Don't Panic!

Timothy Jones
Jeremy Katz
Allison Master
North Carolina School
of Science and Mathematics
Durham, NC 27705

Advisor: Dot Doyle

Assumptions and Hypotheses

- People evacuating in a fire always move towards the nearest exit, regardless of which path to an exit is least crowded and what obstacles are in their way. Thus, a room with multiple exits can be treated as several smaller rooms, each feeding one exit.
- People only become crowded at a finite number of "bottlenecks"—points at which a line develops and evacuating people must wait. In all other areas, people can move freely. However, the line at these points occupies a minimal amount of space.
- Individuals all move through open areas (where there are no bottlenecks) at the same constant rate. This rate depends on the type of occupants in the room and the presence or absence of inanimate obstacles.
- We disregard building construction, panic hardware (such as pushbar doors), and alarm systems.
- The time for an individual to move through the line at a bottleneck follows an exponential probability density function. However, the variation among people is relatively small.

Analysis and Model Development

All of the different reasons to limit the capacity of a building space are more or less independent of one other. For example, during a fire, the sanitation of

The UMAP Journal 20 (3) (1999) 297–309.

a room and the amount of weight that a structure can carry are not immediately important; likewise, when the day-to-day health of a room's occupants is considered, the fact that there might one day be a fire is irrelevant. Thus, we calculate maximum occupancies considering each concern independently and then choose the lowest value.

Simple Rooms in a Fire

In the event of a fire, two elements contribute to the speed with which a room can be evacuated:

- the maximum speed at which the occupants can safely move, and
- the extent of crowding in the room.

We assume that people are free to move except in certain critical areas ("bottlenecks"). At these places, a line builds up, and any individual who reaches the line must wait before moving on to the exit. Thus, in the simplest case—a room with no inanimate obstacles and one exit—the problem can be broken up into two steps: describing how occupants move to the queue at the door, and describing the dynamics of the queue itself. To find how much time is required to evacuate the room, we find the time for the length of the exit queue to drop below 1.

We define a probability density function $P(x, y)$ for the likelihood that an individual is located within a certain region of the room. The coordinate system has the center of the door at the origin and the wall containing the door along the y-axis (**Figure 1**).

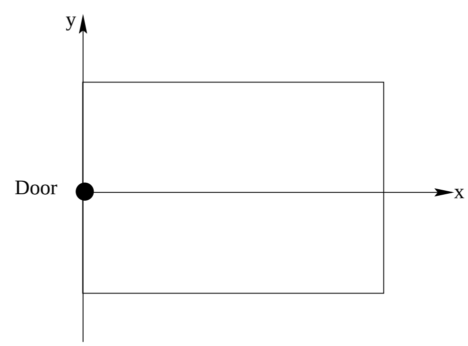

Figure 1. Coordinate system for a room.

We would like to find a second pdf, $q(t)$, to describe the probability that any given individual reaches the door within a certain period of time. Then the probability that an individual reaches the door queue within an interval of time Δt is

$$\int_t^{t+\Delta t} q(t)dt,$$

where t is the time since the alarm first sounded and people began to move to the exits. For simplicity, we call this integral $Q \mid_t^{t+\Delta t}$. If there are initially n people in the room, the number of people entering the exit queue over the interval $(t, t+\Delta t)$ is $n \cdot Q \mid_t^{t+\Delta t}$ and the *rate* at which people are entering the queue is

$$\frac{n \cdot Q \mid_t^{t+\Delta t}}{\Delta t}. \tag{1}$$

Next, we assume that the time that each individual takes to move through the door queue is described by an exponential probability density function of the form $p(t) = \lambda e^{-\lambda t}$. The expected value of the time required for one person to move through the line is $1/\lambda$ and the average rate at which people leave the queue is λ. Because we assume that most people take the same amount of time to move through the doorway, the rate of a steady stream of people moving through the doorway is never much more or less than λ. However, the queue at the door may have so few people that it can empty completely in time Δt, so that the rate that it empties may be less than λ. In other words, if there are fewer than $\lambda\Delta t$ people in the queue at time t, then by time $t + \Delta t$ they will all have left; if there are more than $\lambda\Delta t$ people, only some can leave. We therefore express the rate at which people leave a queue as

$$\rho = \begin{cases} \dfrac{L_{n-1}}{\Delta t}, & \text{for } L_{n-1} < \lambda\Delta t; \\ \lambda, & \text{otherwise,} \end{cases} \tag{2}$$

where L_{n-1} is the number of people waiting in the queue at time t and ρ is the rate at which people are leaving the bottleneck.

Combining **(1)** and **(2)** gives the rate at which the length of the exit queue is changing in the situation of a room with one exit:

$$\frac{n \cdot Q \mid_t^{t+\Delta t}}{\Delta t} - \rho.$$

From this, we can write a system of recursive equations (Euler's method) to approximate the length of the line (L_n) versus time:

$$\begin{aligned} t_n &= t_{n-1} + \Delta t, \\ L_n &= L_{n-1} + \Big(\frac{n \cdot Q \mid_t^{t+\Delta t}}{\Delta t} - \rho\Big) \cdot \Delta t, \end{aligned} \tag{3}$$

where $L_0 = 0$ is the case for a room in which the people are initially distributed throughout the room.

If these equations are iterated until the length of the exit queue is less than 1, we will learn the time required to evacuate the room in an emergency. However, we still must find a general form for $Q \mid_t^{t+\Delta t}$ in terms of $P(x, y)$. To accomplish this, we divide the room into a set of concentric circles centered at the door, such that any person located within a ring-shaped region of the room defined by two of these circles requires between t and $t+\Delta t$ seconds to reach the door. We then divide each of these regions into k smaller segments (**Figure 2**), each of which can be defined in polar coordinates (with the door at the origin) (**Figure 3**). Since each ring corresponds to a certain interval of time, finding the probability that an individual is located within each ring gives the probability that they arrive at the door within that period of time. To approximate this probability, $P(x,y)$ is evaluated at the center of each small wedge-shaped segment and multiplied by the area (ΔA) of the segment; all of these probabilities are then be summed to give an approximate value of $Q\,|_t^{t+\Delta t}$ for each particular t:[1]

$$Q\,|_t^{t+\Delta t} \approx \sum_{i=1}^{k} P(x_i, y_i) \cdot \Delta A. \tag{4}$$

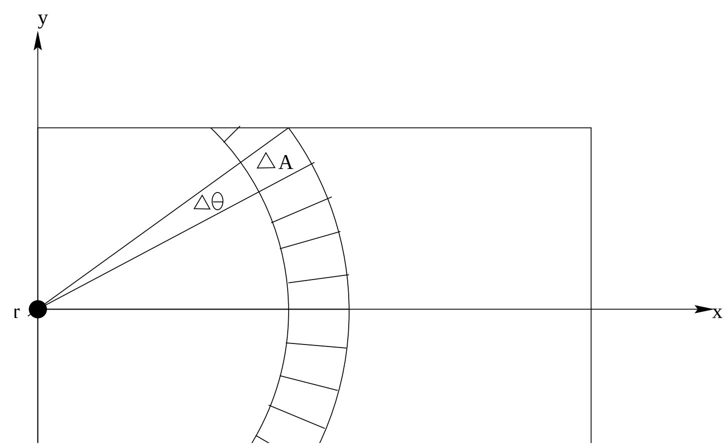

Figure 2. Ring-shaped region divided into k segments.

To compute the sum, we need to express x and y in terms of r (which is constant over the summation) and θ (which varies). From **Figure 3**, we see that

$$x = \frac{r_1 + r_2}{2} \cos\left(\frac{\theta_1 + \theta_2}{2}\right), \qquad y = \frac{r_1 + r_2}{2} \sin\left(\frac{\theta_1 + \theta_2}{2}\right).$$

Since the speed s at which a person can move is known, we can express r_1 and r_2 in terms of t:

$$r_1 = st\,, \qquad r_2 = s(t + \Delta t).$$

Rewriting ΔA in terms of r_1, r_2, and $\Delta\theta$ gives:

$$\Delta A = \frac{\pi (r_2)^2 \Delta\theta}{2\pi} - \frac{\pi (r_1)^2 \Delta\theta}{2\pi} = \frac{(r_2)^2 \Delta\theta}{2} - \frac{(r_1)^2 \Delta\theta}{2}.$$

[1]At the edges of the room (where $P(x,y)$ goes to zero), the small wedge-shaped sections do not accurately follow the wall. As a result, some of the sections "spill over" across the edge; however, since Δt and $\Delta\theta$ are both very small, the effect of this error is minimal.

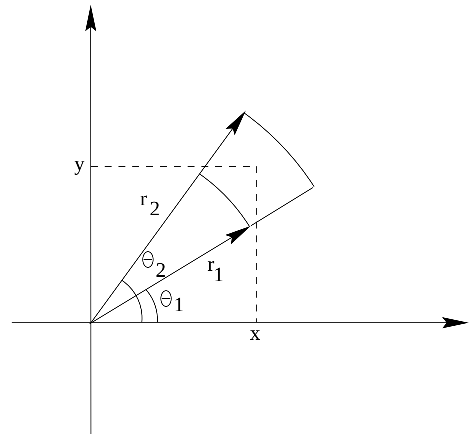

Figure 3. Coordinates of center point.

Thus, **(3)** can be expressed in terms of t, Δt, $\Delta\theta$, and s. (Note that since the wall containing the door is always along the y-axis, θ_i need only be incremented from $-\pi/2$ to $\pi/2$.) Given these parameters, along with $P(x, y)$, we find the value of $Q\mid_t^{t+\Delta t}$ for each value of t. With these values in hand, we can use **(2)** to find the length of the exit queue versus time. Once the length of the queue has dropped below one, the room has been successfully evacuated.

More Complex Evacuation Patterns

Because most situations involve multiple bottlenecks, we need to expand the above model for it to be practical. During normal use, occupants of a room are often distributed throughout one or more "reservoirs"—for instance, aisles in an auditorium, regions served by ladders in a swimming pool, sections of seating at a stadium, or tiers of bleachers in a gymnasium—that are separated from the rest of the room by bottlenecks. As soon as the alarm sounds, people begin to move into the first set of bottlenecks and queue at each one (a "feeder queue" of the main exit). As the first set of feeder queues clears, people move to the second bottleneck and enter the queue there. For instance, in an auditorium people must first leave their own aisles (the first level of feeder queues), proceed to the main exit, and wait in line there.

Since a given group of people head only for one exit, each bottleneck has its own independent reservoir of people; the output rates from all the feeder queues can therefore be added to give the input rate of the final exit queue. The rate at which each feeder queue releases people is given by **(1)**, but since it depends on the value of L_{n-1} the entire set of iterations (**(3)** and **(2)**) used to calculate L_n above must be evaluated for each feeder queue in the more complicated situation. Furthermore, the time for a person to move from the output point of a feeder queue to the main exit queue creates a delay; thus, the final Euler's method equation (which approximates E_n, the length of the exit queue at time t_n) will not depend on the value of L for each feeder queue at time t_n. Rather, we use $L_{n-\beta}$ (the rate at time $t_{n-\beta}$) for each feeder queue, where β represents the number of Euler's method iterations that pass between the time a person leaves a feeder queue and the time she reaches the final exit queue. The value of β for feeder queue j is therefore given by

$$\beta_j = \left\lceil \frac{\sqrt{x_j^2 + y_j^2}}{s\Delta t} \right\rceil,$$

where (x_j, y_j) is the location of the output point of feeder queue j, $\sqrt{x_j^2 + y_j^2}$ is the distance between the output point and the door, $s\Delta t$ is the distance that can be traveled in one iteration, and we take the ceiling (nearest integer at least as large) of the resulting value.

Since people leave the exit queue in the same way as they leave any bottleneck, we can define ρ_E, the rate at which people leave the final exit queue:

$$\rho_E = \begin{cases} \dfrac{E_{n-1}}{\Delta t}, & \text{for } E_{n-1} < \lambda\Delta t; \\ \lambda, & \text{otherwise.} \end{cases}$$

Thus, the Euler's method approximation for E_n, the length of the final exit queue at time t_n with m feeder queues, becomes

$$E_n = E_{n-1} + \left[\left(\sum_{j=1}^{m} \rho_{j-\beta} \right) - \rho_E \right] \cdot \Delta t. \tag{5}$$

As in the previous example, all of the iterative equations **(2)** to **(4)** must be evaluated in parallel to calculate values of E_n vs. time, and the room is considered to be successfully evacuated after the line length drops below 1.

Using a hypothetical square room (10 m $\times$ 10 m) with $P(x,y) = 0.01$ everywhere within the room and $P(x,y) = 0$ everywhere else, $n = 100$, $s = 2$, and $\lambda = 2.8$, we generated a graph of length of exit queue vs. time (**Figure 4**). We measured s and λ empirically for an average person and an average single-width door. Although we have no experimental or theoretical basis for assigning a maximum allowable exit time, we arbitrarily choose 30 s. Since the line has almost disappeared after 30 s, 100 people is just over the maximum capacity for this hypothetical room.

Applications of the Model

Case 1: A Lecture Hall

Our model describes each row of seating in a lecture hall as a bottleneck, the aisle as an open space, and the exit as the final bottleneck. Since the queue at each row forms immediately after the fire alarm sounds and no people enter

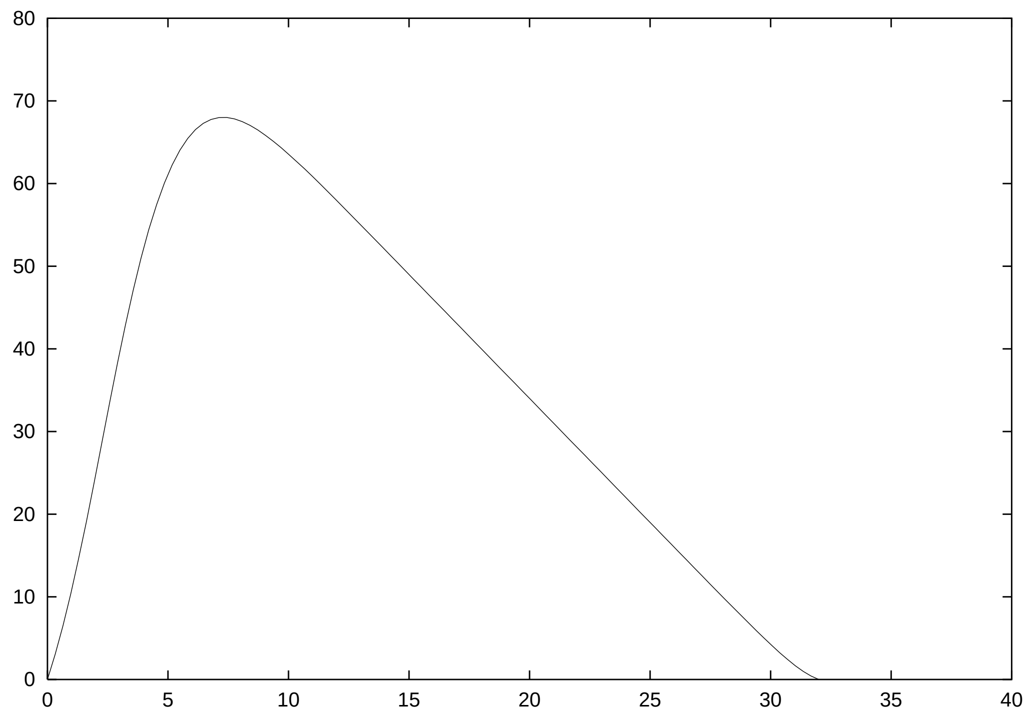

Figure 4. Typical graph of line length vs. time.

the queue thereafter, the value of $Q \mid_t^{t+\Delta t}$ is zero for every feeder queue, and the number of people in the room is given by the summation of L_0 for each feeder queue (in addition to any people on the stage). Lecture halls also typically have a plane of symmetry bisecting the seating and the stage, with each half served by its own exit; thus, only half of the room needs to be considered.

Case 2: A Cafeteria

With the presence of many small obstacles (chairs, tables, etc.), occupants cannot move as fast through the "free" spaces. In other words, the value of s is smaller in a room with movable furniture than in a room without such obstacles. A more thorough discussion of the effects of s on the evacuation time is given in **Sensitivity Analysis**.

Case 3: A Swimming Pool

For a swimming pool, we can assume that the only way out of the pool itself is up the ladders. In this case, the pool represents one feeder queue for each ladder, with each ladder serving a specific region of the pool. To account for people distributed outside of the pool in the event of an alarm, we simply add a $Q \mid_t^{t+\Delta t}$ term to the final exit Euler's method equation **(4)** to represent the rate at which these people enter the exit queue. Thus,

$$E_n = E_{n-1} + \left(\sum_{j=1}^{m} (\rho_j - \beta) + \frac{n_{\text{outside}} \cdot Q \mid_t^{t+\Delta t}}{\Delta t} - \rho_E \right) \cdot \Delta t,$$

where n_{outside} is the number of people outside the pool when the alarm sounds. Another complication is that people move much more slowly in the water; thus, a smaller s-value must be used inside the pool (computing the value of ρ_j) than outside (computing β-values and $Q \mid_t^{t+\Delta t}$ for the people outside). To incorporate people exiting the pool at areas besides the ladder, we could add another term into the rate portion of **(4)**.

Case 4: An Indoor Arena

Since sports arenas usually exhibit some approximate radial symmetry, only one segment of the building (served by one final exit gate) must be considered. Within each segment, there are usually several sets of seating sections, each served by one stairwell and one smaller gate leading to an aisle. These are the first set of bottlenecks. Within each section of seating, the aisle serves a number of different rows—these represent the second set of bottlenecks. Thus, a stadium presents a three-stage evacuation system: rows of seating lead to aisles, aisles pass through a small gate to an open stairwell, and several stairwells feed a main exit gate. A similar method could be used to evaluate the fire risks of any large building; however, the number of computations required rapidly becomes cumbersome.

Testing the Model: A Lecture Hall

Although the lack of a definite maximum allowable exit time prevents us from properly testing our model, we applied it to a lecture hall at our school for which the established occupancy limit is 117. Only half of the lecture hall has to be considered, as each half of the room has an exit. From the blueprints, we measured the distance from the output point of each of the aisle lines to the exit (from which the program calculates each β). Then, using $\lambda = 1.4$ for the queue at each row (arbitrarily chosen to be half the value for a standard door), $\lambda = 2.8$ for the exit, $s = 2$, and $\Delta t = 1$ s, we found that 117 people could be evacuated in about 30 s, confirming our arbitrary choice of 30 s for the maximum allowable time.

Sensitivity Analysis

The major factors that needed to be accounted for are the rate λ at which people can move through a queue, the speed s at which they can walk, the size of the room, and the pdf $P(x, y)$ used to describe the occupants' initial locations.

First, we looked at the effect of changes in λ on the time required for various numbers of people to exit the particular room (4.5 m $\times$ 6.5 m). We iterated the functions until the length of the queue returned to zero, varying both the number of people and the value of λ. (We found $\lambda \approx 2.8$ in our own trials.) Our results show that λ has a greater effect when the number of people in the room is large (see **Table 1**). Therefore, we recommend installing double, or very wide, doors in facilities intended to be used by large numbers of people.

Table 1.
Evacuation time as a function of queue distribution λ and number of people.

$n \backslash \lambda$	1.00	1.25	2.00	2.80	4.00
10	3	3	3	3	3
20	4	4	3	3	3
30	7	4	4	4	4
40	9	6	5	4	4
50	11	7	6	5	4
60	14	8	8	6	5
70	16	9	9	6	5
80	18	10	10	7	6
90	21	11	11	8	6
100	23	13	12	9	7

Second, we looked at the effects of changing the speed of people's movement within the room. For point of reference, in our experiments s ranged from 1 m/s (with obstacles) to 2 m/s (a fast walk in a room with no obstacles). Increased speed does lead to a decreased evacuation time (see **Table 2**)[2]. Even so, our model does not incorporate the negative effects of haste, such as tripping over obstacles or being trampled on by others. Therefore, we feel that a swift but moderate pace should be kept.

We also investigated the effects of increasing area on the evacuation time of a room. Not surprisingly, an increased area (and thus a longer distance to the exit) increases the evacuation time (see **Table 3**).

Finally, we considered the effects of different distributions of people throughout the room. Although we used a uniform distribution for all of our simulations so far, we decided that a normal distribution would make more sense in some cases. Redefining $P(x, y)$ as a three-variable normal pdf, with the hump of the curve in the center of the room, and truncating and renormalizing it over the domain of a 10 m $\times$ 10 m room, we generated graphs of the length of the exit queue versus time for each distribution (**Figure 5**).

Using a distribution in which the majority of people are closer to the door (like the normal pdf used in **Figure 5**) decreases the time to evacuate.

[2]The gaps in the table result because no line ever develops at the exit queue, and the program does not know when to end the simulation. This represents a fundamental weakness of our method of determining evacuation time but does not compromise the overall model.

Table 2.

Evacuation time as a function of speed (m/s) and number of people.

$n\backslash s$	0.50	1.00	2.00	3.50
10			3	2
20		5	3	2
30		7	4	3
40	10	7	4	3
50	12	7	5	4
60	14	7	6	4
70	14	8	6	5
80	14	8	7	5
90	14	9	8	6
100	14	10	9	7

Table 3.

Evacuation time as a function of (square) room size (m^2) and number of people.

$n\backslash A$	25	100	400	2,500	10,000
10	4	6	11		
20	8	7	12	25	
30	11	10	13	27	
40	14	13	15	29	51
50	17	16	18	30	53
60	20	19	20	31	54
70	24	22	24	31	55
80	27	25	26	32	57
90	30	29	29	34	59
100	33	32	32	36	59

A noteworthy feature of all the graphs is that once the line grows to its maximum length, it decreases linearly thereafter. This implies that the rate at which the room can be evacuated depends primarily on the rate at which people can leave the room, since for the entire linear section (from $t = 5$ to $t = 30$ in **Figure 5**) the rate at which people are entering the queue at the door is close to zero and the rate at which they are leaving is close to λ. This confirms our recommendation that rooms intended to hold large numbers of people should use doors as wide as possible.

Strengths and Weaknesses

The greatest strength of our model lies in its flexibility. Without any fundamental changes, our model can be used on any of a number of different kinds of spaces in which maximum occupancy may be an issue, including auditoriums, pools, lecture halls, board rooms, classrooms, cafeterias, and gymnasiums. In addition, the model can be extended to circumstances in which the exits from the room lead only to an intermediary location, such as a hallway, which then

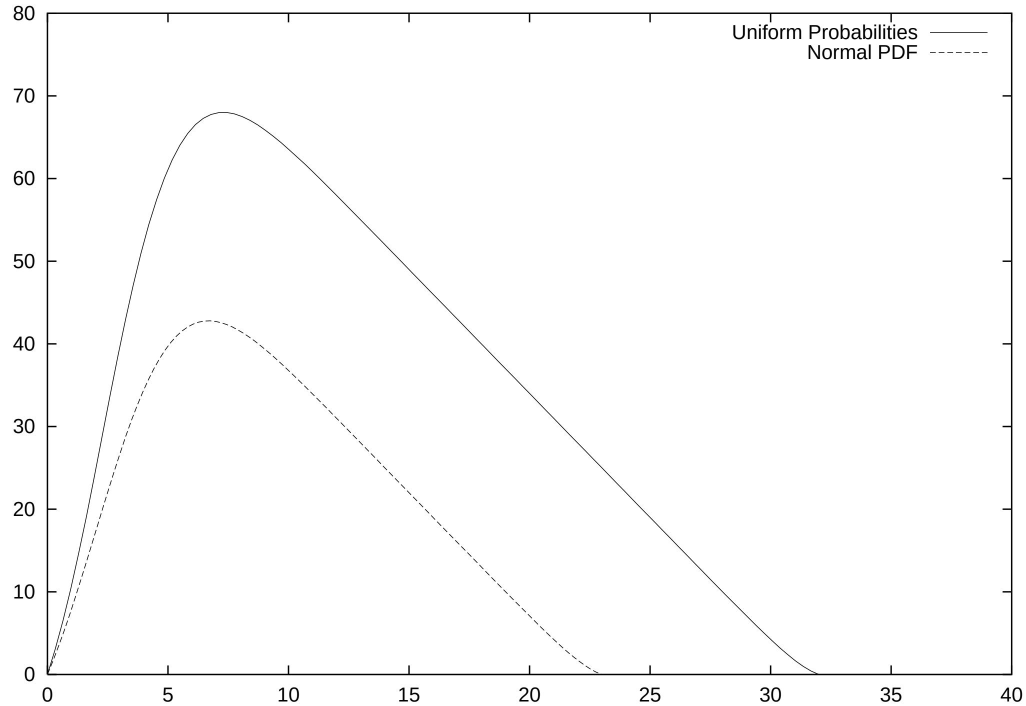

Figure 5. Effect of different pdfs on line length vs. time.

leads into another exit out of the building. This flexibility also leads to one of the larger weaknesses of our model, that it can get overly complex. But, for the majority of situations, our model remains reasonably simple. Furthermore, because the value of $Q\mid_t^{t+\Delta t}$ can be computed numerically for any given $P(x, y)$, a room of any size or shape and with any distribution of occupants can be considered.

Another strength of our model is that the initial parameters and constants are flexible. Though we used values for λ and s that we determined experimentally, repeated experiments could be done with different audiences to determine more accurate values for these parameters. In addition, λ and s can be modified to include many other variables, such as the type of occupant a room usually holds (for instance, small children and adults have different maximum speeds) and door construction (which dictates λ).

A final strength of our model is its consideration of other factors. By picking the minimum of several maxima, we could determine the maximum occupancy of a room considering other factors such as room size, amount of personal space required by the people in the room, sanitation concerns, and weight capacity.

Our model also has several weaknesses. A sensitivity analysis is difficult to perform. As we were unable to find much data on many of the constants which were needed, we had to determine just a few values experimentally, which may not be representative.

A major weakness concerns our assumptions. In many cases, these may be large oversimplifications of the actual circumstances of evacuating a room or building (i.e., the assumption of a constant rate leaving a queue). Furthermore, we have no reasonable basis for determining the maximum permissible time to evacuate a room; without this information, we cannot firmly establish the maximum occupancy of a room or building.

Another weakness of our method is the way in which we determine when everyone has left a room. In general, we do this by seeing when the length of the exit queue returns to zero; unfortunately, some people may still be reaching the door and immediately leaving the room, maintaining the length of the queue at zero but still requiring extra time to evacuate. However, this does not invalidate the overall model, and more sophisticated ways of determining when the room is empty could easily be applied.

References

Allen, Arnold O. 1978. *Probability, Statistics, and Queueing Theory*. Orlando, FL: Academic Press.

Bartkovich, Kevin G., et al. 1996. *Contemporary Calculus Through Applications*. Dedlum, MA: Janson Publications.

Mendenhall, William, Richard L. Scheaffer, and Dennis D. Wackerly. 1986. *Mathematical Statistics with Applications.* Boston, MA: Duxbury Press.

O'Brien-Atkins Architecture. 1995. Education Technology Complex Blueprints. March 24, 1995.

Press, William H., Brian P. Flannery, Saul A. Teukolsky, and William T. Vetterline. 1988. *Numerical Recipes in C: The Art of Scientific Computing*. New York: Cambridge University Press.

Appendix A: Newspaper Article

> A guy walks into a bar and the bartender says, "Hey, man, you can't come in here. We're at maximum capacity already." The man replies, "But there's plenty of room—what's the problem?"

This illustrates one of the most threatening evils plaguing our society today—apathy about maximum occupancy. People just don't realize the importance of being able to evacuate quickly from a facility in the event of an emergency. Sure, it may seem fun to see how many people can squeeze into a phone booth together, but what if the phone booth catches on fire? We won't even consider the possible catastrophes in a clown car.

Seriously, public safety is an important concern, especially when when considering evacuation speed from a facility. During emergency situations, people are likely to panic or not think clearly, so extensive planning for the event of an emergency could save lives. Since businesses like restaurants may be more concerned with making money than with safety, a standard method of determining maximum safe occupancy would be helpful in enforcing this issue.

Obviously, one of the most important considerations during an evacuation procedure is that people tend to get backed up at places like doors and other types of exits. We were able to mathematically incorporate this buildup of people into a model to determine the maximum occupancy for many different public facilities. Using queuing theory and Euler's method, we looked at how long various numbers of people took to vacate a certain room. Given a minimum evacuation time (say, thirty seconds or a minute), we can calculate the maximum safe number of people for that room.

However, additional factors must be considered in addition to evacuation speed. For instance, people need a certain amount of personal space. When people are crowded together for long periods of time, certain health hazards might result, especially in a restaurant or cafeteria. Also, many elevators have a maximum weight capacity, which has more to do with the strength of the cables than with evacuation. Looking at all these different factors enables us to find a reasonable and justifiable maximum capacity. Our model, in accurately representing reality, would be an invaluable tool in predicting the likely outcome of an emergency evacuation. Its ability to handle complex situations and extreme flexibility make it ideal for practical use.

So, the next time you see a sign stating maximum safe occupancy, take the time to consider it carefully. If the number of people in the room exceeds the limit, you may want to consider immediately hurrying towards an exit. Just don't panic.