MAST20005/MAST90058: Week 8 Problems

Some useful information for many of the problems is shown at end of this problem sheet.

1. **Random digits (group activity).** A volunteer in each group should write down a string of 51 random digits, such as:

   $$3 \quad 7 \quad 2 \quad 4 \quad 1 \quad 6 \quad 8 \quad 5 \ldots$$

   For each digit, record whether: (i) the next digit is the same as the preceding one (e.g. '2 2') and (ii) the next digit differs from the preceding one by 1 (e.g. '2 3'). For the purpose of this exercise, assume that the digits 0 and 9 only differ by 1. After the volunteer writes down 51 numbers, the entire group should carry out a hypothesis test at the 5% level of significance to determine whether the volunteer's sequence of numbers is truly random (you will need to think carefully about what this means and how to translated it into an appropriate null hypothesis).

2. Strawberries are being packed for the market. It is claimed that the median weight, m, of these boxes is 40 kilograms.

   (a) Use the following data and a Wilcoxon test statistic at an approximate significance level of $\alpha = 0.05$ to test the null hypothesis $H_0 : m = 40$ against $H_1 : m < 40$.

   41.195, 39.485, 41.229, 36.840, 38.050, 40.890, 38.345, 34.930, 39.245, 31.031, 40.780, 38.050, 30.906

   It may help to complete the following table. Ties are assigned the average rank.

i	x_i	$x_i - m$	Rank	Sign
1	41.195			
2	39.485			
3	41.229			
4	36.840			
5	38.050			
6	40.890			
7	38.345			
8	34.930			
9	39.245			
10	31.031			
11	40.780			
12	38.050			
13	30.906			

   (b) Give limits for the p-value of this test.

   (c) Use the sign test to test the same hypothesis.

   (d) Compare the results of the two tests.

3. In a biology laboratory the mating of two red-eye fruit flies yielded $n = 432$ offspring among which 254 were red-eyed, 69 were brown-eyed, 87 were scarlet-eyed, and 22 were white-eyed. Use these data to test, with $\alpha = 0.05$, the hypothesis that the ratio among the offspring would be 9:3:3:1 respectively.

4. We wish to determine if two groups of nurses distribute their time in six different categories about the same way. That is, the hypothesis under consideration is $H_0\colon p_{i1} = p_{i2}$, $i = 1, \ldots, 6$. To test this, nurses are observed at random throughout several days, each observation resulting in a mark in one of the six categories. The summary data are given in the following frequency table:

	Category						
	1	2	3	4	5	6	Total
Group I	95	36	71	21	45	32	300
Group II	53	26	43	18	32	28	200

Do a chi-squared test with $\alpha = 0.05$.

Some potentially helpful R output:

```
> pbinom(1:6, 13, 0.5)
[1] 0.001708984 0.011230469 0.046142578 0.133422852 0.290527344 0.500000000

> p <- seq(0.9, 0.975, 0.025)
> qnorm(p)
[1] 1.281552 1.439531 1.644854 1.959964
> qchisq(p, 1)
[1] 2.705543 3.170053 3.841459 5.023886
> qchisq(p, 2)
[1] 4.605170 5.180534 5.991465 7.377759
> qchisq(p, 3)
[1] 6.251389 6.904644 7.814728 9.348404
> qchisq(p, 4)
[1]  7.779440  8.496282  9.487729 11.143287
> qchisq(p, 5)
[1]  9.236357 10.008315 11.070498 12.832502
```