A Quasi-Sequential Cellular-Automaton Approach to Traffic Modeling

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Summary

The most popular discrete models to simulate traffic flow are cellular automata, discrete dynamical systems whose behavior is completely specified in terms of its local region. Space is represented as a grid, with each cell containing some data, and these cells act in accordance to some set of rules at each temporal step. Of particular interest to this problem are sequential cellular automata (SCA), where the cells are updated in a sequential manner at each temporal step.

We develop a discrete model with a grid to represent the area around a toll plaza and cells to hold cars. The cars are modeled as 5-dimensional vectors, with each dimension representing a different characteristic (e.g., speed). By discretizing the grid into different regimes (transition from highway, tollbooth, etc.), we develop rules for cars to follow in their movement. Finally, we model incoming traffic flow using a negative exponential distribution.

We plot the average time for a car to move through the grid vs. incoming traffic flow rate for three different cases: 4 incoming lanes and tollbooths, 4 incoming lanes and 4, 5, and 6 tollbooths. In each plots, we noted at certain values for the flow rate, there is a boundary layer in our solution. As we increase the ratio of tollbooths to incoming lanes, this boundary layer shifts to the right. Hence, the optimum solution is to pick the minimum number of tollbooths for which the maximum flow rate expected is located to the left of the boundary layer.

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Introduction

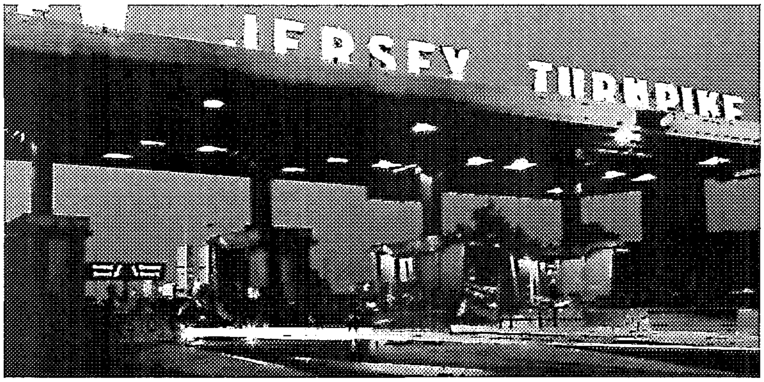


Figure 1. The New Jersey Turnpike (I-95) at night.

Models for traffic flow can be broken down into two basic types.

- The first type treats space and time as a continuum; both cars and time are continuous in nature.
- The second type, discrete models, treats space as a lattice and time discretely. A common discrete model is a cellular automaton, where space is modeled by a lattice and each lattice site represents a state of the system. The lattice sites are updated and their states change. For traffic flow, the states of the lattice sites represent whether a car is present at that spatial location or not.

Near a tollbooth, cars must stop to pay before moving on. Since each car affects the other cars in its direct neighborhood, it is not reasonable to model cars as a continuum. Discrete time also allows us to control the movement of the cars at each individual time step. Finally, discrete models in general are much easier to understand and to implement on modern computing resources.

Assumptions

- Upon nearing a toll plaza, a driver maneuvers based on local congestion to minimize travel time.
- Within 100 ft of the toll plaza, a driver remains in a lane and slows down to an average speed of about 5–10 mph. We base the speed of the cars on what is suggested in most driver's manuals: Car separation should be one car length for every 10 mph of speed.
- Once a driver pays the toll, they maneuver to a highway lane and accelerate to highway speeds.

- Drivers do not cooperate. While the drivers are not directly competing against one other, they are affecting each other and are hence fierce indirect obstacles/opponents.
- Vehicles are of constant length (17.5 ft).
- It takes about 4 s for a tollbooth employee to process a motorist [Chao n.d.].

A Quasi-SCA Model of Toll Plaza Dynamics

Case 1: Equal Numbers of Lanes and Booths

Preliminaries

Cellular automata (CA) are discrete dynamical systems whose behavior is completely specified locally. Space is represented as a uniform grid, with each cell containing data. Time advances in discrete steps, and the laws of the universe are expressed in a look-up table relating each cell to nearby cells to compute its new state. The system's laws are local and uniform.

The basic one-dimensional cellular automata model for highway traffic flow is the CA rule 184, as classified by Wolfram [Nagel et al. 1998; Jiang n.d.; Wolfram 2002]. CA 184 is a discrete time process with state space $\eta \in \{0,1\}^{\mathcal{Z}}$ and the following evolution rule: If $\eta \in \{0,1\}^{\mathcal{Z}}$ is the state of at time n, then the state η' at time $n+1$ is defined by

$$\eta' := \begin{cases} 1, & \text{if } \eta(x) = \eta(x+1) = 1; \\ 1, & \text{if } \eta(x) = 1 - \eta(x+1) = 0; \\ 0, & \text{otherwise,} \end{cases}$$

where $\eta(x)$ denotes the value of $\eta : \mathcal{Z} \to \{0,1\}$ at the coordinate x.

In this model, cars march to the right in a rather uniform manner, and all nodes execute their moves in parallel.

Toll plaza dynamics, while similar to traffic dynamics, are quite different.

- Toll plazas cannot be approximated as covering an infinite domain.
- Drivers must make decisions based on who moves in front of them. In this sense, we use ideas from Sequential Cellular Automata (SCA) [Tosic and Agha n.d.] instead of the classical schemes.
- Cells are updated in a slightly different manner than in classical cellular automata. To model car movement properly, "cars" are moved through cells one at a time.

Our model is like a board game. For these reasons, we dub our model a "Quasi-SCA Model of Toll Plaza Dynamics."

We divide a multilane highway into equally partitioned lanes. Each cell is approximately 25 ft long and contains information on whether it contains a car and, if it does, certain information about the car. Furthermore, there are specialized cell characteristics for different regimes, as shown in **Figure 2**. In our model, we also move forward in discrete time steps. For convenience, this time step is set to be 2 s in length.

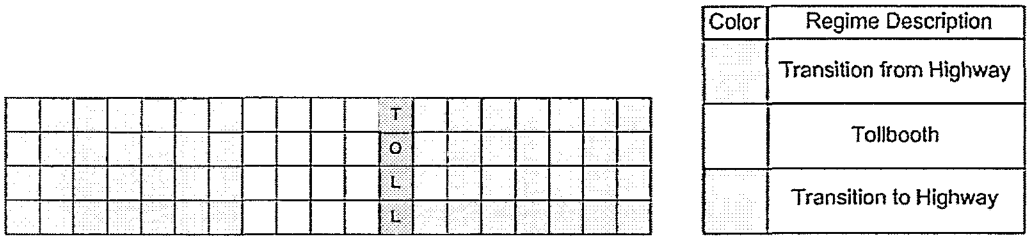


Figure 2. Possible regimes.

To implement our model, we exploit the object-oriented features of C++. We create a car class, with certain variables associated with it, as shown in **Table 1**.

Table 1.
Car class variables in C++.

Car Class Variables	
Occupied	1 = Car, 0 = Null
Congestion	Percent Measure of Local Congestion
Speed	Measure of Car Speed
TotalTimeOnGrid	Counter Measuring Time on Grid
TotalTimeInToll	Counter Measuring Time in Toll

The highway is represented as a large $50 \times n$ array of car variables, where n is the number of lanes. When initialized, this array contains empty grid spaces. As cars enter in from the left, grid spaces are activated and infused with information about the cars. Then, with this information, the state of the system at the next time step can be determined.

Vehicle Speed

The speeds of cars not in the tollbooth regime are dictated by car separation having to be one car length for every 10 mph of speed. Since our model is discrete in both space and time, this criterion must be quantized. Moving one grid space ahead in one temporal step corresponds to a speed of about 8.5 mph. If we approximate one grid space as one car length and 8.5 mph $\sim$ 10 mph, we can generalize the speeds of the cars in the following manner:

$$s(i,j,t) := \begin{cases} 0, & \text{if } \min_{x>i}\{x \mid o(x,j,t)=1\} = i+1; \\ 1, & \text{if } \min_{x>i}\{x \mid o(x,j,t)=1\} = i+2; \\ 2, & \text{if } \min_{x>i}\{x \mid o(x,j,t)=1\} = i+3; \\ 3, & \text{otherwise.} \end{cases}$$

We enforce 25.6 mph as an upper limit to speed, since the vehicles must slow down as they approach the toll. At each time step, the speed for a car is updated just before it initiates movement.

Congestion

Since a driver is far more forward-focused than rearward-focused, we consider congestion to be determined only by the cars immediately in front—in particular, the nearest five cars. We write congestion for the car located in grid cell $\eta(i,j,t)$ as

$$c(i,j,t) := \frac{1}{5}\sum_{k=1}^{5} o(i+k,j),$$

where

$$o(i,j,t) := \begin{cases} 1, & \text{if grid cell } (i,j) \text{ contains a car;} \\ 0, & \text{otherwise} \end{cases}$$

Sequencing

Cells are updated sequentially as opposed to simultaneously, because cars make decisions based on the cars in front. Furthermore, in a given column of our array, that is, one spatial location across four lanes, the car with the largest speed has the first initiative; the car with the second largest speed moves second, etc. In the case of a tie, the car closer to the top of the grid moves first.

Movement

Transition Regimes

Transition regimes are regions where traffic comes in from the highway or leaves to the highway. In these regimes, drivers maneuver in a manner such that they can optimize travel time but minimize effort. Thus, movement possibilities in the transition regimes can be described by **Figure 3**.

The optimal maneuver is to move forward, but a driver will enter a lane to the right or left if the move minimizes congestion.

In two locations of the transition regimes, there are special considerations.

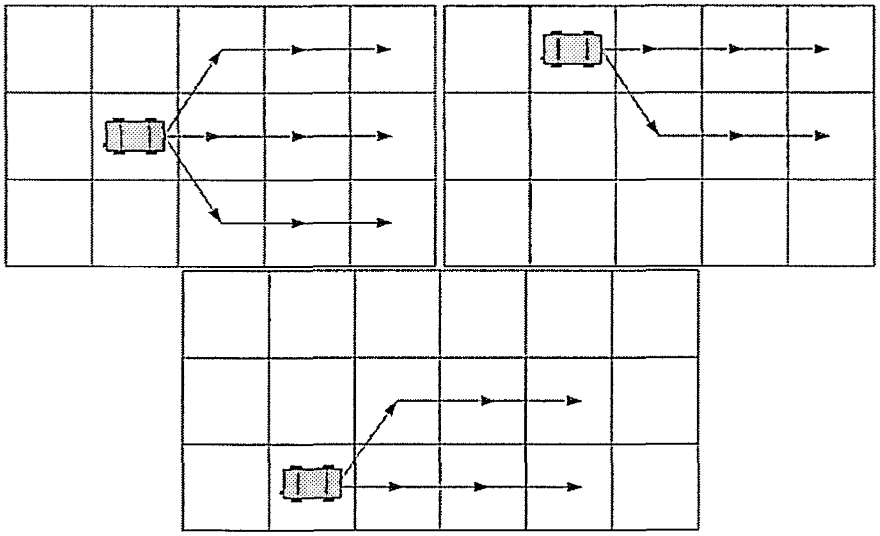


Figure 3. Movement in transition regimes: Center lane, far left lane, far right lane

- The transition from highway regime: There must be some way to depict the arrival of traffic from the highway. We discuss later how we do this.

- The tail end of the transition-to-highway regime: Provided a car has sufficient speed, we eliminate the car from the grid. We also record its TotalTimeOnGrid variable.

Tollbooth Regime

In the tollbooth regime, drivers no longer veer to the right or left. Instead, they move forward in line until they reach the tollbooth. In this region, spanning the 100 ft in front of the tollbooth, cars move at a maximum rate of one grid space per temporal element. Once in the tollbooth, they wait two entire temporal elements solely in the booth (about 4 s) until they move on to the transition regime. This is implemented by incrementing a car's `TotalTimeInToll` variable (initialized to zero when a vehicle enters the map) every temporal step that a car is in the booth (*for the entire step*) and checking if it is greater than 2. Often in this region, lines will form. As soon as a car emerges from the tollbooth, all of the cars behind move forward immediately. The dynamics of this regime are quite a bit different and simpler than the dynamics of the transition regime.

We illustrate this situation in **Figure 4**. The red cars in lanes one and four are stopped, waiting behind cars located in the booth. The green cars ahead of the toll are transitioning to the highway regime. The yellow car is moving into the tollbooth, and the blue car is moving further inside the region. The green car before the toll is just now moving into the tollbooth region. While its current speed is 25.6 mph, once inside the region, it decelerates to 8.5 mph.

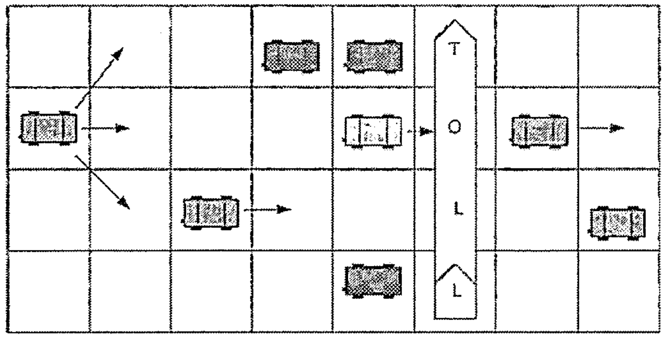


Figure 4. Movement in the tollbooth regime

Modeling the Incoming Traffic Flow

To make our model more accurate, we use a statistical distribution to predict incoming flow. Two commonly-used distributions are the Poisson and the negative exponential. However, the Poisson distribution fits well only for light traffic [Aston 1966]. The negative exponential distribution is a good fit for heavy traffic; it is used to model the variations of gap length in a traffic stream over distance and random arrivals. It has probability density function

$$f(t) = qe^{-qt},$$

where t is the time (s) between arrivals and q is the rate of arrival (cars/s), and cumulative distribution function

$$F(t) = 1 - e^{-qt}. \tag{1}$$

To implement this arrival time into our simulation, we assign it to a site of entry (a space) into the grid. A random number generator creates a random fraction F; using **(1)**, we solve for $t = -\ln R/q$. The value t is assigned to a "spawn site," a place where "cars" are created. We use a counter to keep track of the time between different spawnings of cars. If this counter is greater than F and the "spawn site" is empty (contains a null car), then a car is created at the spawning site. Otherwise, the counter is incremented until one of these two conditions are met. Cars "arrive" in each lane of the simulation using this method. We use a modified q such in units of car per 2 s per lane.

Results

We simulate for varying values of q, the flow rate of cars per 2 s per lane, for a 4-lane highway with 4 tollbooths. We let q vary from 0.01 cars/s/lane (0.02 cars/s overall) to 1 car/s/ lane (2 cars/s overall. **Figure 6** outlines a given time evolution for a small value of q.

The time through which the cars move through the grid (or toll plaza) is an appropriate measure of congestion. Thus, we plot in **Figure 7** the average time

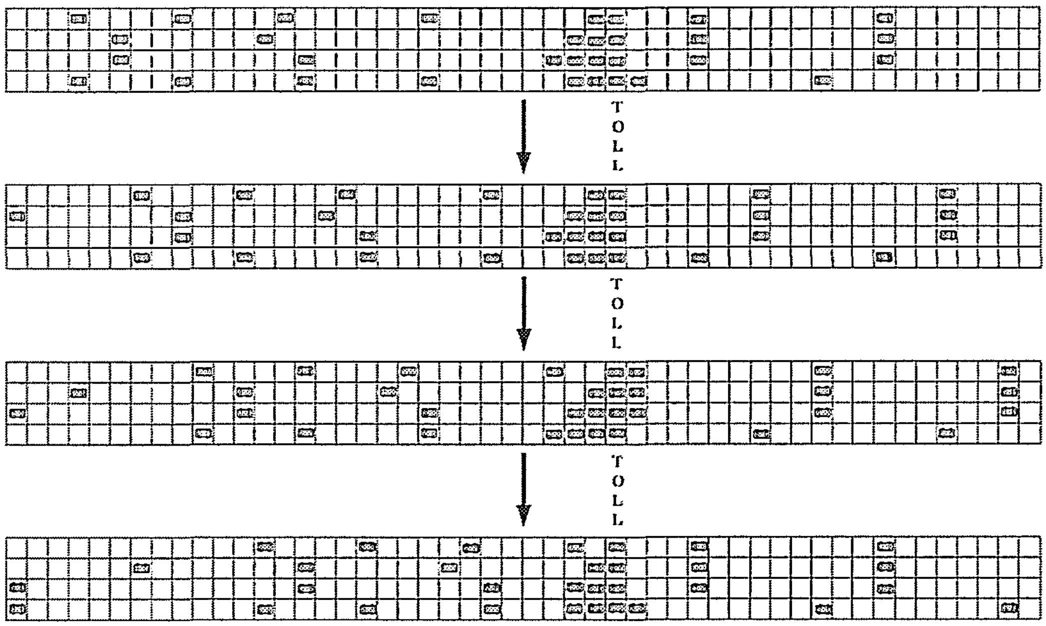


Figure 6. Time evolution of a simulation, four temporal steps.

getting through the grid versus the flow rate. The average time is obtained from a simulation accounting for one hour of traffic. We also plot for each flow value the maximum amount of time that anyone spent getting through the grid.

For q in [0.01,0.37] cars/(2 s)/lane (0.02–0.74 cars/s overall), drivers enjoy an average time through the grid below 50 s. We consider this an optimal situation. However, at around a $q = 0.36$ cars/(2 s)/lane, there appears to be a boundary layer. For $q > 0.37$, it takes drivers an average of 2 min or more to get through the quarter-mile long grid, corresponding to less than 10 mph. We demonstrate later that by adding more tollbooths, we shift the boundary layer and lower the average time for larger q. Thus, a good strategy to determine the number of tollbooths is to estimate the anticipated maximum flow rate and choose a number of lanes for which q is never beyond the boundary layer.

Congestion is at its worst during rush hour, when toll plazas serve as bottlenecks. But what do these congestion levels mean in total time through the plaza? Is the number of tollbooths optimal?

The Hiawassee M/L Toll Plaza in Florida uses a 4-tollbooth plaza. In October 2003, the Eastbound car count 7–8 A.M. was 3403 cars [Orlando-Orange County Expressway Authority 2003], so cars arrived at a rate of 0.945 cars/s/lane. With our assumption that a car is 17.5 ft long, clearly, four tollbooths are not enough to handle this heavy demand.

However, EZPass and other such programs allow one to minimize the time at a tollbooth. If even a small portion of the cars use the EZPass system, the value of q for which the boundary layer results grows vastly. If we were to accurately determine an optimal value of tollbooths for a certain value of q for a highway using such a system, we would have to approach the problem in a

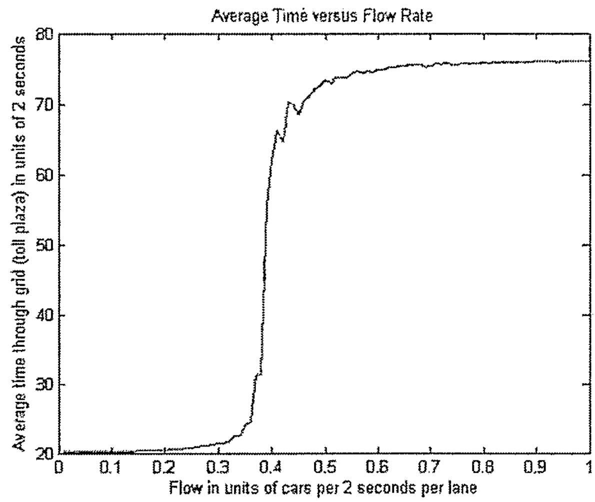


Figure 7. Average time through grid vs. flow rate.

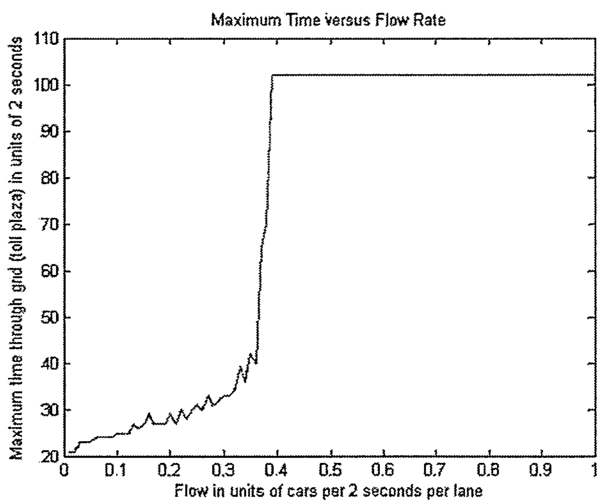


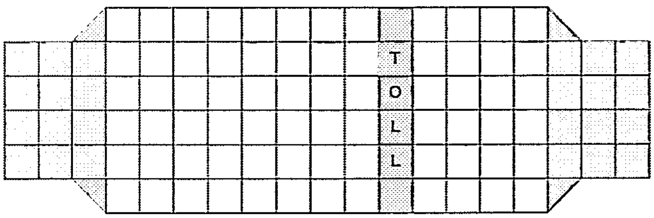
Figure 8. Maximum time through grid vs. flow rate.

slightly different fashion. In particular, we would have to vary the time drivers spend at the booth and designate certain lanes as having a quick pass system.

Case 2: More Tollbooths than Lanes

Preliminaries

The situation changes quite a bit if there are more tollbooths than incoming lanes. Drivers in the far left and right lanes start moving into the new tollbooth lanes. Hence, we introduce a new scheme, as presented in **Figure 9**.



Color	Regime Description
	Transition from Highway
	Expansion
	Tollbooth
	Compression
	Transition to Highway

Figure 9. Possible regimes.

Movement in the Expansion Regime

The expansion regime is where the incoming traffic lanes fan out to a greater number of tollbooth lanes. For the center lanes, movement is identical to the transition regimes. On the outer lanes, however, movement is slightly different. The movement possibilities are outlined in **Figure 10**.

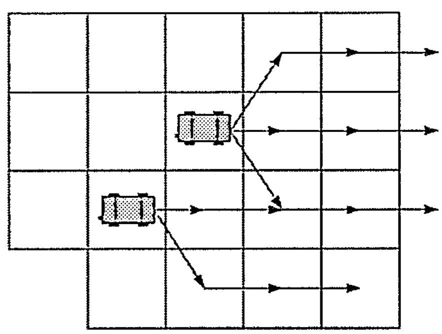


Figure 10. Movement in the expansion regime.

For a driver in the outside lane, the optimal maneuver is to move into one of the newly-created tollbooth lanes, unless the congestion is less in the current lane. Another new addition is that the driver will not try to move into one of the inner lanes—more for psychological reasons than practical reasons. According to the model, the driver assumes that the outside lanes are the least dense (and fastest), since they did not exist on the highway. Drivers on the newly created lanes are allowed to move only forward in our model. While a driver may move to an outside lane just to move back again, we consider the chance of this occurring as very slim.

Movement in the Compression Regime

The compression regime is where a greater number of tollbooth lanes collapse onto a smaller number of highway lanes. We have the movement possibilities presented in **Figure 11**.

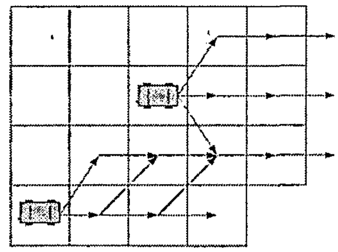


Figure 11. Movement in the compression regime.

A driver in a tollbooth lane that is a highway lane follows the same rules as in the earlier tollbooth regime. A driver not in a highway lane, however, tries to move back onto a highway lanes; if this is not possible, they keep driving forward and trying again until they are forced to stop at the end of the tollbooth lane. This protocol can provide for some hectic situations.

Results

We simulate our second model for varying values of q for 4 highway lanes with 5 and 6 tollbooths. The range for q is the same as our first model. **Figures 12–13** show the results for these two cases, for which we take the expansion and compression regimes to be 125 ft long.

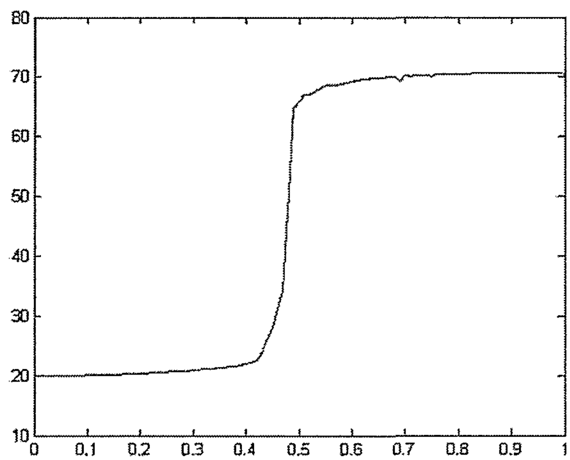


Figure 12. Average time through grid vs. flow rate, for 6 lanes.

The boundary layer is moved to the right as the number of toll lanes increases. Furthermore, the value for q on the right side of the boundary layer decreases with more toll lanes. Thus, as suggested, one should choose a sufficient number of lanes that correlates to this behavior. If the maximum flow rate one expects is a certain value, one can run a simulation for a certain number of

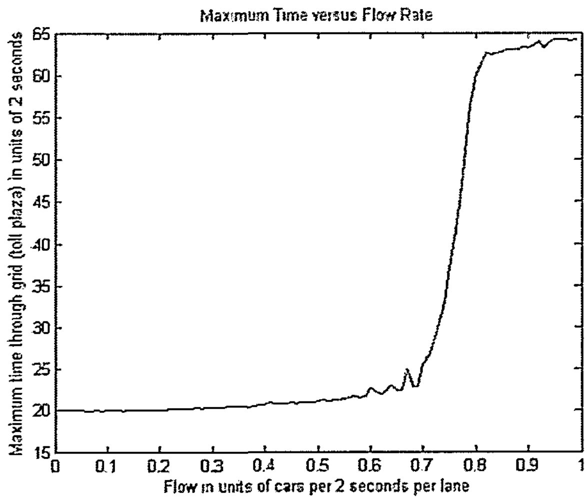


Figure 13. Average time through grid vs. flow rate, for 6 lanes.

tollbooths and choose the least number of tollbooths such that the maximum flow is to the left of the boundary layer.

However, with an increased number of lanes comes an increased maximum individual travel time: At times, people become stuck in the toll lanes and have to wait for an opportune moment to move over. In our model, this is reflected in the fact that while the four-tollbooth case results in a maximum travel time of about 3.4 min, the 5- and 6-lane cases sometimes have a maximum individual travel time near 4 min.

Model Improvements and Discussion

Drivers do not always move in a predictable manner. A probabilistic model taking into account the unpredictable nature of humans could further improve our model.

Our model also does not take into account the possibility of accidents. An accident model would surely improve our model.

While we do take into account the random nature of incoming traffic flow, we could develop an even better model to approximate the flow rate.

Lastly, our model could include a probabilistic model for the time that a car waits at a tollbooth.

Conclusion

We develop a quasi-SCA model for toll plaza dynamics that treats time and space in a discrete manner to capture the motivation and actions of drivers. We use a negative exponential distribution for the incoming flow rate of cars. We compute the average waiting time for different traffic flow rates.

At a certain flow rate, there is a boundary layer at which travel time increases sharply with flow rate. Thus, an optimal solution to the tollbooth problem is to choose the minimum number of tollbooths such that the expected rate of incoming flow corresponds to a point before the boundary layer.

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