第四届“认证杯”数学中国

数学建模国际赛

承 诺 书

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我们的参赛队号为： **1 5 8 4**

我们选择的题目是： **A**

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队员 1：

队员 2：

队员 3：

参赛队教练员 (签名)： （无）

第四届“认证杯”数学中国

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编 号 专 用 页

参赛队伍的参赛队号：（请各个参赛队提前填写好）：

1 5 8 4

竞赛统一编号（由竞赛组委会送至评委团前编号）：

竞赛评阅编号（由竞赛评委团评阅前进行编号）：

The slimline seats on the airplane

Abstract:

Some airlines are introducing new “slimline” seats in economy class,which theoretically allow airlines to increase capacity.However,many passengers have expressed displeasure with these seats.Because the common point of them is a thinner backplate and less padding which significantly affecting passengers’ back prper support,and cause umcomfort during the journey.

In order to make the seat more comfortable and improve service quality,here we accomplish two tasks:

·**Task 1 : Optimize the best seat backplate curve**

When the body back vertebra is in a natural form, the back tension is the lowest, so that passengers will be more comfortable during the voyage. Considering human body in different positions, the size of the compressive force, shear force and bending moment that the everywhere of the vertebra bear,we can determine the bending state of the vertebra. Respectively making mechanical analysis of rigid cylinder vertebrae and flexible cylinder intervertebral disc, then using recursive method, then the shear force ,bending moment and compressive force on each vertebra and intervertebral disk are obtained, respectively summing three physical quantities as total shear force, total bending moment and total compressive force. Then we will summing the total shear force, total bending moment and total compressive force as the value of discomfort at that time. So that we can get a mathematical model to assess passenger discomfort,at this time the model is an optimization problem,we can solving this problem by software Lingo,to get a series of the angle value when back is most comfortable.

Then,we use Matlab, Fourier fitting and cubic spline interpolation to fit the series of angle value,so we can draw an optimal curve,this curve is not only the vertebra’s curve when body feel most comfortable, but also is the optimal seat backplate curve that can make passengers most comfortable.

·**Task 2 : Design a well-looking advertising material**

Showing the best seat backplate curve obtained in Task 1、the design of the “slimline” seats, in the meantime introducing our features and advantages of the "slimline" seats.

While considering the aesthetics of advertising", there are illustrations and text description in the material,the expression of product is vivid,you can intuitively access to the information.

Key words: Ergonomics . Biomechanics . Optimization problem, Lingo , Fourier fitting , Cubic spline interpolation ,Matlab

1. Introduction

1.1 Why some airlines are introducing "slimline" seats

With the development of science and technology, Aviation technology also got great development, therefore, relevant airplane seats technology research has become the focus of attention. For the requirement of the airplane seats, first of all should be safe, the second is to let the passengers sitting comfortable, it has a strict technical requirements and specifications. Thus for airlines, air passenger seat's selection and maintenance are particularly important. Generally, the airlines when choosing seats, in addition to consider safety, comfort, diversity, there are some will tend to choose light, easy to maintain, the seat - "slimline" seats to reduces the order of the passenger seat costs and maintenance costs.

1.2 The characteristics of the "slimline" seats

"Slimline" seats, in addition to weighing less, theoretically allow airlines to increase capacity without significantly affecting passenger comfort. These seats may or may not feature moveable headrests, and generally do not feature adjustable lumbar support. The common point of them is a thinner backplate and less padding.

1.3 The application of human body engineering in the design of the seat backplate

The key to the design of seat backplate is the seat backplate curve. When designing backplate , we usually use the posture that the vertebra in a natural extension of the state as a basic. Not only does seat backplate support the lumbar, but also support the back and head, otherwise it can't maintain a natural posture. Its tilt angle shall not be less than 90 °in any case, with the improvement of its the rest function will increase, usually 95 ° ~ 120 ° is appropriate. Relationship between tilt angle and supporting point can be summarized into the following points:

- When the tilt angle is small, select supporting point in the second to the third lumbar vertebra;
- When the tilt angle is large, Supporting point moves to the lower part of the thoracic spine;
- When the tilt angle exceeds 114 °, it is necessary for the lumbar and lower thoracic and head three points support.

According to the above points, support position is determined,so that we can design the geometry of the seat backplate, we can also adjust the hardness of the backplate, such as make the supporting point larger hardness than some other parts, so we can make the seat backplate more comfortable.

1.4 The application of biomechanics in the design of the seat backplate

Biomechanical analysis shows,when the angle between the seat and the backrest is between 90 ° -120 °;the shear force between seat and Ischial tuberosity can be completely eliminated.At this time buttock don't has sliding trend. In this case, we do force analysis of vertebrae and intervertebral disc will consider three main force, force pressure on vertebra and intervertebral disc, shear force and the stress when intervertebral disc bending. According to the experience, when the vertebra suffered smaller force and bending moment, people will feel more comfortable.

2.The Description of Problem

After researching on the problem, we believe that the key to solving problems is obtaining the optimal curve of the seat backplate, in order to achieve the airplane's goals that they can take advantage of the "slimline" seats without changing the main internal structure and at the same time make the seat more comfortable. Secondly, we will write an advertising material, to introduce the features and advantages of our design of this "slimline" seats.

2.1 The optimal seat backplate curve

Set an appropriate seat backplate curve can reduce back tension, so that passengers will be more comfortable during the voyage.

The body will feel most comfortable when the body back vertebra is in a natural form, to achieve this, the shape of the seat should be consistent with the natural curvature of the vertebra. So, if the seat backplate curve is close to the human ecological curve, it will make the ride comfort greatly improved.

To measure comfort， we consider the human body in different positions, the size of the compressive force, shear force and bending moment that the everywhere of the vertebra bear, established a mathematical model about the resulting physical discomfort. The focus is to analyze the force conditions of the 24 vertebra and Intervertebral disc(in this case the series of angle between them and vertical direction are different): Respectively stress analysis for the rigid cylinder vertebra and elastic disc, at the same time use the recursive method, then the shear force ,bending moment and compressive force on each vertebra and intervertebral disk are obtained ,then respectively summing three physical quantities as total shear force, total bending moment and total compressive force. Then we will summing the total shear force, total bending moment and total compressive force as the value of discomfort at that time. While the lowest value of discomfort corresponding to a set of angle which can be uniquely determined by the bending status where the composition of forces on the vertebra is smallest.

Through finding optimal solution and fitting the data, we can find a smooth curve. This curve is not only the curve of vertebra when it is most comfortable, but also can be seen as the seat back plate curve that can make passengers most comfortable. As a result, we can get the complete mathematical model which not only can assessment the discomfort value of vertebra but also can obtain the optimal seat backplate curve.

2.2 Advertising material

The advertising materials will include the optimal seat backplate curve which we got through the mathematical modeling, the design drawings of the "slimline" seats and our features and advantages of the "slimline" seats

3.Fundamental Assumption

1. Each section vertebrae are rigid, equal length; Intervertebral disc is elastic material.
2. Radius of the spine is the same everywhere.
3. Ignore the weight of intervertebral disc, and the mass of the vertebra are uniformly dispersed.
4. Vertebrae stress deformation does not occur; The deformation of intervertebral disc is pure bending, don’t considering the transverse twist, etc.
5. All passengers are normal adults (24 block vertebrae).
6. Passengers pose is still depends on the back of the chair for a long time.

4.Symbols

Symbol	Meaning
N_{ui}	The compressive force on the upper surface of vertebra
N_{di}	The compressive force on the lower surface ofvertebra
Q_{ui}	The shear force paralleled to the upper surface on the vertebra
Q_{di}	The shear force paralleled to the lower surface on the vertebra
$\alpha_i (i = 1,2, \dots \dots ,24)$	The angle between the vertical direction of the vertebra and The actual vertical direction
F_{x1}	The resultant force on the vertebra in the horizontal direction
F_{y1}	The resultant force on the vertebra in the vertical direction
λ	density of bone
a	The length of each vertebra
g	Acceleration of gravity
F_{x2}	The resultant force on the intervertebral disk in the horizontal direction
F_{y2}	The resultant force on the intervertebral disk in the vertical

	direction
R	The radius of the intervertebral disk's cross section
M	The bending moment acting on the intervertebral disc

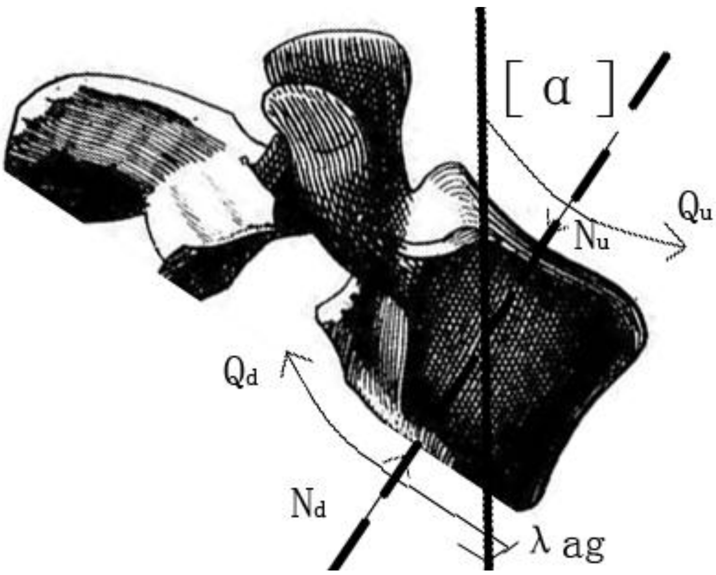
5.Modeling

Based on the assumption, when we do mechanical analysis of vertebrae and intervertebral disc will consider three main force, force pressure on vertebra and intervertebral disc, shear force and the stress when intervertebral disc bending. According to the biomechanics and human body engineering, when the vertebra suffered smaller force and bending moment, people will feel more comfortable. We calculate the bending moment, compressive force and shear force on the vertebra and intervertebral in each unit, then we will summing the total shear force, total bending moment and total compressive force as the value of discomfort at that time. So the value of discomfort will increase with the increase of the sum of total compressive force, total shear and total bending moment.

Firstly we do mechanical analysis for vertebra and intervertebral disk respectively, providing constraints for seeking an optimal curve of the seat backplate.

5.1Model Basic

5.1.1Mechanical analysis of vertebra



(figure 1 : Mechanical analysis of a vertebra）

As shown in figure, make mechanical analysis of vertebra.

According to the assumption, vertebrae are rigid and stress deformation does not occur. So there are 4 force on the vertebra:

1. The compressive force on the upper surface N_{ui}
2. The compressive force on the lower surface N_{di}
3. The shear force paralleled to the upper surface Q_{ui}
4. The shear force paralleled to the lower surface Q_{di}

According to the assumption, passengers are still depends on the back of the chair for a long time. So the force on the vertebra is balanced. At this time, the angle between the vertical direction of the vertebra and the actual vertical direction is $\alpha_i (i = 1,2, \ldots \ldots ,24)$.

Suppose the resultant on the vertebra in the horizontal direction is F_{x1}, while that in vertical direction is F_{y1} ,so that:

$$\begin{cases} F_{x1i} = 0 \\ F_{y1i} = 0 \end{cases} \quad (i = 1,2, \ldots \ldots ,24)$$

$$\begin{cases} \sum_{i=1}^{24} F_{x1i} = 0 \\ \sum_{i=1}^{24} F_{y1i} = 0 \end{cases}$$

Get the first part of the constraint conditions of the optimization problem:

$$\begin{cases} (Q_{ui} - Q_{di}) \cos \alpha_i - (N_{ui} - N_{di}) \sin \alpha_i = 0 \\ (Q_{ui} - Q_{di}) \sin \alpha_i + (N_{ui} - N_{di}) \cos \alpha_i + \lambda \mathrm{ag} = 0 \end{cases}$$
$$(i = 1,2, \ldots \ldots ,24)$$

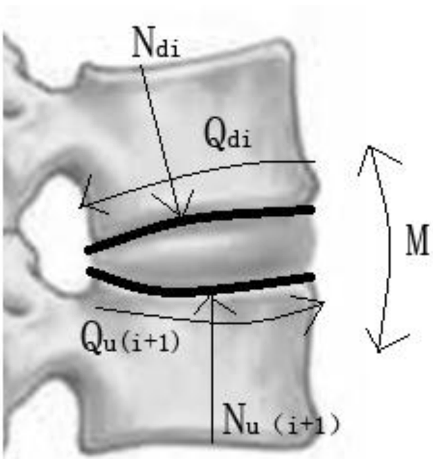
After simplification:

$$\begin{cases} N_{di} - N_{ui} = \lambda \mathrm{ag} \cdot \cos \alpha_i \\ Q_{ui} - Q_{di} = \lambda \mathrm{ag} \cdot \sin \alpha_i \end{cases} \quad (i = 1,2, \ldots \ldots ,24)$$

And according to the assumption:

$$\alpha_3 = \alpha_{12} = \alpha_{21} = 0.$$

5.1.2Mechanical analysis of intervertebral disk



(figure2 : Mechanical analysis of an intervertebral disk）

As shown in figure, make mechanical analysis of intervertebral disk.

According to the assumption, intervertebral disk is rigid and stress deformation does not occur.

1. The compressive force on the upper surface N_{di}
2. The compressive force on the lower surface $N_{u(i+1)}$
3. The shear force paralleled to the upper surface Q_{di}
4. The shear force paralleled to the lower surface $Q_{u(i+1)}$

According to the assumption, passengers are still depends on the back of the chair for a long time. So the force on the intervertebral disk is balanced. At this time, the angle between the vertical direction of the upper surface and lower surface of the intervertebral disk and the actual vertical direction are respectively α_i and $\alpha_{i+1} (i = 1,2, \ldots \ldots ,23)$.

Suppose the resultant on the intervertebral disk in the horizontal direction is F_{x2}, while that in vertical direction is F_{y2} ,so that:

$$\begin{cases} F_{x2i} = 0 \\ F_{y2i} = 0 \end{cases} \quad (i = 1,2, \ldots \ldots ,23)$$

$$\begin{cases} \sum_{i=1}^{23} F_{x2i} = 0 \\ \sum_{i=1}^{23} F_{y2i} = 0 \end{cases}$$

Get the second part of the constraint conditions of the optimization problem:

$$\begin{cases} Q_{di} \cdot \cos\alpha_i + N_{u(i+1)} \cdot \sin\alpha_{i+1} - N_{di} \cdot \sin\alpha_{i+1} - Q_{u(i+1)} \cdot \cos\alpha_i = 0 \\ N_{di} \cdot \cos\alpha_i + Q_{di} \cdot \sin\alpha_i - N_{u(i+1)} \cdot \cos\alpha_{i+1} - Q_{u(i+1)} \cdot \sin\alpha_{i+1} - \lambda \mathrm{ag} = 0 \end{cases}$$

$$(i = 1,2, \ldots \ldots ,23)$$

According to the assumption, the deformation of intervertebral disc is pure bending.
Suppose R is the radius of the cross section, M is the bending moment acting on the intervertebral disc.

$$M_i = \frac{2}{3} E \alpha_i R^3$$

5.1.3Building optimization problem

By this time, we build an optimized model of the angle’s size of the seat backplate throughout:

The objective function:

$$\min Z = \sum_1^{24} Q_{ui} + \sum_1^{24} Q_{di} + \sum_1^{24} N_{ui} + \sum_1^{24} N_{di} + \sum_1^{23} M_i$$

Constraint condition

$$\begin{cases} \alpha_i \in [0,25°] \\ N_{di} - N_{ui} = \lambda \mathrm{ag} \cdot \cos\alpha_i \\ Q_{ui} - Q_{di} = \lambda \mathrm{ag} \cdot \sin\alpha_i \\ Q_{di} \cdot \cos\alpha_i + N_{u(i+1)} \cdot \sin\alpha_{i+1} - N_{di} \cdot \sin\alpha_{i+1} - Q_{u(i+1)} \cdot \cos\alpha_i = 0 \\ N_{di} \cdot \cos\alpha_i + Q_{di} \cdot \sin\alpha_i - N_{u(i+1)} \cdot \cos\alpha_{i+1} - Q_{u(i+1)} \cdot \sin\alpha_{i+1} - \lambda \mathrm{ag} = 0 \\ \alpha_3 = \alpha_{12} = \alpha_{21} = 0 \\ (i = 1,2, \ldots \ldots ,24) \end{cases}$$

Using optimization software Lingo to run the program, we got A set of optimal value α (using angle system) (See Appendix - Annex 1)

$$f_i^{`}(\mathrm{x}) = \tan \alpha_i \,,\ (i = 1,2, \dots \dots ,24)$$

Establish two-dimensional rectangular coordinate system on the basis of seting the top of the vertebtra as coordinate origin and the the direction of vertical downward as x-axis' positive direction.

Let each vertebra length is 25,so we can get the objective function's derivative distribution at 25 points. (See Appendix - Annex 2)

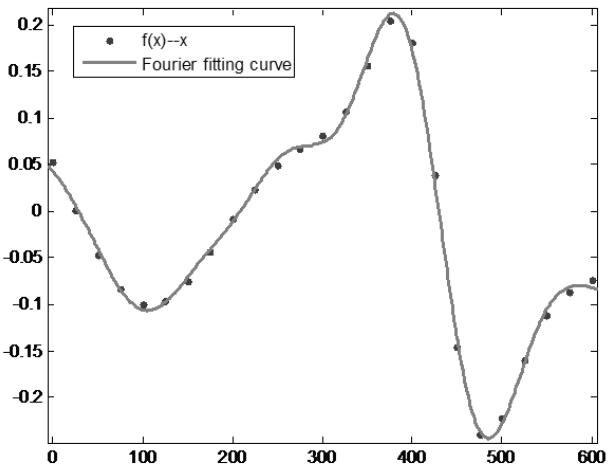
We can use Fourier fitting method to obtain the objective function's curve of derivative function by software Matlab:

$$\mathrm{f(x)} = \sum_{\mathbf{i=1}}^{\mathbf{8}} \mathbf{a}_i \cdot \mathbf{sin}\ (\mathbf{b}_i\,\mathbf{x} + \mathbf{c}_i)$$

The coefficient among them a ,b , c is:

	a	b	c
1	0.07859	0.02094	0.883
2	0.1386	0.01043	-1.978
3	0.07079	0.03142	2.002
4	0.07358	0.005429	2.026
5	0.03204	0.04189	-2.539
6	0.01488	0.05236	-0.4748
7	0.008279	0.06283	1.212
8	0.002986	0.0733	2.156

The graphics is:



(figure2:The derivative function curve of objective function obtained by Fourier fitting method)

Obtained the objective function by integrating the function:

$$F(x) = \int_0^x f(x)dx + C$$

(C is a constant)

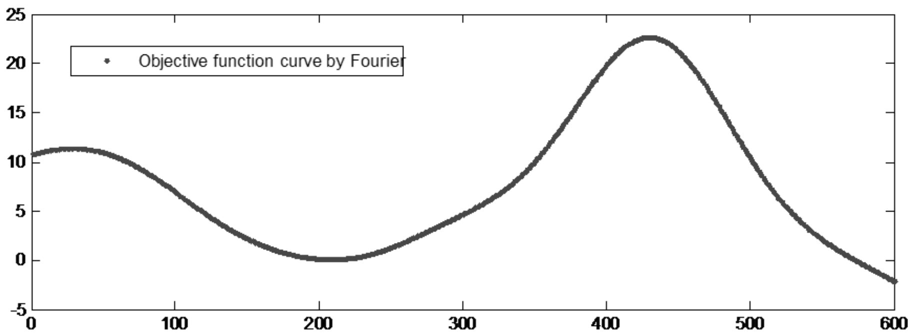
Calculated value of C by undetermined coefficient method and then we obtained the expression of objective function F(x) is:

$$f(x) = \sum_{i=1}^{8} -\frac{a_i}{b_i} \cdot \cos(b_i * x + c_i)$$

The value of coefficient is:

	a	b	c
1	0.07859	0.02094	0.883
2	0.1386	0.01043	-1.978
3	0.07079	0.03142	2.002
4	0.07358	0.005429	2.026
5	0.03204	0.04189	-2.539
6	0.01488	0.05236	-0.4748
7	0.008279	0.06283	1.212
8	0.002986	0.0733	2.156

Draw objective function curve by software Matlab(The algorithm is in the Appendix – Annex3)



(figure3:The objective function's curve obtained by Fourier fitting method)

5.2Error Analysis:

Sum of squares due to error、R-square、Adjusted R-square、RMSE are:

SSE: 0.000874

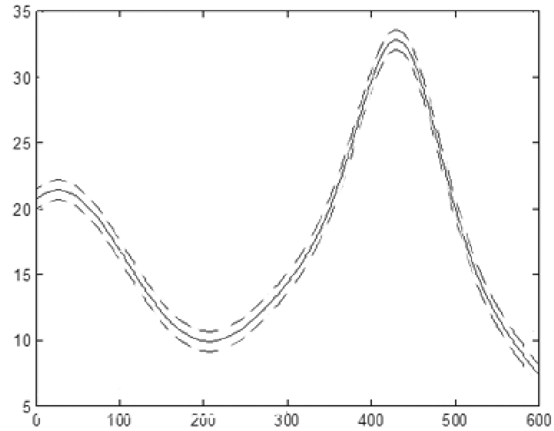
R-square: 0.9974

Adjusted R-square: 0.991

RMSE: 0.01117

The more SSE is close to zero the error is smaller , the more R-square is close to 1 the curve is better fitting, the more RMSSE is close to 0, the curve is better fitting.

Thus SSE>0.0001, RMSE>0.01 the error is relatively large. The error range corresponding to the curve is shown below:



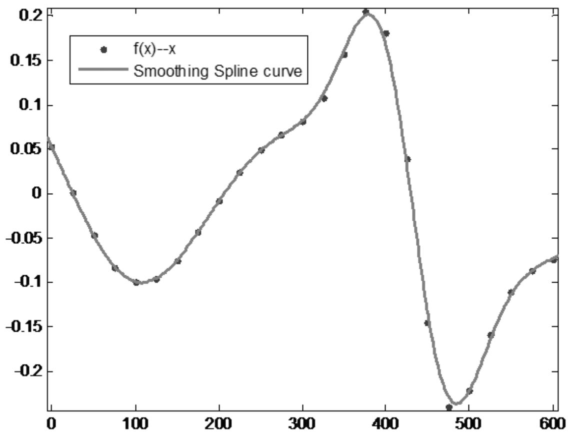
(figure5:The objective function and the banded area formed by error range)

So we need to find an optimization scheme to make SSE<0.0001、RMSE<0.0.

5.3Model Optimization

Our optimization purposes is to find a better optimization method to make RMSE<0.01、SSE<0.0001、Adjusted R-square sufficiently close 1, through analyzing and finding relevant information, we choose smoothing spline fitting method to optimize derivative function curve.（The algorithm of Matlab is in the Appendix – Annex4）

“coefs” representatives interpolated piecewise function coefficient matrix, “breaks” representatives piecewise interval node matrix, the fitting curve images is:



(figure6: The derivative function curve of objective function fitted out by smoothing spline method)

Its' SSE、R-square、Adjusted R-square 、RMSE are:

SSE: 8.409e-005

R-square: 0.9997

Adjusted R-square: 0.9939

RMSE: 0.00917

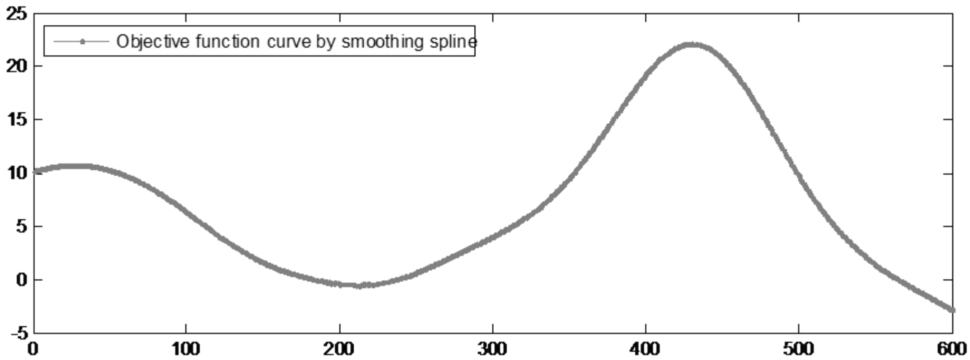
At this time, the case where RMSE<0.0 1 、SSE<0.0001 satisfy error controllable range and R^2 greater than the previous ,the fitting degree is better. （The algorithm is in the Appendix － Annex5）

$$A= sp.coefs$$

$$A2=sp.breaks$$

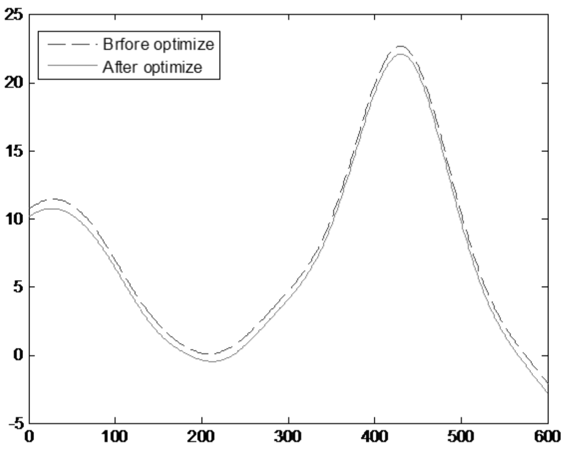
We obtained coefficient matrix B corresponding to the primitive function which the function Corresponding to the coefficient matrix A corresponding to: (The algorithm and matrix B are in the Appendix － Annex6)

So the objective function can be represented as a piecewise polynomial function,And the coefficient matrix of interpolation polynomial on each piecewise interval coefs =B, piecewise interval node matrix A2,number of segments is 25,the order of polynomial is 5, we draw the objective function images with matlab as follows:



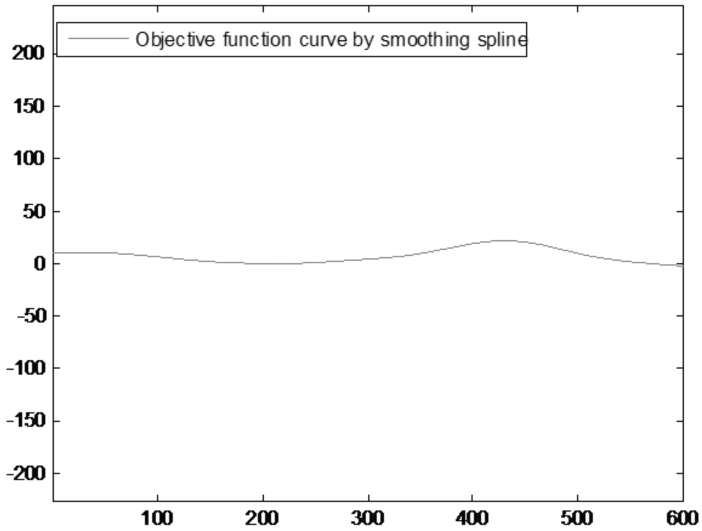
(figure7: The objective function's curve obtained by smoothing spline method)

Comparing optimized Model curve with previous model curve:



(figure8: previous model curve and optimized Model curve)

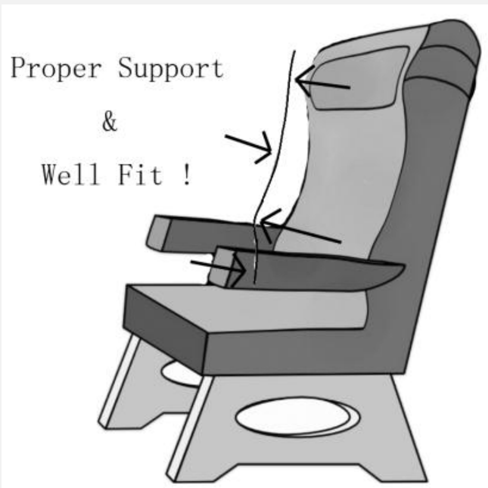
Therefore, we finally obtained a graph as follows(after coordinate transformed in equal proportion):



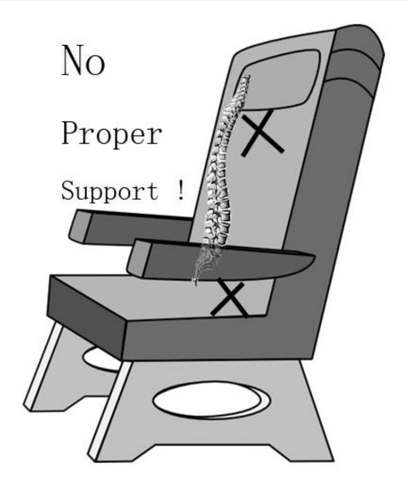
(figure9: The optimal curve in coordinate that transformed in equal proportions)

The function expression is a polynomial function which coefficient matrix coefs =B, piecewise interval node matrix A2， number of segments is 25,the order of polynomial is 5.

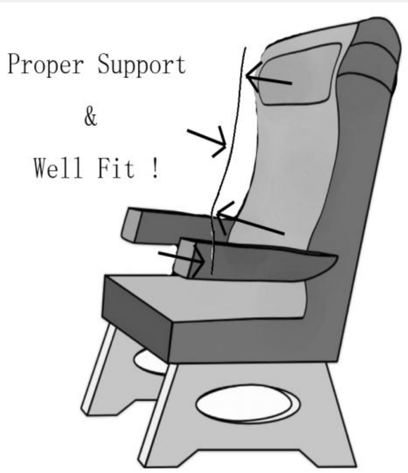
THE NEW SLIMLINE SEATS



Our airlines are now introducing a new "slimline" seats in economy class.

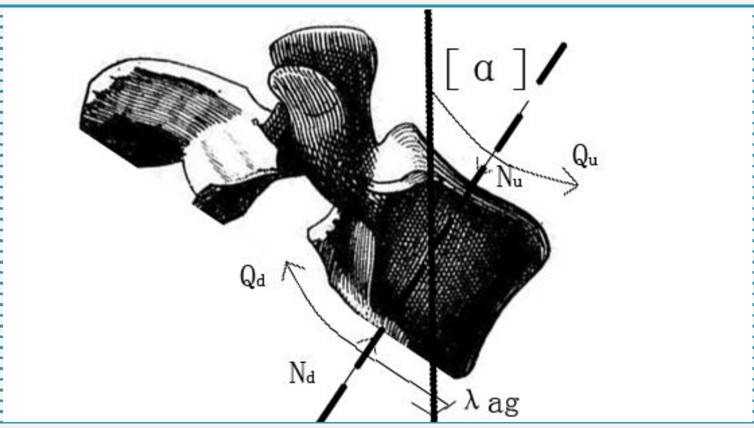


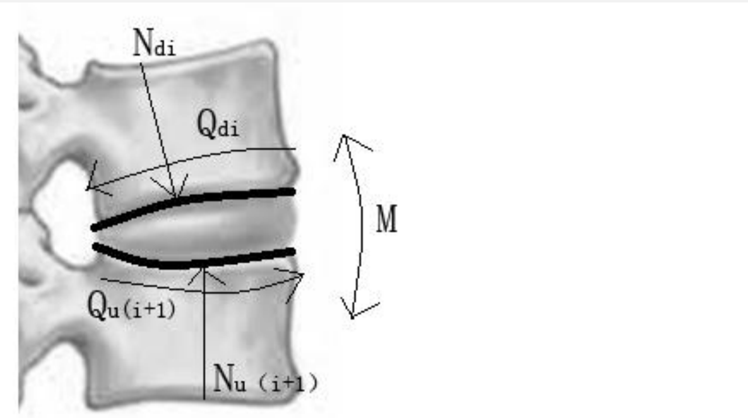
The seat backplate curve which not suitable for the vertebra makes passengers feel tired and uncomfortable

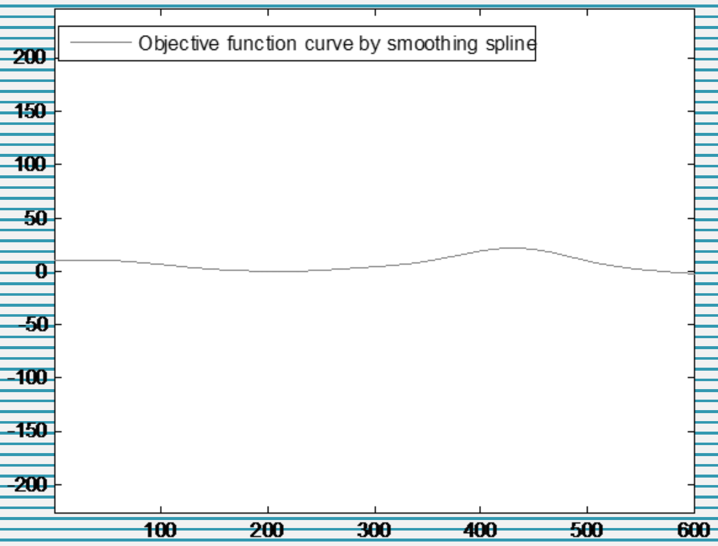


While our new slimline seats will liberating more passenger space,and make you more comefortable.

For it has the optimal seat backplate curve,and new type of fill material.







The application of biomechanics in the design of the seat backplate

- We summing the total shear force, total bending moment and total compressive force as the value of discomfort at that time.

- While the lowest value of discomfort corresponding to a set of angle which can be uniquely determined by the bending status where the composition of forces on the vertebra is smallest.

- Through finding optimal solution and fitting the data, we can find a smooth curve. This curve Is not only the curve of vertebra when it is most comfortable, but also can be seen as the seat back plate curve that can make passengers most comfortable.

All are welcome!!!

Appendix：

Annex 1

Columns 1 through 8

9.3841	0.2142	-8.5110	-14.9356	-17.9228	-17.2622	-13.5392	-7.8644

Columns 9 through 16

-1.5402	4.2764	8.8241	11.9555	14.6021	19.2318	27.9011	36.2675

Columns 17 through 24

32.1341	6.9702	-26.0330	-42.4099	-39.3301	-28.4607	-19.9661	-15.6969

Column 25

-13.4032

Annex 2

Columns 1 through 8

0.0522	0.0012	-0.0473	-0.0832	-0.0999	-0.0962	-0.0754	-0.0437

Columns 9 through 16

-0.0086	0.0238	0.0491	0.0665	0.0813	0.1073	0.1563	0.2043

Columns 17 through 24

0.1804	0.0387	-0.1456	-0.2401	-0.2220	-0.1594	-0.1114	-0.0874

Column 25

-0.0746

Annex 3

```
>> t=1:600;
>>
gx=-((0.07859./0.02094).*cos(0.02094.*t+0.883)+(0.1386./0.01043).*cos(0.01043.*t-1.978)+(0.07079./0.03142).*cos(0.03142.*t+2.002)+(0.07358./0.005429).*cos(0.005429.*t+2.026)+(0.03204./0.04189).*cos(0.04189.*t-2.539)+(0.01488./0.05236).*cos(0.05236.*t-0.4748)+(0.008279/0.06283).*cos(0.006283.*t+1.212)+(0.002986./0.0733).*cos(0.0733.*t+2.156));
>> plot(t,gx)
```

Annex 4

```
>> x=0:25:600;
>> sp=spline(x,k)

sp =

      form: 'pp'
      breaks: [1x25 double]
      coefs: [24x4 double]
      pieces: 24
      order: 4
      dim: 1
```

Annex 5

```
A=
    0.0000    -0.0000    -0.0019     0.0522
    0.0000     0.0000    -0.0021     0.0012
    0.0000     0.0000    -0.0017    -0.0473
    0.0000     0.0000    -0.0011    -0.0832
   -0.0000     0.0000    -0.0003    -0.0999
   -0.0000     0.0000     0.0005    -0.0962
   -0.0000     0.0000     0.0011    -0.0754
   -0.0000     0.0000     0.0014    -0.0437
   -0.0000    -0.0000     0.0014    -0.0086
   -0.0000    -0.0000     0.0012     0.0238
    0.0000    -0.0000     0.0008     0.0491
    0.0000    -0.0000     0.0006     0.0665
    0.0000     0.0000     0.0007     0.0813
```

```
  -0.0000    0.0000    0.0015    0.1073
  -0.0000    0.0000    0.0023    0.1563
  -0.0000   -0.0001    0.0010    0.2043
   0.0000   -0.0001   -0.0034    0.1804
   0.0000   -0.0000   -0.0074    0.0387
   0.0000    0.0001   -0.0062   -0.1456
  -0.0000    0.0001   -0.0013   -0.2401
  -0.0000    0.0000    0.0021   -0.2220
  -0.0000   -0.0000    0.0024   -0.1594
   0.0000   -0.0000    0.0014   -0.1114
   0.0000   -0.0000    0.0006   -0.0874

>> A2=sp.breaks

A2 =

  Columns 1 through 14

     0    25    50    75   100   125   150   175   200   225   250   275   300
325

  Columns 15 through 25

   350   375   400   425   450   475   500   525   550   575   600
```

Annex 6

```
> B=matrix(c(A[,1],0.5*A[,2],A[,3]/3,A[,4]/4),nrow=24,byrow=F)
> C=matrix(0,nrow=24,ncol=1)
> B=cbind(C,B)
      [,1]          [,2]          [,3]          [,4]          [,5]
 [1,]    0  1.160677e-07 -3.359207e-06 -6.480801e-04  0.0130453009
 [2,]    0  1.160677e-07  9.933299e-07 -6.875114e-04  0.0002974359
 [3,]    0  7.096559e-08  5.345867e-06 -5.818581e-04 -0.0118295978
 [4,]    0  1.308865e-08  8.007077e-06 -3.593090e-04 -0.0207916446
 [5,]    0 -3.839639e-08  8.497901e-06 -8.422608e-05 -0.0249753500
 [6,]    0 -7.131894e-08  7.058037e-06  1.750396e-04 -0.0240489807
 [7,]    0 -8.119451e-08  4.383576e-06  3.657331e-04 -0.0188399423
 [8,]    0 -7.003277e-08  1.338782e-06  4.611057e-04 -0.0109297451
 [9,]    0 -4.598990e-08 -1.287447e-06  4.619613e-04 -0.0021392085
[10,]    0 -1.330006e-08 -3.012068e-06  3.903028e-04  0.0059405913
[11,]    0  4.637749e-08 -3.510820e-06  2.815879e-04  0.0122655434
[12,]    0  1.588794e-07 -1.771665e-06  1.935465e-04  0.0166293481
```

[13,]	0	2.037147e-07	4.186312e-06	2.337907e-04	0.0203253229
[14,]	0	-2.126727e-07	1.182561e-05	5.006561e-04	0.0268128806
[15,]	0	-8.933921e-07	3.850385e-06	7.619227e-04	0.0390649330
[16,]	0	-7.450649e-07	-2.965182e-05	3.318988e-04	0.0510644163
[17,]	0	9.248511e-07	-5.759175e-05	-1.122161e-03	0.0451109163
[18,]	0	1.858487e-06	-2.290984e-05	-2.463854e-03	0.0096856806
[19,]	0	1.307471e-07	4.678342e-05	-2.065961e-03	-0.0364111882
[20,]	0	-9.423457e-07	5.168644e-05	-4.247963e-04	-0.0600174008
[21,]	0	-7.052838e-07	1.634848e-05	7.091190e-04	-0.0555113562
[22,]	0	-1.947869e-08	-1.009966e-05	8.132660e-04	-0.0398614892
[23,]	0	1.701205e-07	-1.083011e-05	4.644363e-04	-0.0278449857