
Problems: With what velocity should a body be projected vertically upwards from the surface of the earth so that it may just attain a height of $R/2$. Where R is the radius of the earth. ($R = 6400$ km).

Sol^n: Work done in moving the body from the surface of the earth to a height $R/2$ above the earth's surface

$$W = \int_R^{(R+R/2)=3R/2} \left(\frac{GMm}{x^2}\right) dx$$

$$= GMm \left[-\frac{1}{x}\right]_R^{3R/2}$$

$$= -GMm\left[\frac{1}{3R/2} - \frac{1}{R}\right]$$

$$\therefore W = \frac{GMm}{3R}$$

$$K.E = \frac{1}{2}mv^2$$

$$\text{or, } \frac{1}{2}mv^2 = \frac{GMm}{3R} \quad \text{--- (1)}$$

At the surface of the earth

$$mg = \frac{GMm}{R^2} \quad \text{--- (2)}$$

Deviding (1) ÷ (2)

$$\frac{v^2}{2g} = \frac{R}{3}$$

$$v = \sqrt{\frac{2gR}{3}}$$

$$= \sqrt{\frac{2\times 9.8\times 6.4\times 10^{4}}{3}}$$

$= 6.467$ km/s.

The mass and satellite of Jupitar are 1.9×10^{27} kg and 7×10^{7} m respectively. Calculate the escape velocity.

We know:

$$V_e = \sqrt{2gR}$$

$$= \sqrt{2\times \frac{GM}{R^2}\times R}$$

$$= \sqrt{2\frac{GM}{R}}$$

$$= \left[\frac{2\times 6.67\times 10^{-11}\times 1.9\times 10^{27}}{7\times 10^{7}}\right]^{\frac{1}{2}}$$

$= 6.03\times 10^{4}$ ms^{-1}.

Here,

$G = 6.67\times 10^{-11}$ Nm^{2}kg^{-2}

M = Mass of jupiter $= 1.9\times 10^{27}$ kg

R = radius of the earth $= 7\times 10^{7}$ m.

At what height above the surface of the earth the value of acceleration due to gravity will be 1% of it's value at the earth's surface. [$R = 6.4 \times 10^3$ km]

We know,

$$g' = g\left(\frac{R}{R+h}\right)^2$$

But $g' = \frac{1}{100} g$

$$\therefore \frac{g}{100} = g\left(\frac{R}{R+h}\right)^2$$

or, $$\frac{1}{100} = \left(\frac{R}{R+h}\right)^2$$

or, $$\frac{1}{10} = \frac{R}{R+h}$$

or, $$R+h = 10R$$

or, $$h = 9R = 9 \times 6.4 \times 10^3 \text{ km}$$
$$= 5.76 \times 10^4 \text{ km}.$$

Ans: 5.76×10^4 km

4. An artificial satellite circles of the earth at a distance of 4000 km.

(a) Calculate its orbital velocity

and (b) Time Period of revolution.

[Radius of the earth = 6400 Km ; $g = 9.8 \, ms^{-2}$]

Sol^n:

orbital velocity $v = \sqrt{\frac{gR^2}{(R+h)}}$

$\left[g = \frac{GM}{R^2} \text{ and } \frac{v^2}{(R+h)} = g \right]$

Here, R = radius of earth.

$= 6400 \text{ km} = 64 \times 10^5 \text{ m}$

$\left[a = \frac{v^2}{R} \right]$

h = distance = 4000 km

$g = \frac{v^2}{R+h}$

$= 40 \times 10^5 \text{ m}.$

$\therefore R+h = 104 \times 10^5 \text{ m}$

$\therefore v = \sqrt{\frac{9.8 \, ms^{-2} (64 \times 10^5 \text{ m})^2}{104 \times 10^5 \text{ m}}}$

$= \sqrt{\frac{9.8 \times 64 \times 64 \times 10^5 \times 10^5}{104 \times 10^5}}$

$= 6.212 \times 10^3 \, ms^{-1}.$

$\omega = \frac{2\pi}{T}$

or, $T = \frac{2\pi}{\omega}$, $\omega = \frac{v}{r}$

[Time Period of Sattelite

$T = \frac{2\pi(R+h)}{v}$

$v = \sqrt{\frac{GM}{(R+h)}}$

Time Period,

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$= 2\pi\sqrt{\frac{(104\times10^5)^3}{9.8\times(64\times10^5)^2}}$$

$$= 10512 \text{ s}$$

$$= 175.21 \text{ min.}$$

Ans: (a) $6.212\ kms^{-1}$ (b) 175.21 min.

Prove that the least velocity with which a Particle must be Projected from the surface of a planet of radius R and density ρ in order that it may escape completely is

$$\sqrt{\frac{8\pi G\rho R^2}{3}}.$$

Q7 Calculate the limiting velocity required by an artificial satellite for orbiting round the earth. Given that the radius of earth $= 6.4 \times 10^6$ m and $g = 9.8\ ms^{-2}$.

Here, $v = \sqrt{gR}$ $\left[g = \frac{v^2}{R} \right]$

$g = 9.8\ ms^{-2}$, $R = 6.4 \times 10^6$ m.

$v = \sqrt{9.8 \times 6.4 \times 106}$

$v = 7.92 \times 10^3$ m/s

$v = 7.92$ km/s.