MAST20005/MAST90058: Week 4 Solutions

1. (a) We use the sample mean and sample median as estimators for the population mean. From the R output, $\bar{x} = 10.29$ and $\hat{\pi}_{0.5} = 10.52$.

   (b) From problem 7 in week 3, we know that:

$$\mathrm{sd}(\bar{X}) = \sqrt{\mathrm{var}(\bar{X})} = \frac{\sigma}{\sqrt{n}}$$
$$\mathrm{sd}(\hat{\pi}_{0.5}) = \sqrt{\mathrm{var}(\hat{\pi}_{0.5})} = \sqrt{\frac{\pi}{2}}\frac{\sigma}{\sqrt{n}}$$

   We can approximate both of these by substituing $s = 1.159$ for σ (recall that S is an estimator for σ). This gives standard errors for each of the two estimates:

$$\mathrm{se}(\bar{x}) = \frac{s}{\sqrt{n}} = 0.367$$
$$\mathrm{se}(\hat{\pi}_{0.5}) = \sqrt{\frac{\pi}{2}}\frac{s}{\sqrt{n}} = 0.459$$

2. $73.8 \pm 1.96 \times 5/4 = [71.35, 76.25]$

3. $\bar{X}$ is approximately normally distributed. Using this gives:
   $2.09 \pm 1.96 \times 0.12/4 = [2.03, 2.15]$

4. $\bar{X}$ is approximately normally distributed. Using this gives:
   $11.95 \pm 1.96 \times 11.8/\sqrt{37} = [8.148, 15.75]$
   Some people prefer to use the t-distribution approximation, which is more conservative. Since the necessary quantiles were provided, let's try it:
   $11.95 \pm 2.028 \times 11.8/\sqrt{37} = [8.016, 15.88]$
   Note that the two are fairly similar.

5. $20.9 \pm 2.306 \times 1.858/3 = [19.47, 22.33]$.
   One sensible interpretation of the claim is that the *average* weight of a '22 kg' wheel is 22 kg (rather than claiming that *every* wheel is *exactly* 22 kg in weight). Since 22 is within our confidence interval for the mean, and the interval is relatively narrow, this claim seems to be reasonable given our data.

6. $937.4 - 988.9 \pm 1.96\sqrt{784/56 + 627/57} = [-61.3, -41.7]$.
   This confidence interval is very far from zero, meaning that our data show fairly strong evidence that the mean lifetimes are *not* the same.