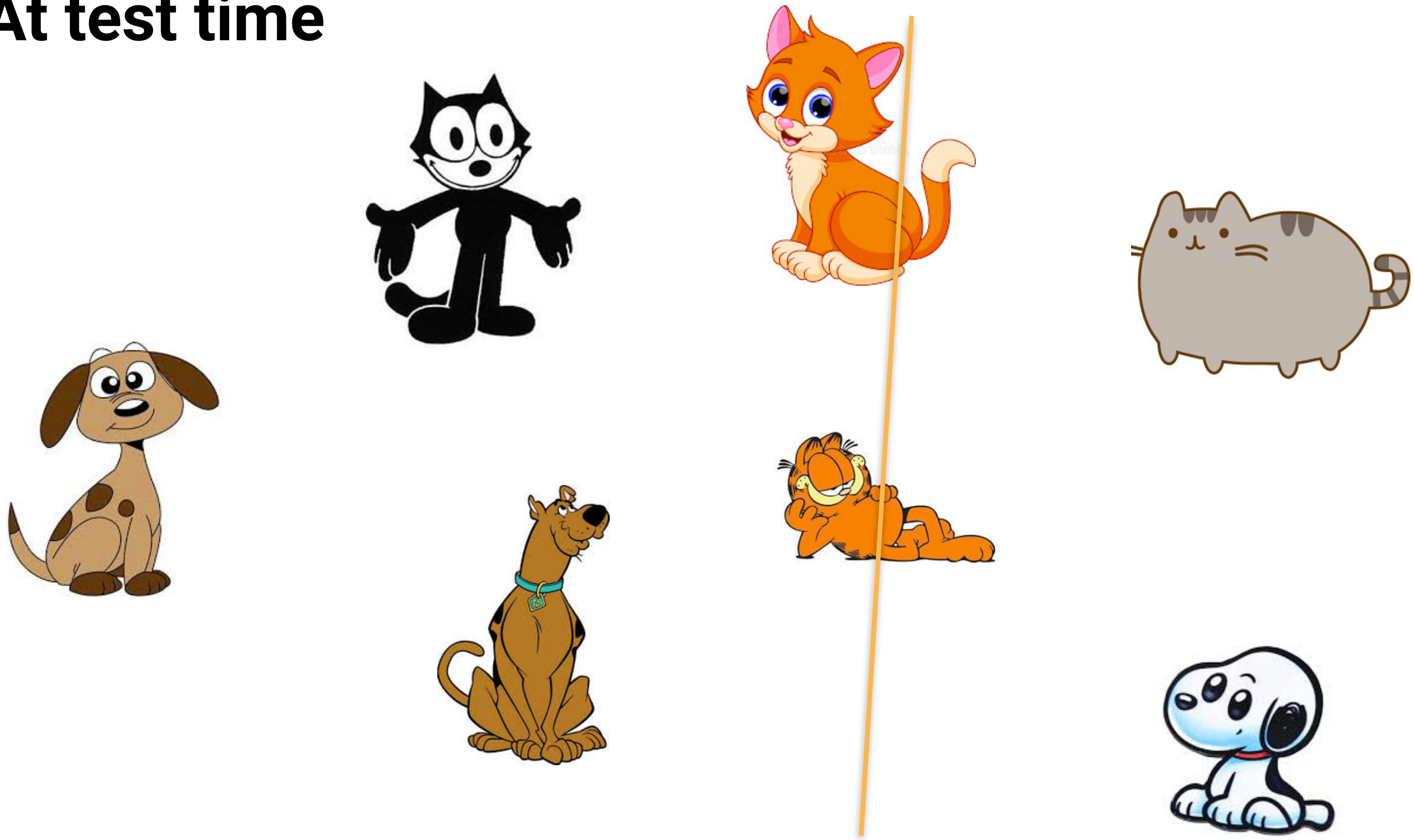
Covariate shift

Training set

At test time



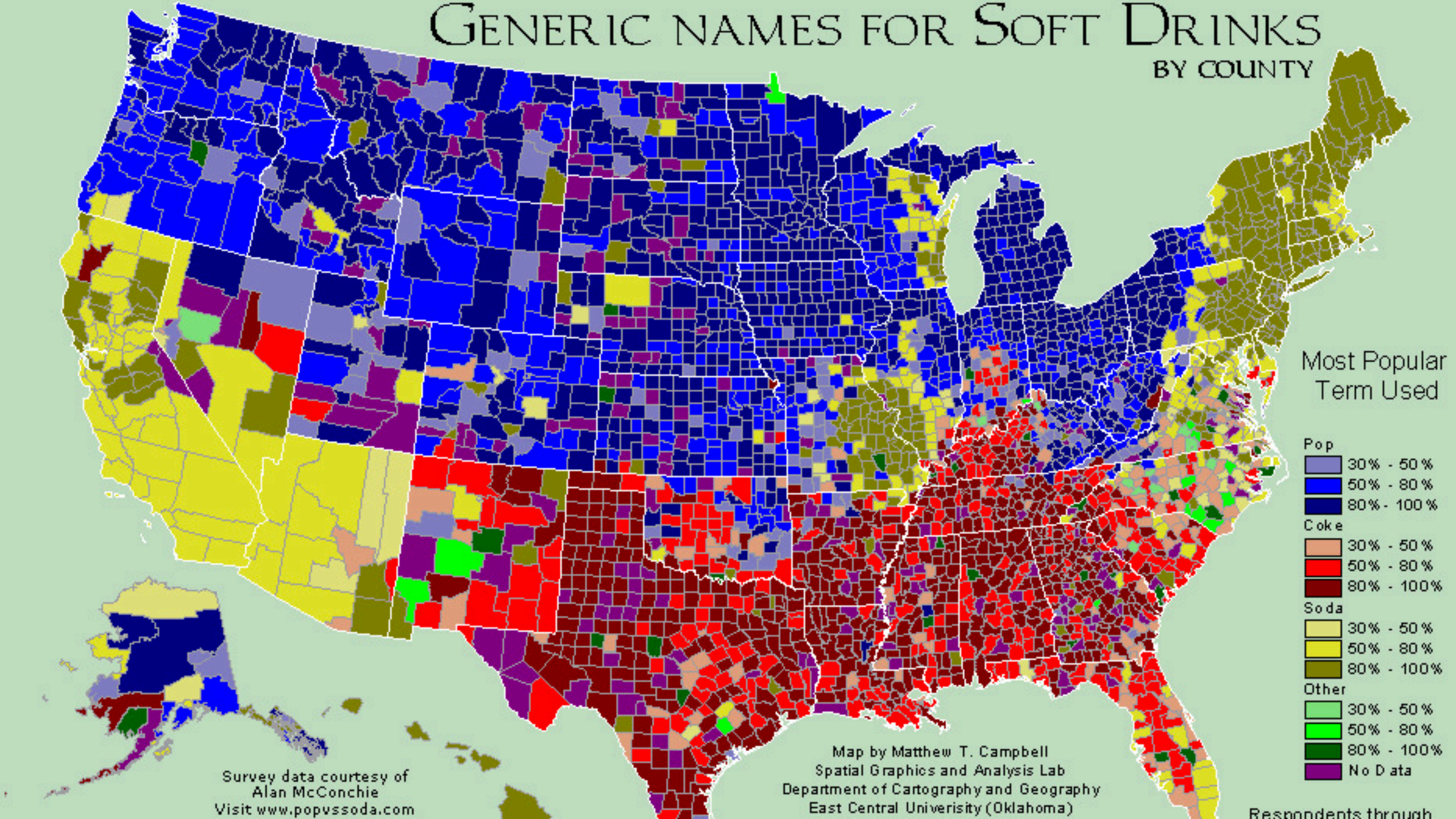
Why would anyone be so stupid?

Covariate Shift

- **Web search**
  - Training - page relevance data for the US market
  - Testing - recommend pages for Canada (UK, Australia)
- **Speech recognition**
  - Training - West coast accent
  - Testing - Southern drawl, Texan, non-native speaker
- **Language**
  - Training - ‘James, bring me a **soda**’
  - Testing - ‘John, bring me a ‘**pop**’ (or coke, etc.)

Covariate Shift

- **Medical**
  - Training - University students + old men with prostate cancer
  - Testing - Potentially sick old men
- **Reinforcement Learning**
  - Training - Data gathered with current policy
  - Testing - Environment reacting to updated policy
- **Databases**
  - Training - DB tuned to 2017 usage pattern
  - Testing - DB deployed on AWS in 2018


Generic names for Soft Drinks
by County
Most Popular Term Used
Pop
30% - 50%
50% - 80%
80% - 100%
Coke
30% - 50%
50% - 80%
80% - 100%
Soda
30% - 50%
50% - 80%
80% - 100%
Other
30% - 50%
50% - 80%
80% - 100%
No Data
Survey data courtesy of
Alan McConchie
Visit www.popvssoda.com
Map by Matthew T. Campbell
Spatial Graphics and Analysis Lab
Department of Cartography and Geography
East Central Univerisity (Oklahoma)
Respondents through

Real Story from CES'19

- Startup with smart vending machine demo that identifies purchases via a camera
- Demo at CES failed
    - Different light temperature
    - Light reflection from table
- The fix
    - Collect new data
    - Buy tablecloth
    - Retrain all night

What is happening? $q(x,y) = q(x)p(y|x)$

- Training Risk

Training data

$$\underset{w}{\text{minimize}} \int dx\, p(x) \int dy\, p(y|x) l(f(x,w),y)$$

$$\text{or rather } \underset{w}{\text{minimize}} \frac{1}{m} \sum_{I=1}^{m} l(f(x_i,w),y_i)$$

- Test Risk is different

Test data

$$\int dx\, q(x) \int dy\, p(y|x) l(f(x,w),y)$$

Fixing it (covariate shift correction)

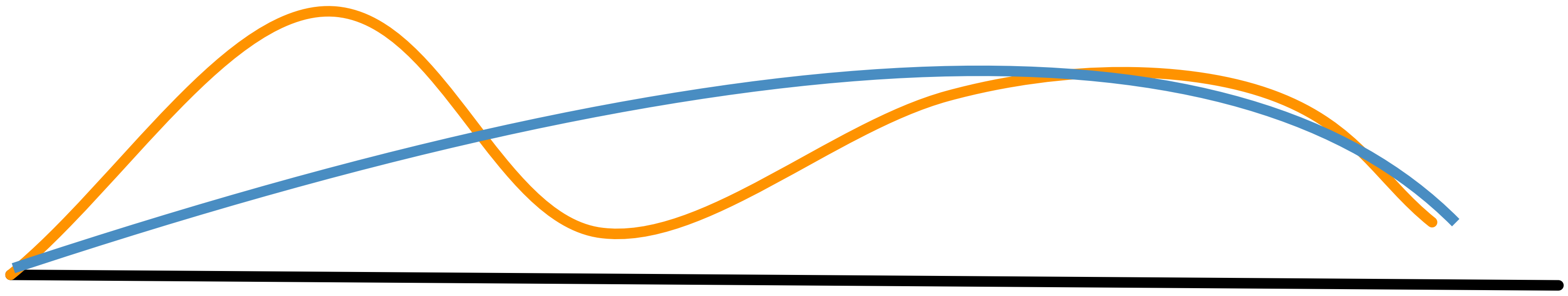
- Density ratio

$$\int dx\, q(x) f(x) = \int dx\, p(x) \underbrace{\frac{q(x)}{p(x)}}_{\alpha(x)} f(x) = \int dx\, p(x) \alpha(x) f(x)$$

- Need to find density ratio, but we don’t have p or q.
- Estimating densities is hard.
  E.g. what to do if density is very high (or very low)?

$$r(x, y) = \frac{1}{2}\left[p(x) \cdot \delta(y, 1) + q(x) \cdot \delta(y, -1)\right]$$

Density Ratio Estimation



- If $p(x)$ and $q(x)$ differ, we can train a classifier to distinguish between them.
- Classification

$$r(x, y) = \frac{1}{2}\left[p(x) \cdot \delta(y,1) + q(x) \cdot \delta(y, -1)\right]$$

Fixing it (covariate shift correction)

$$r(x, y) = \frac{1}{2}\left[p(x) \cdot \delta(y,1) + q(x) \cdot \delta(y, -1)\right]$$

- Conditional class probability

$$r(y = 1|x) = \frac{p(x)}{p(x) + q(x)} \text{ and hence } \alpha = \frac{q(x)}{p(x)} = \frac{r(y = -1|x)}{r(y = 1|x)}$$

- Simple algorithm
  - Train classifier between training and test set
  - Reweight training data with $\frac{r(y = -1 \mid x)}{r(y = 1 \mid x)}$
- Can use Generalization Performance Estimate to decide whether we even have Covariate Shift.

Corrected Training Problem

- Original Problem

$$\underset{w}{\text{minimize}} \sum_{i=1}^{m} \frac{1}{m} l(y_i, f(x_i, w))$$

- Reweighted Problem

$$\underset{w}{\text{minimize}} \sum_{i=1}^{m} \alpha_i l(y_i, f(x_i, w)) \text{ where } \alpha_i = \frac{r(y=-1 \mid x_i)}{r(y=1 \mid x_i)}$$

- Problems
  - Coefficients α_i are estimates. Adds variance & bias.
  - What if p and q are very different?

Problems with p and q

$$\sum_{i=1}^{m} \alpha_i l(y_i, f(x_i, w))$$

Problems with p and q

$$\sum_{i=1}^{m} \alpha_i l(y_i, f(x_i, w))$$

- Effective sample size

$$m^* = \frac{\|\alpha\|_1^2}{\|\alpha\|_2^2}$$

- Unmodified weights
  - Less bias
  - High variance for small m^*
- Clipped weights $\bar{\alpha}_i = \min(\alpha_i, C)$
  - Potentially some bias
  - Smaller variance (larger effective sample size)

Key Takeaways

Covariate Shift assumes

$$p(x, y) = p(x)p(y \mid x) \longrightarrow q(x, y) = q(x)p(y \mid x)$$

- Label dependency for training and test is the same
- Covariate distribution is different

Detection & Fix

- Train binary classifier between training and test set
- If nontrivial accuracy, use it to reweight training data via $\alpha(x) = r(y = 1 \mid x)/r(y = -1 \mid x)$
- Fix effective sample size by clipping $\bar{\alpha}(x) = \min(\alpha(x), C)$