MAST20005/MAST90058: Week 9 Problems

Some useful information for many of the problems is shown at end of this problem sheet.

1. In a one-way ANOVA with I treatments and J observations per treatment, let $\mu = I^{-1}\sum \mu_i$.

   (a) Express $\mathbb{E}(\bar{X}_{..})$ in terms of μ. (Hint: $\bar{X}_{..} = I^{-1}\sum \bar{X}_{i.}$)

   (b) Compute $\mathbb{E}(\bar{X}_{i.}^2)$

   (c) Compute $\mathbb{E}(\bar{X}_{..}^2)$

   (d) Compute $\mathbb{E}(SS(T))$ and then show that

   $$\mathbb{E}(MS(T)) = \sigma^2 + \frac{J}{I-1}\sum(\mu_i - \mu)^2$$

   (e) Using the result of (d), what is $\mathbb{E}(MS(T))$ when H_0 is true?
   When H_0 is false, how does $\mathbb{E}(MS(T))$ compare with σ^2?

2. In an experiment to compare the tensile strengths of five different types of copper wire, four samples of each type were used. In an ANOVA, the between-groups and within-groups mean squares statistics were computed as $MS(T) = 2573.3$ and $MS(E) = 1394.2$ respectively. Use the F-test at a 5% significance level to test $H_0\colon \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ against the alternative $H_1\colon \bar{H}_0$, where μ_i is the mean tensile strength of copper wire of type i.

3. Consider the following partial output from a regression of the average brain and body weights for 62 species of mammals (variables are transformed on the log-scale).

```
lm(formula = brain ~ body)

               Estimate Std. Error
(Intercept)     2.13479    0.09604
Body            0.75169    0.02846
---
Residual standard error: 0.6943 on 60 degrees of freedom
Multiple R-squared:  0.9208,    Adjusted R-squared:  0.9195
F-statistic: 697.4 on 1 and 60 DF,  p-value: < 2.2e-16
```

   (a) Test the null hypothesis of no association between body and brain weights at the $\alpha = 0.01$ level of significance.

   (b) Use the following approximate distribution to obtain a test of size α for the null hypothesis $H_0\colon \rho = 0$ against $H_1\colon \rho \neq 0$ based on R, the sample correlation coefficient.

   $$\frac{1}{2}\ln\left(\frac{1+R}{1-R}\right) \approx \mathrm{N}\left(\frac{1}{2}\ln\left(\frac{1+\rho}{1-\rho}\right), \frac{1}{n-3}\right)$$

   (c) What is the sample correlation coefficient for these data?

   (d) Apply the procedure in (b) to the mammals data using the significance level $\alpha = 0.01$.

   (e) Based on the above results, state your conclusion about the relationship between body and brain weight of mammals.

4. Let $X_1, \ldots, X_n$ be a random sample from $\mathrm{N}(\mu, \sigma^2)$, where μ and σ^2 are unknown. We wish to test $H_0\colon \sigma^2 = \sigma_0^2$ against $H_1\colon \sigma^2 \neq \sigma_0^2$.

   (a) Find L_0 and L_1, the maximised likelihoods under H_0 and H_1, that are required in order to write a likelihood ratio.

   (b) Show that the likelihood ratio test rejects H_0 if $w > c_1$ or $w < c_2$ (for some constants c_1 and c_2), where $w = \sum_i (x_i - \bar{x})^2/\sigma_0^2$.

Some potentially helpful R output:

```
> p <- c(0.95, 0.975, 0.99, 0.995)
> qnorm(p)
[1] 1.644854 1.959964 2.326348 2.575829
> qt(p, 60)
[1] 1.670649 2.000298 2.390119 2.660283
> qchisq(p, 60)
[1] 79.08194 83.29767 88.37942 91.95170
> qf(p, 4, 15)
[1] 3.055568 3.804271 4.893210 5.802907
> qf(p, 5, 20)
[1] 2.710890 3.289056 4.102685 4.761574
> qf(p, 15, 4)
[1]  5.857805  8.656541 14.198202 20.438268
> qf(p, 20, 5)
[1]  4.558131  6.328555  9.552646 12.903488
```