GROUP k-SCHEMES

1. Transporters

Definition 1.1. Let $\mathfrak{G}$ be a monoid k-functor and let $\alpha : \mathfrak{G} \times \mathfrak{X} \to \mathfrak{X}$ be an action of $\mathfrak{G}$ on a k-functor $\mathfrak{X}$. Suppose that $\mathfrak{Y}_1, \mathfrak{Y}_2$ are k-subfunctors of $\mathfrak{X}$. For every k-algebra A we define

$$\mathrm{Transp}_{\mathfrak{G}}\left(\mathfrak{Y}_1, \mathfrak{Y}_2\right)(A) = \left\{g \in \mathfrak{G}(A) \,\middle|\, \alpha_g\left(\mathfrak{Y}_1(A)\right) \subseteq \mathfrak{Y}_2(A)\right\}$$

where as usual α_g is a slice of α along g. Then $\mathrm{Transp}_{\mathfrak{G}}\left(\mathfrak{Y}_1, \mathfrak{Y}_2\right)$ is a k-subfunctor of $\mathfrak{G}$. It is called *the tranporter of $\mathfrak{Y}_1$ into $\mathfrak{Y}_2$ with respect to α.*

2. Chevalley's Theorem

Now are ready to prove the following fundamental result.

Theorem 2.1 (Chevalley's theorem)**.** *Let k be a field and let $\mathbf{M}$ be an affine monoid k-scheme of finite type over k. Suppose that $\mathbf{N}$ is a closed monoid k-subscheme of $\mathbf{M}$. Then there exists a finitely dimensional vector space V over k and a morphism of monoid k-functors $\rho : \mathfrak{P}_{\mathbf{M}} \to \mathcal{L}_V$ such that the following assertions hold.*

(1) *ρ is closed immersion of k-functors.*

(2) *There exists a subspace W of V such that $\mathbf{N} = \mathrm{Transp}_{\mathbf{M}}(W, W)$.*

For the proof we need the following elementary result.

Lemma 2.1.1. *Let I be an ideal of $\mathbf{N}$ in $k[\mathbf{M}]$. We consider $k[\mathbf{M}]$ as a representation of $\mathbf{M}$. Then*

$$\mathbf{N} = \mathrm{Transp}_{\mathbf{M}}(I, I)$$

Proof. □