Problem Title: The Longest Lasting Sandcastle(s)

Year: 2020

Student Level: Undergraduate

Source: MCM

Commentary: Yes (1)

Student Papers: Yes (5)

Problem

Wherever there are recreational sandy ocean beaches in the world, there seem to be children (and adults) creating sandcastles on the seashore. Using tools, toys, and imagination, beach goers create sandcastles that range from simple mounds of sand to complicated replicas of actual castles with walls, towers, moats, and other features that mimic real castles. In all these, one typically forms an initial foundation consisting of a single, nondescript mound of wetted sand, and then proceeds to cut and shape this base into a recognizable 3-dimensional geometric shape upon which to build the more castle-defining features.

Inevitably, the inflow of ocean waves coupled with rising tides erodes sandcastles. It appears, however, that not all sandcastles react the same way to waves and tides, even if built roughly the same size and at roughly the same distance from the water on the same beach. Consequently, one wonders if there exists a best 3-dimensional geometric shape to use for a sandcastle foundation.

Requirements

1. Construct a mathematical model to identify the best 3-dimensional geometric shape to use as a sandcastle foundation that will last the longest period of time on a seashore that experiences waves and tides under the following conditions:

- built at roughly the same distance from the water on the same beach, and
- built using the same type of sand, roughly the same amount of sand, and the same water-to-sand proportion.

2. Using your model, determine an optimal sand-to-water mixture proportion for the castle foundation, assuming you use no other additives or materials (e.g. plastic or wooden supports, stones, etc.).

3. Adjust your model as needed to determine how the best 3-dimensional sandcastle foundation you identified in requirement 1 is affected by rain, and whether it remains the best 3-dimensional geometric shape to be used as a castle foundation when it is raining.

4. What other strategies, if any, might you use to make your sandcastle last longer?

5. Finally, write an informative, one- to two-page article describing your model and its results for publication in the vacation magazine: Fun in the Sun, whose readers are mainly non-technical.

Your submission should consist of:

- One-page Summary Sheet
- Table of Contents
- One- to Two-page Article
- Your solution of no more than 20 pages, for a maximum of 24 pages with your summary, table of contents, and article.

Note: Reference List and any appendices do not count toward the page limit and should appear after your completed solution. You should not make use of unauthorized images and materials whose use is restricted by copyright laws. Ensure you cite the sources for your ideas and the materials used in your report.

Commentary

Judges' Commentary: The Sandcastle Problem

William P. Fox
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Student Papers

Team 2007698: The Longest Lasting Sandcastle(s)

Wuhan University, China

Team 2010821: The Longest Lasting Sandcastle(s)

North University of China, China

Team 2011873: The Longest Lasting Sandcastle(s)

Northwestern Polytechnical University, China

Team 2013836: The Longest Lasting Sandcastle(s)

Xi'an Jiaotong University, China

Team 2019696: The Longest Lasting Sandcastle(s)

Beijing Normal University, China