.}$$

Example 8.4. *An observer on a railway platform noticed that when a train passed through the station, at a speed of 72* km/hr, *the frequency of the whistle appeared to drop by 500* Hz. *Calculate the actual frequency of the note given by the whistle. Velocity of sound in air = 340* m/s. [Delhi, 1986]

Here, $V = 340$ m/s

$a = 72$ km/hr $= 20$ m/s

$n = ?$

$n_1 - n_2 = 500$

Here n_1 is the apparent pitch when the train is approaching the observer and n_2 is the apparent pitch when the train is moving away from the observer.

$$\therefore \quad n_1 = n\left[\frac{V}{V-a}\right]$$

and
$$n_2 = n\left[\frac{V}{V+a}\right]$$

$$\therefore \quad n_1 - n_2 = n\left[\frac{V}{V-a}\right] - n\left[\frac{V}{V+a}\right]$$

$$n_1 - n_2 = n\left[\left(\frac{V}{V-a}\right) - \left(\frac{V}{V+a}\right)\right]$$

$$500 = n\left[\left(\frac{340}{340-20}\right) - \left(\frac{340}{340+20}\right)\right]$$

or
$$n = \frac{500}{\left(\frac{340}{320}\right) - \left(\frac{340}{360}\right)}$$

$$n = \mathbf{4235{\cdot}3\ Hz.}$$

Example 8.5. *A person is standing on a railways platform. An engine while approaching the platform blows a whistle of pitch 660* hertz. *The speed of the engine is 72* km/hr, *velocity of sound = 350* m/s. *Calculate the apparent pitch of the whistle as heard by the person.*

Here $\because V = 350$ m/s

$a = 72$ km/hr $= 20$ m/s

$n = 660$ hertz.

The source is moving ~~away from~~ towards a stationary observer.

$$\therefore \qquad n' = \left(\frac{V}{V-a}\right)n$$

$$n' = \left(\frac{350}{350-20}\right) \times 660$$

$$n' = \mathbf{700\ hertz.}$$

Example 8.6. *A person is standing on a platform. A railway engine moving away from the person with a speed of 72* km/hr *blows a whistle of pitch 740* hertz. *Calculate the apparent pitch of the whistle as heard by the person. The velocity of sound = 350* m/s.

Here $\qquad V = 350$ m/s

$a = 72$ km/hr $= 20$ m/s

$n = 740$ hertz

The source is moving away from a stationary observer.

$$\therefore \qquad n' = \left(\frac{V}{V+a}\right)n$$

$$n' = \left(\frac{350}{350+20}\right) \times 740$$

$$n' = \mathbf{700\ hertz.}$$

Example 8.7. *A person is standing near a railway track and a train moving with a speed of 36*·km/hr *is approaching him. The apparent pitch of the whistle as heard by the person is 700* hertz. *Calculate the actual frequency of the whistle. Velocity of sound = 350* m/s.

Here, $\qquad V = 350$ m/s

$a = 36$ km/hr $= 10$ m/s

$n' =$ **700 hertz**

$n = ?$

The source is moving towards a stationary observer

$$\therefore \qquad n' = \left(\frac{V}{V-a}\right)n$$

$$n = n'\left(\frac{V-a}{V}\right)$$

$$n = 700\left(\frac{350-10}{350}\right);$$

$$n = \mathbf{680\ hertz.}$$

Example 8.8. *Two aeroplanes A and B are approaching each other with a speed of 360* km/hr. *The frequency of the whistle emitted by A is 1000* hertz.

Calculate the apparent pitch of the whistle as heard by the passengers of aeroplane B. Velocity of sound in air = 350 m/s.

Here,
$$V = 350 \text{ m/s}$$
$$a = 360 \text{ km/hr} = 100 \text{ m/s}$$
$$b = 360 \text{ km/hr} = 100 \text{ m/s}$$
$$n = 1000 \text{ hertz}$$

The source and the observer are moving towards each other

$$\therefore \quad n' = n\left[\frac{V+b}{V-a}\right]$$

$$\therefore \quad n' = 1000\left(\frac{350+100}{350-100}\right); \qquad n' = \mathbf{1800 \text{ hertz.}}$$

2

Example 8.9. *Two aeroplanes A and B are moving away from one another with a speed of 720* km/hr. *The frequency of the whistle emitted by A is 1100* hertz. *Calculate the apparent frequency of the whistle as heard by the passengers of the aeroplane B. Velocity of sound in air = 350* m/s.

Here,
$$V = 350 \text{ m/s}$$
$$a = 720 \text{ km/hr} = 200 \text{ m/s}$$
$$b = 720 \text{ km/hr} = 200 \text{ m/s}$$
$$n = 1100 \text{ hertz}$$

The source and the observer are moving away from one another

$$\therefore \quad n' = n\left(\frac{V-b}{V+a}\right)$$

$$n' = 1100\left(\frac{350-200}{350+200}\right); \qquad n' = \mathbf{300 \text{ hertz.}}$$

Example 8.10. *Two aeroplanes A and B are approaching each other and their velocities are 108* km/hr *and 144* km/hr *respectively. The frequency of a note emitted by A as heard by the passengers in B is 1170* hertz. *Calculate the frequency of the note heard by the passengers in A. Velocity of sound = 350* m/s.

(Delhi, 1992)

Here,
$$V = 350 \text{ m/s}$$
$$a = 108 \text{ km/hr} = 30 \text{ m/s}$$
$$b = 144 \text{ km /hr} = 40 \text{ m/s}$$
$$n' = 1170 \text{ hertz}$$
$$n = ?$$

The source and the observer are approaching each other.

$$\therefore \quad n' = n\left(\frac{V+b}{V-a}\right)$$

$$1170 = n\left(\frac{350+40}{350-30}\right)$$

$$n = \frac{1170 \times 320}{390};$$

$$n = \mathbf{960\ hertz.}$$

Example 8.11. *Two aeroplanes pass each other in opposite directions and one of them is blowing a whistle of frequency 540* hertz. *Calculate the frequencies of the notes heard in the other aeroplane (i) before and (ii) after they have passed each other. Velocity of either of the aeroplanes is 540* km-hr^{-1} *and the velocity of sound = 350* ms^{-1}. (Delhi, 1991)

Here,
$$V = 350 \text{ m/s}$$
$$a = 540 \text{ km/hr} = 150 \text{ m/s}$$
$$b = 540 \text{ km/hr} = 150 \text{ m/s}$$
$$n = 540 \text{ hertz}$$

(*i*) In the first case, the source and the observer are moving towards each other. Therefore,

$$n' = n\left[\frac{V+b}{V-a}\right]$$

$$n' = 540\left[\frac{350+150}{350-150}\right];$$

$$n' = \mathbf{1350\ hertz.}$$

(*ii*) In the second case, the source and the observer are moving away from each other.

$$n' = n\left[\frac{V-b}{V+a}\right]$$

$$n' = 540\left[\frac{350-150}{350+150}\right];$$

$$n' = \mathbf{216\ hertz.}$$

Example 8.12. *An observer on a railway platform observed that as a train passed through the station at 90* km/hr, *the frequency of the whistle appeared to drop by 400* hertz. *Find the frequency of the whistle. Velocity of sound in air = 350* m/s. (Bombay, 1991)

Here,
$$V = 350 \text{ m/s}$$
$$a = 90 \text{ km/hr} = 25 \text{ m/s}$$
$$n = ?$$
$$n_1 - n_2 = 400$$

Here n_1 is the apparent pitch when the train is approaching the observer and n_2 is the apparent pitch when the train is moving away from the observer.

$$\therefore \quad n_1 = n\left[\frac{V}{V-a}\right] \text{ and } n_2 = n\left[\frac{V}{V+a}\right]$$

$$\therefore \quad n_1 - n_2 = n\left[\frac{V}{V-a}\right] - n\left[\frac{V}{V+a}\right]$$

$$n_1 - n_2 = n\left[\left(\frac{V}{V-a}\right) - \left(\frac{V}{V+a}\right)\right]$$

$$\therefore \quad 400 = n\left[\left(\frac{350}{350-25}\right) - \left(\frac{350}{350+25}\right)\right]$$

$$n = \frac{400}{\left(\frac{350}{325}\right) - \left(\frac{350}{375}\right)}$$

$$n = \mathbf{2785{\cdot}7 \text{ hertz.}}$$

Example 8.13. *The frequency of the horn of a car is observed to drop from 272 hertz to 256 hertz as the car passes a stationary observer. What is the speed of the car. Velocity of sound in air = 346·5* m/s.

Here, $v = 346{\cdot}5$ m/s

$$\frac{n_1}{n_2} = \frac{272}{256} = \frac{17}{16}$$

$$a = ?$$

(*i*) When the car approaches a stationary observer, the apparent pitch

$$n_1 = \left(\frac{v}{v-a}\right) n \qquad \ldots (1)$$

(*ii*) When the car moves away from the stationary observer, the apparent pitch

$$n_2 = \left(\frac{v}{v+a}\right) n \qquad \ldots (2)$$

Dividing (*i*) by (*ii*)

$$\frac{n_1}{n_2} = \frac{v+a}{v-a}$$

or

$$\frac{17}{16} = \frac{346{\cdot}5 + a}{346{\cdot}5 - a}$$

$$a = \mathbf{10{\cdot}5 \text{ m/s}}$$

The speed of the car is 10·5 m/s.