

Assumptions: Each cell only contains 1 digit

Each number can only be used once

Every number from 1 to 9 must be used

Model: In order for the sum of two neighbouring cells to be odd, odd and even numbers need to follow a checkerboard pattern, and since there are 5 odd numbers and 4 even numbers from 1 to 9, it needs to follow such a pattern:

odd	even	odd
even	odd	even
odd	even	odd

Solution: For the sum of the numbers in the gray squares to be the largest possible, the 3 largest odd numbers and largest even number from 1-9 must be used to fill the gray squares, namely 8, 5, 7, and 9. Thus, the numbers left to fill the white squares are 1, 2, 3, 4, and 6. Therefore, the sum of the numbers written on the white cells is 16.

$$1 + 2 + 3 + 4 + 6 = 16.$$

Discussion: The assumptions made above are important, as it is not mentioned that each cell can only contain 1 digit. If digits can be placed next to each other to form larger numbers such as 987, there will not be a finite number of solutions.

Furthermore, while the question states that Olivia wants to number from 1 to 9 the cells of the board, it is not explicitly mentioned that each number can only be used 1 time and that every number must be used, otherwise other solutions will be possible.