PEM Fuel Cell system analysis

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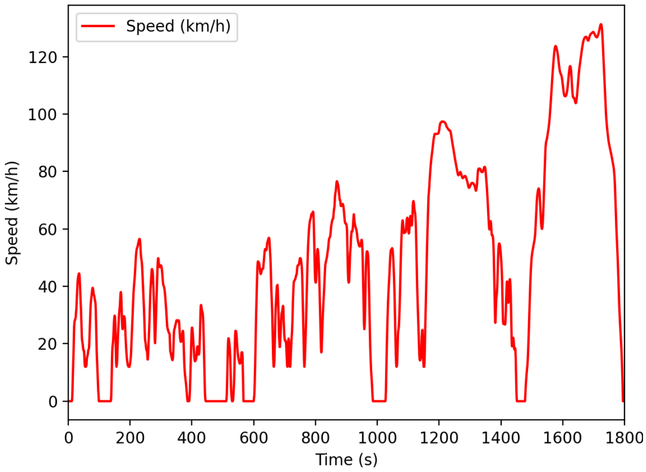
Contents

1 Calculation of the power demand inside the vechicle

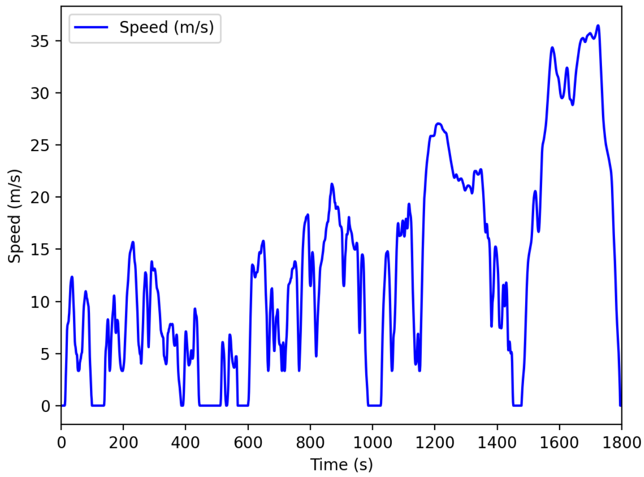
- The specification of the vehicle are the following:
  - Weight $M = 2000kg$
  - Front area $A = 2.25m^2$
  - Drag coefficient (or air penetration coefficient) $C = 0.29$
  - Rolling Resistance coefficient $C_r = 0.0115$
- The efforts applied on the vehicle in the rolling direction have to following expression:
  - Air penetration :
    $$F(t) = \frac{1}{2}\rho_{air} v_{(t)}^2 CA \tag{1.1}$$
    with $\rho_{air} = 1.2kg/m^3$
  - Rolling resistance :
    $$F(t) = MgC_r \cos\alpha \tag{1.2}$$
    with $g = 9.81m/s^2$ and α the slope angle
  - Climbing or descent :
    $$F(t) = Mg \sin\alpha \tag{1.3}$$

1.1 Calculate and plot the instant power provided by the vechicle powertrain for the road cycles " WLTC "

⋆ Consider a flat road ($\alpha = 0$)



(a) Speed Profile in km/h



(b) Speed Profile in m/s

Figure 1: Graph of the Speed Profile in km/h and Speed in m/s

To calculate the **Instant Power**, we need to study of the *force* that have action on the car. By using second Newton's law with the Figure (2) shown below, we can assume that there are 4 forces that have action on thecar while driving.

- The first force is to make the car move in direction. It called the force from motor or machine of the car (F_{motor})
- The second force is the rolling force from the car wheels. It called **Rolling Resistance** ($F_{rolling}$)

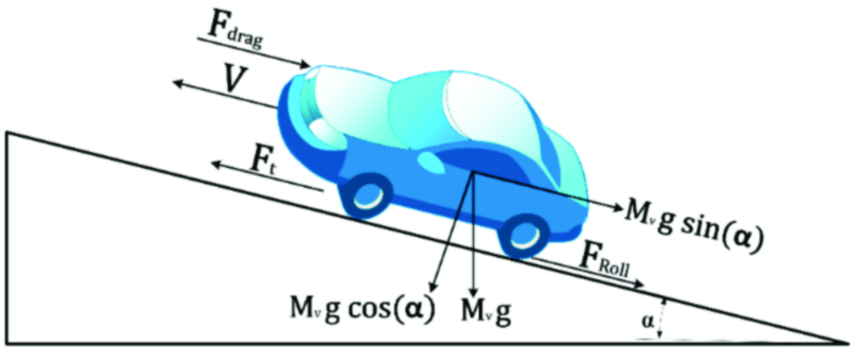


Figure 2: Speed (km/h) of the vechicle powertrain by time (s)

- The third force is the climbing or descent force (F_{climb})
- The fourth force is the force from the air friction. we can called it Air penetration (F_{air}).

Using second Newton's law, we can written :

$$\overrightarrow{F}_{motor} - \overrightarrow{F}_{rolling} - \overrightarrow{F}_{climb} - \overrightarrow{F}_{air} = m\overrightarrow{a} \tag{1.4}$$

$$\overrightarrow{F}_{motor} = \overrightarrow{F}_{rolling} + \overrightarrow{F}_{climb} + \overrightarrow{F}_{air} + m\overrightarrow{a} \tag{1.5}$$

since a is the acceleration of the vechical in time t, as we written :

$$\overrightarrow{a} = \frac{\mathrm{d}\overrightarrow{v}}{\mathrm{d}t} = \frac{\Delta v}{\Delta t} \tag{1.6}$$

By using Equation (1.6), we can get the result of acceleration on the Figure (3)

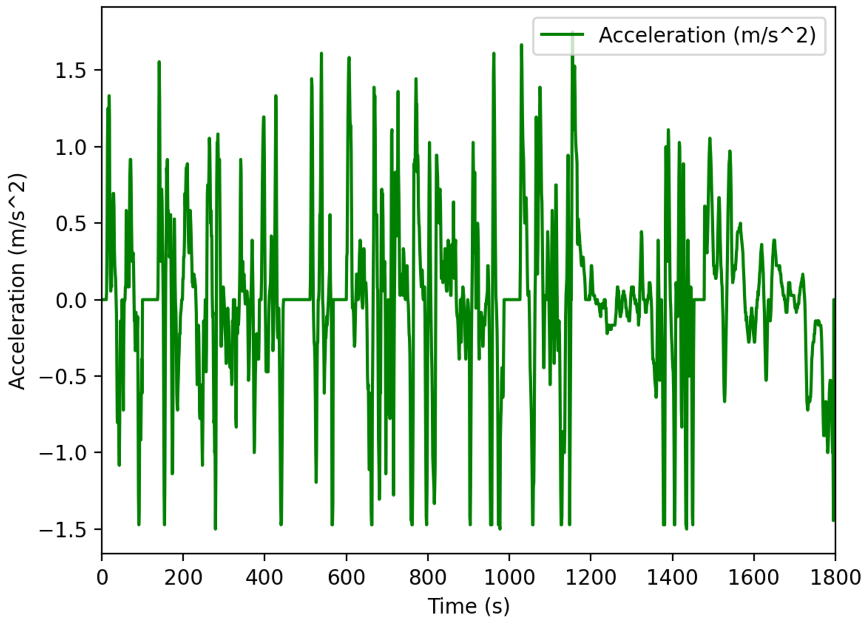


Figure 3: Acceleration (m/s^2) of the vechicle powertrain by time (s)

According to the graph, it shown that the vechical did not have the stable speed drive on the. At the some time (t), the vechical increasing the speed immediately. In contrast, at some time (t), the vechical reducing the speed quickly as shown in the Figure (3).

- For calculate F_{air} by using Equation (1.1) , we got :

  F$_{air}(t) = \frac{1}{2}\rho_{air}v^2CA = \frac{1}{2} \times 1.2 \times v^2_{m/s,t} \times 0.29 \times 2.25$(1.7)In this section, to calculate F_{air} we need to get the speed in each time t in m/s^2 to analyze in the Equation (1.7) By using the Equation (1.1), we got the result of the force air penetration by shown in below graph.

- For calculate $F_{rolling}$, we will be using the Equation (1.2), we got :

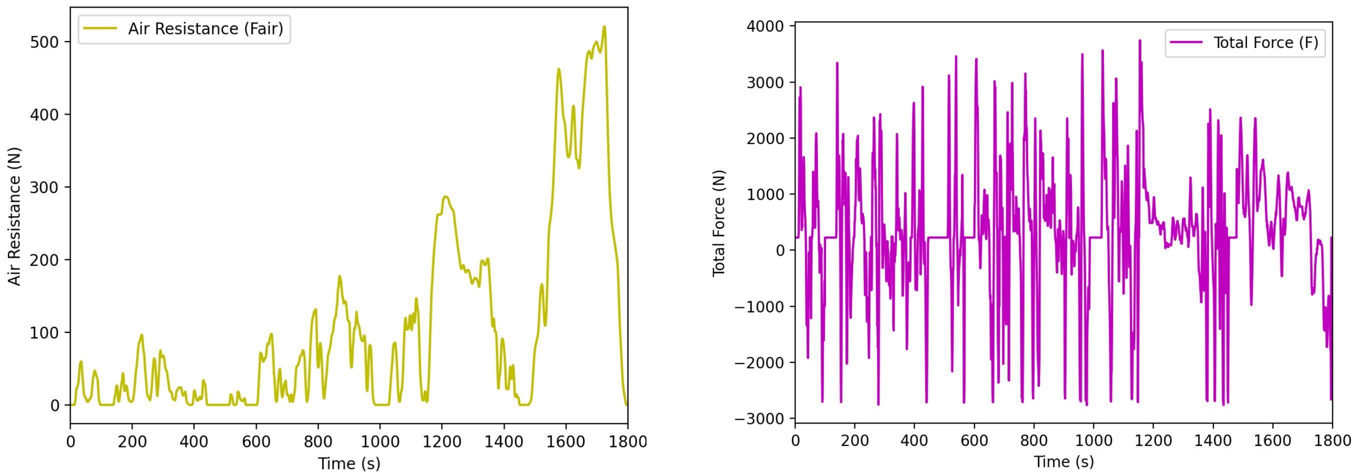
$$F_{rolling}(t) = MgC_r\cos(\alpha) = 2000kg \times 9.81m/s^2 \times 0.0115 \times \cos(0°) = \mathbf{225.630} \tag{1.8}$$

  For this force, it will be constant in time (t) becuase there is not any parameter in the Equation (1.8) will change in which time.

- For calculate F_{climb}, we will use Equation (1.4) then we got:

$$F_{climb}(t) = Mg\sin(\alpha) = 2000kg \times 9.81m/s^2 \times \sin(0°) = 0 \tag{1.9}$$

By using Equation (1.6), (1.7), (1.8) and (1.9) substitution into Equation (1.5). The result of total force was shown by the graph in Figure (4).



(a) Force of Air Resistance (b) Total Force

Figure 4: Graph of the Force of Air Resistance and Total Force

To calculate Instant power we are using :

$$P = F \times v \tag{1.10}$$

The result of the instant power calcuation will be show at Figure (5)b. The instant power of the vechical are depend on two parameter :

- The total force from the vechical action (N).
- The speed that make the vechical go forward (m/s).

As now, we can write that

$$P_t = F_t \times v_t \tag{1.11}$$

The result of the instant power will show in the Figure (5).

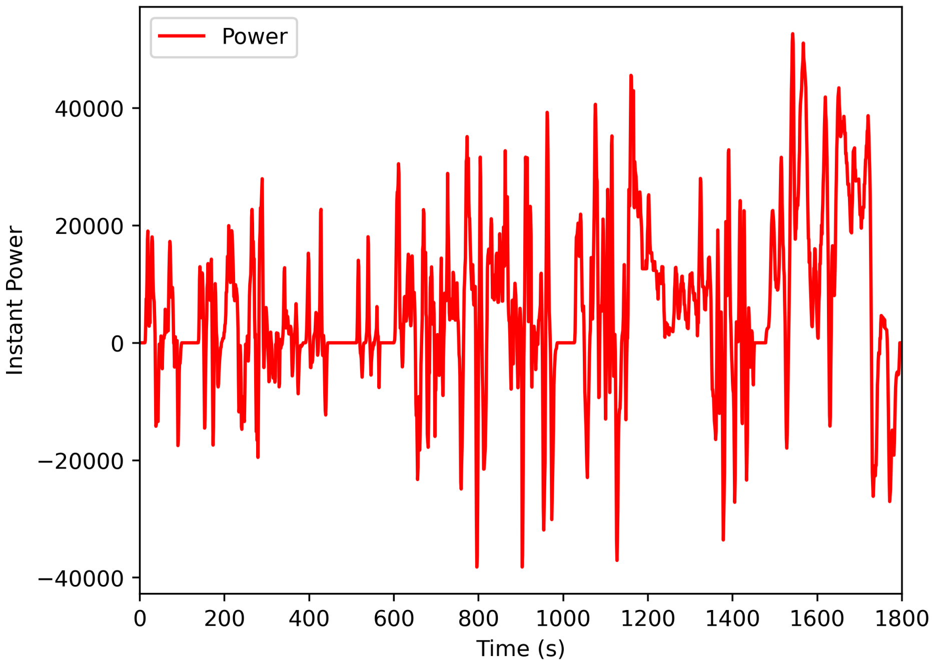


Figure 5: Instant Power of the Vechicle (W)

1.2 Calculate the instant power provide (positive) or received (negative) by the electric hybrid power source.

The vehicle auxiliaries consume an electrical power of 300W (no air conditioning, minimum consumption of all the equipment of the vehicle: sensor, supervisor,etc.) The DC/DC converter efficiency is assumed constant at 90% both direction.

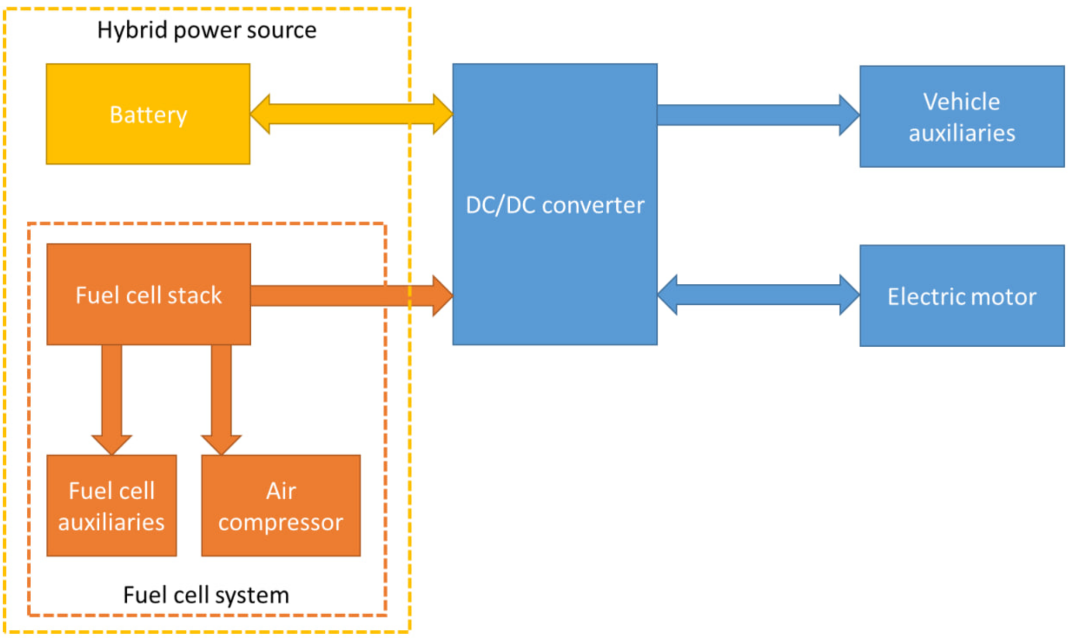


Figure 6: Hybrid system in the vehicle

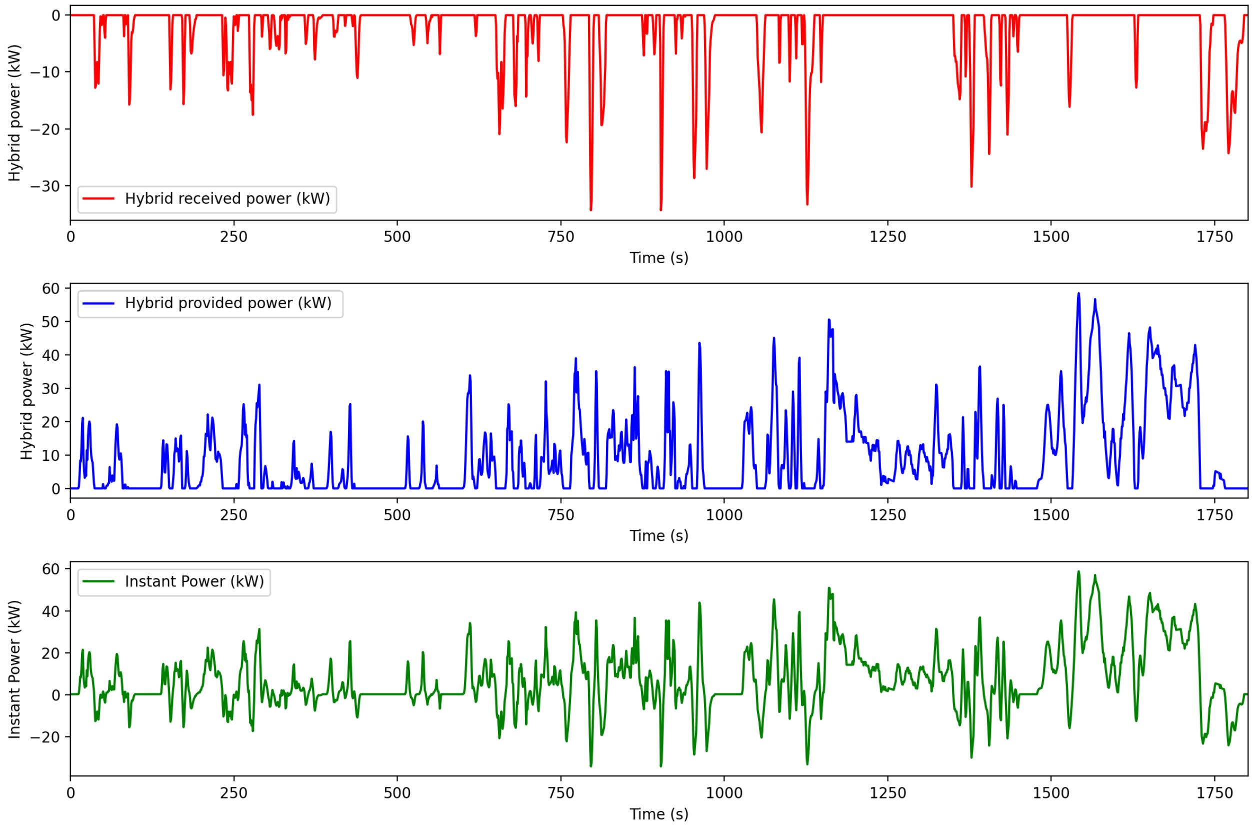


Figure 7: Hybrid working system in power transfer

To find the transfer power in the hybrid system in order to find the provided (positive) or recevied (negative), we have use instant power from pervious question as data in the Figure (5) with the efficieny of the DC/DC converter. As shown in the Figure (7): At the **first figure** had been show the **RED line graph** represented the received power (negative) to the hybrid power with the maximum received is **34.365 kW**. Moreover, as shown in the **second figure** shown the **Blue line graph** represented the provided power (positive) from the hybrid system to the motor and auxiliaries. The maximum provided power to the motor was around **58.488 kW**. According to data from the calculation, we can assumed that the vechicle mostly consum power from the hybrid and less provided power to hybrid system based on the data of speed that provided.

1.3 Calculate and Plot as a function of time : The power of the battery (kW), The power of the fuel cell system (kW), The SOC battery (%)

The energy management strategy of the hybridization between the battery and the fuel cell system is not disclosed by Tooyta.

- The battery technology is Li-ion, with a stored energy of **1.24kWh**.
- The test results of Mirai 1 indicate that the battery State of Charge (SoC) is comprised between **50%** and **65%**
- The power delivered by the battery is often close to **5%** of the total power provided by the hybrid power source when $SoC < 55\%$ or **30%** when $SoC > 55\%$
- Discharging power of the battery is approximately **12.4 kW** (or 10C), while the charging power depends on the battery SoC: **10C** if $SoC < 55\%$ or **6C** if $SoC > 55\%$
- The battery provides $0\%, 5\%, 30\%$ of the total power depending on its SoC and in the limit of its maximum discharging power.

- The EMS avoids values of SoC below 50% and above 65%
- The fuel cell system provides the rest of the power reuqired, excpet if the power demand is too low: the power provided by the fuel cell system can't be lower than **2.5kW**

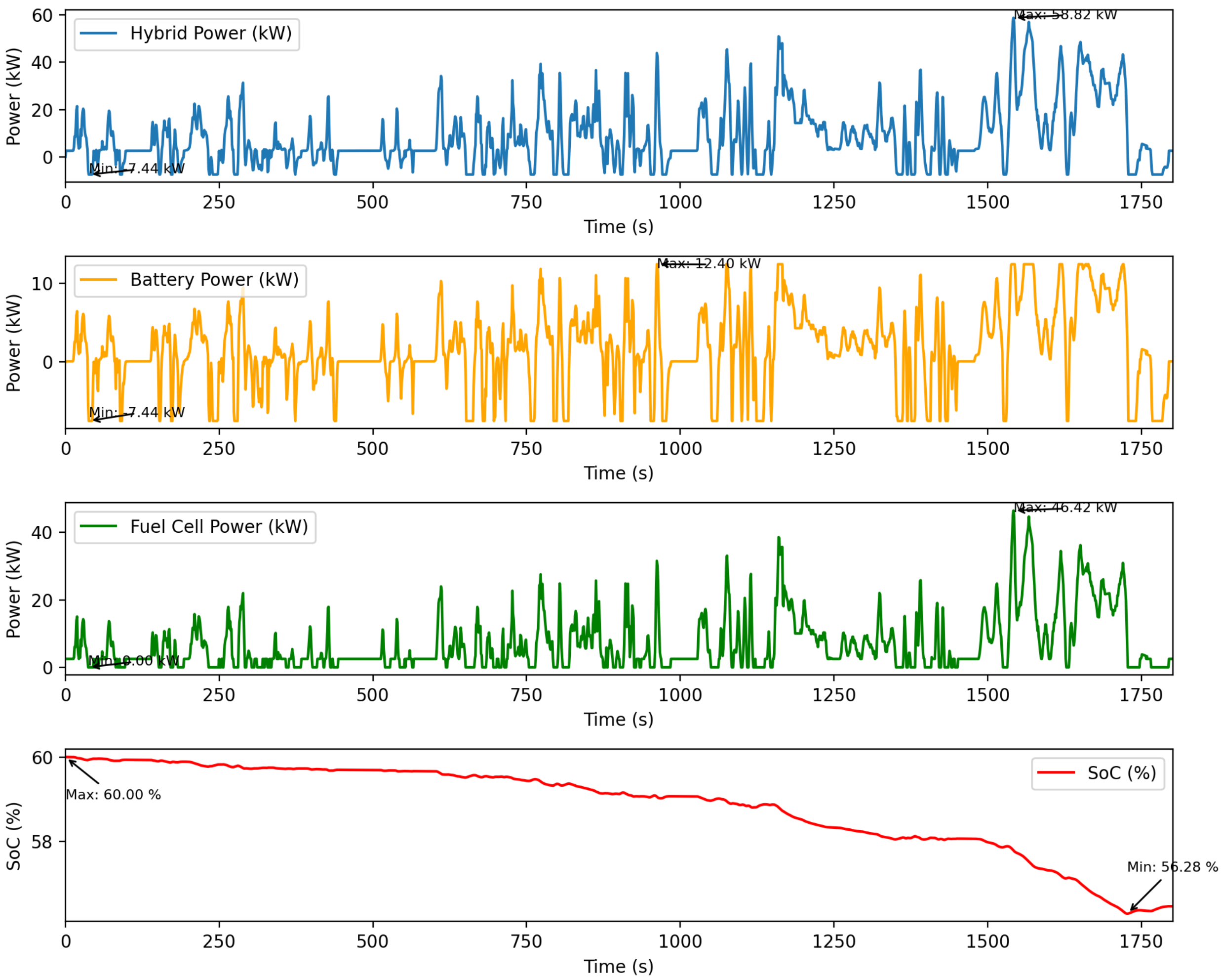


Figure 8: Hybrid system, Battery Management, Fuel Cell power and State of Charge in the Vehicle

As the result that shown in the Figure (8),

- The first plot of the Figure (8) (represented by the by blue curve) show about the **Hybrid Power** working in the system.
- The second plot of the Figure (8) (represented by the by yellow curve) show the characteristic of the charging and discharging of the Battery in the vehicle. The maximum power discharging is **12.4 kW** and the maximum power charging is **7.44 kW**. The status of charge and discharge ws depend on the time and speed per second.
- The third plot of the Figure (8) (represented by the by green curve) show the fuel cell consumption that will be use in the system. The maximum fuel cell consumption is **46.42 kW**. The consumption of the fuel cell variable depend on the time.
- The fourth plot in Figure (8) (represented by the by red curve) illustrates the State of Charge (SoC) of the battery system. By using formula to determine State of Charge in the system, we do it by :

$$\mathrm{SoC}_i = \min\left(\mathrm{SoC}_{\max}, \max\left(\mathrm{SoC}_{\min}, \mathrm{SoC}_{i-1} - \frac{\text{power bat}_i}{3600 \cdot \text{batcapacity}} + \frac{\text{demand}_i}{3600 \cdot \text{batcapacity}}\right)\right) \tag{1.12}$$

It starts with an initial SoC of approximately **60%** and gradually decreases to **56.28%** over time, depending on the driving characteristics. The SoC can fluctuate, increasing or decreasing based on the motor's operation and the dynamics of the hybrid system.

1.4 Make the same analysis with the road cycle "130 kmh". Consider two different cases : $\alpha = 0°$ and $\alpha = 2°$

1.4.1 Analysis $\alpha = 0°$

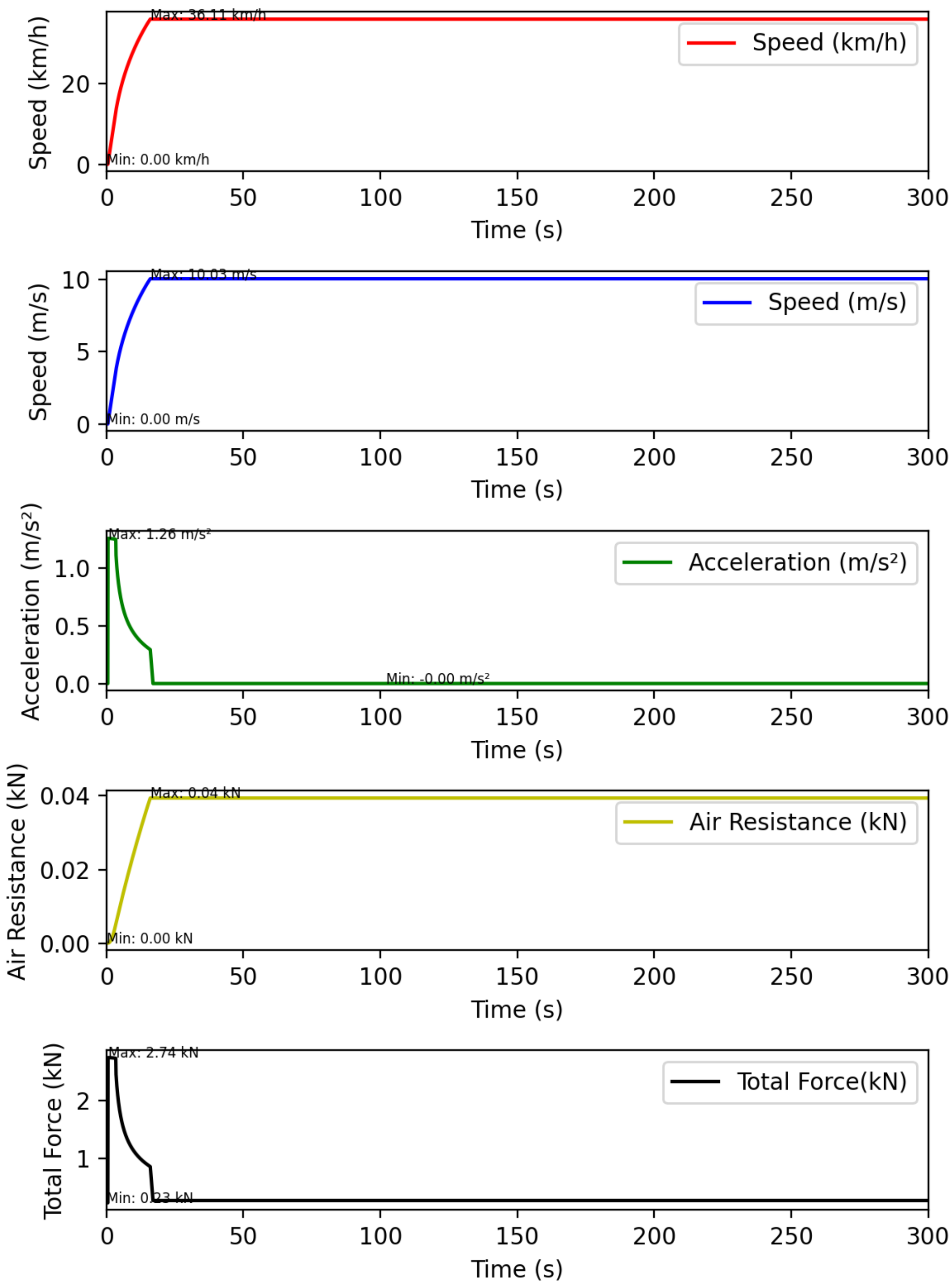


Figure 9: Plots related to Speed, acceleration, air resistance, and Total Force over time

In the Figure (9), it was shown the data curve about the speed within km/h and m/s, acceleration in m/s^2, Air Resistance and Rolling Resistance.

- The first red and blue curve in the Figure (9) represented to the speed of the vehicle drive over time in km/h and m/s.
- The green curve in the Figure (9) presented to the acceleration of the vehicle that show the characteristic of the vehicle drive. (Increase or Decrease Speed)
- The air resistance calculated by using Equation (1.1) and (1.8) . The result of the calculate shown in the yellow curve of the Figure (9).
- The total Force or Motor Force determined by usig Equation (1.5) and the result of the calculation displayed at the black curve shown that the maximum motor force is **2.74 kN**.

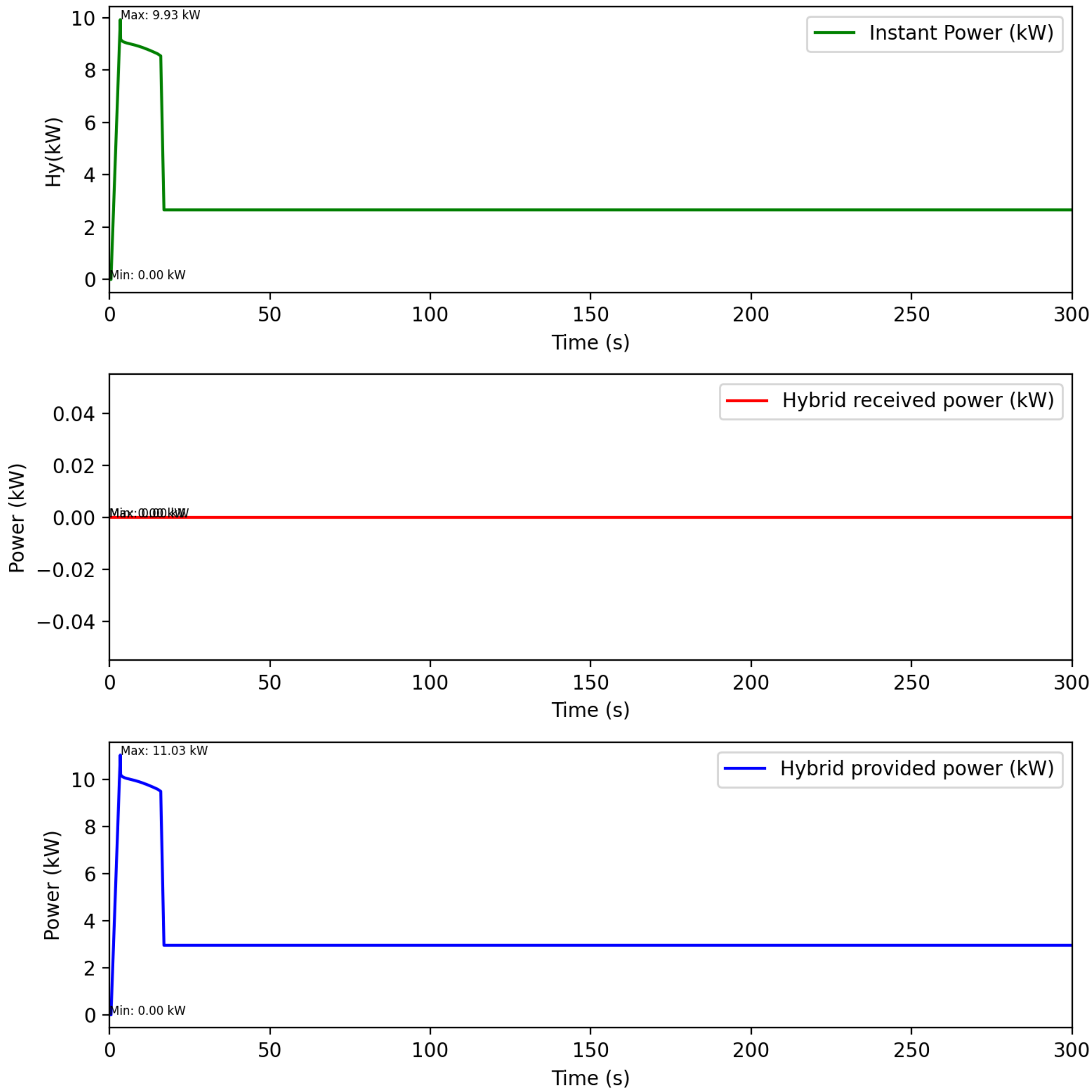


Figure 10: Hybrid working in Power transfer in Vehicle 130kmh

As show in Figure (10), we can assume that vehicle at this speed characteristic shown that the Hybrid system did not receive any power from the motor but it provided full power to the motor.

- **Hybrid Power (kW)**:
  - This plot shows the total hybrid power delivered by the system.

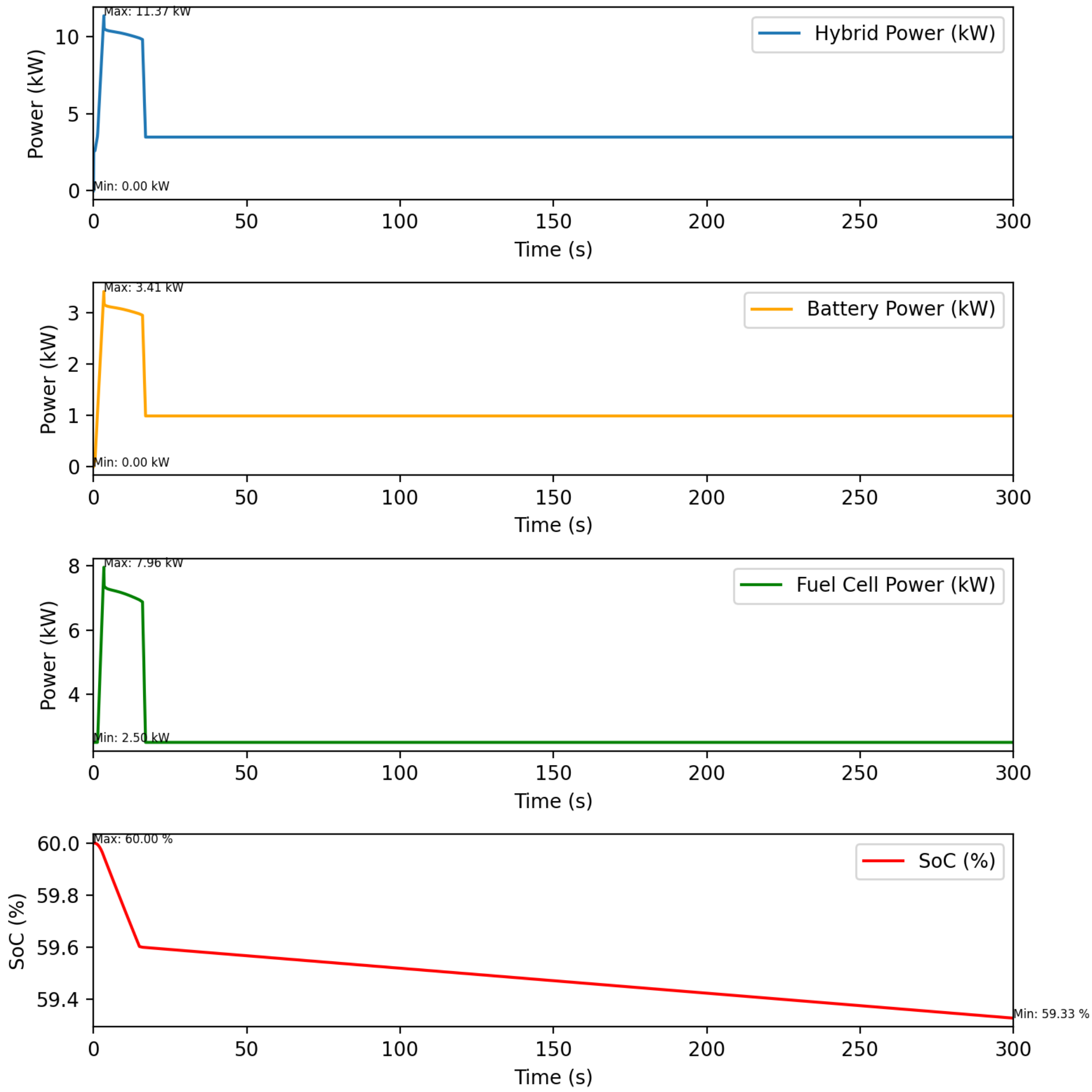


Figure 11: Hybrid system, Battery Management, Fuel Cell power and State of Charge in the Vehicle

- The power reaches a peak of approximately 3.17 kW at the start.
- It rapidly drops to 0 kW after around 25 seconds and remains flat for the rest of the 300-second period.

- **Battery Power (kW)**:
  - This plot shows the power supplied by the battery alone.
  - Initially, the battery power spikes to a peak of around 3.31 kW.
  - It sharply decreases to 0 kW after about 25 seconds, staying at that level for the rest of the duration.

- **Fuel Cell Power (kW)**:
  - This plot shows the power generated by the fuel cell.
  - The fuel cell power reaches a maximum of about 2.76 kW early in the period.
  - It drops rapidly to 2.5 kW within 25 seconds and remains constant at this level for the remainder of the time.

- **State of Charge (SoC) (%)**:
  - This plot represents the battery's state of charge (SoC) over time.
  - The SoC starts at approximately 60% and decreases gradually over time.
  - By the end of the 300-second period, it reaches around 59.3%.

1.4.2 Analysis $\alpha = 2°$

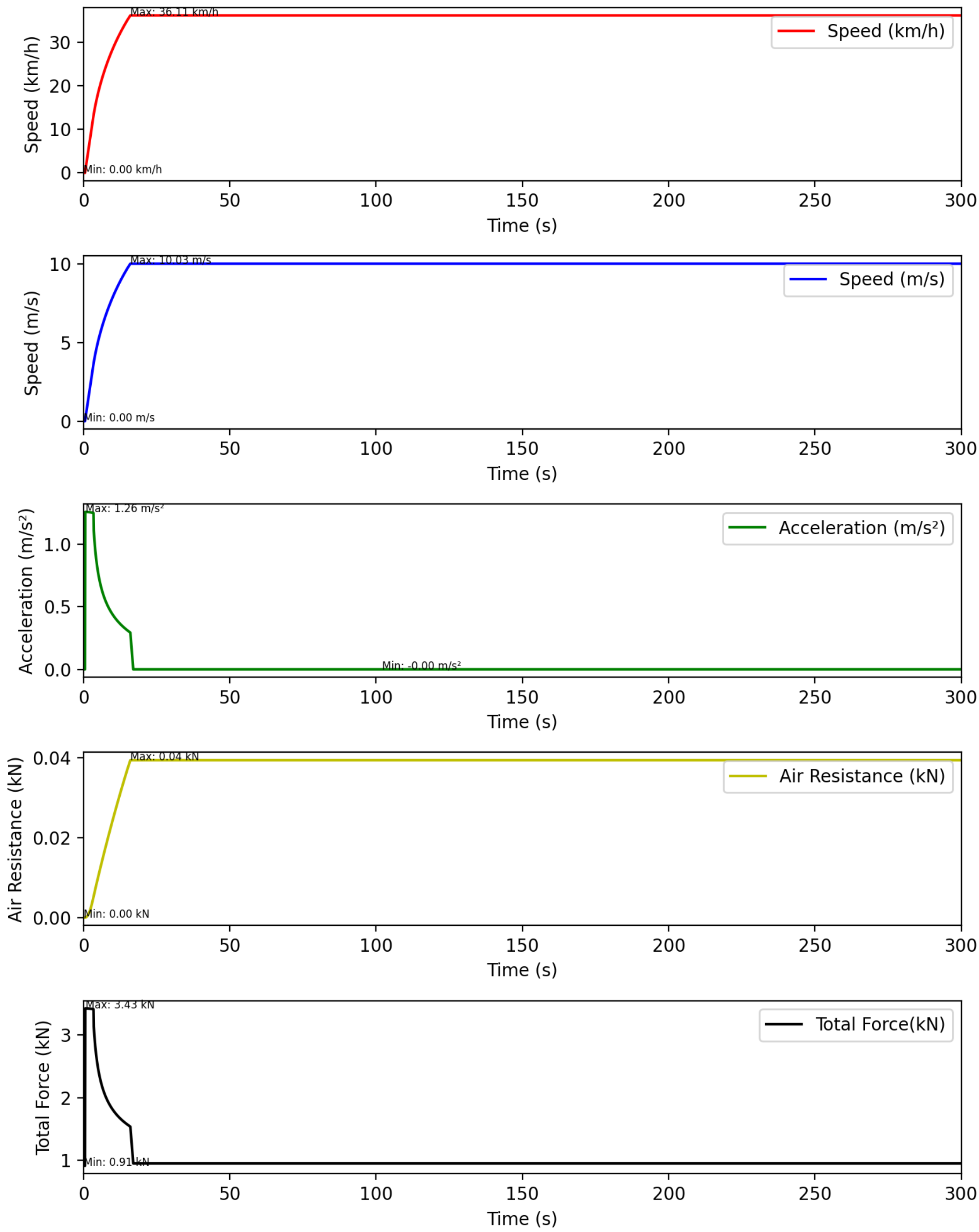


Figure 12: Plots related to Speed, acceleration, air resistance, and Total Force over time

The speed and acceleration are the same from the pervious but there are some change on the air resistance, rolling resistance and climbing resistance.

- For the Air resistance using Equation (1.1), we got the result shown in the Figure (12).
- The Rolling Resistance using Equation (1.2), as the result we can write :

$$F_{rolling}(t) = 200kg \times 9.81m/s^2 \times 0.0115 \times \cos(0^\circ) = \mathbf{225.4926\ N} \tag{1.13}$$

- The Climbing Resistance using Equation (1.3), as the result we computed :

$$F_{climb}(t) = Mg\sin(\alpha) = 2000kg \times 9.81m/s^2 \times \sin(2^\circ) = \mathbf{684.811\ N} \tag{1.14}$$

- The total Force or Motor Force determined by usig Equation (1.5) and the result of the calculation displayed at the black curve shown that the maximum motor force is **3.43 kN**.

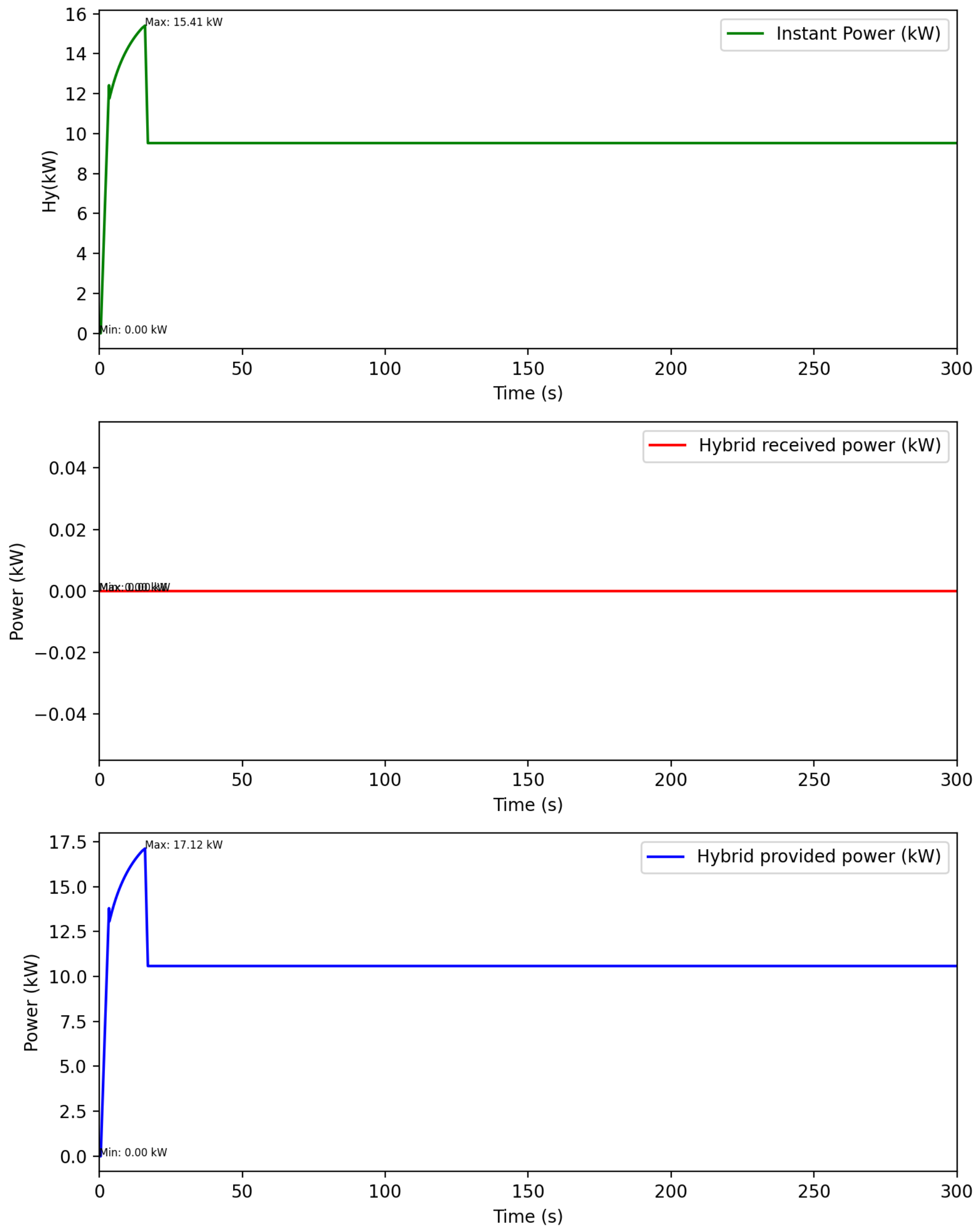


Figure 13: Hybrid working in Power transfer in Vehicle 130kmh

As show in Figure (13), we can assume that vehicle at this speed characteristic shown that the Hybrid system did not receive any power from the motor but it provided full power to the motor.

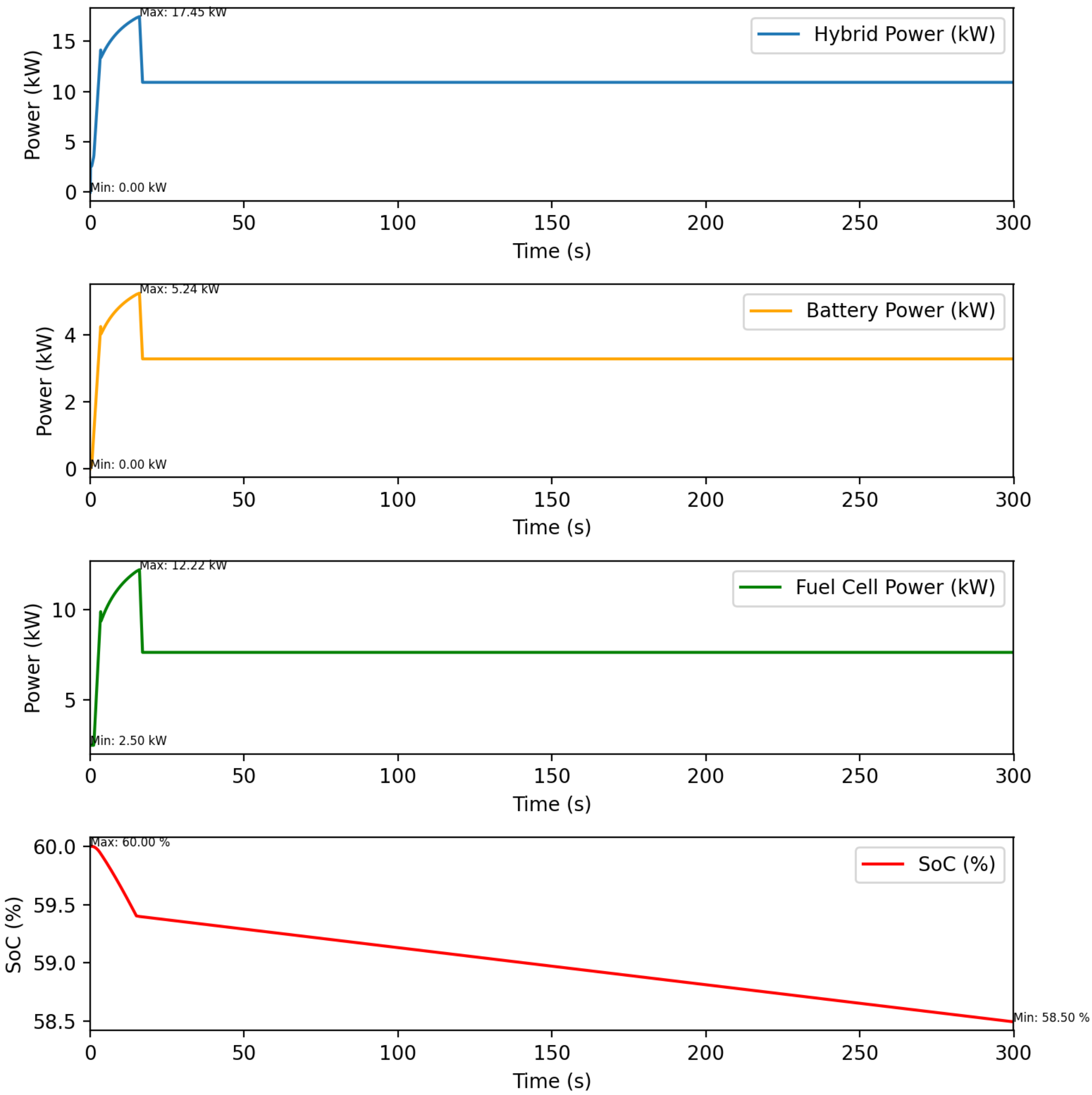


Figure 14: Hybrid working in Power transfer in Vehicle 130kmh

- **Hybrid Power (kW) - Blue Curve (Top Plot):**
  - The hybrid power increases sharply from 0 to about **17.45 kW** within the first few seconds.
  - After this peak, the power stabilizes around **10.917 kW** and remains constant for the rest of the 300-second interval.
- **Battery Power (kW) - Orange Curve (Second Plot):**
  - The battery power rises rapidly to a maximum value of **5.24 kW** at the beginning and then drops slightly, stabilizing around **3.27 kW**.
  - The battery maintains this output throughout the time period shown.
- **Fuel Cell Power (kW) - Green Curve (Third Plot):**
  - The fuel cell power ramps up initially to about **12.22 kW**.

  - After reaching this maximum value, the fuel cell output drops slightly and stabilizes at around **7.64 kW** for the remainder of the time.

- **State of Charge (SoC %) - Red Curve (Bottom Plot):**
  - The SoC starts at **60.0%** and decreases gradually over time.
  - By the end of the 300-second period, the SoC has dropped to **58.5%**.

2 Analysis of the fuel cell system with WLTC profile

We assume that the fuel cell system of the vehicle has the following characteristics, based on some specifications of the Mirai 2 and an experimental study of the Mirai 1:

- Hydrogen storage: 5.6 kg stored at 700 bars
- Molar mass of dihydrogen: 2.016 g/mol
- Enthalpy of hydrogen combustion in oxygen:
  - LHV: 242 kJ/mol
  - HHV: 285 kJ/mol
- The hydrogen loss (mainly in the purge) represents 2% of the consumed hydrogen
- Air stoichiometry is 1.5
- Atmospheric pressure is 1 bar
- Oxygen molar fraction in ambient air is 21%
- Air compressor's efficiency is 60% (comprising both compression efficiency and electric motor efficiency)
- The stack is composed of 330 cells of 273 cm^2

2.1 Calculate and Plot as a function of the stack current:

2.1.1 The electrical power consumed by the air compressor

To calculate of the compression power are proportional to the current:

$$P_{comp}(t) = \frac{1}{\eta_{sys}(t)} q_{m_{air}(t)} c_p T_e (\tau^{\frac{\gamma-1}{\gamma}} - 1) \tag{2.1}$$

Where we got :

- Air mass Flow :

$$q_{m_{air}}(t) = M_{air} F_{air}(t) = M_{air} \frac{v_{air}}{x_{O_2}} \frac{N_{cell} I(t)}{4\mathscr{F}} = M_{air} \frac{v_{air}}{x_{O_2}} \frac{P_{sys}(t)}{2 v_{H_2}(t) \rho_{sys} \Delta H} \tag{2.2}$$

- Heat Capacity:

$$c_p = \frac{\gamma}{\gamma - 1} \frac{R}{M} \tag{2.3}$$

- Compression ratio:

$$\tau = \frac{p_{out}}{p_{in}} \tag{2.4}$$

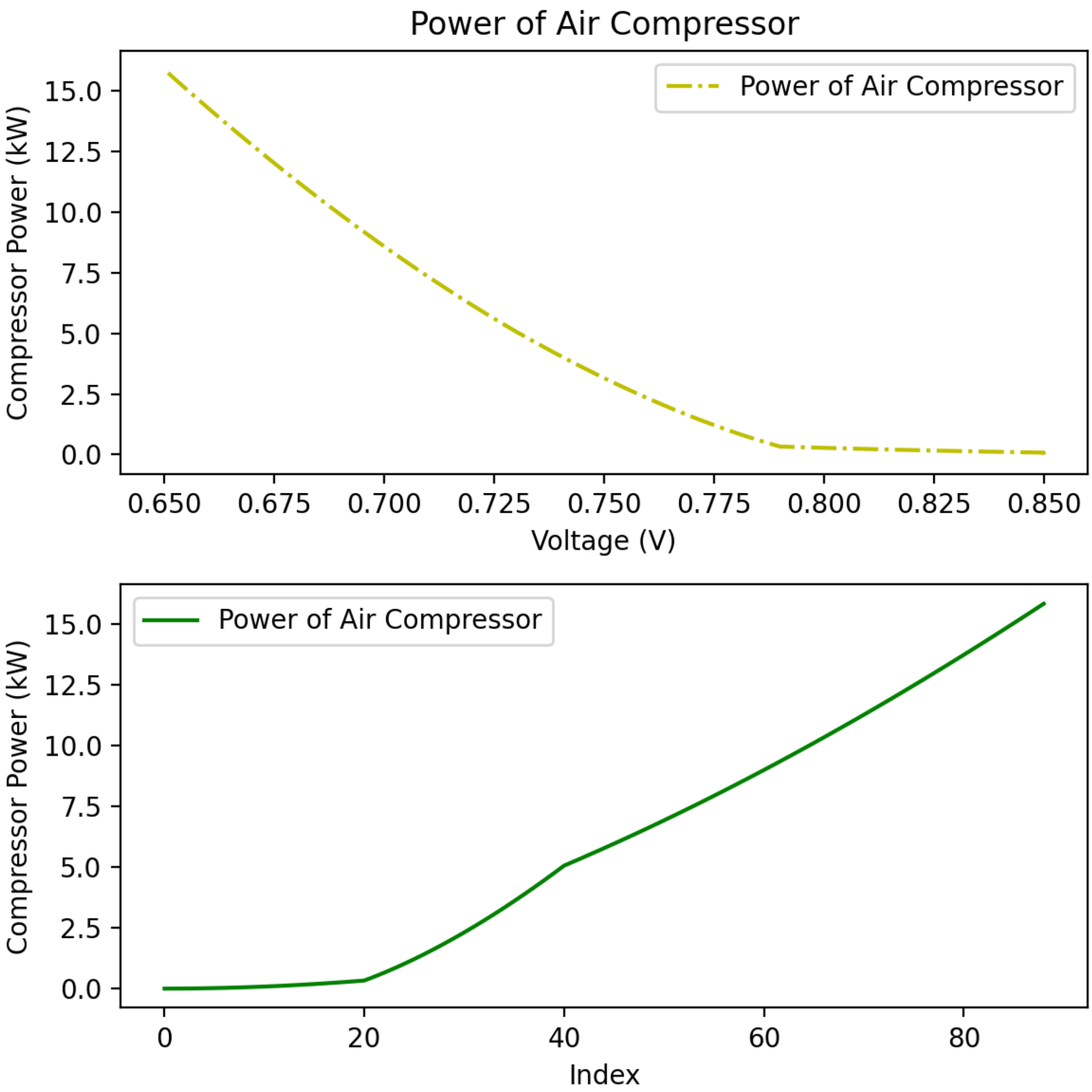


Figure 15: Power of Air Compressor

Get Equation (2.2), Equation (2.3), Equation (2.4) Substitution into Equation (2.1)

$$P_{comp}(t) = \frac{1}{\eta_{sys}(t)} \frac{v_{air}}{x_{O_2}} \frac{N_{cell} I(t)}{4\mathscr{F}} \frac{\gamma}{\gamma - 1} R T_e (\tau^{\frac{\gamma-1}{\gamma}} - 1) \tag{2.5}$$

- **Figure (15) Top Plot: Power vs. Voltage (Yellow Dashed Curve)**
  - The curve shows a downward trend as the voltage increases. It starts high on the left (at around 15 kW for a voltage near 0.65 V) and gradually declines, approaching zero as the voltage nears 0.85 V.
  - The decrease is non-linear, with a steep drop at first and then leveling off as the voltage approaches higher values. This suggests that as voltage increases, the power output of the air compressor significantly reduces, particularly more rapidly at lower voltage values.
- **Figure (15) Bottom Plot: Power vs. Index (Green Solid Curve)**
  - The curve exhibits a steady upward trend as the index increases. Starting close to 0 kW at a low index, it rises continuously, reaching around 15 kW at the highest index value.

- The increase appears almost linear, with a constant or slightly accelerating rate of power increase as the index progresses.

2.1.2 The electrical power delivered by fuel cell system.

To calculate the fuel cell that consumption in the system, we used: To find the hydrogen consumption, we will use:

$$P_{comp}(t) = U_{cell} \times i(t) \times N_{cell} \times A_{cell} \tag{2.6}$$

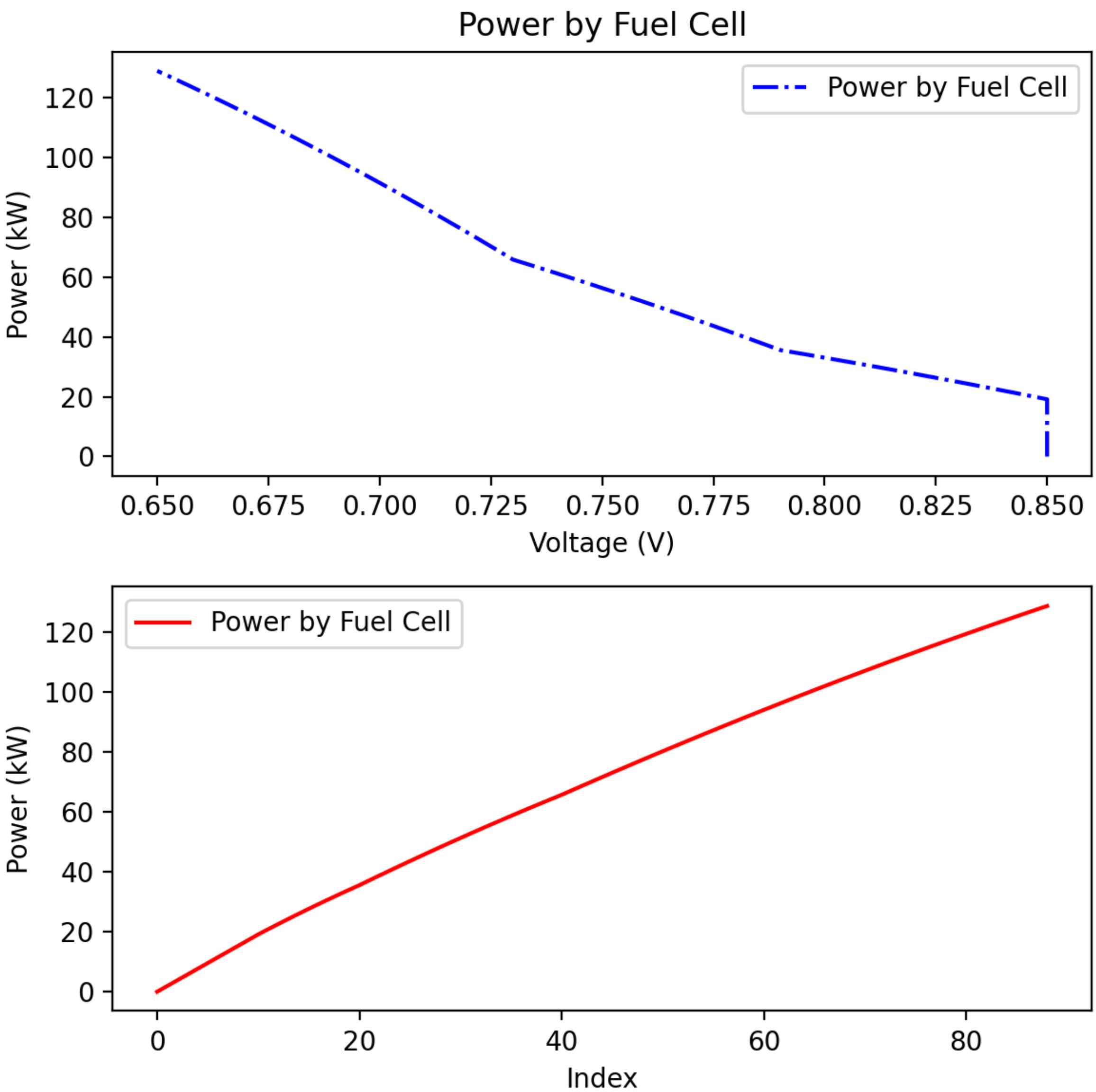


Figure 16: The electrical power delivered by fuel cell system

- **Figure (16) : Top Plot**
  - The x-axis represents the **Voltage (V)** of the fuel cell, ranging from approximately 0.65V to 0.85V.
  - The y-axis represents the **Power output** in watts, ranging from 0 to around 120 watts.
  - The plot shows a **decreasing trend** as voltage increases, suggesting that power output drops as voltage rises. The line is **dash-dotted and blue**, indicating a measured or calculated trend of power with respect to voltage.

- **Figure (16) : Bottom Plot**
  - The x-axis represents the **Index**, possibly indicating the step number or data point (e.g., time or sequence index), ranging from 0 to 90.
  - The y-axis again represents **Power output** in watts, similar to the top plot.
  - The plot shows a **steady increase** in power as the index increases, potentially reflecting the increase in power over time or due to increasing load. The line is **solid red**, representing a direct correlation between index and power output.

2.1.3 The hydorgen consumption by the duel cell system.

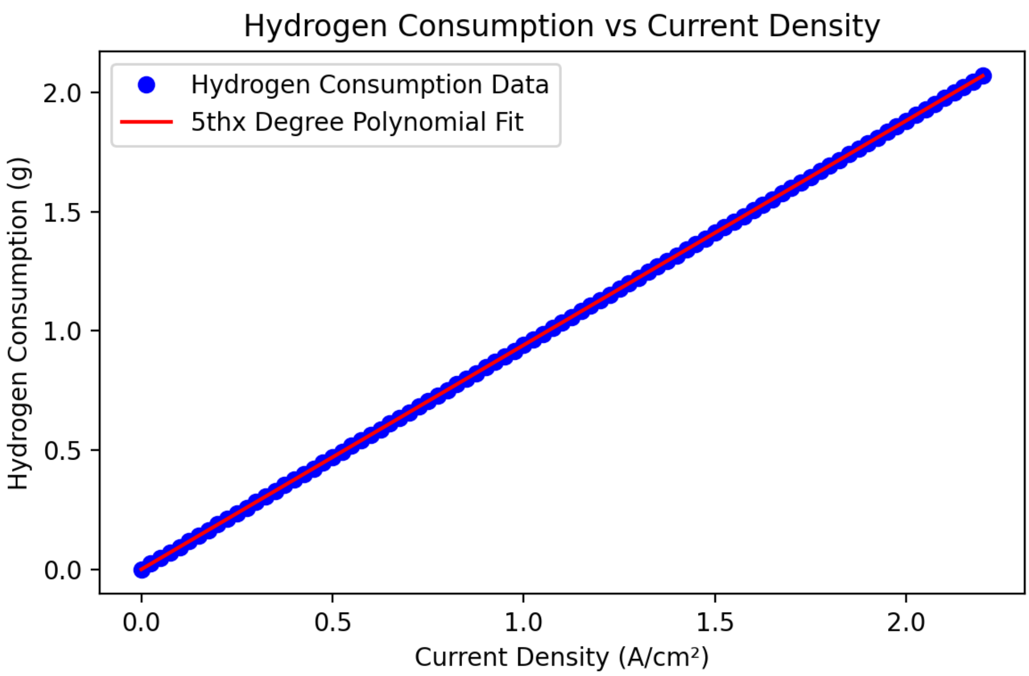


Figure 17: Hydrogen Consumption vs Current Density fit with fifth degree polynomial

- Find the mole flow in the system :

$$\dot{m}_{H_2}(t) = \frac{I(t)}{2\mathscr{F}} \times M_{H_2} \times N_{Cell} \times A_{cell} \tag{2.7}$$

- Fifth-Degree Polynomial Equation

$$P(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 \tag{2.8}$$

- Hydrogen Consumption as a Function of Current Density:

$$P(i) = a_5(i)^5 + a_4(i)^4 + a_3(i)^3 + a_2(i)^2 + a_1(i) + a_0 \tag{2.9}$$

- $(a_0, a_1, a_2, a_3, a_4, a_5)$: Coefficients of the polynomial, determined through the fitting process. These coefficients define the shape of the polynomial curve.

This graph displays in Figure (17) the relationship between hydrogen consumption (in grams) and current density (in A/cm²)

- **Data Points (Blue)**:
  - The blue markers represent the experimental hydrogen consumption data at various current densities.
  - A clear linear trend is observed, where hydrogen consumption increases as the current density increases.
- **Polynomial Fit (Red Curve)**:
  - The red line represents a 5th-degree polynomial fit to the data.
  - Despite being a higher-order polynomial, the fit closely resembles a straight line, indicating a strong linear relationship between current density and hydrogen consumption.
- **Linear Trend**:
  - The plot shows that hydrogen consumption increases almost linearly with the current density in the range of 0 to 2 A/cm**²**.
  - This is expected in electrochemical systems, where fuel consumption is proportional to the current density.
- **Accuracy of Fit**:
  - The data points closely align with the polynomial fit, indicating a high degree of accuracy.
  - The minimal deviation between the data and the fit suggests that a linear or near-linear model accurately describes the relationship.
- **Practical Implication**:
  - Increasing the current density results in a proportional increase in hydrogen consumption.
  - This insight is important for fuel cell applications, where controlling current density helps manage fuel consumption.

The graph effectively communicates that hydrogen consumption increases linearly with the current density over the given range.

2.2 Considering the WLTC drive cycle, calculate and plot the instant hydrogen consumption in kg/s as a function of time.

To Calculate the value in this question, we will use servals equation :

- To calculate the mass flow rate of hydrogen, we obtained :

$$\dot{m}_{H_2}(t) = \frac{I_{cell}(t) \times N_{cell} \times M_{H_2}}{2\mathscr{F}} \tag{2.10}$$

- To determine the Current of the cell, we used

$$I_{cell}(t) = \frac{\text{Power demand}(t)}{U_{cell}(t)} \tag{2.11}$$

- To obtained the Cell voltage, we use one of optimization method to fine the optimal voltage of the each cell depend on speed, time and power demand of the vehicle. We will Minimize Scalar Optimization to do it.

- Let $f(x)$ be a scalar function defined as:

$$f : \mathbb{R} \to \mathbb{R} \tag{2.12}$$

- The goal is to find x^* such that:

$$x^* = \arg\min_{x\in[a,b]} f(x) \tag{2.13}$$

- We may define constraints in optional bounds for x:

$$a \leq x \leq b \tag{2.14}$$

- The optimization seeks to minimize the function:

$$\min_{x\in[a,b]} f(x) \tag{2.15}$$

- Several methods can be used in the `minimize_scalar` function:
  * **Brent's Method**: A robust method that combines the bisection method, secant method, and inverse quadratic interpolation.
  * **Golden Section Search**: A technique that systematically narrows the interval containing the minimum point.
  * **Bounded Methods**: Enforces the constraints a and b while searching for the minimum.
- If the function f is differentiable, the first derivative condition is applied:

$$f'(x^*) = 0 \tag{2.16}$$

- To confirm that x^* is a local minimum, we use the second derivative condition:

$$f''(x^*) > 0 \tag{2.17}$$

- The optimization algorithm will typically terminate when:
  * The change in the objective function value is less than a specified tolerance level, ϵ:

$$|f(x_{k+1}) - f(x_k)| < \epsilon \tag{2.18}$$

  * The change in x is less than a specified tolerance level, δ:

$$|x_{k+1} - x_k| < \delta \tag{2.19}$$

- To find the total hydorgen consumed, we can use :

$$\text{Total Hydrogen Consumption} = \int_0^T \dot{m}_{H_2}(t)dt \tag{2.20}$$

- To detemine System Efficiency, we do :

$$E_{\text{from H2}} = \frac{m_{\text{hydro}}[t] \cdot LHV_{\text{hydrogen}}}{M_{\text{olar mass H2}}} \quad \text{if } m_{\text{hydro}}[t] > 0 \tag{2.21}$$

$$\eta[t] = \begin{cases} \frac{P_{\text{fuel cell}}}{E_{\text{from H2}}} & \text{if } E_{\text{from H2}} > 0 \\ 0 & \text{if } E_{\text{from H2}} \leq 0 \end{cases} \tag{2.22}$$

- To calculate the range operation of the vehicle:

$$d_{\text{traveled}} = v_s[1:] \cdot \Delta t \tag{2.23}$$

$$D_{\text{total}} = \frac{\sum d_{\text{traveled}}}{1000} \tag{2.24}$$

$$\text{Consumption of Hydrogen Average} = \begin{cases} \frac{\text{Consumption of Hydrogen total}\cdot 100}{D_{\text{total}}} & \text{if } D_{\text{total}} > 0 \\ 0 & \text{if } D_{\text{total}} \leq 0 \end{cases} \tag{2.25}$$

$$R_{\text{operation}} = \begin{cases} \frac{\text{Capacity tank}}{\text{Consumption of Hydrogen Average}}/100 & \text{if Consumption of Hydrogen Average} > 0 \\ 0 & \text{if Consumption of Hydrogen Average} \leq 0 \end{cases} \tag{2.26}$$

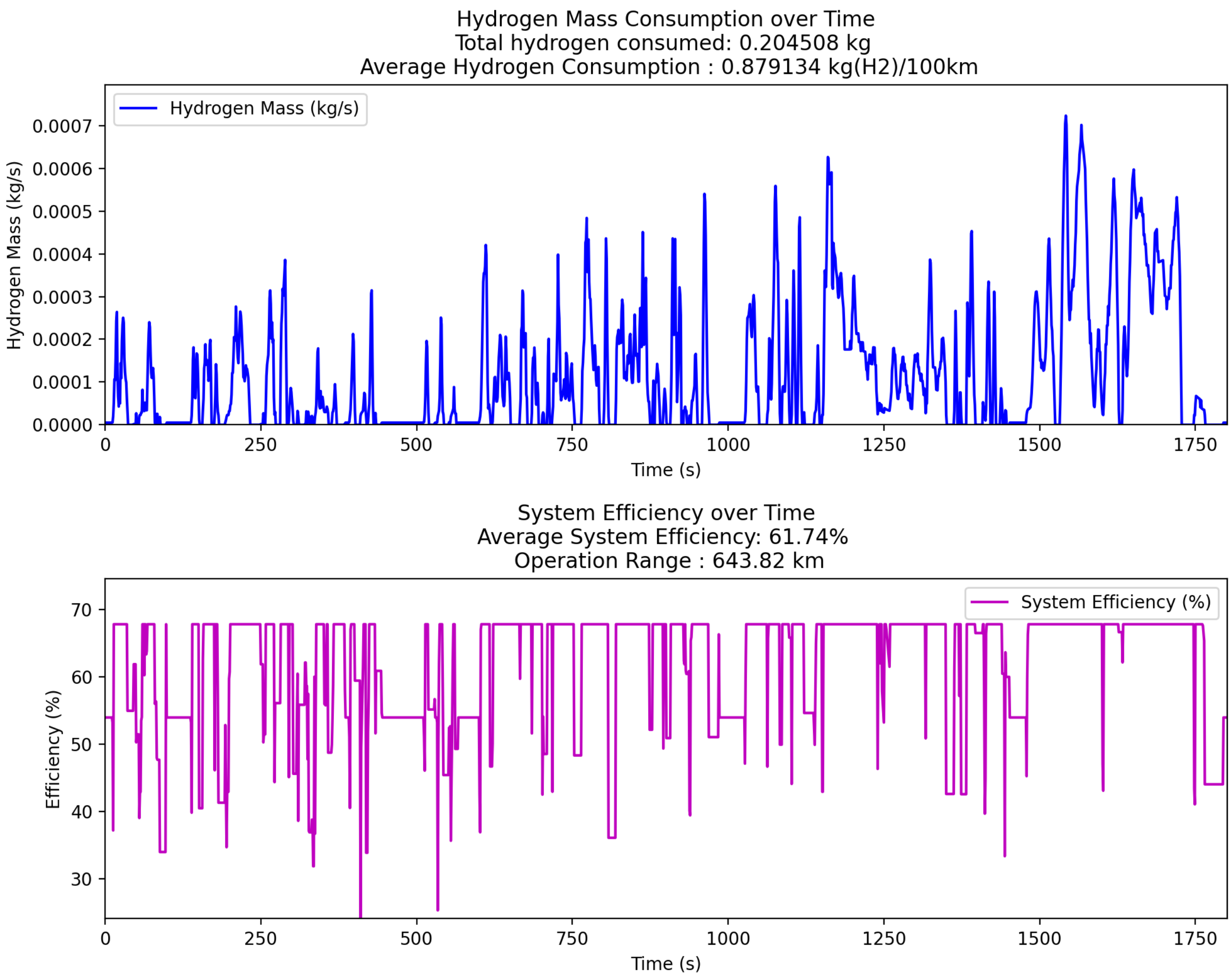


Figure 18: Mass Flow rate of the Hydrogen and Efficiency of the System

From Figure (18), the plot presents two graphs that examine hydrogen mass consumption and system efficiency over time, each shedding light on operational performance.

- The first graph shows hydrogen mass consumption in kilograms per second across a duration of **0 to 1800** seconds. It displays considerable fluctuations, with distinct spikes occurring at various points. This variability indicates that the system undergoes changing operational demands, potentially driven by factors such as load variations or environmental influences.The total hydrogen consumed is **0.204508 kg**, indicating the total usage during the observed timeframe and the average hydrogen consumption is approximately **0.879134 kg(H_2)/100 km**, serving as a useful reference for assessing efficiency relative to distance traveled.
- The second graph focuses on system efficiency, represented as a percentage, within the same time frame. Efficiency varies between **30%** and **70%**, suggesting that the system doesn't operate at a steady level. With an average system efficiency of around **61.74%**, the data reflects a moderate level of performance throughout the observed period. Additionally, the operation range is reported to be **643.82 km**, which may relate to the effective distance achieved based on hydrogen usage and efficiency. The fluctuations in efficiency highlight potential areas where operational improvements could be made.
- The relationship between hydrogen consumption and system efficiency is crucial to this analysis. The changes in hydrogen consumption can directly affect efficiency, with higher consumption periods possibly linked to lower efficiency levels. Identifying these patterns and understanding their underlying causes can offer valuable insights for optimizing system performance. Analyzing the factors that lead to low efficiency during certain periods will be vital for refining operational strategies. Collecting further data on load conditions and external factors during these fluctuations could deepen the analysis and help improve overall performance in hydrogen-dependent applications.

2.3 Sensitivity analysis - observe the effect of the following variations on the results obtained in the previous question

- Varation of C:
  - C = 0,25
  - C = 0.40
- Varation of Cr:
  - Cr = 0.01
  - Cr = 0.014
- Variation of bateery charging Power:
  - Battery Charging = 5C
  - Battery Charging = 20C
- Variation of the battery capacity :
  - Battery Capacity = 620 Wh or 0.62 kWh
  - Battery Capaity = 1860 Wh or 1.86 kWh

In this question , we will devided the input parameter into two main part:

- First : $C = 0.25$, $Cr = 0.01$, Battery Charging = $5C$,$20C$ and Battery Capacity = $0.62kWh$
- Second : $C = 0.40$, $Cr = 0.014$, Battery Charging = $5C$,$20C$ and Battery Capacity = $1.86kWh$

2.3.1 First : C = 0.25 , Cr = 0.01, Battery Charging = 5C,20C and Battery Capacity = 0.62 kWh

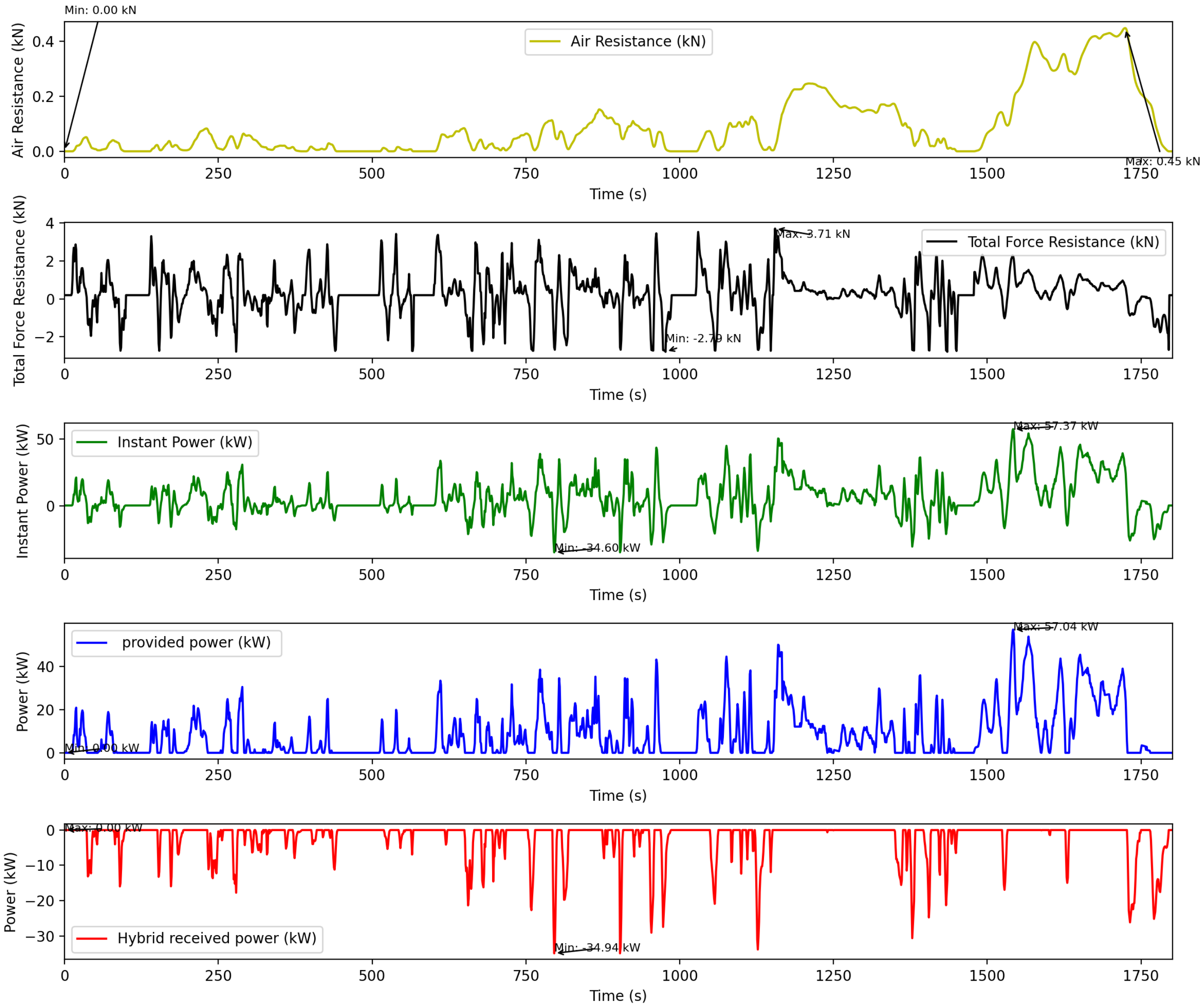


Figure 19: Curve of Air Resistance, Total Force, Instant Power, Hybrid Provided and Received Power

As, we used to do at the previous Section(1) we assumed that:

- By looking between the Figure (4)a with Figure (19) yellow curve, we can see that air resistance likely decrease while the maximum air resistance force at Figure (4)a was around **500 N** but in Figure (19) is around **450 N**.
- For the total resistance force of vehicle from previous question in Figure (4)b slightly the same value of maximum and minimum of the value that we calculate in this question shown on the Figure (19) black curve.
- As the result of the Total force resistance of previous question between this question calculation likely the same. We can assumed that the Instant power, Hybrid power received and provided of pervious question shown by Figure (7) with this question calculation shown by Figure (19) (Green, Blue and Red curve) are slightly the same characteristic as pervious one.
  - The maximum, minimun of Instant Power of this question around **57.37 kW** and **34.60 kW** . So, we can said that the maximum of hybrid system will provided to the system is around **57.37 kW** and received **34.60 kW**.

- The maximum, minimun of Instant Power of this question around **58.488 kW** and **34.365 kW**

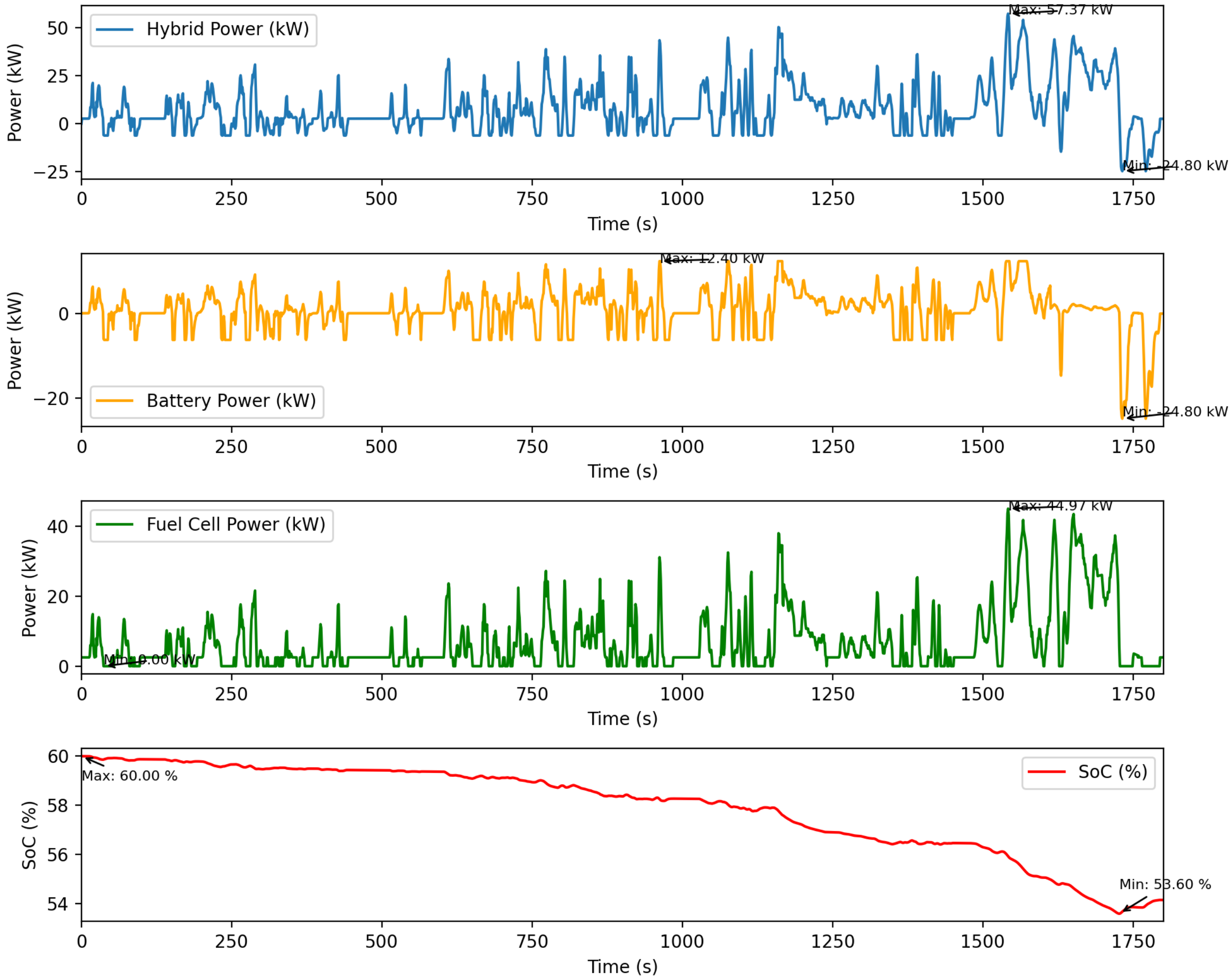


Figure 20: Hydrid System, Battery Manangement, Fuel Cell Power and State of Charge of the Vehicle

- **Hybrid Power (kW):**
  - Figure (20): The range goes up to maximum about **57.37 kW**, with frequent fluctuations between 0 and 60 kW, and a few drops below 0 kW to the minimum of **24.80 kW**. Moreover, this figure is more dynamic behavior where power transitions between positive and negative more frequently, showing a more active power management system that involves energy recovery (negative power).
  - Figure (8): The hybrid power fluctuates significantly between 0 and **58.82 kW** and the minimum is just **7.44 kW** below 0. There are high peaks with fewer negative values, implying a system that mostly generates or draws power in positive ranges.
- **Battery Power (kW):**
  - Figure (20): Similarly, the maximum range of charging is **24.8 kW** and and discharging range is **12.4 kW**. While in Figure (8): the maximum chargine is juat **7.44 kW** and maximum range of discharge almost **12.40 kW**.

  - For the Figure (8) lightly broader range with more focus on discharging power. On the other hand, For the Figure (20) is clearer fluctuations between charging and discharging, emphasizing a balance in battery management between supplying and recovering energy.

- **Fuel Cell Power (kW):**
  - Figure (20): The data shows similar behavior, with sharp spikes and a max value of **44.97 kW**. The power stays within the same range.
  - Figure (8): Tuel cell power remains mostly positive, fluctuating between 0 and max value of **46.42 kW**. The curve is characterized by sharp peaks followed by rapid declines.

- **State of Charge (SoC):**
  - Figure (8): he SoC starts around **60%** and drops steadily over time, reaching a minimum of **56.28%**. The curve shows a consistent decline, with a larger total drop in SoC.
  - Figure (20): The SoC also starts at **60%** but drops more slowly, reaching a minimum of **53.60%**. The decline is less pronounced compared to the Figure (8) and SoC has drop deeply than Figure (8) because of the capacity of battery is 0.62 kWh compare with Figure (8) is 1.24kWh

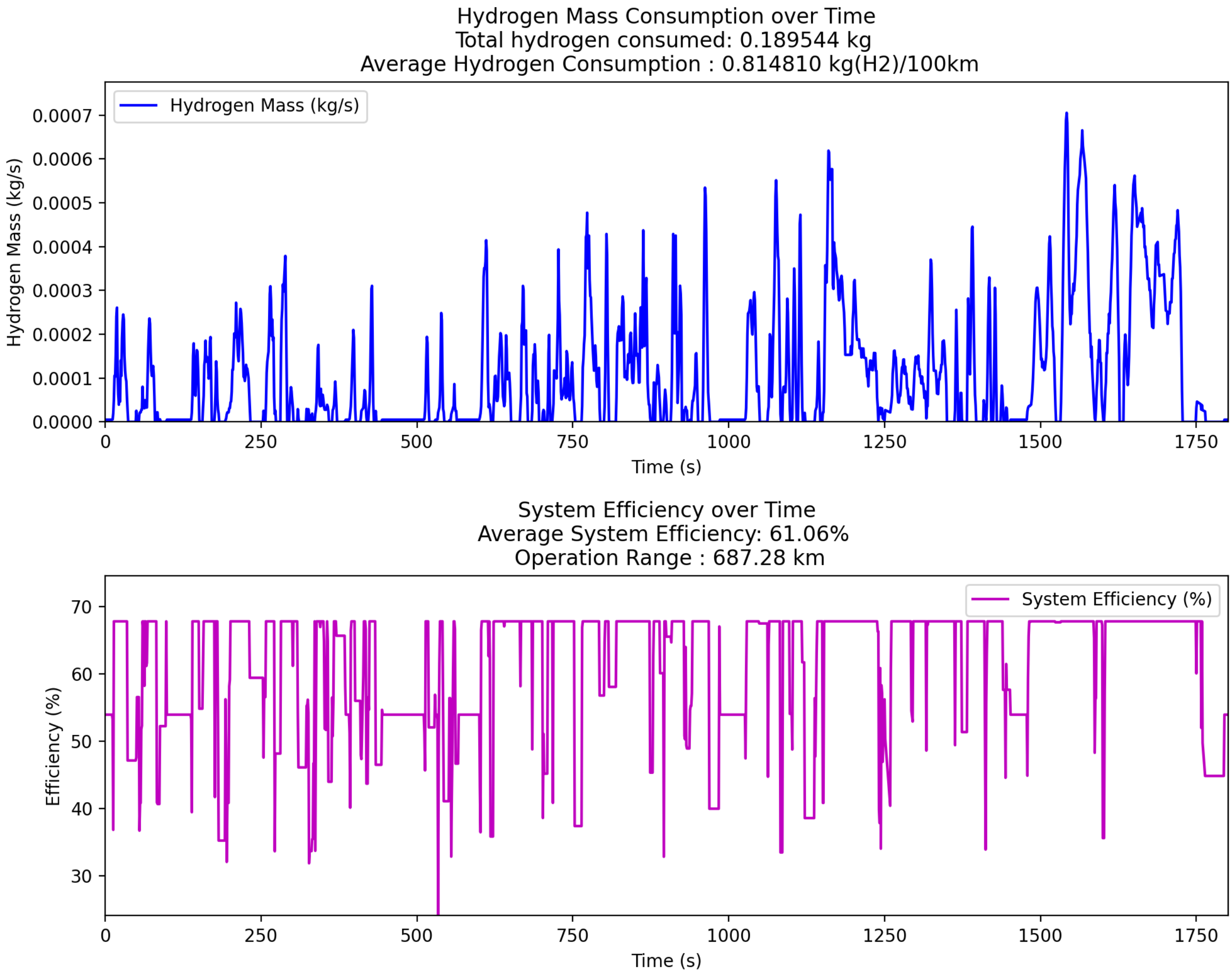


Figure 21: Plot of the hydrogen consumption in kg/s and Systen Efficiency in %

At the Previous question that shown in the Figure (18) and our calculation at the moment in the Figure(21):

- For the Total Hydrogen Consumed at the Figure (18) is around **0.2045 kg** with **0.8791 kg(H_2)/100km** while the Total Hydrogen Consumed at this question is around **0.189 kg** with **0.8148 kg(H_2)/100km** and it is coming from the Power of the Hybrid system based on time decreased.

- Average System Efficiency is **61.74%** with **643.82km** Operation Range at the previous question. In contract, at moment the Average System Efficiency is slightly dropped to **61.06%** but the operation range is approximately increasing to **687.28 km** because of the Power demand decreasing becuase of the total force and other main reasons.

- The efficiency of the system comes from the characteristics, behavior of the driver, characteristics of component of the system and any varible parameters that we input or do calculation.

2.3.2 Second : C = 0.40 , Cr = 0.014, Battery Charging = 5C,20C and Battery Capacity = 1.86 kWh

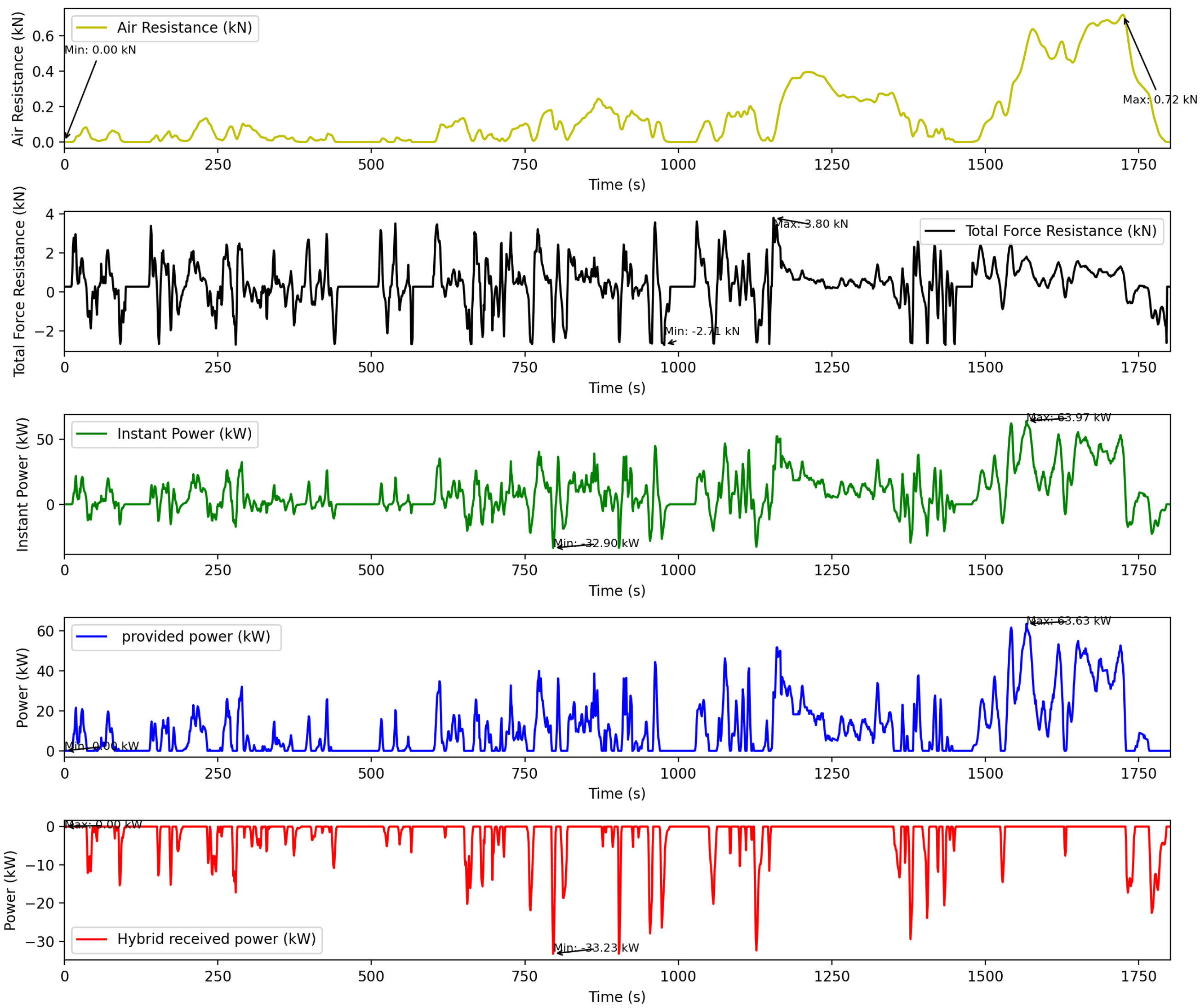


Figure 22: Curve of Air Resistance, Total Force, Instant Power, Hybrid Provided and Received Power

As, we used to do at the previous Section(1) we assumed that:

- By looking between the Figure (4)a with Figure (22) yellow curve, we can see that air resistance likely increasing deeply while the maximum air resistance force at Figure (4)a was around **500 N** but in Figure (22) is around **720 N**.

- For the total resistance force of vehicle from previous question in Figure (4)b slightly the lower value of maximum and minimum of the value that we calculate in this question shown on the Figure (22) black curve.

- According to sightly increasing amount of the total force resistance in the system because increasing the value of drag coefficient (C) and Rolling Resistance coefficient $C(_r)$ so, it make the power demand that represented the green curve in the Figure (22) are increasing significantly. The maximum and minimum value are about **63.97 kW** and **32.90 kW** below 0 while in previous question just around **58.48 kW** max and **34.365 kW** min.

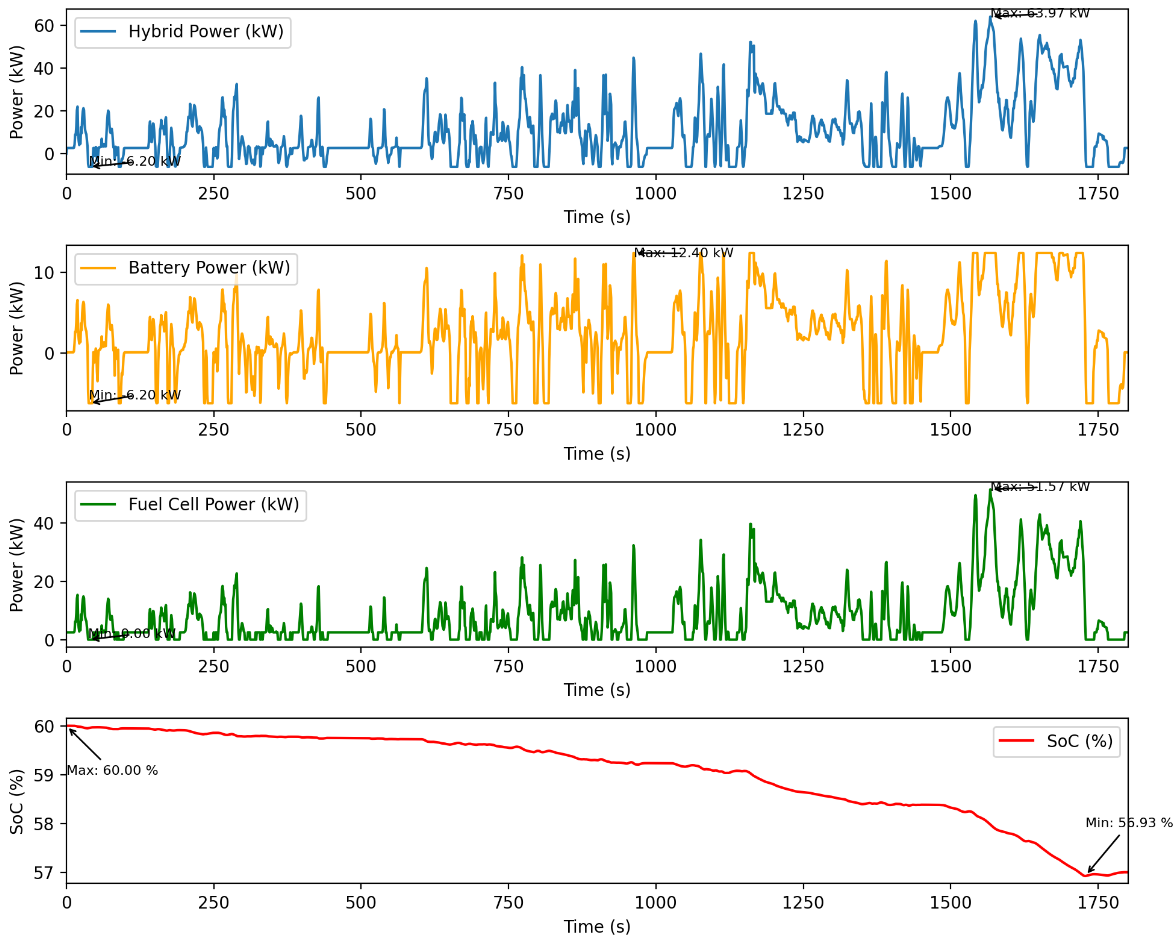


Figure 23: Hydrid System, Battery Manangement, Fuel Cell Power and State of Charge of the Vehicle

- **Hybrid Power (kW):**
  - Figure (23): The range goes up to maximum about **63.97 kW**, with frequent fluctuations between 0 and 65 kW, and a few drops below 0 kW to the minimum of **6.2 kW**. While Figure (8): The hybrid power fluctuates significantly between 0 and **58.82 kW** and the minimum is just **7.44 kW** below 0. There are high peaks with fewer negative values, implying a system that mostly generates or draws power in positive ranges.
  - For the maximum Hybrid power consumption was much increasing from the previous one in term of change parameter of the coefficient.

- **Battery Power (kW):**
  - Figure (23): Similarly, the maximum range of charging is just **6.23 kW** and and discharging range is **12.4 kW**. While in Figure (8): the maximum chargine is juat **7.44 kW** and maximum range of discharge almost **12.40 kW**.

  - For the Figure (8) lightly broader range with more focus on discharging power. On the other hand, For the Figure (23) is clearer fluctuations between charging and discharging, emphasizing a balance in battery management between supplying and recovering energy.

- **Fuel Cell Power (kW):**
  - Figure (23): The data shows similar behavior, with sharp spikes and a max value of **51.57 kW**. The power stays within the same range.
  - Figure (8): Tuel cell power remains mostly positive, fluctuating between 0 and max value of **46.42 kW**. The curve is characterized by sharp peaks followed by rapid declines.

- **State of Charge (SoC):**
  - Figure (8): he SoC starts around **60%** and drops steadily over time, reaching a minimum of **56.28%**. The curve shows a consistent decline, with a larger total drop in SoC.
  - Figure (20): The SoC also starts at **60%** but drops more slowly, reaching a minimum of **56.93%**. SoC has a little bit higher than Figure (8) event we have increasing the capacity of battery to **1.86 kWh**. The reason of this problem because of coefficient is highly in crease to make the power demand increasing significantly.

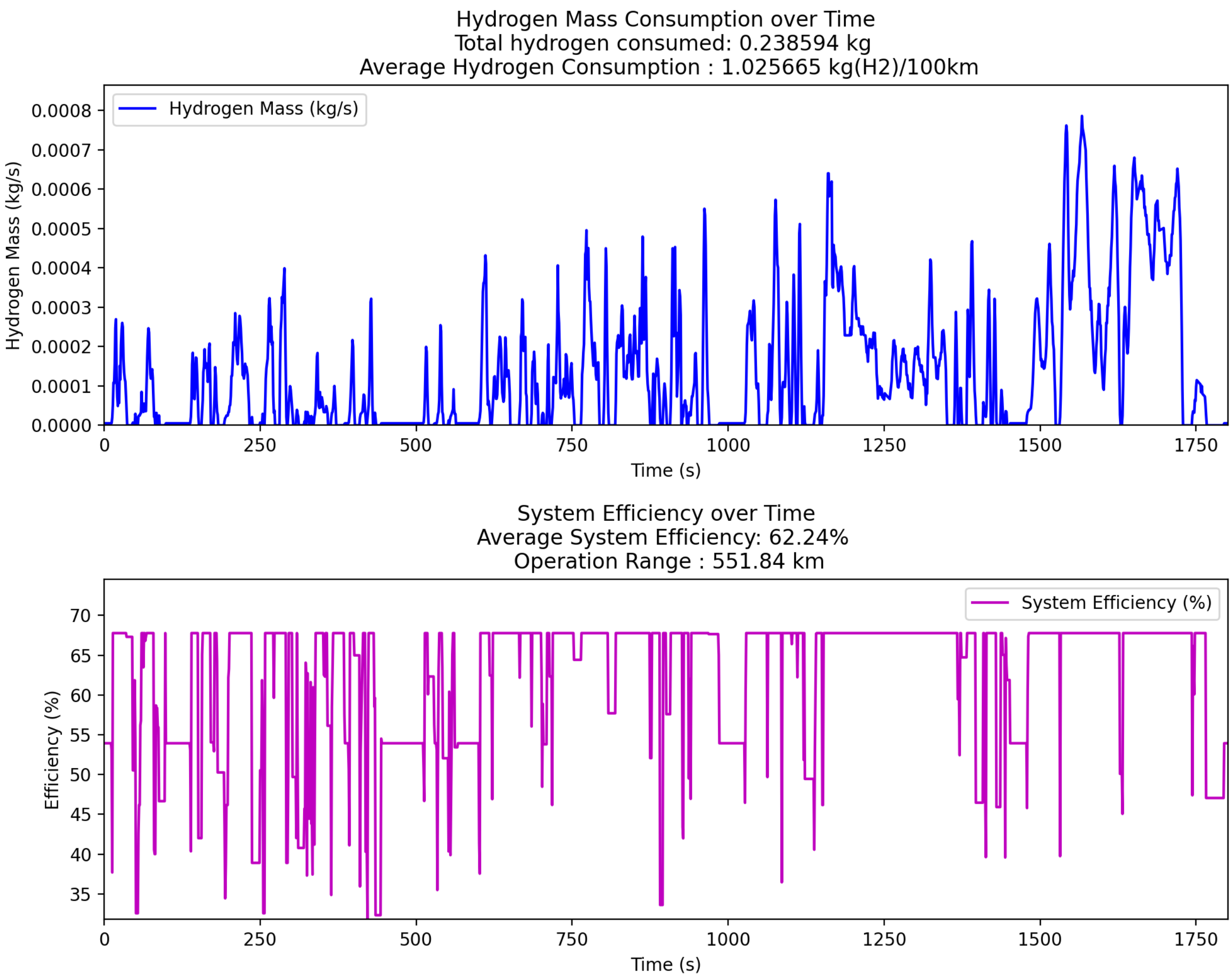


Figure 24: Plot of the hydrogen consumption in kg/s and Systen Efficiency in %

At the Previous question that shown in the Figure (18) and our calculation at the moment in the Figure(24):

- For the Total Hydrogen Consumed at the Figure (18) is around **0.2045 kg** with **0.8791 kg(H_2)/100km** while the Total Hydrogen Consumed at this question is around **0.23859 kg** with **1.0256 kg(H_2)/100km** and it is coming from the sum of Power of the Hybrid system based on time decreased.
- Average System Efficiency is **61.74%** with **643.82km** Operation Range at the previous question. In contract, at moment the Average System Efficiency is slightly dropped to **62.23%** but the operation range is approximately increasing to **551.84 km** because of the Power demand increasing deeping becuase of the total force and other main reasons.
- The efficiency of the system comes from the characteristics, behavior of the driver, characteristics of component of the system and any varible parameters that we input or do calculation.

In Summary after we change varible of the vehicle or system, we saw that the system of the vehicle change as much as the amount of parameter you changed and the main parameter that mostly effect to the vehicle are **Drag Coefficient** and **Rolling Resistance Coefficient**.

3 Analysis of the fuel cell system at 130 km/h

3.1 Calculate and plot as a function of time for both slope angle values:

3.1.1 First: C = 0.25, Cr = 0.01, Battery Charging = 5C, 20C, Battery Capacity = 0.62 kWh, $\alpha = 0°$

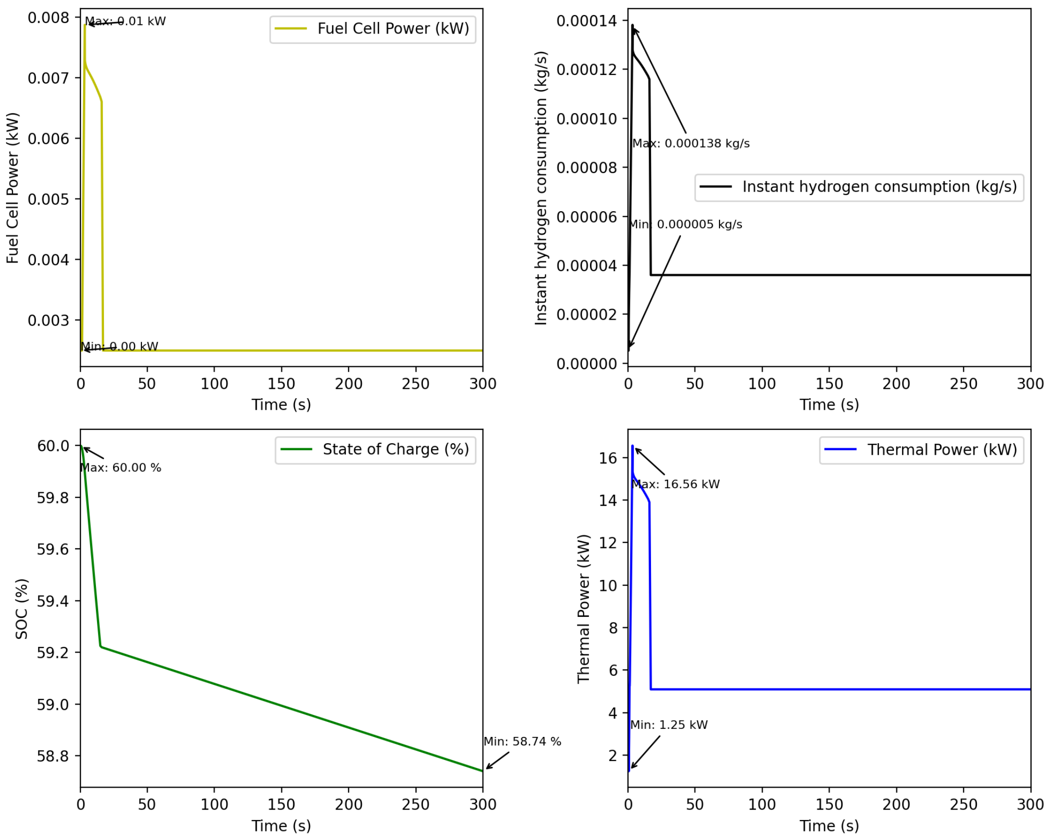


Figure 25: Fuel Cell Power, Instant hydro consumption SOC, and Thermal Power

3.1.2 First: C = 0.25, Cr = 0.01, Battery Charging = 5C, 20C, Battery Capacity = 0.62 kWh, $\alpha = 2°$

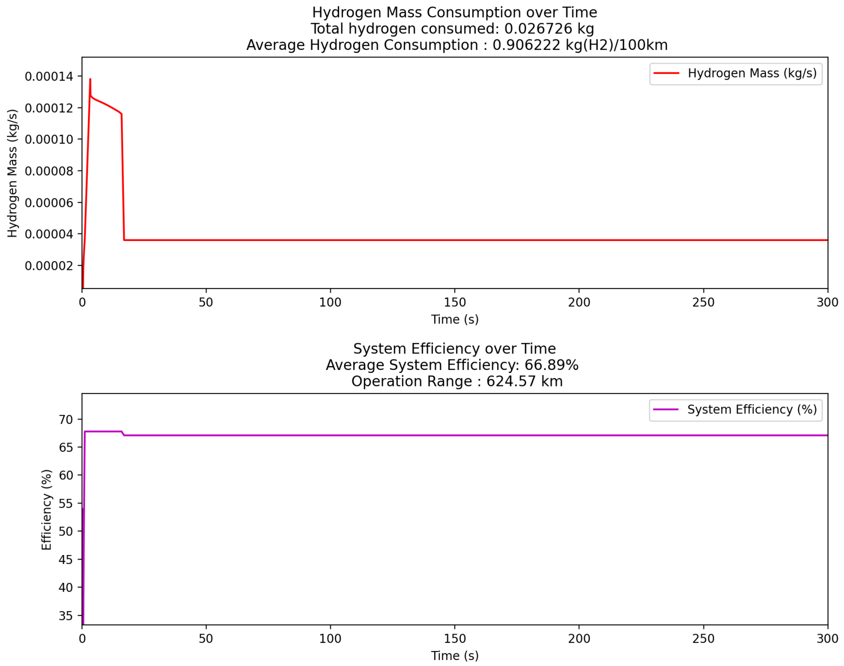


Figure 26: Plot of hydrogen consumption in kg/s and System Efficiency in %

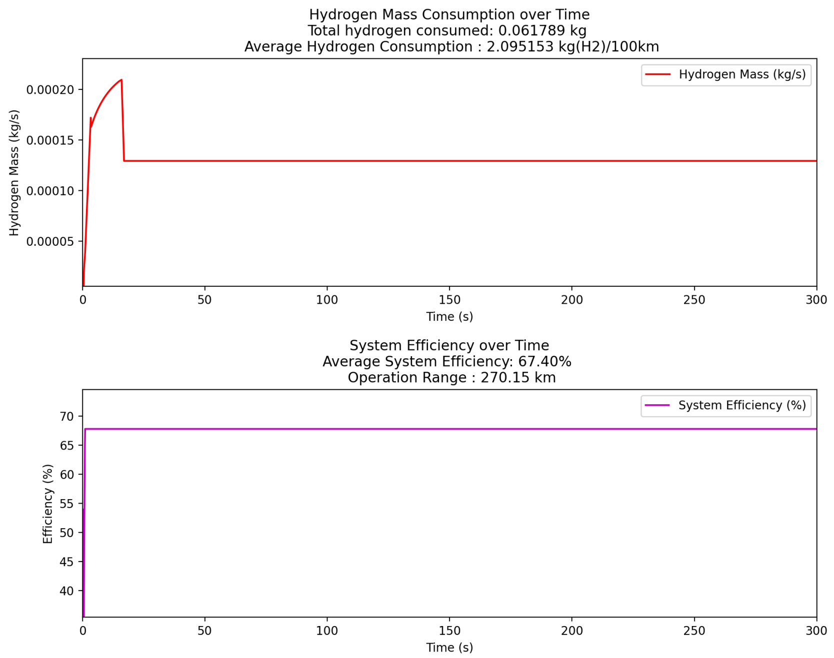


Figure 27: Plot of hydrogen consumption in kg/s and System Efficiency in %

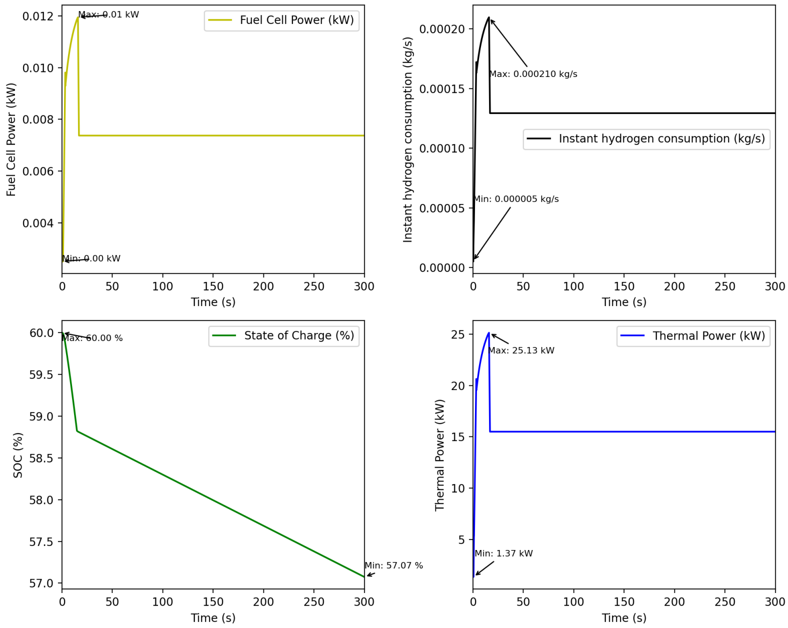


Figure 28: Fuel Cell Power, Instant hydro consumption SOC, and Thermal Power

3.1.3 Second: C = 0.40, Cr = 0.014, Battery Charging = 5C, 20C, Battery Capacity = 1.86 kWh, $\alpha = 0°$

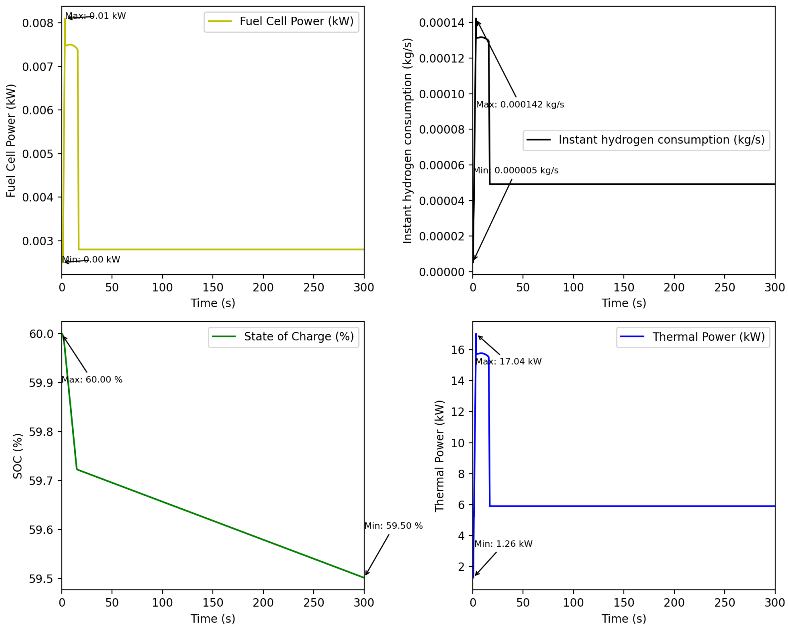


Figure 29: Fuel Cell Power, Instant hydro consumption SOC, and Thermal Power

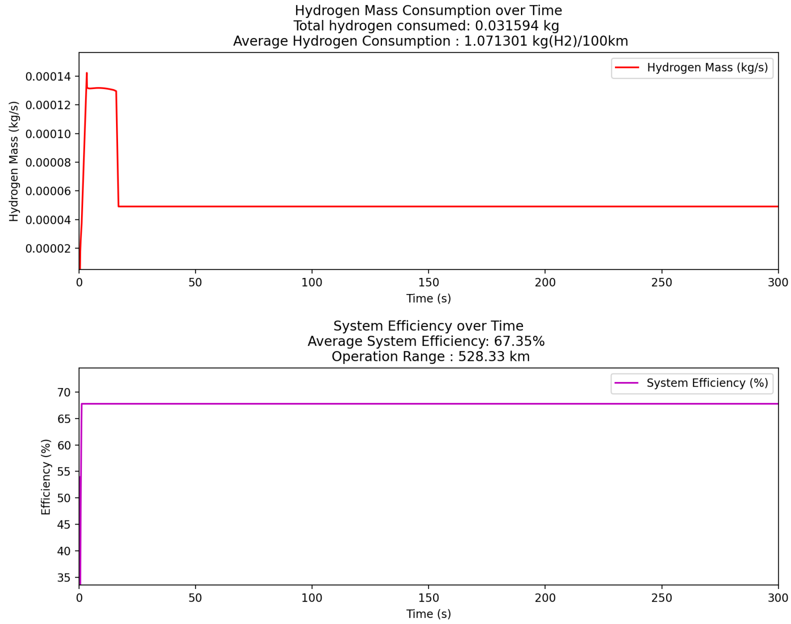


Figure 30: Plot of hydrogen consumption in kg/s and System Efficiency in %

3.1.4 Second: C = 0.40, Cr = 0.014, Battery Charging = 5C, 20C, Battery Capacity = 1.86 kWh, $\alpha = 2°$

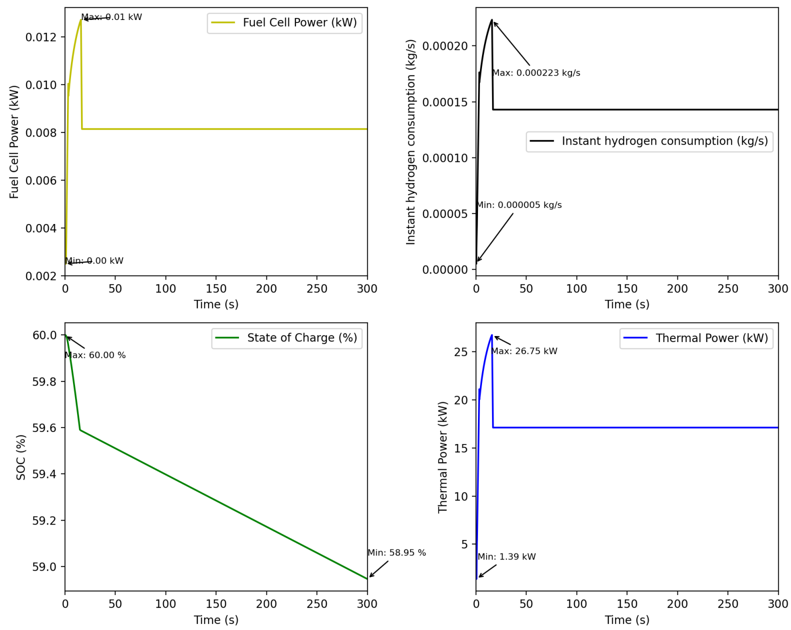


Figure 31: Fuel Cell Power, Instant hydro consumption SOC, and Thermal Power

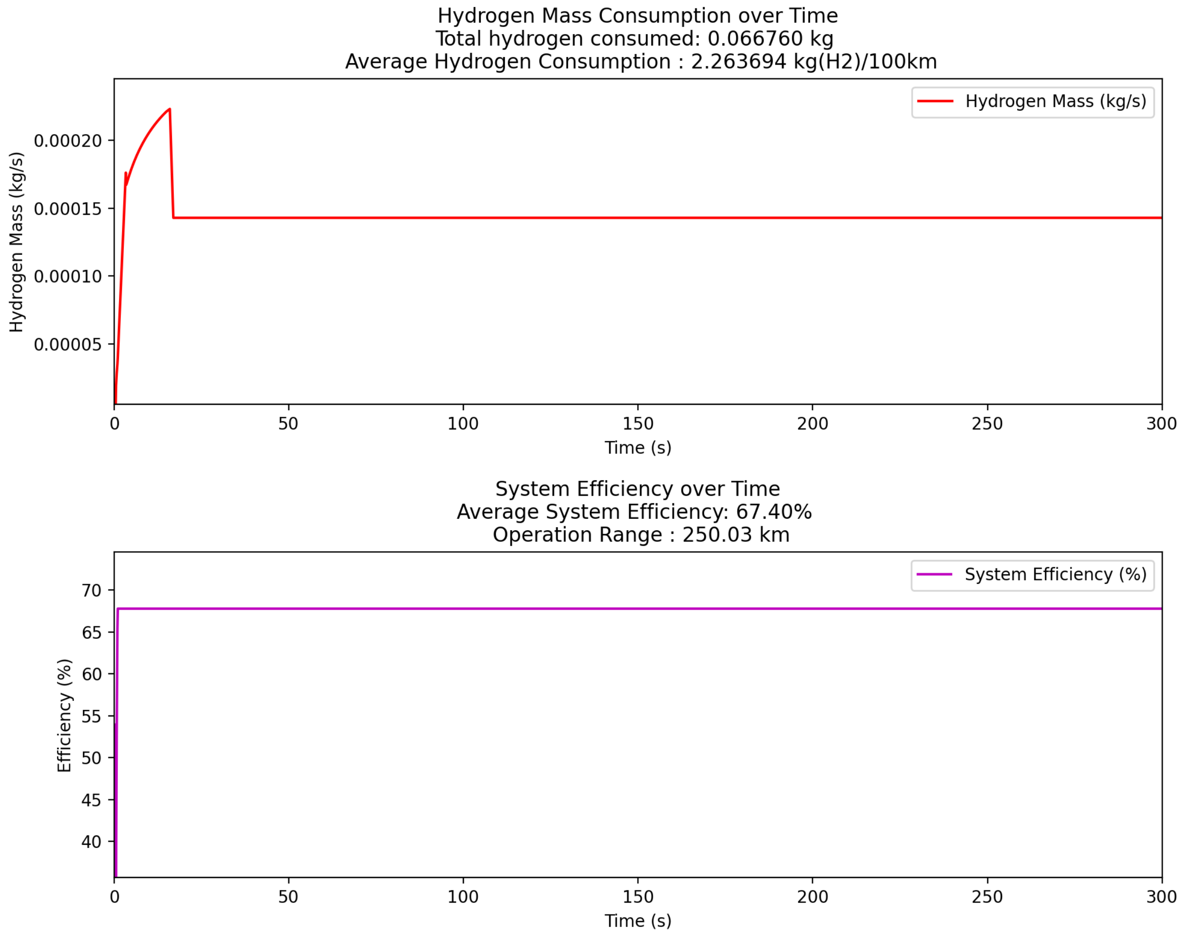


Figure 32: Plot of hydrogen consumption in kg/s and System Efficiency in %

3.1.5 Conclude on the effect of the slope of the road on the vehicle consumption.

After testing the case study of slope road, we can assumed that the slope angel of the road will have a huge effect to the vehicle consumption. If the slope of road got bigger and bigger so, the consumption will got higher and higher.

4 Conclusion

Hydrogen consumption is closely linked to power demand and the stack current in the fuel cell, with the relationship between current and hydrogen use being nearly linear. During the WLTC drive cycle, hydrogen consumption spikes when the vehicle is under higher power loads, and system efficiency varies between 30% and 70%, averaging around 61.74%. This results in an operational range of about 643.82 km. Sensitivity analysis shows that factors like the drag coefficient and rolling resistance have a significant effect on power demand—higher values for either lead to more hydrogen use and reduced driving range. Larger battery capacity helps manage energy better by keeping the SoC more stable, which can slightly improve efficiency. Road conditions, especially the slope, also play a major role; steeper roads require more power, reducing both fuel efficiency and range compared to flat roads. The performance of a PEM fuel cell hybrid vehicle is shaped by multiple factors, primarily related to power demand and energy distribution. The vehicle's power demand arises from forces such as air resistance, rolling resistance, and the climbing force due to road inclines. Power fluctuates depending on driving conditions, like flat or sloped roads, and is calculated based on speed, acceleration, and these resistance forces. A steeper road grade, for instance, dramatically increases the required power, thereby raising hydrogen consumption and lowering fuel efficiency. In such conditions, the vehicle's fuel cell and battery hybrid system work in tandem, distributing power effectively to ensure smooth operation. The battery, typically a Li-ion type, supplements the fuel cell during periods of high power demand, especially during acceleration or steep climbs, while the fuel cell delivers the bulk of power during sustained high-demand scenarios. This ensures that the battery's state of charge (SoC) remains within the safe range of 50%-65%. When the SoC dips too low, the fuel cell compensates, safeguarding battery longevity while maximizing vehicle performance.

Code of the report and Exercise

`https://github.com/Monychot-SARY/PEMFC`