Judge's Commentary: The Outstanding Discussion Groups Papers

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Making the An Tostal situation particularly open-ended is the fact that a "good mix" of board members is not clearly defined. This also makes it a particularly realistic problem. In practice, it is not uncommon that those requesting the solution don't know exactly what they want or what is possible. They look to the modeler for these answers and related suggestions. While the problem statement provides some guidance of the mixing desired, it allows for a lot of interpretation by the modeler. Thus, in order to evaluate any set of board member assignments to the seven sessions it is imperative that some sort of measure of the "goodness" of a particular solution be identified.

To establish this measure, or objective function, it is necessary to make several assumptions. These assumptions can be made by answering questions such as:

- Is the third meeting of two board members worse than the second?
- Is the second meeting of two in-house members worse than the second meeting of two regular members?
- How should the second meeting of an in-house member with a regular member be evaluated?
- Does increasing the time between sessions in which two members are in the same group reduce the "cost?"
- How does the "cost" of having two board members fail to meet compare to that of having them meet more than once?

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The name "An Tostal" has no real meaning as far as the problem is concerned. It was the name of the spring-quarter weekend of celebration just before final exams at Kent State University, where I studied as an undergraduate.

- Is common membership in the form of A-B-C worse than common membership of the form A-B and C-D?

It should not come as a surprise that wide variations in these assumptions and thus in how to measure success were employed by the modeling teams. The quality and justification of such assumptions were weighted heavily by the judges in their evaluation of papers. While varied assumptions were reasonable, others considered unreasonable include:

- It is better to have a member skip a session than to be in a session with the same member again.

- To minimize common membership, the number of groups for the afternoon sessions may be increased from four to five.

- To ensure that everyone meets at least once, there will be only one group for the seventh session.

A common weakness of papers in past competitions has been their failure to provide both a functional model and a solution. Frequently, they have provided either a "brute force" solution or a simulated solution without a model. At the other extreme are those providing creative models without demonstrating their functionality in solving the problem at hand.

In the statement of this year's problem, we attempted to avoid these pitfalls by calling for a solution to the current problem as well as a simple algorithm that can be used in the event that the problem parameters are changed. Thus, papers providing only a "brute force" solution for the existing problem were screened out early in the process. The judges were unanimous in their opinion that the general quality of the entries was improved over those of a year ago. There seemed to be an increased understanding and a willingness to discuss what form a realistic solution might take as well as the related mathematics and bounds on solutions.

While methods of solution varied from the brute-force listing, with creative matrix methods of accounting, to orthogonal Latin squares, the most common methods were simulated annealing and the greedy algorithm. However, few teams addressed the generalized problem for future meetings with any or all parameters changed. One team that used the greedy algorithm noted that local optimization—that is, optimization at the session level—does not guarantee global optimization, and thus it may be desirable to allow a second encounter of two members early in the day.

Several teams used a different algorithm to make last-minute changes than to make the initial assignments. One team doing this made a conscious effort to alter drastically the schedule of a few people rather than modify slightly several schedules. One judge commented that the team must have contained a social scientist, while another retorted, "Or someone who had worked with college faculties." Another team noted that in measuring success it is desirable to consider balance (balance of membership of groups) and mixture (pairs that meet a disproportionate number of times). To accomplish this, the geometric mean of these two measures was minimized. Another team decided that if a pair of members must meet twice during the day, this would be less "costly" if the time between these two meetings were maximized.

The four papers judged outstanding had many similarities. All decided it would be desirable to keep the size of the groups for each session as equal as possible. With this agreement, most observed that 532 pairings would need to be made in order to complete the day's schedule, a value achieved in their final solution only by the team from East China University of Science and Technology. Some argued effectively that uneven group sizes increase the number of pairings needed. All four teams recognized that 406 pairings are necessary if each board member were to meet each other board member. That is, for a group of 29 people it would take 406 handshakes for each to shake hands with every other person. In fact, all these teams reported the number of times each of these pairings (handshakes) occurred in their final or "best" solution, as shown in **Table 1**. For example, the University of Toronto team reported that 33 of the pairs never met, 226 of the pairs met exactly once, 134 of the pairs met twice, and 14 of the pairs met three times.

Table 1.

Occurrences of numbers of pairs in final solutions.

Team	Number of pairs			
	0	1	2	3
East China Univ. of Science and Tech.	26	253	102	25
Macalester College	40	214	138	14
Rose Hulman Inst. of Tech.	32	218	152	4
University of Toronto	33	226	134	14

In spite of the fact that the teams used quite different objective functions, these results are quite similar. Some of the differences might be predicted from the difference in objective functions. The team from Rose-Hulman Institute of Technology got only four groups meeting three times, since their penalty for this situation was modeled as powers of four. The team from Macalester College had the largest number of pairs that never met, 40. This is because they were the only one of the four teams that looked beyond individual pairs in developing their objective function. They even reported that for their solution, "No two discussion groups have more than two members in common."

Three of the four Outstanding papers used a form of the greedy algorithm to obtain a solution. The other, from Macalester College, used simulated annealing. This paper stood out for its explanation of how simulated annealing is used and for its strong objective function. This function was the sum of four objectives and included a penalty for more than one repeat pairing in a group. In addition, the paper contained a nice proof that if no two pairs of board members are together more than two times, then some pair is never together.

The paper from the Rose-Hulman team stood out for its comparison of three solution methods and the fact that it was well written for the intended audience. Schedules made randomly, with the greedy algorithm and a modified greedy algorithm, were compared statistically. The random method could then be used as a baseline for comparison of the other methods.

Finally, the paper from the University of Toronto team provided some excellent proofs on bounds for solutions. These results were proven in general and then demonstrated for the An Tostal situation.

About the Author

Donald Miller is Associate Professor and of Chair of Mathematics at Saint Mary's College. He has served as an associate judge of the MCM for five years and prior to that mentored two Meritorious teams. He has done considerable consulting and research in the areas of modeling and applied statistics. His current research, with a colleague in political science, involves the statistical analysis of the politics related to the adoption of state lotteries and state approval of other forms of gambling. He is currently a member of SIAM's Education Committee and Past President of the Indiana Section of the Mathematical Association of America.