MAST20005/MAST90058: Week 7 Lab Solutions

1. (a) Only a small number of simulations were run (200). This makes the resulting estimates quite variable.

   (b) We can improve the estimates by substantially increasing the number of simulations. For example, here is a repeat of the simulation for estimating α:

```
T <- rbinom(20000, 10, 0.5)  # simulate under H0
alpha <- sum(T < 3.5) / length(T)  # estimate
alpha

## [1] 0.1724
```

   The result is much closer to the true value (0.171875). The corresponding confidence interval is now much narrower:

```
alpha + c(-1 ,1) * 1.96 * sqrt(alpha * (1 - alpha) / 20000)

## [1] 0.167165 0.177635
```

   The repeat of the simulation for β is left as an exercise.

2. (a)

```
prop.test(23, 209, p = 0.07, alternative = "greater")

##
##  1-sample proportions test with continuity correction
##
## data:  23 out of 209, null probability 0.07
## X-squared = 4.5522, df = 1, p-value = 0.01644
## alternative hypothesis: true p is greater than 0.07
## 95 percent confidence interval:
##  0.07727412 1.00000000
## sample estimates:
##         p
## 0.1100478
```

   The p-value is 0.016. Therefore, we would reject if $\alpha = 0.05$ but not reject if $\alpha = 0.01$.

   (b) First, we need to find the critical region for this test. It will be of the form $X \geqslant c$ for some c. Using direct calculation of the binomial probabilities to find this:

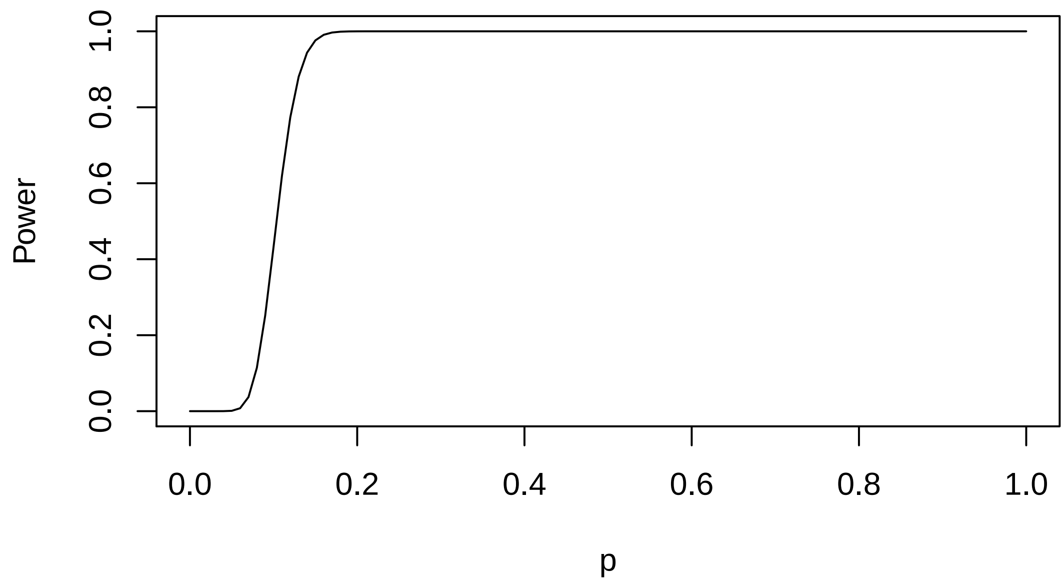
```
qbinom(0.95, 209, 0.07)

## [1] 21

1 - pbinom(20:21, 209, 0.07)

## [1] 0.06138392 0.03710675
```

   Therefore, the critical region will need to be $X \geqslant 22$ and has an actual significance value of 0.037 (if we changed it to $X \geqslant 21$ then the actual significance value will be greater than 0.05). Now we define the power function:

```r
K1 <- function(p)
    1 - pbinom(21, 209, p)
K1(0.07)  # significance level (for reference)

## [1] 0.03710675
```

Then we can plot it:

```r
curve(K1, from = 0, to = 1, xlab = "p", ylab = "Power")
```




3. ```r
   prop.test(c(124, 70), c(894, 700))

   ##
   ##  2-sample test for equality of proportions with continuity
   ##  correction
   ##
   ## data:  c(124, 70) out of c(894, 700)
   ## X-squared = 5.1453, df = 1, p-value = 0.02331
   ## alternative hypothesis: two.sided
   ## 95 percent confidence interval:
   ##  0.005691986 0.071712935
   ## sample estimates:
   ##    prop 1    prop 2
   ## 0.1387025 0.1000000
   ```

   The p-value is less than 0.05 (the chosen significance level) so we reject H_0. A 95% confidence interval for the difference in proportions is shown in the R output above. Note that the solutions for the tutorial problems are shown without continuity correction. For reference, here is how to obtain those calculations in R:
   ```

```r
prop.test(c(124, 70), c(894, 700), correct = FALSE)

##
##  2-sample test for equality of proportions without continuity
##  correction
##
## data:  c(124, 70) out of c(894, 700)
## X-squared = 5.5014, df = 1, p-value = 0.019
## alternative hypothesis: two.sided
## 95 percent confidence interval:
##  0.006965556 0.070439366
## sample estimates:
##    prop 1    prop 2
## 0.1387025 0.1000000
```

4. (a) We use the normalised sample proportion as the test statistic. We reject H_0 if:

$$z = \frac{\hat{p} - 0.5}{\sqrt{0.5 \times 0.5/900}} > 1.2816.$$

Note that $\alpha = 0.1$ and the critical value can therefore be obtained by:

```r
qnorm(0.9)

## [1] 1.281552
```

(b) We need to specify the rejection rule in a form that makes it easier to calculate the power. We could do it in terms of binomial probabilities like in the earlier examples, but given the large sample size in use here, it is probably easier to do it in terms of normal distributions. We know that $\hat{p} \approx \mathrm{N}(p, p(1-p)/900)$. Also, the rejection rule simplifies to:

$$\hat{p} > 0.52136.$$

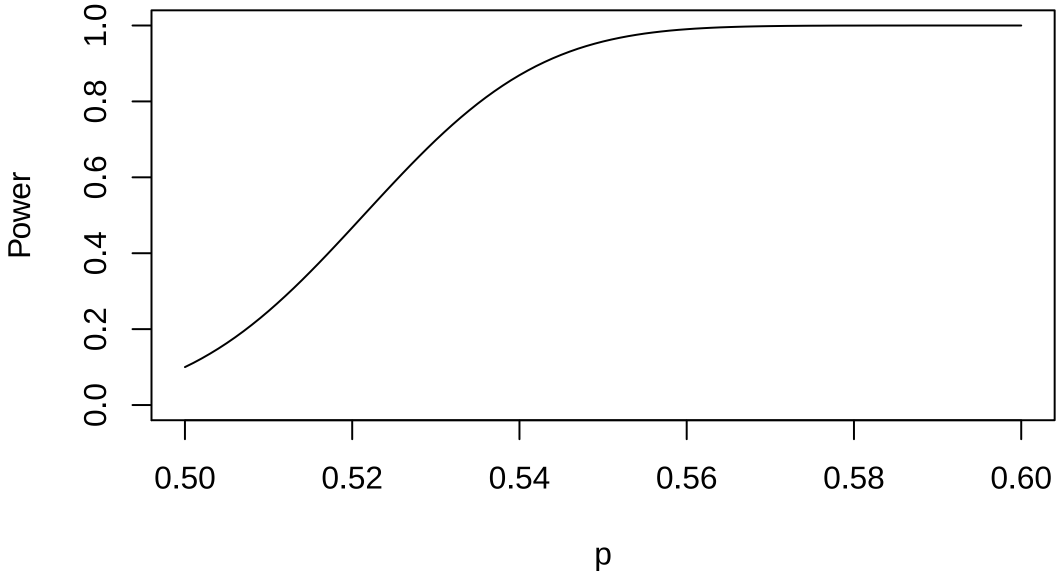
Therefore, the power function is:

```r
K2 <- function(p)
    1 - pnorm(0.52136, p, sqrt(p * (1 - p) / 900))
K2(0.5)  # check the significance level is correct

## [1] 0.0999915
```

Calculate the desired power:

```r
K2(0.52)

## [1] 0.4674564
```

(c)
```r
curve(K2, from = 0.5, to = 0.6, xlab = "p", ylab = "Power", ylim = 0:1)
```



(d)
```r
prop.test(465, 900, alternative = "greater")

##
##  1-sample proportions test with continuity correction
##
## data:  465 out of 900, null probability 0.5
## X-squared = 0.93444, df = 1, p-value = 0.1669
## alternative hypothesis: true p is greater than 0.5
## 95 percent confidence interval:
##  0.4887039 1.0000000
## sample estimates:
##         p
## 0.5166667
```

We cannot reject H_0. Although a majority of people were in favour of voting for this party, it was too close to 50% for us to know more decisively whether or not the party has majority support in the population.

(e) Survey a larger sample of people.