2000 年美国大学生数学建模竞赛 MCM、ICM 试题

2000 MCM A: Air Traffic Control

To improve safety and reduce air traffic controller workload, the Federal Aviation Agency (FAA) is considering adding software to the air traffic control system that would automatically detect potential aircraft flight path conflicts and alert the controller. To that end, an analysit at the FAA has posed the following problems.

Requirement A: Given two airplanes flying in space, when should the air traffic controller consider the objects to be too close and to require intervention?

Requirement B: And airspace sector is the section of three-dimensional airspace that one air traffic controller controls. Given any airspace sector, how do we measure how complex it is from an air traffic workload perspective? To what extent is complexity determined by the number of aircraft simultaneously passing through that sector

1. at any one instant?

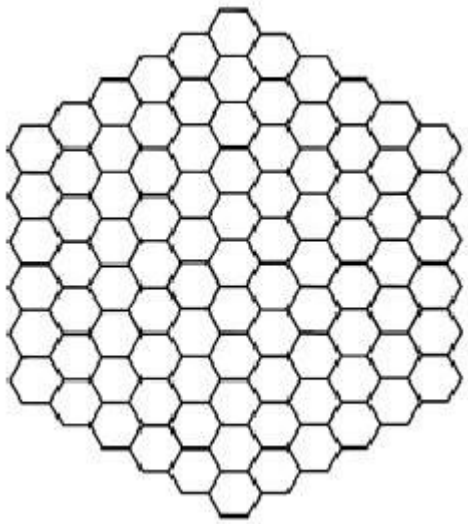
2. during any given interval of time?

3. during a particular time of day?

How does the number of potential conflicts arising during those periods affect complexity? Does the presence of additional software tools to automatically predict conflicts and alert the controller reduce or add to this complexity? In addition to the guidelines for your report, write a summary (no more than two pages) that the FAA analyst can present to Jane Garvey, the FAA Administrator, to defend your conclusions.

2000 MCM B: Radio Channel Assignments

We seek to model the assignment of radio channels to a symmetric network of transmitter locations over a large planar area, so as to avoid interference. One basic approach is to partition the region into regular hexagons in a grix (honeycomb-style), as shown in Figure 1, where a transmitter is located at the center of each hexagon.



An interval of the frequency spectrum is to be alloted for transmitter frequencies. The interval will be divided into regularly spaced channels, which we represent by integers 1,2,3, … . Each transmitter wil be assigned one positive integer channel. The same channel can be used at many locations, provided that interference from nearby transmitters is avoided.

Our goal is to minimize the width of the interval in the frequency spectrum that is needed to assugn channels subject to some constraints. This is achieved with the concept of a span. The span is the minimum, over all assignments satisfying the constraints, of the largest channel used at any location. It is not required that every channel smaller than the span be used in an assignment that attains the span.

Let s be the length of a side of one of the hexagons. We concentrate on the case that there are two levels of interference.

Requirement A: There are several contrainsts on the frequency assignments. First, no two transmitters within distance 4s of each other can be given the same channel. Second, due to spectral spreading, transmitters within distance 2s of each other must not be given the same or adjacent channels: Their channels must differ by at least 2. Under these contraints, what can we say about the span in Figure 1?

Requirement B: Repeat Requirement A, assuming the grid in the example spreads arbitrarily far in all directions.

Requirement C: Repeat Requirements A and B, except assume now more generally that channels for transmitters within distance 2s differ by at least some given integer k, while those at distance at most 4s must still differ by at least one. What cna we say about the span and about efficient strategies for designing assignments, as a function of k?

Requirement D: Consider generalizations of the problem, such as several levels of interference or irregular transmitter placements. What other factors may be important to consider?

Requirement E: Write an article (no more than 2 pages) for the local newspaper explaining your findings.

2000 ICM: Elephants: When is Enough, Enough?

“Ultimately, if a habitat is undesirably changed by elephants, then their removal should be considered -even by culling.” National Geographic (Earth Almanac) –December 1999 A large National Park in South Africa contains approximately 11,000 elephants. Management policy requires a healthy environment that can maintain a stable herf of 11,000 elephants. Each year park rangers count the elephant population. During the past 20 years whole herds have been removed to keep the population as close to 11,000 as possible. The process involved shooting (for the most part) and occasionally relocating approximately 600 to 800 elephants per year.

Recently, there has been a public outcry against the shooting of these elephants. In addition, it is no longer feasible to relocate even a small population of elephants each year. A contraceptive dart, however, has been developed that can prevent a mature elephant cow from conceiving for a period of two years.

Here is some information about eh elephants in the Park:

- There is very little emigration of immigration of elephants.
- The gender ratio is very close to 1:1 and control measures have endeavored to maintain parity.
- The gender ratio of newborn calves is also about 1:1. Twins are born about 1.35% of the time.
- Cows first conceive between the ages of 10 and 12 and produce, on average, a calf every 3.5 years until they reach an age of about 60. Gestation is approximately 22 months.
- The contraceptive dart causes an elephant cow to come into oestrus every month (but not conceiving). Elephants usually have courtship only once in 3.5 years, so the monthly cycle can cause additional stress.
- A cow can be darted every year without additional detrimental effects. A mature elephant cow will not be able to conceive for 2 years after the last darting.

- Between 70% and 80% of newborn calves survive to age 1 year. Thereafter, the survival rate is uniform across all ages and is very high (over 95%), until about age 60; it is a good assumption that elephants die before reading age 70.

- There is no hunting and negligible poaching in the Park.

The park management has a rough data file of the approximate ages and gender of the elephants they have transported out of the region during the past 2 years. This data is available on website: icm2000data.xls. Unfortunately no data is available for the elephants that have been shot or remain in the Park.

Your overall task is to develop and use models to investigate how the contraceptive dart might be used for population control. Specifically:

Task 1: Develop and use a model to speculate about the likely survival rate for elephants aged 2 to 60. Also speculate about the current age structure of the elephant population.

Task 2: Estimate how many cows would need to be darted each year to keep the population fixed at approximately 11,000 elephants. Show how the uncertainty in the data at your disposal affects your estimate. Comment on any changes in the age structure of the population and how this might affect tourists. (You may want to look ahead about 30-60 years.)

Task 3: If it were feasible to relocate between 50 and 300 elephants per year, how would this reduce the number of elephants to be darted? Comment on the trade-off between darting and relocation.

Task 4: Some opponents of darting argue that if there were a sudden loss of a large number of elephants (due to disease or uncontrolled poaching), even if darting stopped immediately, the ability of the population to grow again would be seriously impeded. Investigate and respond to this concer.

Task 5: The management in the Park is skeptical about modeling. In particular, they argue that a lack of complete data makes a mockery of any attempt to use models to guide their decision. In addition to your technical report, include a carefully crafted report (3-page maximum) written explicitly for the park management that responds to their concerns and provides advice. Also suggest ways to increase the park managers confidence in your model and your conclusions.

Task 6: If your model works, other elephant parks in Africa would be interested in using it. Prepare a darting plan for parks of various sizes (300-25,000 elephants), with slightly different survival rates and transportation possibilities.