Week 2

① getwd(), setwd("M:/.../.../")

② install.packages("....")

③ seq(1, 2, by=0.1) # 1.0 1.1 1.2 ... 2.0

A ← matrix(c(2,-1,4,2,-1,3), nrow=2)

A[1,2] ⇐ 1st row, 2nd column

A[1,] ⇐ first row

④ max(), min(), range(), sort(), length(), sum()
prod(), dim(), mean()

⑤ Graphics: plot(x, y, xlab="...", ylab="...")

boxplot(x, y, names=c("x","y")) horizontal = TRUE

hist(x, freq=FALSE, nclass=10)

QQplot: qqnorm(x) qqline(x)   qqnorm(x, main="...")

⑥ dat ← data.frame(x=c(1,2,3), y=c(30,20,10))

⑦ Missing data ⇒ na.rm = TRUE

⑧ function

```
myfun ← function(x){
  y ← x^2
  return(y)
}
```

⑨ descriptive statistic: summary(x), var(x), sd(x)
IQR(x), quantile(x, type=6)

Week 3

sd(x, na.rm=TRUE)

m1 = yields[yields$machine == "A", "yields"]

① A ← read.csv("A.csv")

s_1 ~~year~~ ← A[,1] ⇐ 索引给 s_1

hist(s_1, freq=FALSE, xlab="...", col=8)

smooth.density = density(s_1) # fits a smooth curve

lines(smooth.density, lty=2, lwd=2, col=2)

② d- ⇒ pdf or pmf   dnorm(0, mean=4, sd=2)

p- ⇒ cdf   pnorm(0, mean=2, sd=3, lower.tail=FALSE)

q- ⇒ inverse ~~cumul~~ cdf (quantiles)   qnorm(c(0.25, 0.5, 0.75), mean=1, sd=2)

r- ⇒ randomly generated numbers   rnorm(4, mean=3, sd=3

③ Normal MLE: $\hat{\mu} = \bar{x}$, $\hat{\sigma}^2 = (n-1)s^2/n$

sd = $s/\sqrt{n}$

mu.hat = mean(s_1)

n = length(s_1)

sigma.hat = sqrt((n-1) * var(s_1) / n)

q ← qnorm(0.25, mean=40, sd)

p ← pnorm(q, mean=35, sd)

power

Gumbel pdf   h ← read.table("H...txt", header=TRUE)

dgumbel ← function(x, mu, sigma)
exp((mu-x)/sigma - exp((mu-x)/sigma))/sigma

gumbel.fit ← fitdistr(x=s_1, densfun= dgumbel,
start = list(mu=50, sigma=10))

normal.fit ← fitdistr(x=s_1, densfun= "normal")

Week 4 CI

① Find quantiles for distributions

P ← c(0.9, 0.95, 0.975)

Week 4 ANOVA

model1 ← lm(Force ~ factor(Position), data=data1)

anova(model1)

H0: No difference
H1: difference

model ← lm(X ~ factor(A) + factor(B), data) anova(model)

interaction 很重

interaction plot

with(data, interaction.plot(B, A, X, col=...

$N(0,1)$ ⇒ qnorm(p)

$N(5,3^2)$ ⇒ qnorm(p, 5, 3)

t_5 ⇒ qt(p, 5)

χ_1^2 ⇒ qchisq(p, 1)

χ_5^2 ⇒ qchisq(p, 5)

$F_{12,4}$ ⇒ qf(p, 12, 4)

~~ser~~ discrete distri...

sample

days <- c("Mon", ... "Sun")
sample(days, 1)

sample(c("tail", "head"), 10, prob=c(0.2, 0.8), replace = TRUE)

② 90% CI   t.test(x, conf.level = 0.90)

95% CI $\mu_x - \mu_y$ t.test(x, y, var.equal = TRUE,

check variance: var.test(x, y)

③ 有符号 txt ⇒ read.table(...)
没符号 ⇒ read.delim(...)

④ x3 ← inva[inva$condition == 3, 2]
x4 ← .... == 4, 2]

x3.bar ← mean(x3)   x4.bar ← mean(x4)

95% CI for x3   x3.bar + c(-1,1) * 1.96 * sqrt(x3.ba... / length...

⑤ $\bar{x} \pm c\frac{\sigma}{\sqrt{n}}$ ⇐ CI normal case

1 Normal, single mean, known σ

$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

$\mu \neq \mu_0$ t = 0.75 < c = 1.86

Pr(|T| ≥ 0.75) = 2Pr(T ≥ ...
= 2 × (1 - pt(0.7...

2 Normal, single mean, unknown σ

t.test(x, conf.level = 0.9)

3 Normal, two means, known σ

$\bar{x} - \bar{y} \pm c\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}$

④ Normal, two means, unknown σ, many samples

$\bar{x} - \bar{y} \pm c\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}$

⑤ Normal, two means, unknown σ, common variance
small sample

$\bar{x} - \bar{y} \pm c \cdot s_p\sqrt{\frac{1}{n} + \frac{1}{m}}$

$s_p = \sqrt{\frac{(n-1)s_x^2 + (m-1)s_y^2}{n-m+2}}$

⑥ Normal, two means, unknown σ, different variance

Welch's t.test(x, y, conf.level = 0.95)

pooled value
t.test(x, y, conf.level=0.95, var.equal = TRUE)

⑦ Normal, paired

$\bar{d} \pm c\frac{s_d}{\sqrt{n}}$ $D_i = X_i - Y_i$
$D_i \sim N(\mu_D, \sigma_D^2)$

⑧ Normal, single variance

$\frac{(n-1)s^2}{\sigma^2} \sim \chi_{n-1}^2$ qchisq( ) σ^2

100 × (1-α)% CI $\left[\frac{(n-1)s^2}{b}, \frac{(n-1)s^2}{a}\right]$

single mean, unknown σ $a = \Phi(\frac{\alpha}{2})$ $b = \Phi^{-1}(1-\frac{\alpha}{2})$

H0	H1	Critical region
$\mu = \mu_0$	$\mu > \mu_0$	$t \geq F^{-1}(1-\alpha)$
---	$\mu < \mu_0$	$t \leq F^{-1}(\alpha)$

$\mu \neq \mu_0$ $|t| \geq F^{-1}(1-\frac{\alpha}{2})$

直接用 t.test(data, mu=9, alternative...

9) Normal, two variance $100 \times (1-\alpha)\%$ CI for $\boxed{\sigma_x^2/\sigma_y^2}$

$\boxed{\begin{matrix} X_1 \dots X_n \\ Y_1 \dots Y_m \end{matrix}}$ 化简 $\boxed{F_{m-1,\,n-1}}$

$\left[c\frac{s_x^2}{s_y^2},\ d\frac{s_x^2}{s_y^2}\right]$ $c = \Phi^{-1}(\frac{\alpha}{2})$

$d = \Phi^{-1}(1-\frac{\alpha}{2})$

$\boxed{\text{df}}$

10 Single proportion $\hat{p} \pm c\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ qnorm $\sim N(0,1)$

11) Two proportions $\hat{p}_1 - \hat{p}_2 \pm c\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

Week 5

never use Wald when n is small

1) Wald estimate the margin of error. 90% CI

```
n <- 20, p.hat <- 6/n  z0 <- qnorm(0.95)
p.hat + c(-1,1) * z0 * sqrt(p.hat * (1-p.hat)/n)
```

when n large

Quadratic avoid margin of error

```
prop.test(x=6, n=20, conf.level=0.9, correct=FALSE)
```

Exact method 两个都可以

```
binom.test(x=6, n=20, conf.level=0.9)
```

2) Two sample CI 95% CI for $d = p_A - p_B$

```
prop.test(x = c(141,170), n=c(460,440), correct=FALSE)
```

alternative = "less"

3) $X \sim N(\mu, \sigma^2)$

① point estimator and 90% CI for μ

```
mean(x)    mean(x) + c(-1,1) * qt(0.95, n-1) * sd(x)/sqrt(n)
```

② point estimator and 95% CI for σ

```
sd(x)    sqrt((n-1)/qchisq(c(0.975, 0.025), df=n-1)) * sd(x)
```

3) 90% PI

```
mean(x) + c(-1,1) * qt(0.95, n-1) * sd(x) * sqrt(1+1/n)
```

④ PI : $\bar{x} \pm c\sqrt{1+\frac{\sigma^2}{n}}$ CI: $\bar{x} \pm c \cdot \frac{\sigma}{\sqrt{n}}$

wider

data indicate the difference in mean and this is picked up by Wilcox test. the presence of outliers has large influence on t-test

5) Sample size $n = \left(\frac{c\sigma}{\epsilon}\right)^2$

for proportion $n = c^2\frac{\hat{p}(1-\hat{p})}{\epsilon^2} \approx \frac{c^2}{4\epsilon^2}$

Week 6 Regression

$\boxed{Y_i = \alpha + \beta x_i + \varepsilon_i}$

1) $\hat{\alpha}_0 = \bar{Y}$, $\hat{\beta} = \frac{\sum_{i=1}^n (x_i-\bar{x})Y_i}{\sum_{i=1}^n (x_i-\bar{x})^2} = \frac{\sum_{i=1}^n (x_i-\bar{x})(Y_i-\bar{Y})}{\sum_{i=1}^n (x_i-\bar{x})^2}$

$\hat{\alpha} = \hat{\alpha}_0 - \hat{\beta}\bar{x}$

$\hat{\mu}(x) = \bar{Y} + \hat{\beta}(x-\bar{x})$ $Var(\hat{\alpha}_0) = \frac{\sigma^2}{n}$

$K = \sum_{i=1}^n (x_i-\bar{x})^2$ $Var(\hat{\beta}) = \frac{\sigma^2}{K}$

$Var(\hat{\alpha}) = (\frac{1}{n} + \frac{\bar{x}^2}{K})\sigma^2$

$\boxed{\hat{\sigma}^2 = \frac{1}{n-2}D^2}$ $Var(\hat{\mu}(x)) = (\frac{1}{n} + \frac{(x-\bar{x})^2}{K})\sigma^2$

$D^2 = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$

$\hat{\beta} \sim N(\beta, \frac{\sigma^2}{K})$ $\frac{\hat{\beta}-\beta}{\hat{\sigma}/K} \sim t_{n-2}$

$\frac{(n-2)\hat{\sigma}^2}{\sigma^2} \sim \chi^2_{n-2}$ $\frac{\hat{\mu}(x)-\mu(x)}{\hat{\sigma}\sqrt{\frac{1}{n}+\frac{(x-\bar{x})^2}{K}}} \sim t_{n-2}$

95% CI for β $\hat{\beta} \pm c\frac{\hat{\sigma}}{\sqrt{K}}$

② PI $\hat{\mu}(x^*) \pm c\hat{\sigma}\sqrt{1+\frac{1}{n}+\frac{(x^*-\bar{x})^2}{K}}$

R

```
model <- lm(y ~ x)
summary(model1)
# CI for mean parameters
confint(model1)

data2 <- data.frame(x=3)
predict(model1, newdata = data2, interval = "confidence")
# CI for mu(3)    level=0.9
predict(model1, newdata = data2, interval = "prediction")
# PI when x=3
# plot data and fitted model
plot(x, y, col="blue")
abline(model1, col="blue")
```

③ Model check `plot(model1, 1:2)`

① Equal var residuals vs fitted and ② Normal QQ plot

③. plot(...) abline(...) => Linear model

④ `cor.test(x, y)` $\Leftarrow R^2 \Leftarrow$ `cor(x,y)^2`

Pearson's correlation

$R = R_{xy} = \frac{S_{xy}}{S_x S_y} = \frac{cov(x,y)}{\sqrt{Var\,x\,Var\,y}} = \frac{\sum_{i=1}^n (x_i-\bar{x})(Y_i-\bar{Y})}{\sqrt{\sum_{i=1}^n (x_i-\bar{x})^2 \sum_{i=1}^n (Y_i-\bar{Y})^2}}$

Remedy subset

⑤ 取 log `ldata2 = log(data[, c("murder", "style")])`

Week 7 Hypothesis testing

The same

① $\alpha = Pr(\text{reject } H_0 \mid H_0 \text{ true})$

$\beta = Pr(\text{don't reject } H_0 \mid H_0 \text{ false})$

② Single proportion

```
prop.test(29, 40, p=0.5, alternative = "greater", correct = FALSE)
binom.test(29, 40, p=0.5, alter...)
```

Two proportion

binary factor 手不sb

```
t.test(Thickness ~ Group, data = gun)
```

Week 8

distribution free

① Sign test 样出来 `sum(x>10)`

```
binom.test(11, 20, alternative = "less")
```

Wilcoxon Signed Rank test

```
wilcox.test(x, mu=3.7, alter...
            exact = TRUE)
```

V=40 n=10

```
1 - psignrank(39, 10)
```

Wilcoxon rank sum test (two sample `wilcox.test(x, y)`

② $Q_{k-1} = \sum_{i=1}^k \frac{(O_i-E_i)^2}{E_i} \approx \chi^2_{k-1}$ k outcomes $\boxed{E_i = np \geq 5}$

```
x <- c(26, 15, 32, 7)  p <- c(0.25, 0.15, 0.5, 0.1)
chisq.test(x, p=p)
```

counts prob x.bar = mean (table)

```
p1 <- sum(dpois(0:3, x.bar))    p5 = 1 - (p1 + ... p4)
chisq.test(x, p=p)  # X^2 = 2.734
pchisq(2.734, 4)
```

→ df = k - p - 1

Test independence

```
③ x <- rbind(male = c(34, 74), female = c(20, 22))
chisq.test(x, correct = FALSE)
```

M1

① $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$ $E(X^k) = \int_{-\infty}^{\infty} x^k f(x)\,dx$

⇒ $E(g(x)) = \int_{-\infty}^{\infty} g(x) \cdot f(x)\,dx$ $E(cx) = c\,E(x)$ $Var(cx) = c^2 Var(x)$

② **Variance** $Var(x) = E(x^2) - E(x)^2$

$Var(X+Y) = Var(X) + Var(Y) \pm 2cov(X,Y)$ $Var(\bar{x}) = \frac{1}{n^2}\sum Var(x_i) = \frac{Var(x)}{n}$

$cov(X,Y) = E(XY) - E(X)E(Y) = E\{(X-E(X))(Y-E(Y))\}$

③ correlation $\rho = cor(X,Y) = \frac{cov(X,Y)}{sd(X)\,sd(Y)} = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2\sum(y_i-\bar{y})^2}}$

④ **Distribution** (1) **Bernoulli** $X \sim Be(p)$

$p(x) = p^x(1-p)^{1-x}$ $E(x) = p$ $Var(x) = p(1-p)$ $x \in \{0,1\}$

(2) **Binomial** $X \sim Bi(n,p)$ $p(x) = \binom{n}{x}p^x(1-p)^{n-x}$ $x \in \{0,1,\dots,n\}$

$E(x) = np$ $Var(x) = np(1-p)$

(3) **Poisson** $X \sim Pn(\lambda)$ $p(x) = e^{-\lambda}\frac{\lambda^x}{x!}$ $x \in \{0,1,\dots\}$ $E(x) = Var(x) = \lambda$

(4) **Exponential** $X \sim Exp(\lambda)$ $f(x) = \lambda e^{-\lambda x}$ $x \in [0,\infty)$

$E(x) = \frac{1}{\lambda}$ $Var(x) = \frac{1}{\lambda^2}$ $F(x) = 1 - e^{-\lambda x}$

(5) **Gamma** Gamma(α,θ) $f(x|\alpha,\theta) = \frac{1}{\Gamma(\alpha)\theta^{\alpha}}x^{\alpha-1}e^{-\frac{x}{\theta}}$

$E(x) = \frac{\alpha}{\theta}$ $Var = \sigma^2 = \frac{\alpha}{\theta^2}$

(6) **uniform** unif(a,b) $f(x) = \frac{1}{b-a}$ $E(x) = \frac{a+b}{2}$

$Var(x) = \frac{(b-a)^2}{12}$ cdf $= \frac{x}{\theta}$ pdf $= \frac{1}{\theta}$

(7) **Normal** $X \sim N(\mu,\sigma^2)$ $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$E(x) = \mu$ $Var(x) = \sigma^2$ $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

(8) **Chi-square** $T \sim \chi_k^2$ $E(T) = k$ $Var(T) = 2k$

$Z_i \sim N(0,1) \Rightarrow T \Rightarrow Z_1^2 + \dots + Z_n^2 \sim \chi_k^2$

sample Normal $\frac{(n-1)s^2}{\sigma^2} \sim \chi_k^2$

(9) **t-dist** $E(T) = 0$ $Var(T) = \frac{k}{k-2}$ $T = \frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t_{n-1}$

(10) **F-dist** $W \sim F_{m,n}$ $F = \frac{U/m}{V/n}$ $U \sim \chi_m^2$ $V \sim \chi_n^2$

⑤ **Quantiles** $p = \int_{-\infty}^{x_p} f(x)\,dx = F(x_p) \Rightarrow x_p = F^{-1}(p)$

$x_p = X_{(k)}$, $k = 1+(n-1)p \Rightarrow$ Type 7

$p = \frac{k}{n+1}$, $\Rightarrow$ Type 6

if $\frac{k-1}{n} < p \le \frac{k}{n} \Rightarrow$ Type 1

M2

① MM: (1) kth moments $\mu_k = E(X^k)$

(2) kth sample moments $M_k = \frac{1}{n}\sum X_i^k$

(3) $\mu_k = M_k$

② MLE: ① pdf → $L(p) = \prod_{i=1}^{n} f(x_i)$

② $\ln L(p)$ ③ $\frac{\partial \ln L(p)}{\partial p} = 0$

σ^2 CI $(1-\alpha)$ ☆ $\left[\frac{(n-2)\hat{\sigma}^2}{F^{-1}(1-\frac{\alpha}{2})}, \frac{(n-2)\hat{\sigma}^2}{F^{-1}(\frac{\alpha}{2})}\right]$

M3

① $Pr(-c < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < c) = 1-\alpha$

$\Rightarrow Pr(\bar{x} - c\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + c\frac{\sigma}{\sqrt{n}}) = 1-\alpha$ CI

② CI ☆ (1) **Normal, single mean, known σ** $100\cdot(1-\alpha)\%$

$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim N(0,1) \Rightarrow \Phi^{-1}$ ☆

CI: $\bar{x} \pm c\frac{\sigma}{\sqrt{n}}$

H_0	H_1	critical region
$p = p_0$	$p > p_0$	$Z > \Phi^{-1}(1-\alpha)$
	$p < p_0$	$Z < \Phi^{-1}(\alpha)$
	$p \ne p_0$	$Z > \Phi^{-1}(1-\frac{\alpha}{2})$

(2) **Normal, single mean, unknown σ**

$T = \frac{\bar{x}-\mu}{s/\sqrt{n}} \sim t_{n-1} \leftarrow \Phi^{-1}$ CI $\bar{x} \pm c\cdot\frac{s}{\sqrt{n}}$ $T_\beta = \frac{\hat{\beta}-\beta}{\hat{\sigma}/\sqrt{K}} \sim t_{n-2}$

(3) **Normal, two means, known σ** $X_n \sim N(\cdot)$ $Y_m \sim N(\cdot)$

CI: $\bar{x} - \bar{y} \pm c\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}$

(4) **Normal, two means, unknown σ, many samples**

CI: $\bar{x} - \bar{y} \pm c\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}$

(5) **Normal, two means, unknown σ, same Variance**

CI: $\bar{x} - \bar{y} \pm c\cdot s_p\sqrt{\frac{1}{n}+\frac{1}{m}}$ $s_p = \sqrt{\frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}}$

$T = \frac{\bar{x}-\bar{y}}{s_p\sqrt{\frac{1}{n}+\frac{1}{m}}} \sim t_{m+n-2}$ ☆ Φ^{-1}

(6) **two means, unknown σ, different Var**

Welch approximation $T = \frac{\bar{x}-\bar{y}-(\mu_x-\mu_y)}{\sqrt{\frac{s_x^2}{n}+\frac{s_y^2}{m}}} \approx t_r$

$r = \frac{\left(\frac{s_x^2}{n}+\frac{s_y^2}{m}\right)^2}{\frac{s_x^4}{n^2(n-1)} + \frac{s_y^4}{m^2(m-1)}}$

$a = \Phi^{-1}(\frac{\alpha}{2})$ $b = \Phi^{-1}(1-\frac{\alpha}{2})$

(7) **Normal paired ⇒ 差值, single mean** σ^2

(8) **Normal, single Variance** CI $\left[\frac{(n-1)s^2}{b}, \frac{(n-1)s^2}{a}\right]$

$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$ Φ^{-1} ☆ $\chi^2 > F^{-1}(1-\alpha)$ $\chi^2 < F^{-1}(\alpha)$

(9) **Normal, two Variance** $\chi^2 < F^{-1}(\frac{\alpha}{2})$ or $\chi^2 > F^{-1}(1-\frac{\alpha}{2})$

CI $\left[c\frac{s_x^2}{s_y^2}, d\frac{s_x^2}{s_y^2}\right]$ $c = \Phi^{-1}(\frac{\alpha}{2})$ $d = \Phi^{-1}(1-\frac{\alpha}{2})$ ⇐ $\frac{\sigma_x^2}{\sigma_y^2}$ $F_{m-1,n-1}$ 倒过来

X_n. Y_m For Hypothesis testing

$F = \frac{(n-1)s_x^2/\sigma^2/(n-1)}{(m-1)s_y^2/\sigma^2/(m-1)} = \frac{s_x^2}{s_y^2} \sim F_{n-1,m-1}$ Φ^{-1} 判断与 go single value 一样

(10) **single proportion** CI: $\hat{p} \pm c\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ 变为 Normal

$Z = \frac{y/n - p_0}{\sqrt{p_0(1-p_0)/n}}$ Φ^{-1}(Normal)

(11) **two proportion** CI: $\hat{p}_1 - \hat{p}_2 \pm c\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1}+\frac{1}{n_2})}}$ $\hat{p}_1 = \frac{y_1}{n_1}$ $\hat{p}_2 = \frac{y_2}{n_2}$ $\hat{p} = \frac{y_1+y_2}{n_1+n_2}$ Φ^{-1}(Normal)

M4

① **CI for MLE** ① $V(\theta) = -\frac{\partial^2 \ln L_p}{\partial\theta^2}$ ② $se(\hat{\theta}) = \frac{1}{\sqrt{V(\hat{\theta})}}$

③ $\Rightarrow \hat{\theta} \pm \frac{c}{\sqrt{V(\hat{\theta})}}$ $c = \Phi^{-1}(1-\frac{\alpha}{2})$

② PI $\bar{x} \pm c\sqrt{1+\frac{\sigma^2}{n}}$ wider

CI $\bar{x} \pm c\sqrt{\frac{\sigma^2}{n}}$

③ **sample size** $n = \left(\frac{c\sigma}{\epsilon}\right)^2$ for proportion $n = \frac{c^2\hat{p}(1-\hat{p})}{\epsilon^2}$ ⇐ $n = \frac{c^2}{4\epsilon^2}$

$\hat{\alpha}_0 = \hat{\alpha} + \hat{\beta}\bar{x}$

M5 Regression

① $E(Y|x) = \alpha + \beta x = \alpha_0 + \beta(x - \bar{x})$

$\hat{\alpha}_0 = \bar{Y}$, $Var(\hat{\alpha}_0) = \frac{\sigma^2}{n}$ $se(\hat{\alpha}_0) = \frac{\sigma}{\sqrt{n}}$ $\frac{1}{n-1}\sum(x_i-\bar{x})^2 = s^2$

$\hat{\beta} = \frac{\sum(x_i-\bar{x})Y_i}{\sum(x_i-\bar{x})^2} = \frac{\sum(x_i-\bar{x})(Y_i-\bar{Y})}{\sum(x_i-\bar{x})^2}$ $K = (n-1)s^2$

$Var(\hat{\beta}) = \frac{\sigma^2}{K}$ $se(\hat{\beta}) = \frac{\sigma}{\sqrt{K}}$ $K = \sum(x_i-\bar{x})^2$

$\hat{\mu}(x) = \bar{Y} + \hat{\beta}(x-\bar{x})$ $Var(\hat{\mu}(x)) = \left(\frac{1}{n} + \frac{(x-\bar{x})^2}{K}\right)\sigma^2$

$se(\hat{\mu}(x)) = \sqrt{Var(\hat{\mu}(x))}$

$Var(\hat{\alpha}) = \left(\frac{1}{n} + \frac{\bar{x}^2}{K}\right)\sigma^2$

$\hat{\sigma}^2 = \frac{1}{n-2}D^2$, $D^2 = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$ $\frac{(n-2)\hat{\sigma}^2}{\sigma^2} \sim \chi^2_{n-2}$

$= \sum_{i=1}^{n}(y_i-\bar{y})^2 - \frac{[\sum(x_i-\bar{x})(y_i-\bar{y})]^2}{\sum(x_i-\bar{x})^2}$

CI $\hat{\beta} \sim N(\beta, \frac{\sigma^2}{K})$

$\hat{\beta} \pm c\cdot\frac{\hat{\sigma}}{\sqrt{K}} \sim t_{n-2}$ $\hat{\mu}(x) \pm c\cdot se(\hat{\mu}(x))$ $\sim t_{n-2}$

M6 ① critical region reject H_0.
② $\alpha = Pr(\text{reject } H_0 \mid H_0 \text{ true})$
$\beta = Pr(\text{fail to reject } H_0 \mid H_0 \text{ false})$
power $1-\beta$

M7 ① sign test : $Y \sim Bi(n, 0.5)$ positive signs. reject H_0 if Y too small
$Pr(Y \le 6) = 0.057 \Leftarrow$ pbinom(6, 20, 0.5)

② Wilcoxon signed rank test $\begin{cases} W = \text{全加起符号的} \\ V = \text{只加正的} \end{cases}$
$E(W)=0$, $Var(W) = \frac{n(n+1)(2n+1)}{6}$, $E(V) = \frac{n(n+1)}{4}$, $Var(V) = \frac{n(n+1)(2n+1)}{24}$
$Z = \frac{W-0}{\sqrt{Var(W)}}$ 判断 $Z > \Phi^{-1}(1-\alpha)$ normal 上推 看 H_1 N

③ two sample W = (signed Y) 全加起来
$H: m_X > m_Y \Rightarrow W$ should be smaller $\Rightarrow W \le c$, reject H_0.
$E(W) = \frac{n_Y(n_X+n_Y+1)}{2}$ $Var(W) = \frac{n_X n_Y(n_X+n_Y+1)}{12}$
$Z = \frac{W-E(W)}{\sqrt{Var(W)}} \le c$

④ Goodness of fit (χ^2) $Q_{k-1} = \sum_{i=1}^{k} \frac{(O_i-E_i)^2}{E_i} \approx \chi^2_{k-1}$ $E_i = np_i \ge 5$
reject H_0, if $Q_{k-1} > c$, where $c = \Phi^{-1}(1-\alpha)$ from χ^2_{k-1}

M8 ANOVA ① ANOVA Table 遇到 pmf 时 $k-p-1$ →

$F = \frac{SS(T)/(k-1)}{SS(E)/(n-k)}$

Source	df	SS	MS	F
Treatment	$k-1$	$SS(T)$	$MS(T)=\frac{SS(T)}{k-1}$	$\frac{MS(T)}{MS(E)}$
Error	$n-k$	$SS(E)$	$MS(E)=\frac{SS(E)}{n-k}$	
Total	$n-1$	$SS(TO)$		

reject H_0 if $F > c$.

$\hat\sigma^2 = MS(E)$

总 $SS(T) = \sum_{i=1}^{n} (\bar Y_i - \bar Y)^2$

two-way

FA	$a-1$	$SS(A)$	$\frac{SS(A)}{a-1}$	$\frac{MS(A)}{MS(E)}$
FB	$b-1$	$SS(B)$	$\frac{SS(B)}{b-1}$	$\frac{MS(B)}{MS(E)}$
Error	$(a-1)(b-1)$	$SS(E)$	$\frac{SS(E)}{(a-1)(b-1)}$	
Total	$ab-1$	$SS(TO)$		

interaction

FAB	$(a-1)(b-1)$	$SS(AB)$	$\frac{SS(AB)}{(a-1)(b-1)}$	$\frac{MS(AB)}{MS(E)}$
Error	$ab(c-1)$	$SS(E)$	$\frac{SS(E)}{ab(c-1)}$	
Total	$abc-1$	$SS(TO)$		

② LRT $\lambda = \frac{L_0}{L_1} = \frac{\max_{\theta\in\Omega_0} L(\theta)}{\max_{\theta\in\Omega} L(\theta)}$ $X_i \sim$ 某个 distribution $Be(p)$. $\lambda\uparrow \Rightarrow H_0\uparrow$
$\Rightarrow$ reject H_0, $\lambda \le k$
$\Rightarrow$ reject H_0, $Y \ge c$ $Y = \sum_{i=1}^{n} X_i \sim Bi(n,p)$

M9 ① distribution of $X_{(k)}$. cdf $F(x)$ pdf $f(x)$
cdf: $G_k(x) = Pr(X \le x) = \sum_{i=k}^{n} \binom{n}{i} F(x)^i (1-F(x))^{n-i}$
pdf: $g_k(x) = k\binom{n}{k} F(x)^{k-1}(1-F(x))^{n-k} f(x)$
$g_1(x) = n(1-F(x))^{n-1} f(x)$
$g_n(x) = n F(x)^{n-1} f(x)$
$\binom{n}{k} = \frac{n!}{k!(n-k)!}$
$Pr(X_{(1)} > x) = (1-F(x))^n$
$Pr(X_{(n)} \le x) = F(x)^n$ cdf

② Asymptotic distribution $\hat\pi_p \approx N\left(\pi_p, \frac{p(1-p)}{n f(\pi_p)^2}\right)$ $f \Rightarrow$ pdf
for median $M = \pi_{0.5}$ $\hat M \approx N\left(m, \frac{1}{4n f(m)^2}\right)$

$X \sim N(\mu, \frac{\sigma^2}{n})$. $\hat M \approx N\left(m, \frac{1}{4n f(m)^2}\right)$
normal $f(m) = f(\mu) = \frac{1}{\sigma\sqrt{2\pi}}$
$\Rightarrow \hat M \approx N\left(\mu, \frac{\pi\sigma^2}{2n}\right)$

③ CI for median
$Pr(X_{(i)} < m < X_{(j)}) = Pr(i \le W \le j-1)$ Pbinom
$= \sum_{k=i}^{j-1} \binom{n}{k} (\frac{1}{2})^k (\frac{1}{2})^{n-k} \approx 1-\alpha$
$Pr(1 \le W \le 9) = 1 - 2Pr(W \le 0)$ table
W: number of below m

④ CI for quantiles
$1-\alpha = Pr(X_{(i)} < \pi_p < X_{(j)})$
$= Pr(i \le W \le j-1)$
$= \sum_{k=i}^{j-1} \binom{n}{k} p^k (1-p)^{n-k}$

M10 ① $Pr(B|A) = \frac{Pr(A|B)Pr(B)}{Pr(A)}$
improper prior $\mu_1 = y = \bar x$
$f(\theta|y) \propto f(y|\theta) f(\theta)$ $\prod$ pdf
$P(B_i|A) = \frac{Pr(A|B_i)P(B_i)}{\sum^{k} Pr(A|B_i)Pr(B_i)} \propto Pr(A|B_i)Pr(B_i)$
① likelihood $Pr(A|B_i)$ data $\Rightarrow$ 看 dist pdf $\prod$ pdf
prior dist $Pr(B_i)$ = 已知
② Beta dist $p \in [0,1]$ $\alpha, \beta > 0$ $P \sim Beta(\alpha,\beta)$
$f(p) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}$, $\Gamma(n) = (n-1)!$
$E(p) = \frac{\alpha}{\alpha+\beta}$ mode$(p) = \frac{\alpha-1}{\alpha+\beta-2}$ ① dist ③ $\int \Rightarrow$ normally const
$Var(p) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ pseudo $(\alpha+\beta=60)$ $\frac{\alpha}{\alpha+\beta} = 0.4$
③ Prior : $\theta \sim Beta(\alpha,\beta) \Rightarrow$ posterior $\theta | X=x \sim Beta(x+\alpha, n-x+\beta)$
④ credible interval $0.95 = Pr(a<\theta<b \mid X)$
$\theta|X=x \sim Beta(16,6) \Rightarrow$ qbeta(c(0.025, 0.975), 16, 6)
$\Rightarrow$ 0.528 0.887
⑤ Normal single mean, known σ $X_1 \dots X_n \sim N(\theta, \sigma^2)$
$Y = \bar x \sim N(\theta, \sigma^2/n)$ prior : $\theta \sim N(\mu_0, \sigma_0^2)$
$f(\theta|y) \propto f(y|\theta) f(\theta)$
$\Rightarrow \mu_1 = \left(\frac{\frac{1}{\sigma_0^2}}{\frac{1}{\sigma_0^2}+\frac{1}{\sigma^2/n}}\right)\mu_0 + \left(\frac{\frac{1}{\sigma^2/n}}{\frac{1}{\sigma_0^2}+\frac{1}{\sigma^2/n}}\right) y$
$\frac{1}{\sigma_1^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma^2/n}$
posterior $\theta|y \sim N(\mu_1, \sigma_1^2)$
95% CI : $\mu_1 \pm 1.96\sigma_1$

① Efficiency eff$(T) = \frac{1/I(\theta)}{Var(T)} \le 1$

M11 ① $l(\theta) = \ln L(\theta)$ ← likelihood
score function $U(\theta) = \frac{\partial l}{\partial \theta}$ 一次导 MLE $\Rightarrow U(\theta) = 0$
$V(\theta) = \frac{\partial^2 l}{\partial\theta^2}$ 二次导 取负 $\Rightarrow E[-V(\theta)] = I(\theta)$
Cramer-Rao lower bound : $\frac{1}{I(\theta)}$
③ sufficient statistic $\Rightarrow$ likelihood $\Rightarrow$ 看 $e^{\text{指数}}$ 找 $\sum X_i$
Asymptotic distribution $\hat\theta \approx N\left(\theta, \frac{1}{I(\theta)}\right)$
$se(\hat\theta) = \sqrt{\frac{1}{I(\theta)}} = \sqrt{\frac{1}{V(\theta)}}$
95% CI $\hat\theta \pm \Phi^{-1}(1-\frac{\alpha}{2}) \cdot se(\hat\theta)$