

Abstract

We build a mathematical model to determine the optimal curve of slimline seats on the airplane.

In the first question, we take two main factors into consideration, one is the standard deviation of spine equivalent stress, while the other is the difference between the seat back curve and the natural curve of spine. We propose a simplified model which is a system of 24 rigid rod and 25 mass point and distribute the weight of upper body on each rods equally. Then carry out force analysis on the rods and the mass point. Based on the above discussion, we build an optimization model to find a curve to make passenger most comfortable-uniformly forced. Finally, we use Matlab software to figure out the optimal seat back curve.

In the second question, we put emphasis on both the passengers' space and comfort. First, we study the space which is needed to be left for the back passengers. Then we assume that the backplate curve coincides with the back seat curve. To let the back passengers receive more space, we need to adjust the backplate curve and of course have to keep the back seat curve invariant. Thus, we deal with backplate curve along with the padding's shape. Obviously, the space for the back passengers gets better if the backplate curve is adjusted and as a result the padding will be deeply cut down. However, we keep the padding thicker than 5cm, as the minimum, in case the front passengers feel bad if the padding is too thin. Consequently, how to eliminate this contradiction is our main purpose. Fortunately, we find a deep study of the mechanical analysis and build an optimization model. With the help of Matlab, we finally figure out the answer.

Many mathematical model lay emphasis on the answer, while our model focus on the way of solving this problem. It has a wider application. At the same time, we connect the natural spine curve with the mechanical analysis. It is more scientific.

In despite of this, there are still some shortcomings. The weakness of our model are need to further investigate, which is shown in the paper.

Key words: optimal curve padding Matlab optimization model
mechanical analysis

Contents

1.Introduction.....	1
1.1Background of the problem.....	1
1.2 Some solutions.....	1
2.The Description of the Problem.....	1
2.1 What do we need to do?.....	1
2.2 The preparation of the problem.....	2
3.Models.....	3
3.1 Terms, Definitions and Symbols.....	3
3.2 Assumptions.....	4
3.3 The Foundation of First Model.....	4
3.3.1The quantization of aspects.....	4
3.3.2 mechanical analysis model.....	6
3.3.3 optimization model.....	8
3.3.4 Solution and Result.....	9
3.3.5 Analysis of the Result.....	12
3.4The Foundation of Second Model.....	12
3.4.1Enlarge space.....	12
3.4.2 Mechanical factor.....	13
3.4.3 Optimization model.....	13
3.5 Strength and Weakness.....	15
4.Future Work.....	16
4.1 The Improvement of Model.....	16
4.2 Extra Advice.....	17
5.Advertising material.....	18
6.References.....	19

I. Introduction

In order to indicate the origin of the slimlines seats problem, the following background is worth mentioning.

1.1 Background of the problem

Airplane is a good choice for a long-term journey because of its high-speed and comfort. An airline seat is a seat on an airliner in which passengers are accommodated for the duration of the journey. In the past, the airline seats are very big and thick. When passengers sit on the seat, their weight will cause the deformation of backrest to adapt to their figure which will help passengers get the more comfortable condition. However, the traditional seat has some disadvantages such as its big size. It will affect the seating capacity and the profile of airlines. At the same time, it will still cause the discomfort of passengers' lumbar vertebra if they have to sit on it for a long time.

1.2 Some solutions

To solve this problem, some people have come up with some ideas.

Alireza yaghoub, a Malaysia student, who needs to take an eight-hour air journey between home and school, invented a kind of brand-new airline seat which can make passengers in economy class have the same comfortable feeling with those in first class cabin and he named it AirGo airline seat. The design of independent AirGo airline seat and the improvement of foot pedal are both creative features to improve the satisfaction of passengers. However, it occupies 16% more space than traditional airline seats.

Some airlines are now introducing new slimline seats in economy class. These seats, in addition to weighing less, theoretically allow airlines to increase capacity. Contrary to its advantages, its inferiority may raise more concern. Many passengers expressed displeasure with these seats.

In a word, it is the right time for us to invent a kind of comfortable slimline seat.

II. The Description of the Problem

2.1 What do we need to do?

Premise: These seats may or may not feature moveable headrests, and generally do not feature adjustable lumbar support. The refined seats can liberate more passenger space.

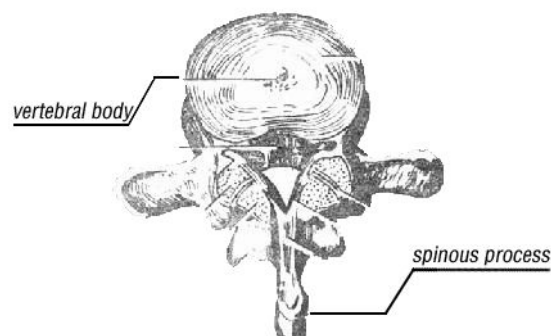
we should find a mathematical model to solve the following problems.

- Without changing the structure of the premise, design a suitable seat back curve, in order to make the seat more comfortable.
- Without changing the main internal structure, optimize the seat backplate curve and padding , in order to make the seat more comfortable.
- Write an advertising material with 2~3 pages for the airline to describe our design features ad advantages concisely.

2.2 The preparation of the problem

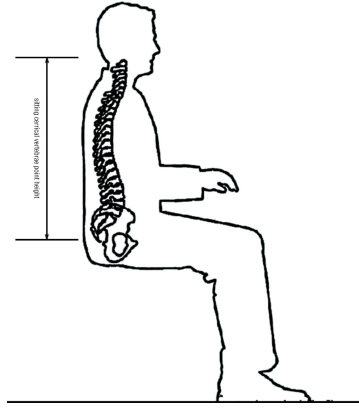
To solve this problem , we need to know some knowledge about human body:

- the constitute of spine: human's spine is constituted by 33vertebras, which was connected by borrow ligament, arthrosis and intervertebral disc.
- physiological bend: spine has four bends, which looks like a “S” from edgewise, they are cervical lordosis, postural curvature of thoracalspine, lumbar spinal lordosis and postural curvature of ssacral vertebral.
- equivalent stress: in the aspect of prompting material lose efficacy, it has the same single-track stress with the complex stress or condition stress.
- the natural curve of spine: the curve that make human's spine most comfortable when human sitting
- spinous process: the spinulose in the middle of spinal marrow bow or the leng scale formed ministry of uplift on the back. As picture 1:



Picture 1

- sitting cervical vertebrae point height: the distance between cervical spine point and seat, please look at picture2:



Picture 2

III. Models

3.1 Terms, Definitions and Symbols

The signs and definitions are mostly generated from queuing theory.

signs	definitions
i	The number of vertebral body, ranging from 0 to 24
θ_i	The angle between the i-th vertebral body and horizontal plane
β_i	The angle between the i-th vertebral body and the (i+1)-th vertebral body
F_{n2}^i	The normal force of the lower end of i-th vertebral body
F_{n1}^i	The normal force of the upper end of i-th vertebral body
$F_{\tau1}^i$	The tangential force of the upper end of i-th vertebral body
$F_{\tau2}^i$	The tangential force of the lower end of i-th vertebral body
N_i	The seats back supporting force to i-th vertebral body

R_1	The length 3cm longer than the length of thigh
R_2	Half of the height of upper body
R_3	The height of upper body

3.2 Assumptions

- The passengers considered in this thesis are Chinese (Similarly, we can deal with them from other countries)
- Ignore the deformation of backrest in the first question
- Each node of the spine is assumed as a cylindrical rigid body while each node of the intervertebral disc with the same basal diameter is assumed as a cylinder made of elasticity material
- Ignore the changing of spine length caused by gravity
- Weight distributes averagely on every vertebral body according to their length
- Chairs are supported on the centroid
- Seat back is a rigid body without deformation
- Ignore the effect on the spine caused by clothes and seat back material

3.3 The Foundation of First Model

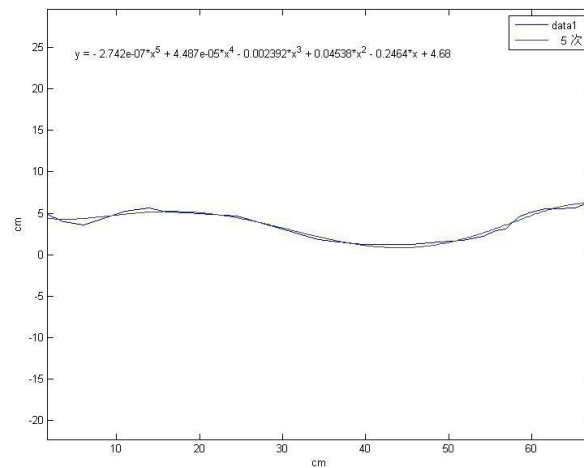
3.3.1 The quantization of aspects

People's heights are affected by so many factors ,such as age, sexy, race. It is hard for us to find a representative natural spine curve. So thesis places emphasis on analysing the way of calculating seat back curve instead of getting a accurate curve. Now we need to quantify spine equivalent stress, spinous process curve and the natural spine curve.

- spinous process curve: to get the information of spine process, we select some reference points from spinous process.

After 90°Clockwise rotation ,spine will form four physiological bend. So we need to use Matlab to read their coordinates and giving a five times matching. Finally, we can

get the curve(Picture 3) and it can be described as a approximate formula.



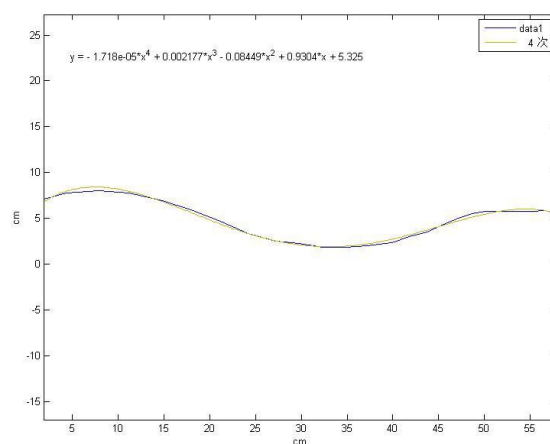
Picture 3

The approximate formula:

$$y_1 = -2.742 \times 10^{-7} x^5 + 4.487 \times 10^{-5} x^4 - 0.002392 x^3 + 0.04538 x^2 - 0.2464 x + 4.68 \quad (1)$$

●the natural spine curve:

In the solving of the natural spine curve, spine will form four physiological bend. While we ignore the auricular surface, so four times matching is enough. The natural spine curve can be obtained with the help of Matlab. Then according the standard Chinese adults sitting cervical vertebrae point, we enlarge curve to the realistic size(Picture 4).



Picture 4

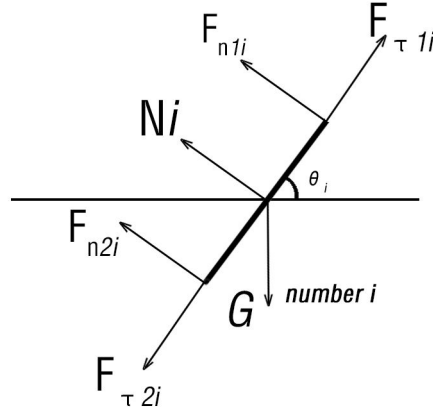
The approximate formula:

$$y_1 = -1.718 \times 10^{-5} x^4 + 0.002177 x^3 - 0.08449 x^2 + 0.9304 x + 5.325 \quad (2)$$

So the relationship between spinous process curve and the natural spine curve can be figured out.

3.3.2 mechanical analysis model

Spine have 24 scleromeres, so we considered spine as a 24 segments hinge. Then distribute the weight of the upper body without head and limbs to the hinge averagely. Look at this force analysis (Picture 5):



Picture 5

Every scleromeres are in equilibrium condition of forces, so no parallel translation exit. It means resultant force is zero. The formula is:

$$\sum F_{(\tau)} = \sum F_{(n)} = 0 \quad (3)$$

So:

$$F_{\tau 1}^{(i)} = F_{\tau 2}^{(i)} + G_i \sin \theta_i \quad (4)$$

$$F_{n 1}^{(i)} + F_{n 2}^{(i)} + N_i = G_i \cos \theta_i \quad (5)$$

According to mechanical knowledge, when Every scleromeres in equilibrium condition of force, no rotation exit. So the sum of moment is zero:

$$\sum M = 0 \quad (6)$$

Take Force analysis into this formula:

$$F_{n1}^{(i)} = F_{n2}^{(i)} \quad (7)$$

Simultaneous equations is:

$$\begin{cases} F_{\tau 1}^{(i)} = F_{\tau 2}^{(i)} + G_i \sin \theta_i \\ F_{n1}^{(i)} + F_{n2}^{(i)} + N_i = G_i \cos \theta_i \\ F_{n1}^{(i)} = F_{n2}^{(i)} \end{cases} \quad (8)$$

We use θ_i to represent other variables:

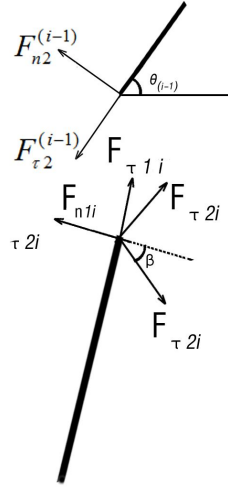
$$\begin{cases} N_i = G_i \cos \theta_i - 2F_{n1}^{(i)} \\ F_{n2}^{(i)} = F_{n1}^{(i)} \\ F_{\tau 2}^{(i)} = F_{\tau 1}^{(i)} - G_i \sin \theta_i \end{cases} \quad (9)$$

For the first vertebral body:

$$i = 0$$

$$F_{\tau 1}^{(i)} = F_{n1}^{(i)} = 0 \quad (10)$$

now, we can infer N_i , F_{n2} and $F_{\tau 2}$ when $i = 0$.



Picture 6

Look at picture(6), we analyze the force between the upper end vertebral body and the lower end. They are a pair of force and reacting force, so they have the same size and opposite direction. We defined:

$$\beta_i = \theta_i - \theta_{i-1} \quad (11)$$

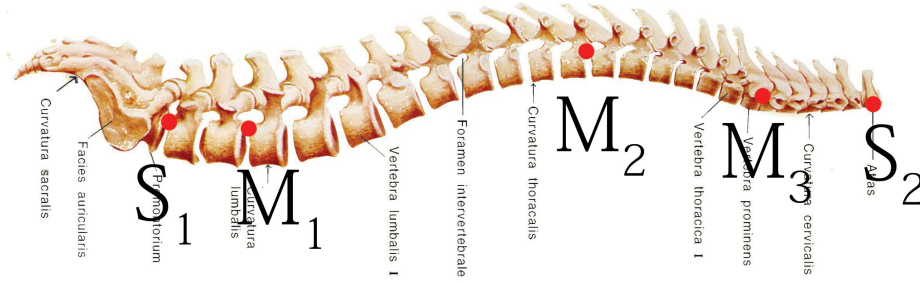
so

$$\begin{aligned} \begin{pmatrix} F_{\tau 1}^{(i)} \\ F_{n1}^{(i)} \end{pmatrix} &= \begin{pmatrix} \cos \beta_i & -\sin \beta_i \\ -\sin \beta_i & -\cos \beta_i \end{pmatrix} \begin{pmatrix} F_{\tau 2}^{(i-1)} \\ F_{n2}^{(i-1)} \end{pmatrix} \\ \Rightarrow \begin{cases} F_{\tau 1}^{(i)} = F_{\tau 2}^{(i-1)} \cos \beta_i - F_{n2}^{(i-1)} \sin \beta_i \\ F_{n1}^{(i)} = -F_{\tau 2}^{(i-1)} \sin \beta_i - F_{n2}^{(i-1)} \cos \beta_i \end{cases} \end{aligned} \quad (12)$$

In this way, every vertebral body's stress can be figured out. It is an mechanical analysis mathematical model.

3.3.3 optimization model

To make people feel comfortable, we need to find a balance between average distribution of stress and the natural spine curve. So after an mechanical analysis mathematical model, we need to build a optimization model to find this balance.



Picture 7

As picture(7), we suppose that S1 and S2 is fixed on the seat. Adjust three selected points M1.M2.M3 to make :

$$\min \left(\sum_{i=1}^n \left(F_{n1}^{(i)} - \overline{F_{n1}} \right) + \sum_{i=1}^n \left(F_{\tau 1}^{(i)} - \overline{F_{\tau 1}} \right) + \sum_{i=1}^n \left(N_{(i)} - \overline{N} \right)^2 \right) \quad (13)$$

$$i = 0, 1, \dots, 24$$

When i is 2, 11, 18, the vertebral bodies' space is $\Delta h_2, \Delta h_{11}, \Delta h_{18}$. According to these three points, We can ensure the approximate shape of curve. The angle between seat back and horizontal line is 103° which is the least angle of the range of the most comfortable angle:

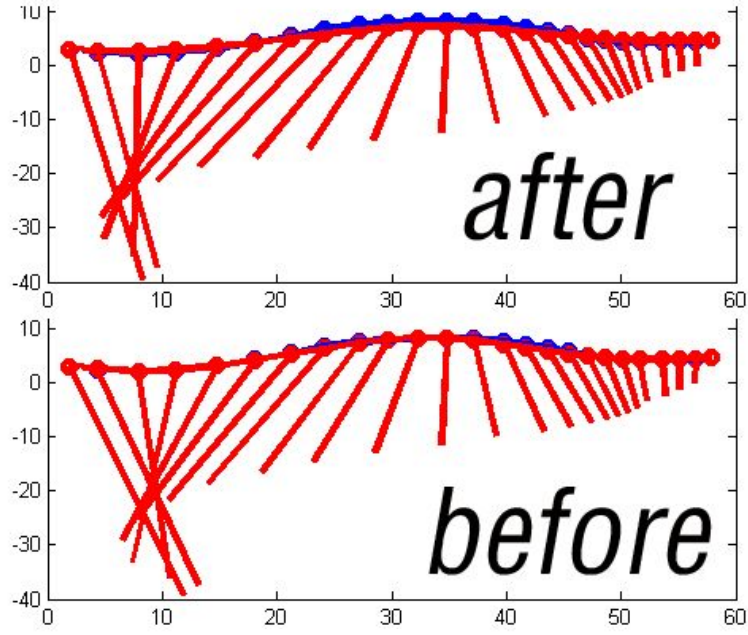
$$\begin{cases} N_i \geq 0 \\ |\Delta h_2| < 0.5, |\Delta h_{11}| < 1, |\Delta h_{18}| < 0.4 \\ \theta_0 = 87^\circ \end{cases} \quad (14)$$

Then we use Matlab to have spline interpolation and figure out the curve.

PS: This restriction is an example that can be promoted widely.

3.3.4 Solution and Result

With the help of Matlab, we can figure out this curve(Picture 8). The upper curve is the optimized result, while the other is the natural spine curve.



Picture 8

These long string stand for the stress. The variance of the stress distribution of the natural is spine is:

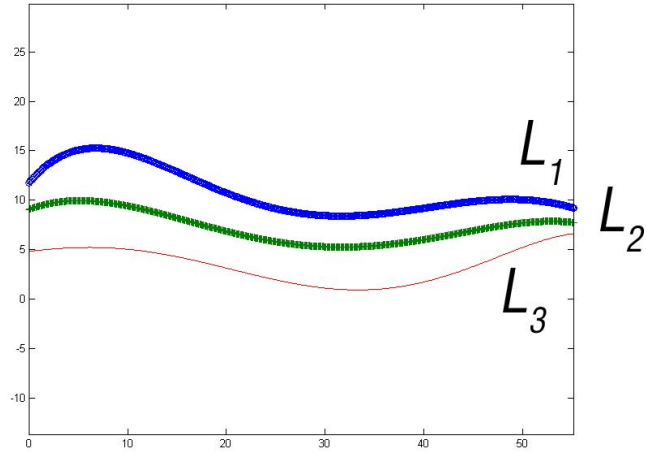
$$\sum S_1^2 = 7.5145 \times 10^3 \quad (15)$$

While our optimized curve's variance is:

$$\sum S_2^2 = 5.6827 \times 10^3 \quad (16)$$

It means the optimized curve is more comfortable than the natural spine curve.

So we can figure out the final back seat curve according to the relationship between the spine curve and the spinous process curve(Picture 9):



Picture 9

L_3 is the original back curve. L_2 is the optimized vertebral curve which is similar to the curve of this expression:

$$y_2 = -1.093 \times 10^{-5} x^4 + 0.001308 x^3 - 0.04616 x^2 + 0.3768 x \quad (17)$$

L_1 is the optimized back seat curve which is similar to the curve of this expression:

$$y_1 = -2.087 \times 10^{-5} x^4 + 0.002447 x^3 - 0.089 x^2 + 0.899 x \quad (18)$$

The detailed coordinates and change are following:

i	1	2	3	4	5	6	7	8	9
x	1.8832	4.2881	7.9209	11.145	14.777	18.103	21.276	24.141	27.16
y	7.1229	7.3489	7.4927	7.2552	6.6323	5.8506	5.0217	4.2847	3.6011
y_0	7.1229	7.7369	7.9927	7.7369	6.9182	5.8437	4.6157	3.3365	2.5178
Δh	0	0.388	0.5	0.4817	0.2859	-0.0069	-0.406	-0.9482	-1.0833
i	10	11	12	13	14	15	16	17	-
x	29.667	32.277	34.835	37.137	39.798	41.743	43.636	45.478	-
y	3.1644	2.8908	2.8527	3.0387	3.4521	3.8353	4.2336	4.6111	-
y_0	2.2108	1.8527	1.8527	2.0062	2.3643	2.9783	3.5412	4.3087	-
Δh	-0.9536	-1.0381	-1	-1.0325	-1.0878	-0.857	-0.6924	-0.3024	-
i	18	19	20	21	22	23	24	25	-
x	47.064	48.599	50.083	51.618	53.613	55.046	56.479	57.911	-
y	4.9034	5.1367	5.3027	5.4186	5.5027	5.5291	5.5375	5.5367	-
y_0	5.025	5.5367	5.7414	5.7414	5.7925	5.7414	5.8437	5.5367	-
Δh	0.1216	0.4	0.4387	0.3228	0.2898	0.2123	0.3062	0	-

It is our final answer. Similarly, we can deal with other countries.

3.3.5 Analysis of the Result

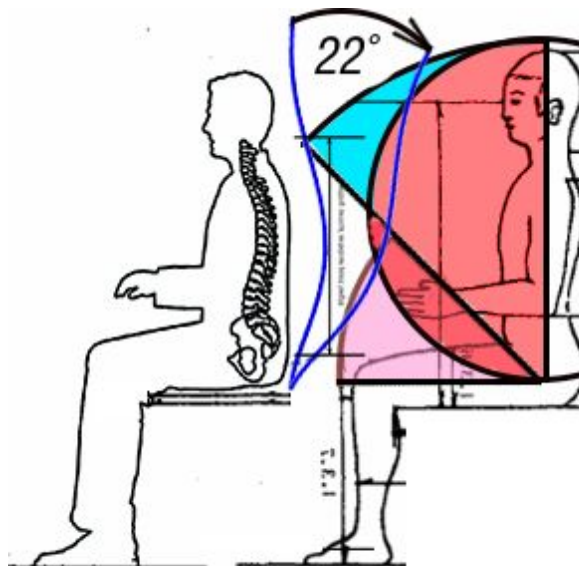
It make a great progress in the average distribution of stress. So the comfort is improved to a great extent.

However, due to the limit of time, we ignored some aspects. If take these aspects into consideration, the result will be more accurate.

3.4The Foundation of Second Model

In this question, we put emphasis on both the passengers' space and comfort.

3.4.1Enlarge space

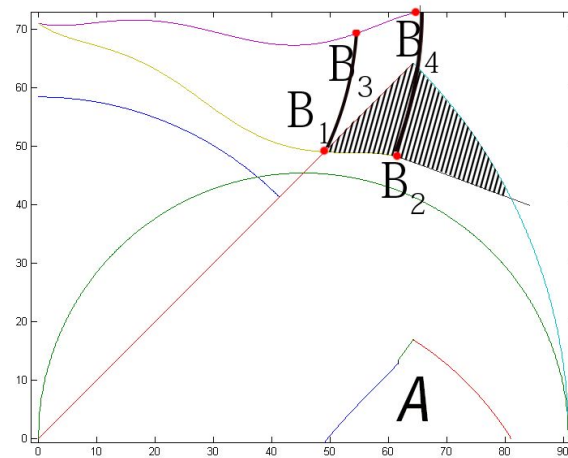


Picture 10

Look at the above schematic diagram(Picture 10), the red field is the space that half of the upper body need. It is the most important space for us to do most of things, such as reading and eating. The pink field is used to contain passengers' thigh. It plays a very important role in the effect of comfort. The last field is the blue which provide space for passengers' upper body. It is the least important.

When the back seat perpendicular to the ground, the angle is 90° . According to the 《 ERGONOMICS》 ,the most comfortable angle is 112° . So the angle of the chair rotation is 22° . Like the above graph.

We select several important points and use Matlab software to draw the coordinate graph(Picture 11):



Picture 11

we assume that the backplate curve coincides with the back seat curve. To let the back passengers receive more space, we need to adjust the backplate curve and of course have to keep the back seat curve invariant. Thus, the only thing we can do is try to deal with the padding.

Now, look at the graph. B_2 and B_4 are piecewise functions. The shadow region is

passengers' space that embezzled by the backplate. We use A to represent the difference between these two piecewise functions. While the head do not need too much support, so we can cut down the padding in the part of head.

However, the front passengers will feel bad if the padding is too thin. So, we should make a mechanical analysis about the spine to ensure the comfort.

3.4.2 Mechanical factor

In the first problem, we aimed at distribute stress on the spine averagely. However, with the limit of nature spine curve, there must be some vertebral body have more stress. If padding be cut too much, the passengers will feel bad. So we need to find a good balance between the padding and the space.

3.4.3 Optimization model

Based on the above analysis, we can build an optimization model. According to the requirement of airline seat, the thickness of the padding ranging from 5cm to 8cm. Our padding use the same standard. In the process of building model, from the aspect

of comfort and requirement, the thickness must thicker than 5cm. While from the aspect of saving space and material, our thickness is thinner than 8cm.

We defined the backplate curve as f_1 ($f_1 = y_1 + c$, c is a constant. In our model, c is the most narrow distance of the neighbouring airline seats which provided by Spirit Airlines Company. $c=71\text{cm}$). f_2 is the curve of the biggest angle between horizontal line and the backplate which can be get after a 22° rotation of f_1 .

The intersection point of f_1 and space is B_1 . B_2 and its above is determined by the head support. So we just need to optimize the backplate curve between B_1 and B_2 .

We defined the curve above the shadow as f_3 , $g = f_3 - f_2$. We can use g to present the embezzlement condition of the passengers' space. A is the changing of g .

We defined k as the embezzlement condition of each unitage which equal to the cutting of the padding. So :

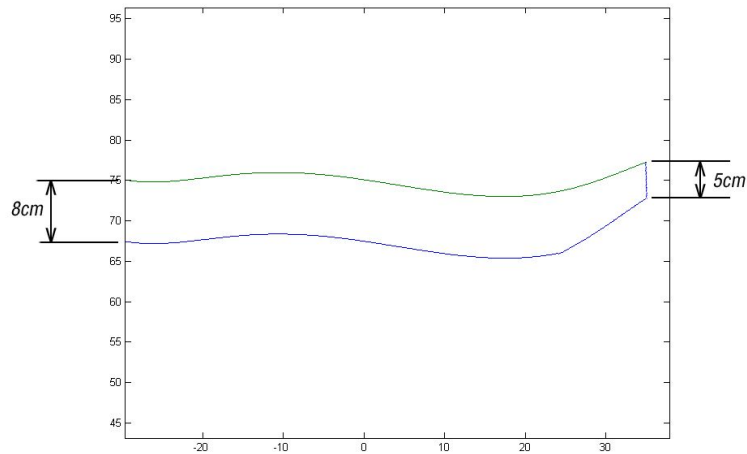
$$k = \frac{8-5}{[g(B_{2x}) - g(B_{1x})]} \quad (19)$$

B_3, B_4 can be get from B_1, B_2 after a counterclockwise rotation which centered by the intersection point of f_1 and y-axes. f_{bp} is the optimized backplate curve. So:

$$f_{bp}(x) = \begin{cases} f_1, & 0 < x \leq B_{3x} \\ f_1 + k[Bx_1 + (x - x_3) \frac{x_2 - x_1}{x_4 - x_3}], & B_{3x} < x \leq B_{4x} \end{cases} \quad (20)$$

We can get f_3 by a 8cm upward translation of f_1 . The backplate curve is combined by f_2 and f_3 .

With the help of Matlab, we can figure out this curve(Picture 12):



Picture 12

3.5 Strength and Weakness

- **Strength:** In despite of this, the model has proved much more reasonable than the traditional model. Traditional airline seat curve just try to attach the natural spine curve. While our model take mechanical analysis into consideration to prevent the discomfort caused by stress accumulation. It can deal with long journey easily.
- Moreover, we have drawn some useful conclusions about our model application. We put emphasis on the way instead of the answer. So the model is fit for different areas.
- **Weakness:** Although the model is very reasonable, there still have some disadvantages.
- The model is still an approximate on a large scale. In the processing of building model, we ignore some effects such as the deformation of back seat, the effect of muscles, clothes and so on. At the same time, we considered vertebral body as a rigid body, but the fact isn't. It will leads to errors.
- At the same time, our study object is the most of passengers, it can't fit everyone's needs.

IV. Future Work

4.1 The Improvement of Model

Due to the limit of time, there are still some weakness exit. We can make the following improvement:

- 1.Take the properties of the spine material instead of regarding it as a hinge. Spine's properties decided that the deformation of vertebral body is much less than the intervertebral disc. In order to make the result more accurate, we can assume that vertebral body is a rigid body, while the intervertebral disc is a elastic material. In this way, the combination of a segment of vertebral body and intervertebral disc is a basic unit. Then we can make mechanical analysis on the unit.
- 2.In this model, we just think about the effect of gravity and the spine. In fact, we should consider about the effect of muscles.
3. In this model, we use the data of standard adults height. In the improvement of model, we can take more data into consideration to make the optimized curve cater to more passengers' need.
- 4.Due to the limit of time, we regard the padding as rigid body and ignore the deformation. At the same time,we haven't make a choice of the padding material. In the improvement of model, we should take the deformation into consideration. Build a improved model to discuss the effect of material.
- 5.Some aspects such as the permeability and temperature also play a very important role.
- 6.If passengers sit on the seat for a long time, they may slip from the seat. Thus, in the improving model,we should think about it. Such as adding thwartwise stripes to improve the seats' skid resistance.
- 7.We can expand the model into 3D space and consider the distribution of forces in more directions.

The Improved Model aims to make up for the neglect. The result seems to declare that this model is more reasonable than the Basic Model and much more effective than the existing design.

4.2 Extra Advice

Except for the adjustment of the backplate curve and back seat curve. We still have some advice to make the airline seat more comfortable.

- 1) When people sit on the seat for a long time ,people may slip descent, so we can increase a nonskid device, such as a foot support.
- 2) Different peoples leg length is different ,so we can invent a adjustable foot support to cater to different people's needs
- 3) No chair is perfect, what we can do is just improving its comfort. Sitting on the seat for a long time must cause discomfort, while if the seat have massaging function, mild massage is enough , the journey will become more pleasant.

V. Advertising material

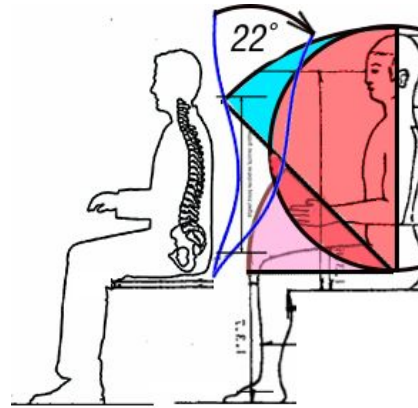
Airplanes are where the comfort received from the airline seats affects a lot.

As we know, while traveling by air most of the activities of passengers would take place on the seat. Thus, the passengers' feeling is well affected by the seat comfort.

Slimline seats are widely used in economy class to increase airplanes' capacity, as a result, many passengers expressed displeasure with these seats and more and more disputes have been caused. There is no doubt that company's benefit and reputation will be declining as well.

We are now recommending this kind of aero seats, which contains our new study achievement.

After a careful observation of the current status of airline seats, we can easily find the lack of consideration related to the waist support and the unreasonable design of the



backrest's curve which contradicts to human being's natural posture curve. Usually, reasonable design of waist support and backrest's curve will reduce passengers' fatigue during the journey which is also good for their physical and mental health.

Our design focuses on the optimization of the back seat curve and the padding without changing the structure. Not only the slimline seat can liberate more passenger space, but also can it ensure the comfort degree.

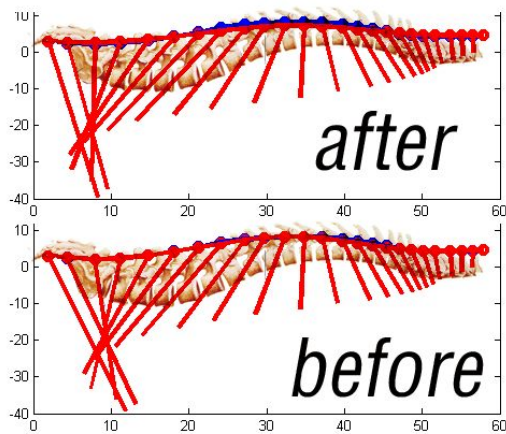
To design a qualified airline seat, first of all, we searched lots of literature, such as the lowest requirement airline seat. Thus, we can know the height requirement, distance requirement between two seats and the shape requirement. Based on these requirements, we use knowledge of ergonomics to figure out an optimized curve to cater to human's needs.

Why we dare to say the new slimline seat is very comfortable?

Because we take three main factors into consideration: the back seat curve, the backplate curve and the padding. So we ensure the seat comfort from all directions.

1. First of all, we considered about the average distribution of stress. We build a model to figure out the most comfortable curve which both makes the spine uniformly forced and stay in its natural state.

2. Then, we concerned about activity space for passengers on the front seat and back seat. Without decreasing comfort of the backrest, keeping the back curve to match passengers' spine curve, we correct the backplate curve, mainly at the position of head, to spare more space for the passenger behind. Thickness of padding is also taken into consideration. Through consulting data, we ensure the thickness ranging from 5cm to 8cm. There



efore,we gain the optimised seat backrest.

**Compared with conventional airline seat,
Where are our advantages?**

1.Apart from matching the curve of spine,our seat can make release the pressure of spine too.We concerned about the interval reason ,force.Only taking outer shape like curve as standard is not scientific and comprehensive.Fatigue results from longtime static force of tendon and muscle,so keeping proper posture is an

efficient method to reduce static force.

2.We choose to cut down padding to spare more activity space for passengers.Nowadays,that's a common occasion where two passengers dispute for putting down backrest.So we rule the back angle and adjust the backplate curve to spare room to solve problems like this.

VI. References

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