

PEM Fuel Cell system analysis

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1 Calculation of the power demand inside the vehicle

- The specification of the vehicle are the following:

- Weight $M = 2000kg$
- Front area $A = 2.25m^2$
- Drag coefficient (or air penetration coefficient) $C = 0.29$
- Rolling Resistance coefficient $C_r = 0.0115$

- The efforts applied on the vehicle in the rolling direction have to following expression:

- Air penetration :

$$F(t) = \frac{1}{2} \rho_{air} v_{(t)}^2 C A \quad (1.1)$$

with $\rho_{air} = 1.2kg/m^3$

- Rolling resistance :

$$F(t) = MgC_r \cos \alpha \quad (1.2)$$

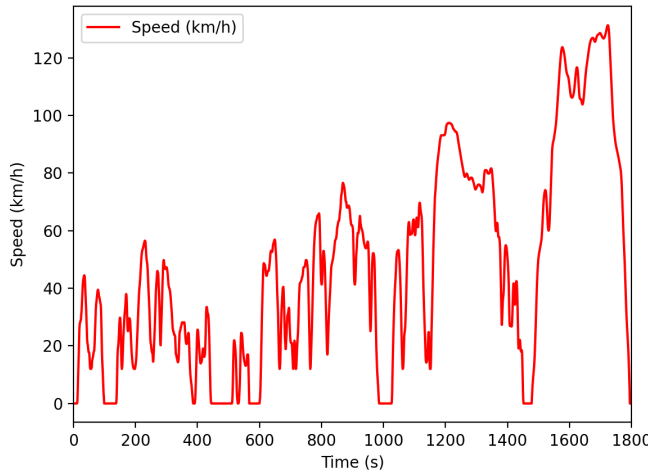
with $g = 9.81m/s^2$ and α the slope angle

- Climbing or descent :

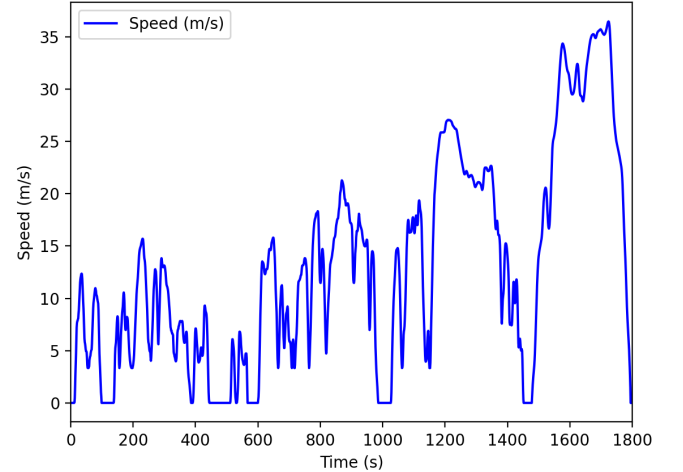
$$F(t) = Mg \sin \alpha \quad (1.3)$$

1.1 Calculate and plot the instant power provided by the vehicle powertrain for the road cycles " WLTC "

- ★ Consider a flat road ($\alpha = 0$)



(a) Speed Profile in km/h



(b) Speed Profile in m/s

Figure 1: Graph of the Speed Profile in km/h and Speed in m/s

1.1.1 Calculation

To calculate the **Instant Power**, we need to study of the *force* that have action on the car. By using second Newton's law with the Figure (2) shown below, we can assume that there are 4 forces that have action on thecar while driving.

- The first force is to make the car move in direction. It called the force from motor or machine of the car (F_{motor})

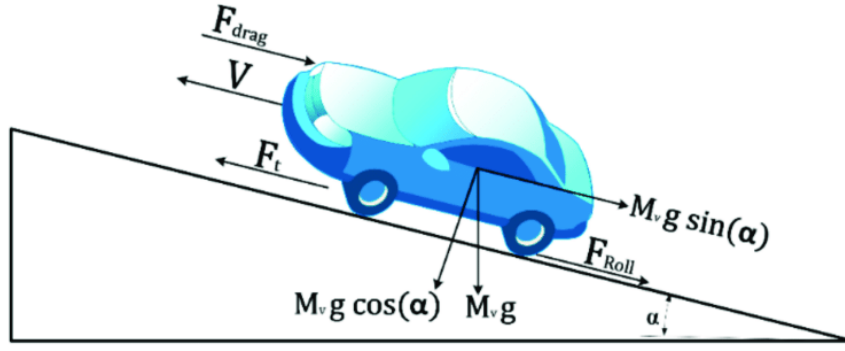


Figure 2: Speed (km/h) of the vehicle powertrain by time (s)

- The second force is the rolling force from the car wheels. It called **Rolling Resistance** ($F_{rolling}$)
- The third force is the climbing or descent force (F_{climb})
- The fourth force is the force from the air friction. we can called it Air penetration (F_{air}).

Using second Newton's law, we can written :

$$\vec{F}_{motor} - \vec{F}_{rolling} - \vec{F}_{climb} - \vec{F}_{air} = m \vec{a} \quad (1.4)$$

$$\vec{F}_{motor} = \vec{F}_{rolling} + \vec{F}_{climb} + \vec{F}_{air} + m \vec{a} \quad (1.5)$$

since a is the acceleration of the vehical in time t , as we written :

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{\Delta v}{\Delta t} \quad (1.6)$$

By using Equation (1.6), we can get the result of acceleration on the Figure (3)

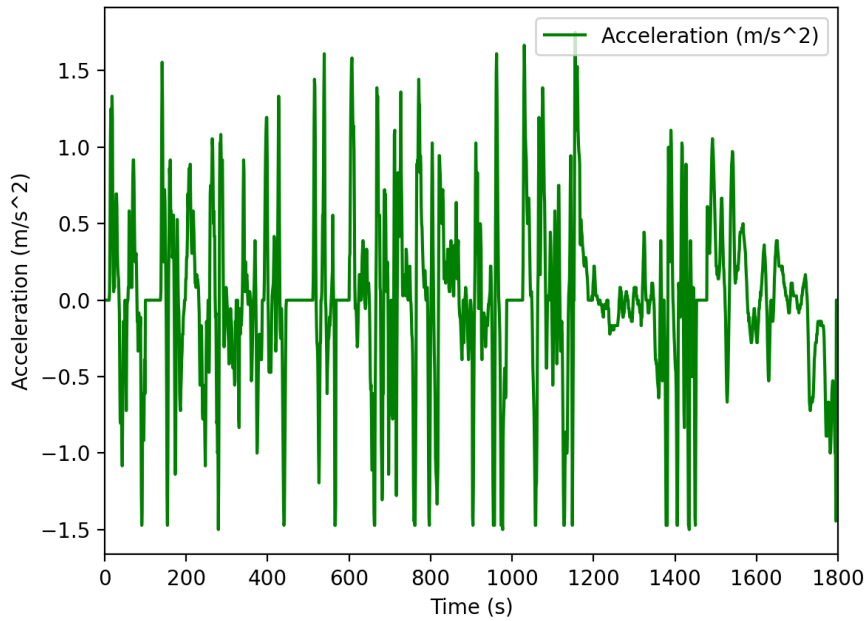


Figure 3: Acceleration (m/s^2) of the vehicle powertrain by time (s)

According to the graph, it shown that the vehical did not have the stable speed drive on the. At the some time (t), the vehical increasing the speed immediately. In contrast, at some time (t), the vehical reducing the speed quickly as shown in the Figure (3).

- For calculate F_{air} by using Equation (1.1) , we got :

$F_{air}(t) = \frac{1}{2}\rho_{air}v^2CA = \frac{1}{2} \times 1.2 \times v_{m/s,t}^2 \times 0.29 \times 2.25$ (1.7) In this section, to calculate F_{air} we need to get the speed in each time t in m/s^2 to analyze in the Equation (1.1.1) By using the Equation (1.1), we got the result of the force air penetration by shown in below graph.

- For calculate $F_{rolling}$, we will be using the Equation (1.2), we got :

$$F_{rolling}(t) = MgC_r \cos(\alpha) = 2000kg \times 9.81m/s^2 \times 0.0115 \times \cos(0^\circ) = \mathbf{225.630} \quad (1.8)$$

For this force, it will be constant in time (t) becuase there is not any parameter in the Equation (1.8) will change in which time.

- For calculate F_{climb} , we will use Equation (1.4) then we got:

$$F_{climb}(t) = Mg \sin(\alpha) = 2000kg \times 9.81m/s^2 \times \sin(0^\circ) = 0 \quad (1.9)$$

By using Equation (1.6), (1.1.1), (1.8) and (1.9) substitution into Equation (1.5). The result of total force was shown by the graph in Figure (4).

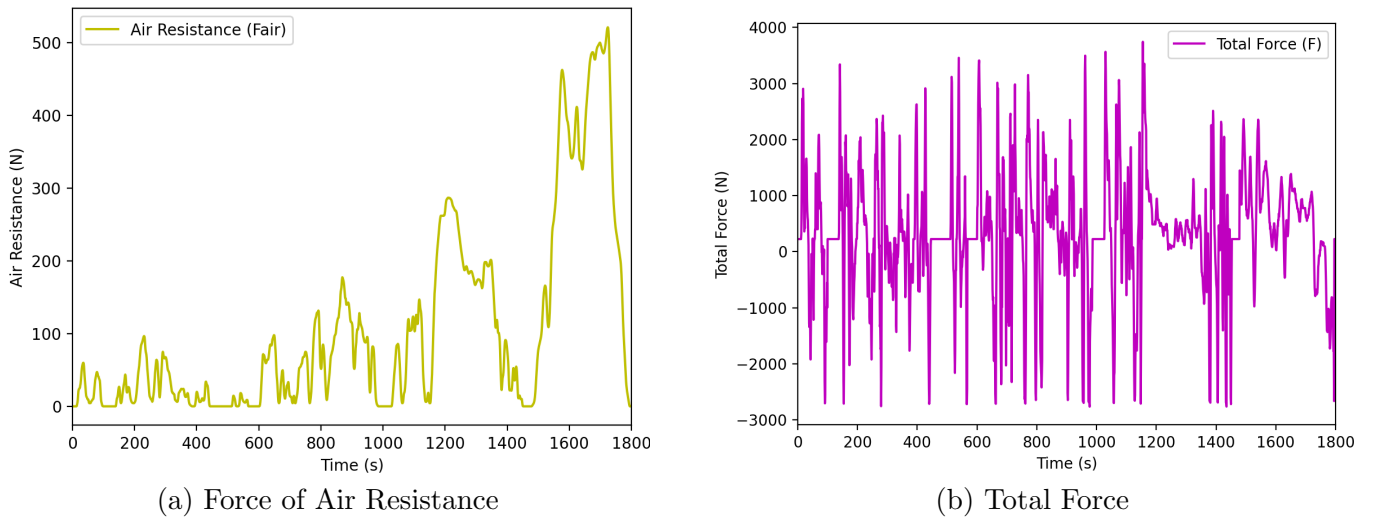


Figure 4: Graph of the Force of Air Resistance and Total Force

To calculate Instant power we are using :

$$P = F \times v \quad (1.10)$$

The result of the instant power calculation will be show at Figure (5)b. The instant power of the vehical are depend on two parameter :

- The total force from the vehical action (N).
- The speed that make the vehical go forward (m/s).

As now, we can write that

$$P_t = F_t \times v_t \quad (1.11)$$

The result of the instant power will show in the Figure (5).

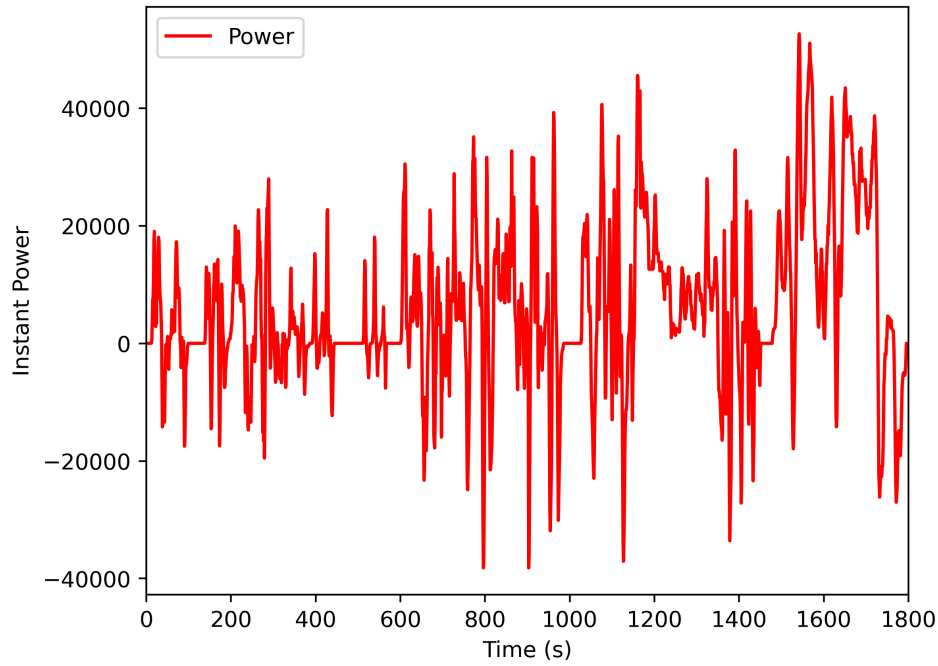


Figure 5: Instant Power of the Vehicle (W)

1.2 Calculate the instant power provide (positive) or received (negative) by the electric hybrid power source.

The vehicle auxiliaries consume an electrical power of 300W (no air conditioning, minimum consumption of all the equipment of the vehicle: sensor, supervisor, etc.) The DC/DC converter efficiency is assumed constant at 90% both direction.

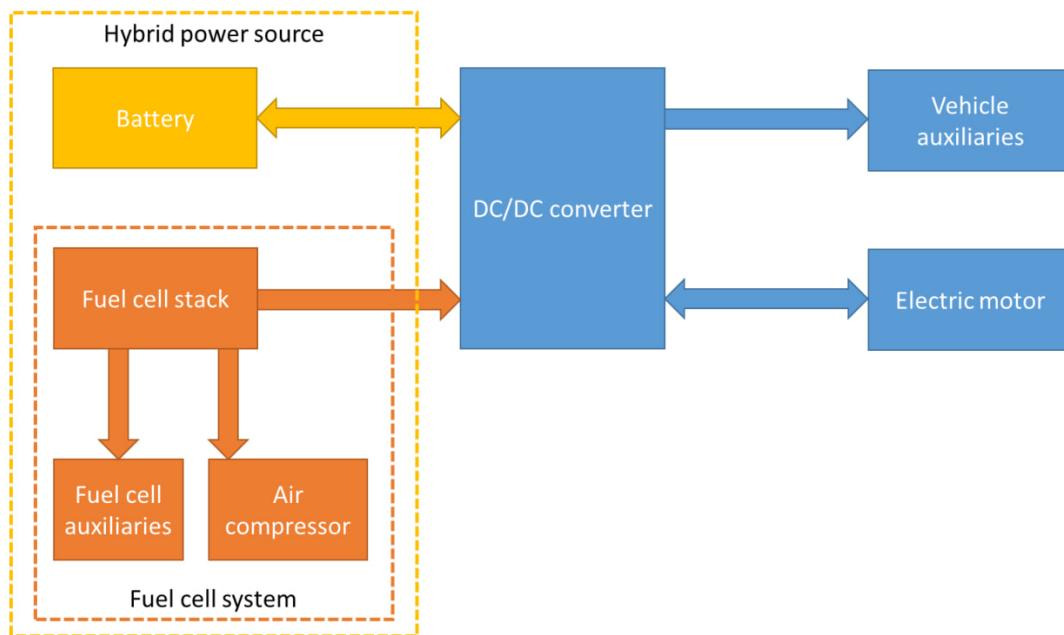


Figure 6: Hybrid system in the vehicle

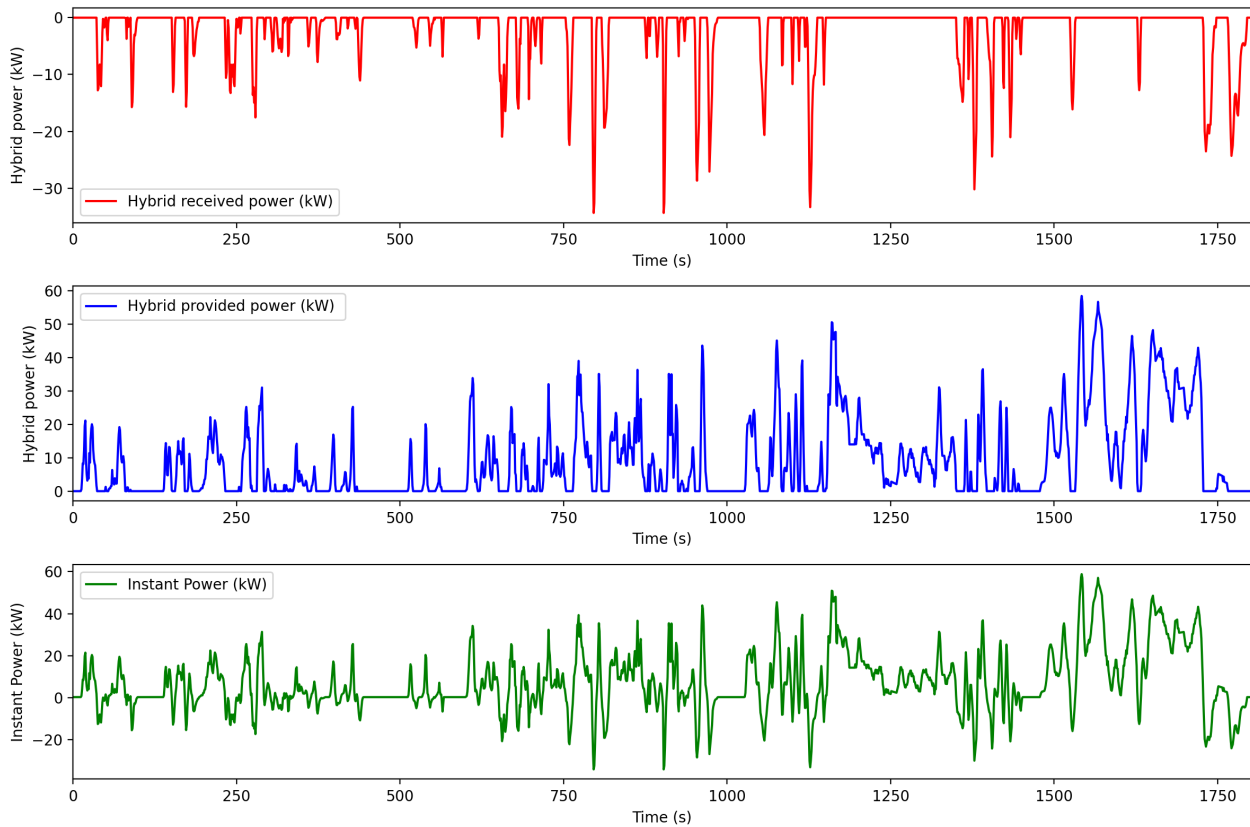


Figure 7: Hybrid working system in power transfer

To find the transfer power in the hybrid system in order to find the provided (positive) or received (negative), we have used instant power from the previous question as data in the Figure (5) with the efficiency of the DC/DC converter. As shown in the Figure (7): At the **first figure** had been shown the **RED line graph** represented the received power (negative) to the hybrid power with the maximum received is **34.365 kW**. Moreover, as shown in the **second figure** shown the **Blue line graph** represented the provided power (positive) from the hybrid system to the motor and auxiliaries. The maximum provided power to the motor was around **58.488 kW**. According to data from the calculation, we can assume that the vehicle mostly consumes power from the hybrid and less provided power to the hybrid system based on the data of speed that provided.

1.3 Calculate and Plot as a function of time : The power of the battery (kW), The power of the fuel cell system (kW), The SOC battery (%)

The energy management strategy of the hybridization between the battery and the fuel cell system is not disclosed by Toyota.

- The battery technology is Li-ion, with a stored energy of **1.24kWh**.
- The test results of Mirai 1 indicate that the battery State of Charge (SoC) is comprised between **50%** and **65%**
- The power delivered by the battery is often close to **5%** of the total power provided by the hybrid power source when $SoC < 55\%$ or **30%** when $SoC > 55\%$
- Discharging power of the battery is approximately **12.4 kW** (or 10C), while the charging power depends on the battery SoC: **10C** if $SoC < 55\%$ or **6C** if $SoC > 55\%$
- The battery provides 0%, 5%, 30% of the total power depending on its SoC and in the limit of its maximum discharging power.

- The EMS avoids values of SoC below 50% and above 65%
- The fuel cell system provides the rest of the power required, except if the power demand is too low: the power provided by the fuel cell system can't be lower than **2.5kW**

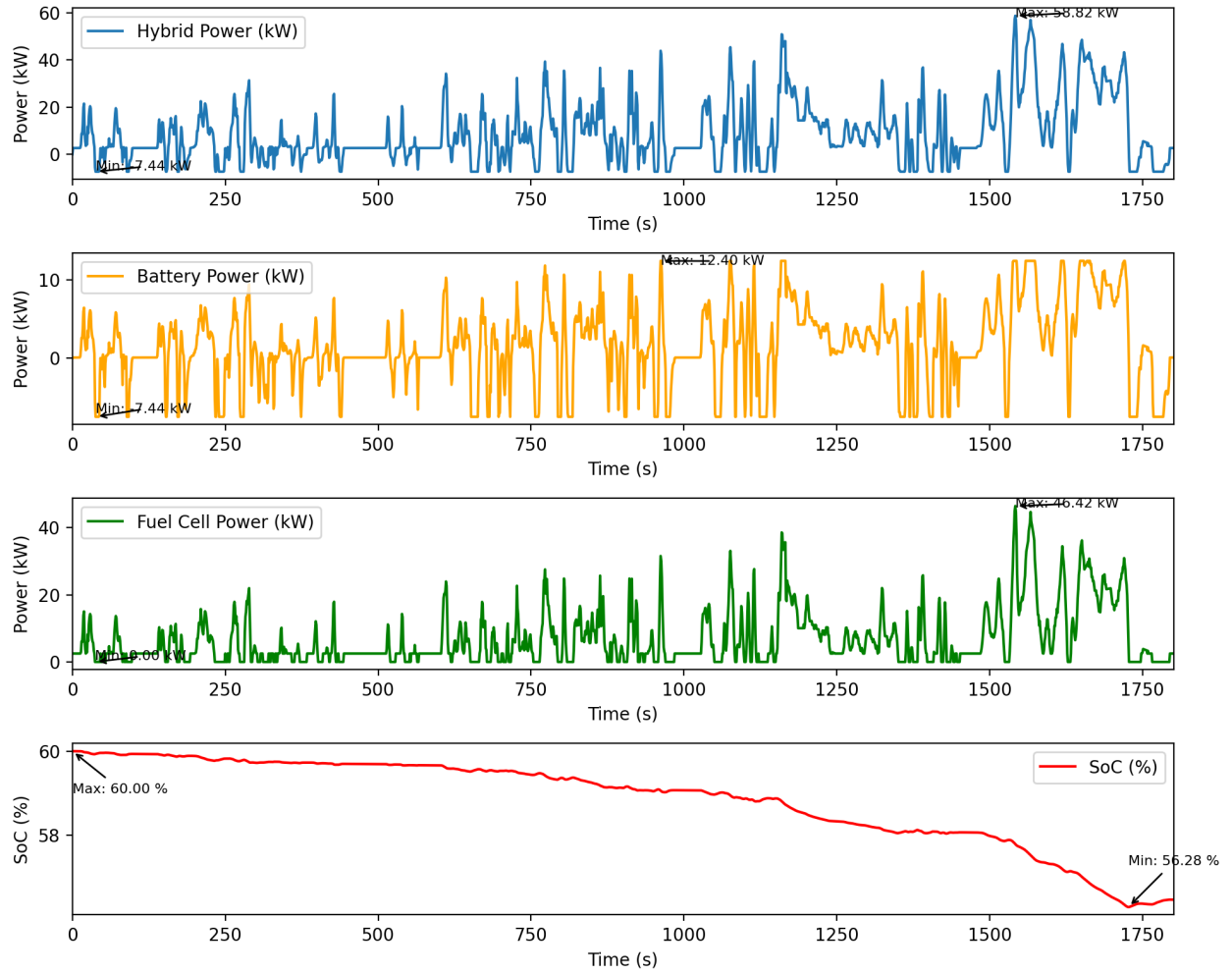


Figure 8: Hybrid system, Battery Management, Fuel Cell power and State of Charge in the Vehicle

As the result that shown in the Figure (8),

- The first plot of the Figure (8) (represented by the by blue curve) show about the **Hybrid Power** working in the system.
- The second plot of the Figure (8) (represented by the by yellow curve) show the characteristic of the charging and discharging of the Battery in the vehicle. The maximum power discharging is **12.4 kW** and the maximum power charging is **7.44 kW**. The status of charge and discharge ws depend on the time and speed per second.
- The third plot of the Figure (8) (represented by the by green curve) show the fuel cell consumption that will be use in the system. The maximum fuel cell consumption is **46.42 kW**. The consumption of the fuel cell variable depend on the time.
- The fourth plot in Figure (8) (represented by the by red curve) illustrates the State of Charge (SoC) of the battery system. By using formula to determine State of Charge in the system, we do it by :

$$SoC_i = \min \left(SoC_{\max}, \max \left(SoC_{\min}, SoC_{i-1} - \frac{\text{power bat}_i}{3600 \cdot \text{batcapacity}} + \frac{\text{demand}_i}{3600 \cdot \text{batcapacity}} \right) \right) \quad (1.12)$$

It starts with an initial SoC of approximately **60%** and gradually decreases to **56.28%** over time, depending on the driving characteristics. The SoC can fluctuate, increasing or decreasing based on the motor's operation and the dynamics of the hybrid system.

1.4 Make the same analysis with the road cycle "130 kmh". Consider two different cases : $\alpha = 0^\circ$ and $\alpha = 2^\circ$

1.4.1 Analysis $\alpha = 0^\circ$

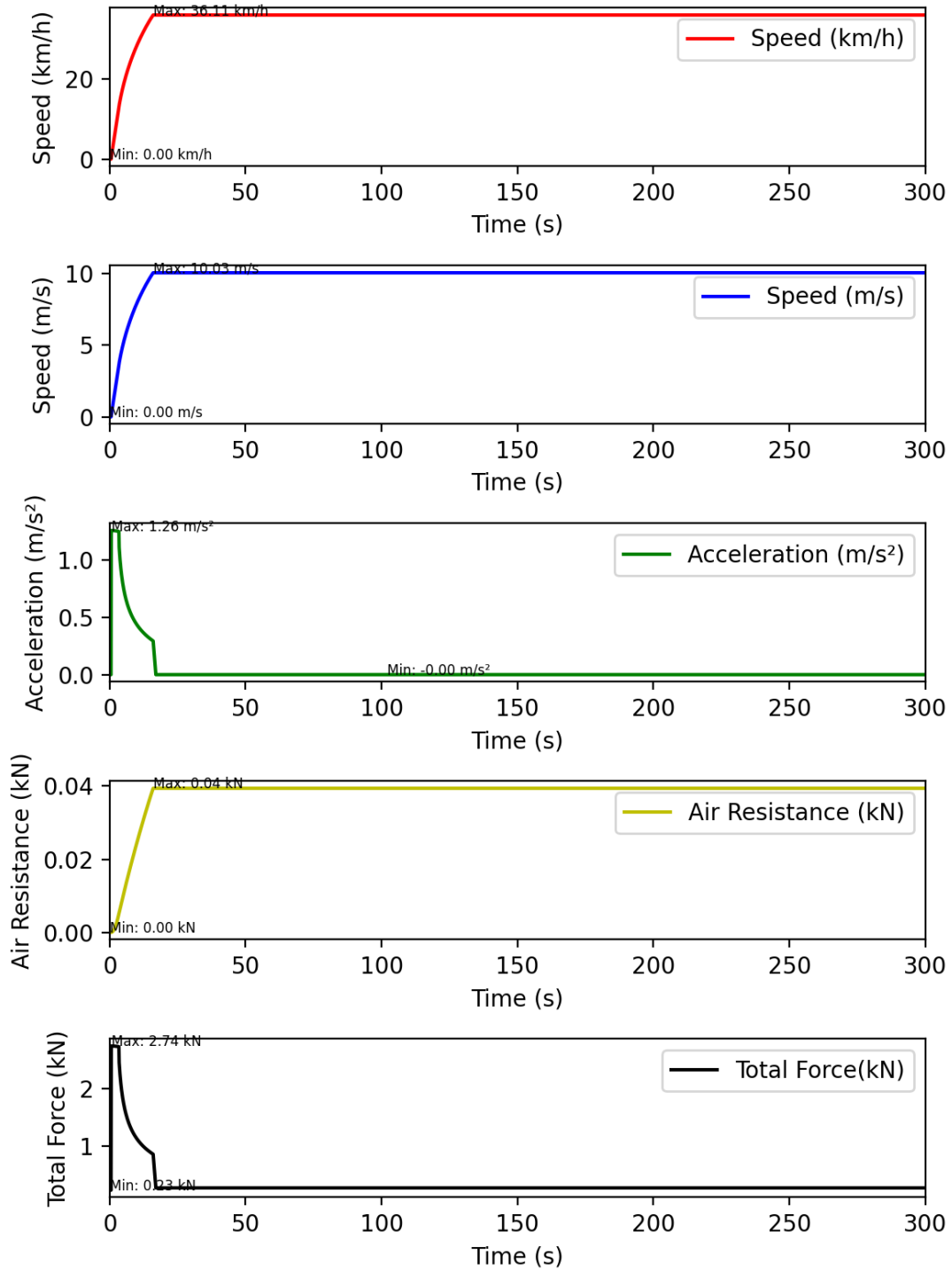


Figure 9: Plots related to Speed, acceleration, air resistance, and Total Force over time

In the Figure (9), it was shown the data curve about the speed within km/h and m/s , acceleration in m/s^2 , Air Resistance and Rolling Resistance.

- The first red and blue curve in the Figure (9) represented to the speed of the vehicle drive over time in km/h and m/s .
- The green curve in the Figure (9) presented to the acceleration of the vehicle that show the characteristic of the vehicle drive. (Increase or Decrease Speed)
- The air resistance calculated by using Equation (1.1) and (1.8) . The result of the calculate shown in the yellow curve of the Figure (9).
- The total Force or Motor Force determined by usig Equation (1.5) and the result of the calculation displayed at the black curve shown that the maximum motor force is **2.74 kN**.

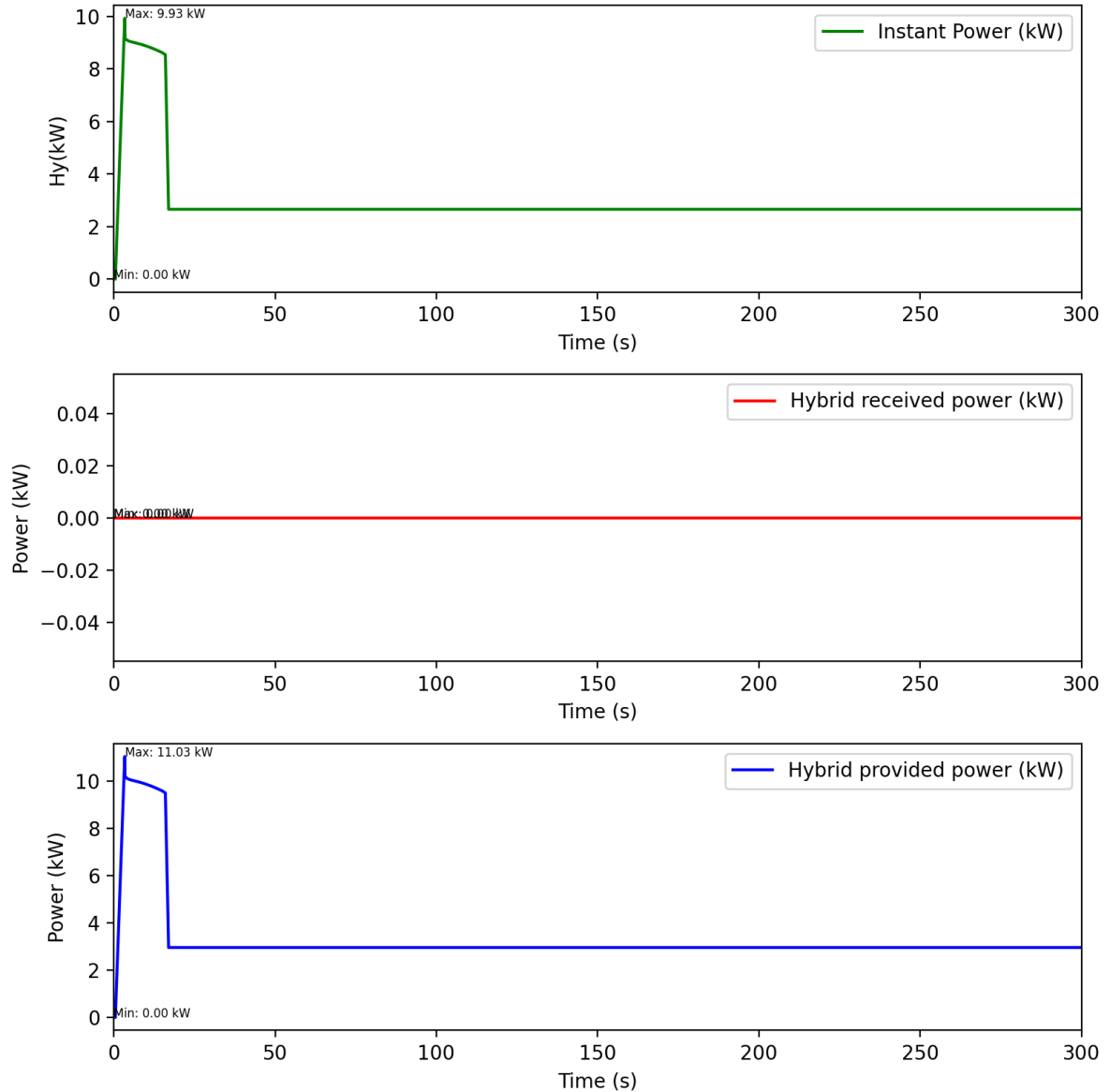


Figure 10: Hybrid working in Power transfer in Vehicle 130kmh

As show in Figure (10), we can assume that vehicle at this speed characteristic shown that the Hybrid system did not receive any power from the motor but it provided full power to the motor.

- **Hybrid Power (kW):**

- This plot shows the total hybrid power delivered by the system.

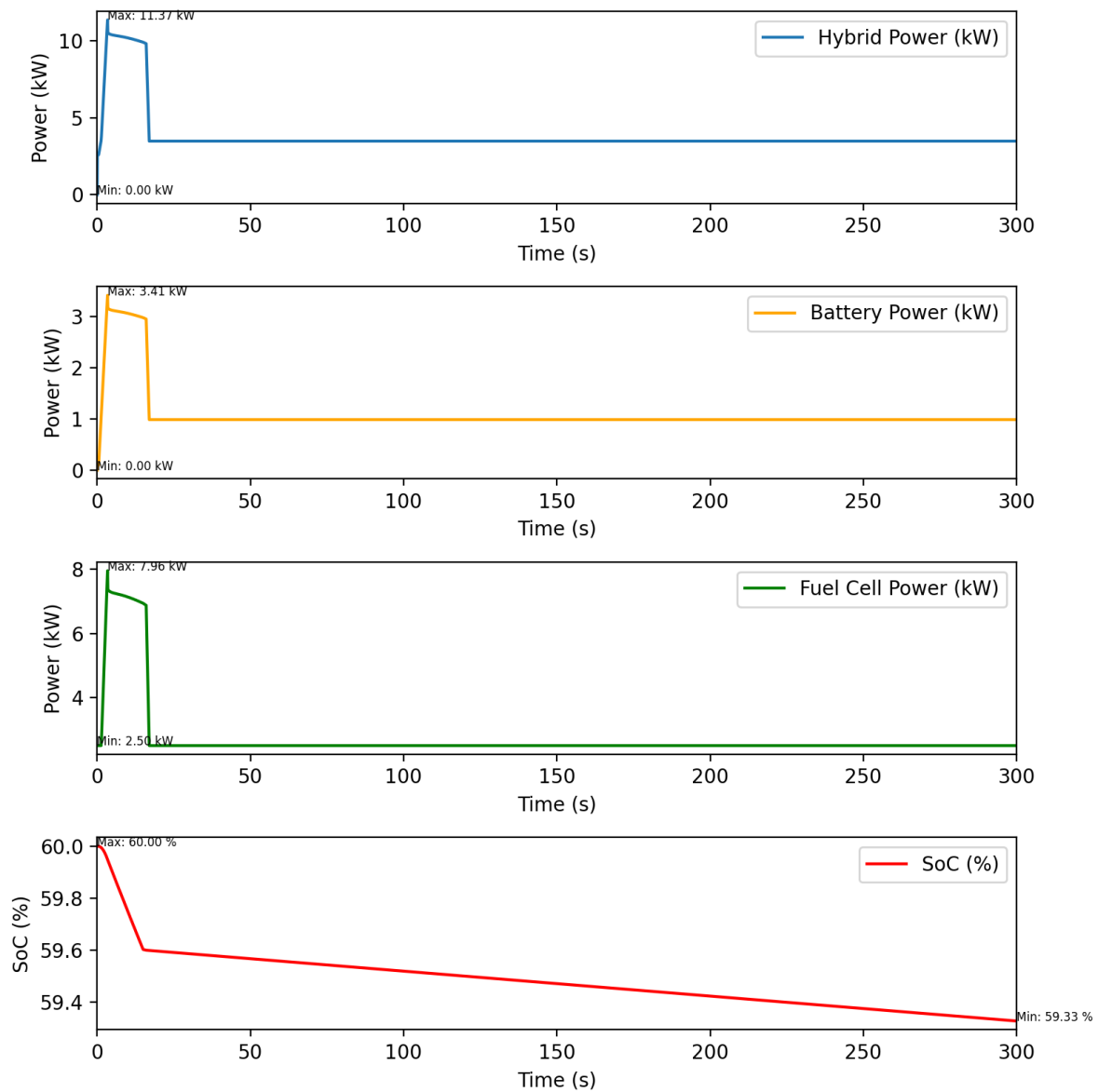


Figure 11: Hybrid system, Battery Management, Fuel Cell power and State of Charge in the Vehicle

- The power reaches a peak of approximately 3.17 kW at the start.
- It rapidly drops to 0 kW after around 25 seconds and remains flat for the rest of the 300-second period.
- **Battery Power (kW):**
 - This plot shows the power supplied by the battery alone.
 - Initially, the battery power spikes to a peak of around 3.31 kW.
 - It sharply decreases to 0 kW after about 25 seconds, staying at that level for the rest of the duration.
- **Fuel Cell Power (kW):**
 - This plot shows the power generated by the fuel cell.
 - The fuel cell power reaches a maximum of about 2.76 kW early in the period.
 - It drops rapidly to 2.5 kW within 25 seconds and remains constant at this level for the remainder of the time.

- **State of Charge (SoC) (%)**:

- This plot represents the battery's state of charge (SoC) over time.
- The SoC starts at approximately 60% and decreases gradually over time.
- By the end of the 300-second period, it reaches around 59.3%.

1.4.2 Analysis $\alpha = 2^\circ$

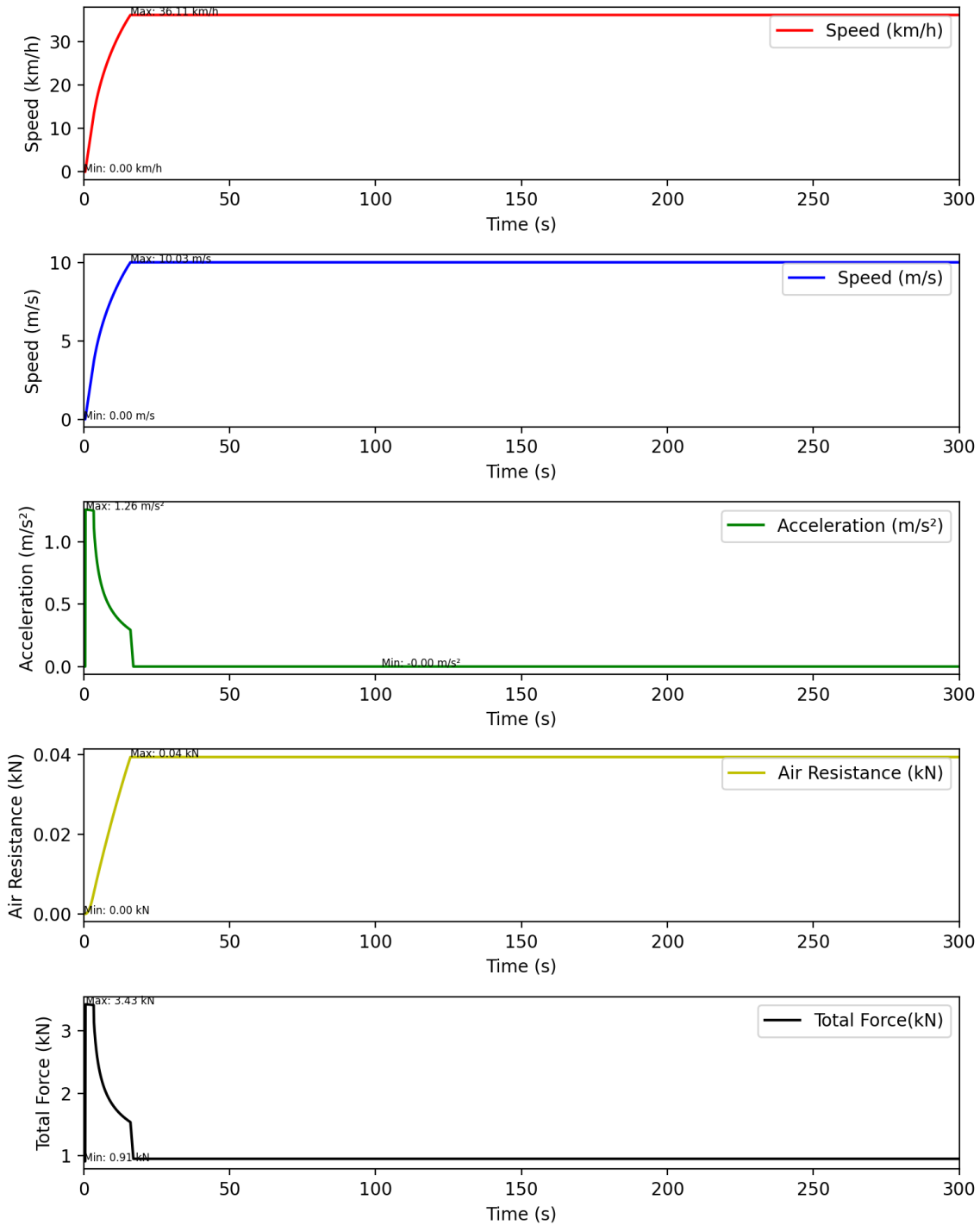


Figure 12: Plots related to Speed, acceleration, air resistance, and Total Force over time

The speed and acceleration are the same from the pervious but there are some change on the air resistance, rolling resistance and climbing resistance.

- For the Air resistance using Equation (1.1), we got the result shown in the Figure (12).
- The Rolling Resistance using Equation (1.2), as the result we can write :

$$F_{rolling}(t) = 200kg \times 9.81m/s^2 \times 0.0115 \times \cos(0^\circ) = \mathbf{225.4926 \text{ N}} \quad (1.13)$$

- The Climbing Resistance using Equation (1.3), as the result we computed :

$$F_{climb}(t) = Mg \sin(\alpha) = 2000kg \times 9.81m/s^2 \times \sin(2^\circ) = \mathbf{684.811 \text{ N}} \quad (1.14)$$

- The total Force or Motor Force determined by usig Equation (1.5) and the result of the calculation displayed at the black curve shown that the maximum motor force is **3.43 kN**.

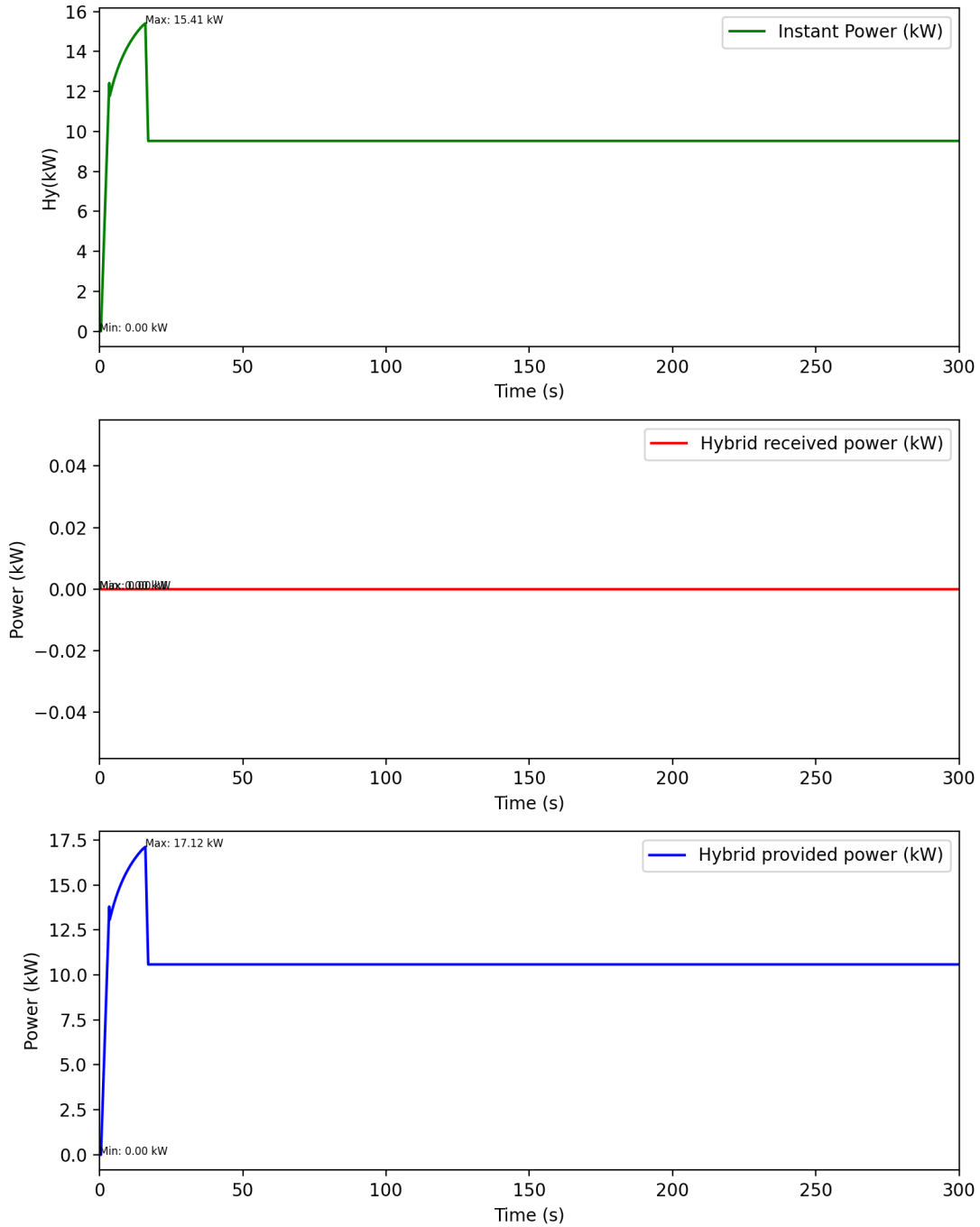


Figure 13: Hybrid working in Power transfer in Vehicle 130kmh

As show in Figure (13), we can assume that vehicle at this speed characteristic shown that the Hybrid system did not receive any power from the motor but it provided full power to the motor.

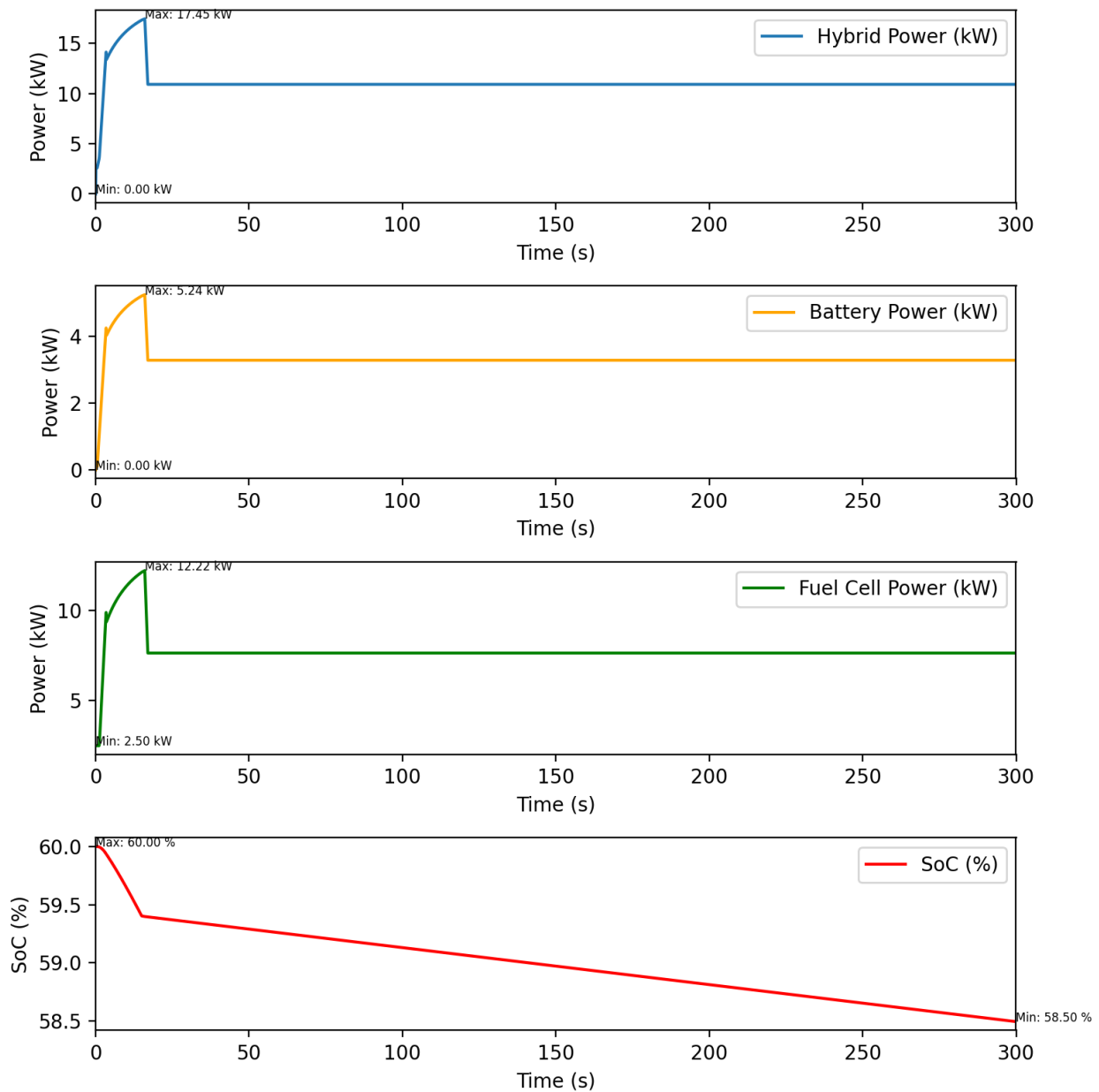


Figure 14: Hybrid working in Power transfer in Vehicle 130kmh

- **Hybrid Power (kW) - Blue Curve (Top Plot):**

- The hybrid power increases sharply from 0 to about **17.45 kW** within the first few seconds.
- After this peak, the power stabilizes around **16 kW** and remains constant for the rest of the 300-second interval.
- This curve represents the total power output from the hybrid system.

- **Battery Power (kW) - Orange Curve (Second Plot):**

- The battery power rises rapidly to a maximum value of **5.24 kW** at the beginning and then drops slightly, stabilizing around **4.5 kW**.
- The battery maintains this output throughout the time period shown.
- This shows the power contribution from the battery over time.

- **Fuel Cell Power (kW) - Green Curve (Third Plot):**

- The fuel cell power ramps up initially to about **12.22 kW**.
- After reaching this maximum value, the fuel cell output drops slightly and stabilizes at around **10 kW** for the remainder of the time.
- This reflects the power contribution from the fuel cell system.
- **State of Charge (SoC %) - Red Curve (Bottom Plot):**
 - The SoC starts at **60.0%** and decreases gradually over time.
 - By the end of the 300-second period, the SoC has dropped to **58.5%**.
 - This represents the decline in the battery's charge level over the duration of operation.

2 Analysis of the fuel cell system with WLTC profile

We assume that the fuel cell system of the vehicle has the following characteristics, based on some specifications of the Mirai 2 and an experimental study of the Mirai 1:

- Hydrogen storage: 5.6 kg stored at 700 bars
- Molar mass of dihydrogen: 2.016 g/mol
- Enthalpy of hydrogen combustion in oxygen:
 - LHV: 242 kJ/mol
 - HHV: 285 kJ/mol
- The hydrogen loss (mainly in the purge) represents 2% of the consumed hydrogen
- Air stoichiometry is 1.5
- Atmospheric pressure is 1 bar
- Oxygen molar fraction in ambient air is 21%
- Air compressor's efficiency is 60% (comprising both compression efficiency and electric motor efficiency)
- The air pressure inside the stack is variable, depending on the current density (data available in the file "BEfuelcellsystem.xlsx")
- The parasitic power consumed by the system auxiliaries depends on the stack current (data available in the file "BEfuelcellsystem.xlsx")
- The stack is composed of 330 cells of 273 cm²
- The cell polarization curve has been studied during the vehicle operation. A simplified version is given below (data available in the file "BEfuelcellsystem.xlsx"):

2.1 Calculate and Plot as a function of the stack current:

2.1.1 The electrical power consumed by the air compressor

To calculate of the compression power are proportional to the current:

$$P_{comp}(t) = \frac{1}{\eta_{sys}(t)} q_{m_{air}(t)} c_p T_e \left(\tau^{\frac{\gamma-1}{\gamma}} - 1 \right) \quad (2.1)$$

Where we got :

- Air mass Flow :

$$q_{m_{air}}(t) = M_{air} F_{air}(t) = M_{air} \frac{v_{air}}{x_{O_2}} \frac{N_{cell} I(t)}{4\mathcal{F}} = M_{air} \frac{v_{air}}{x_{O_2}} \frac{P_{sys}(t)}{2v_{H_2}(t) \rho_{sys} \Delta H} \quad (2.2)$$

- Heat Capacity:

$$c_p = \frac{\gamma}{\gamma - 1} \frac{R}{M} \quad (2.3)$$

- Compression ratio:

$$\tau = \frac{p_{out}}{p_{in}} \quad (2.4)$$

Get Equation (2.2), Equation (2.3), Equation (2.4) Substitution into Equation (2.5)

$$P_{comp}(t) = \frac{1}{\eta_{sys}(t)} \frac{v_{air}}{x_{O_2}} \frac{N_{cell} I(t)}{4\mathcal{F}} \frac{\gamma}{\gamma - 1} R T_e \left(\tau^{\frac{\gamma-1}{\gamma}} - 1 \right) \quad (2.5)$$