Proposer's Commentary: The Outstanding Scanner Papers

Yves Nievergelt
Department of Mathematics, MS 32
Eastern Washington University
526 5th Street
Cheney, WA 99004–2431
yves.nievergelt@mail.ewu.edu

Once again, the problem came from the laboratory of Dr. Mark F. Dubach, who is studying the effects of intracerebral drug injections on monkeys with brain diseases at the University of Washington's Regional Primate Research Center in Seattle, WA.

This year, the striking novelty about the winning solutions of this purely mathematical modeling problem is the student teams' mastery of several electronic tools, which they used very adeptly along with mathematics.

- The first such electronic tool is the World Wide Web, which teams used in various degrees to find general medical information about Magnetic Resonance Imaging, real and simulated three-dimensional data sets for the human brain, and mathematical algorithms for two-dimensional interpolation. Two teams, however, found all the information they needed in printed form, and then adroitly generated their own test data.

- The second electronic tool lies in computer graphics, which all teams employed efficiently to communicate their results. For this problem, as one team noted, there does not seem to exist any numerical estimate of performance—such as a root-mean square or any other norm—that can substitute for the final visual medical diagnostic, and hence graphics may remain the best way to compare algorithms to reality.

- The third electronic tool consists of computer programming, which the teams utilized for the change of coordinates, in effect an isometric parametrization of a plane in space, and for three-dimensional interpolation.

- The fourth electronic tool, used appropriately by all teams, is the preparation of a final document containing prose, mathematical formulae, and graphics.

The UMAP Journal 19 (3) (1998) 277–278.

All these tools helped, of course, with the essential part of the problem, namely, mathematics. Within mathematics, the teams demonstrated a good command of concepts and details. As a first example, one crucial place for concepts is at the start, where all teams realized that the practical problem could be cast as a mathematical problem in three-dimensional interpolation. As a second example, one place where detail became important is in the generalization from one- or two-dimensional to three-dimensional interpolation. While one team (Tsinghua University) already knew the result, other teams (Eastern Oregon University, Harvey Mudd College) offered excellent explanations and proofs of their mathematical generalizations.

Finally, all teams demonstrated an efficient use of their time in balancing time devoted to searches and time devoted to in-house production for such items as data and algorithms. Such a balancing act between finding and reinventing the wheel can be critical in practice to deliver a working computer program in time. For example, none of the teams appears to have used a three-dimensional interpolation computer program from the World Wide Web, perhaps because it is not obvious where to get one. Indeed, a search of Netlib at `http://netlib2.cs.utk.edu` for "three-dimensional interpolation" shows such one- and two-dimensional routines as `toms/474` (bicubic interpolation) but does not reveal any specifically three-dimensional routines. Nevertheless, such routines exist, but finding them and using them may demand more time than available. For instance, there is a multidimensional (with an unlimited number of dimensions) interpolation routine using nonuniform rational B-splines (NURBS) at `http://dtnet33-199.dt.navy.mil/dtnurbs/about.htm` .

About the Author

Yves Nievergelt graduated in mathematics from the École Polytechnique Fédérale de Lausanne (Switzerland) in 1976, with concentrations in functional and numerical analysis of PDEs. He obtained a Ph.D. from the University of Washington in 1984, with a dissertation in several complex variables under the guidance of James R. King. He now teaches complex and numerical analysis at Eastern Washington University.

Prof. Nievergelt is an associate editor of *The UMAP Journal*. He is the author of many UMAP Modules, a bibliography of case studies of applications of lower-division mathematics (*The UMAP Journal* 6 (2) (1985): 37–56), and *Mathematics in Business Administration* (Irwin, 1989).