AI Planning for Autonomy

2. Search Algorithms

Basic Stuff You're Gonna Need to Search for a Solution
Where To Search Next?

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Basic State Model: Classical Planning

Ambition:

Write one program that can solve all classical search problems.

State Model $\mathcal{S}(P)$**:**

- finite and discrete state space S
- a known initial state $s_0 \in S$
- a set $S_G \subseteq S$ of goal states
- actions $A(s) \subseteq A$ applicable in each $s \in S$
- a deterministic transition function $s' = f(a, s)$ for $a \in A(s)$
- positive action costs $c(a, s)$

$\rightarrow$ A **solution** is a sequence of applicable actions that maps s_0 into S_G, and it is **optimal** if it minimizes **sum of action costs** (e.g., # of steps)

$\rightarrow$ Different **models** and **controllers** obtained by relaxing assumptions in **blue** . . .

Solving the State Model: Path-finding in graphs

Search algorithms for planning exploit the **correspondence** between **(classical) states model** $\mathcal{S}(P)$ and **directed graphs**:

- The **nodes** of the graph represent the **states** s in the model
- The edges (s, s') capture corresponding transition in the model with same cost

In the **planning as heuristic search** formulation, the problem P is solved by **path-finding** algorithms over the **graph** associated with model $\mathcal{S}(P)$

Classification of Search Algorithms

Blind search vs. heuristic (or informed) search:

- Blind search algorithms: Only use the basic ingredients for general search algorithms.
  - *e.g., Depth First Search (DFS), Breadth-first search (BrFS), Uniform Cost (Dijkstra), Iterative Deepening (ID)*
- Heuristic search algorithms: Additionally use heuristic functions which estimate the distance (or remaining cost) to the goal.
  - *e.g., A*, IDA*, Hill Climbing, Best First, WA*, DFS B&B, LRTA*, . . .*

Systematic search vs. local search:

- Systematic search algorithms: Consider a large number of search nodes simultaneously.
- Local search algorithms: Work with one (or a few) candidate solutions (search nodes) at a time.

→ This is not a black-and-white distinction; there are *crossbreeds* (e.g., enforced hill-climbing).

What works where in planning?

Blind search vs. heuristic search:

- For satisficing planning, heuristic search vastly outperforms blind algorithms pretty much everywhwere.
- For optimal planning, heuristic search also is better (but the difference is less pronounced).

Systematic search vs. local search:

- For satisficing planning, there are successful instances of each.
- For optimal planning, systematic algorithms are required.

→ Here, we cover the subset of search algorithms most successful in planning. Only some Blind search algorithms are covered. (refer to Russel & Norvig Chapters 3 and 4 for that).

Search Terminology

Search node n: Contains a *state* reached by the search, plus information about how it was reached.

Path cost $g(n)$: The cost of the path reaching n.

Optimal cost g^*: The cost of an optimal solution path. For a state s, $g^*(s)$ is the cost of a cheapest path reaching s.

Node expansion: Generating all successors of a node, by applying all actions applicable to the node's state s. Afterwards, the *state* s itself is also said to be expanded.

Search strategy: Method for deciding which node is expanded next.

Open list: Set of all *nodes* that currently are candidates for expansion. Also called frontier.

Closed list: Set of all *states* that were already expanded. Used only in graph search, not in tree search (up next). Also called explored set.

World States vs. Search States

Reminder: Search Space for Classical Search

A (classical) search space is defined by the following three operations:

- start(): Generate the start (search) state.
- is-target(s): Test whether a given search state is a target state.
- succ(s): Generates the successor states (a, s') of search state s, along with the actions through which they are reached.

Search states $\neq$ world states:

- Progression: Yes, search states = world states.
- Regression: No, search states = sets of world states, represented as conjunctive sub-goals.

$\rightarrow$ We consider progression in the entire course, unless explicitly stated otherwise. We use "s" to denote world/search states interchangeably.

Search States vs. Search Nodes

- **Search states** s: States (vertices) of the search space.
- **Search nodes** σ: Search states, plus information on where/when/how they are encountered during search.

What is in a search node?

Different search algorithms store different information in a search node σ, but typical information includes:

- *state*(σ): Associated search state.
- *parent*(σ): Pointer to search node from which σ is reached.
- *action*(σ): An action leading from *state*(*parent*(σ)) to *state*(σ).
- $g(\sigma)$: Cost of σ (cost of path from the root node to σ).

For the root node, *parent*(σ) and *action*(σ) are undefined.

Criteria for Evaluating Search Strategies

Guarantees:

Completeness: Is the strategy guaranteed to find a solution when there is one?

Optimality: Are the returned solutions guaranteed to be optimal?

Complexity:

Time Complexity: How long does it take to find a solution? (Measured in generated states.)

Space Complexity: How much memory does the search require? (Measured in states.)

Typical state space features governing complexity:

Branching factor b: How many successors does each state have?

Goal depth d: The number of actions required to reach the shallowest goal state.

Before We Begin

Blind search vs. informed search:

- Blind search does not require any input beyond the problem.

  → Pros and Cons? Pro: No additional work for the programmer. Con: It's not called "blind" for nothing . . . same expansion order regardless what the problem actually is. Rarely effective in practice.

- Informed search requires as additional input a heuristic function h (**Next Chapter**) that maps states to estimates of their goal distance.

  → Pros and Cons? Pro: Typically more effective in practice. Con: Somebody's gotta come up with/implement h.

  → Note: In planning, h is generated automatically from the declarative problem description

Before We Begin, ctd.

Blind search strategies we'll discuss:

- Breadth-first search. Advantage: time complexity.
  Variant: Uniform cost search.
- Depth-first search. Advantage: space complexity.
- Iterative deepening search. Combines advantages of breadth-first search and depth-first search. Uses depth-limited search as a sub-procedure.

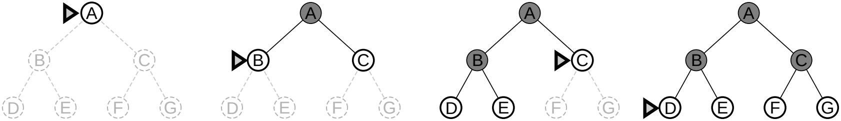
Blind search strategy we won't discuss:

- Bi-directional search. Two separate search spaces, one forward from the initial state, the other backward from the goal. Stops when the two search spaces overlap.

Breadth-First Search: Illustration and Guarantees

Strategy: Expand nodes in the order they were produced (FIFO frontier).

Illustration:



Guarantees:

- Completeness? Yes.
- Optimality? Yes, for uniform action costs. Breadth-first search always finds a shallowest goal state. If costs are not uniform, this is not necessarily optimal.

Breadth-First Search: Complexity

Time Complexity: Say that b is the maximal branching factor, and d is the goal depth (depth of shallowest goal state).

- Upper bound on the number of generated nodes? $b + b^2 + b^3 + \cdots + b^d$: In the worst case, the algorithm generates all nodes in the first d layers.
- So the time complexity is $O(b^d)$.
- And if we were to apply the goal test at node-expansion time, rather than node-generation time? $O(b^{d+1})$ because then we'd generate the first $d + 1$ layers in the worst case.

Space Complexity: Same as time complexity since all generated nodes are kept in memory.

Breadth-First Search: Example Data

Setting: $b = 10$; 10000 nodes/second; 1000 bytes/node.

Yields data: (inserting values into previous equations)

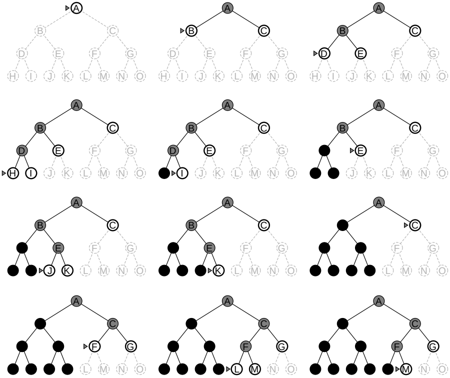
Depth	Nodes	Time		Memory	
2	110	.11	milliseconds	107	kilobytes
4	11110	11	milliseconds	10.6	megabytes
6	10^6	1.1	seconds	1	gigabyte
8	10^8	2	minutes	103	gigabytes
10	10^{10}	3	hours	10	terabytes
12	10^{12}	13	days	1	petabyte
14	10^{14}	3.5	years	99	petabytes

→ So, which is the worse problem, time or memory? Memory. (In my own experience, typically exhausts RAM memory within a few minutes.)

Depth-First Search: Illustration

Strategy: Expand the most recent nodes in (LIFO frontier).

Illustration: (Nodes at depth 3 are assumed to have no successors)



Depth-First Search: Guarantees and Complexity

Guarantees:

Quizz@SpeakUp: A) Complete and optimal B) Complete but may not be optimal C) Optimal but may not be complete D) Neither complete nor optimal

- Optimality? No. After all, the algorithm just "chooses some direction and hopes for the best". (Depth-first search is a way of "hoping to get lucky".)
- Completeness? No, because search branches may be infinitely long: No check for cycles along a branch!

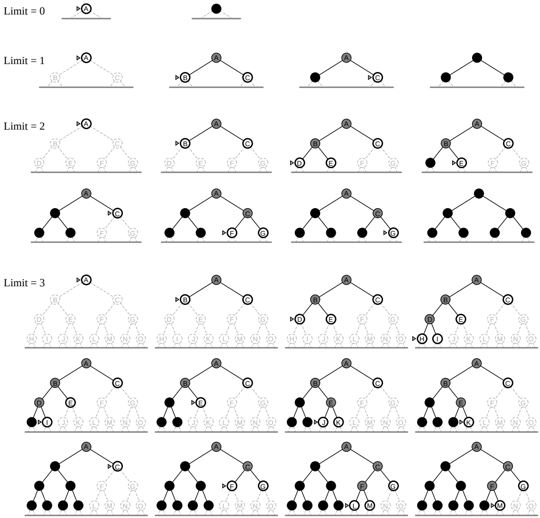
  $\rightarrow$ Depth-first search is complete in case the state space is acyclic, e.g., Constraint Satisfaction Problems. If we do add a cycle check, it becomes complete for finite state spaces.

Complexity:

- Space: Stores nodes and applicable actions on the path to the current node. So if m is the maximal depth reached, the complexity is $O(b\,m)$.
- Time: If there are paths of length m in the state space, $O(b^m)$ nodes can be generated. Even if there are solutions of depth 1!

  $\rightarrow$ If we happen to choose "the right direction" then we can find a length-l solution in time $O(b\,l)$ regardless how big the state space is.

Iterative Deepening Search: Illustration



Iterative Deepening Search: Guarantees and Complexity

"Iterative Deepening Search=
Keep doing the same work over again until you find a solution."

BUT: Optimality? Yes! Completeness? Yes! Space complexity? $O(b\,d)$.

Time complexity:

Breadth-First-Search	$b + b^2 + \cdots + b^{d-1} + b^d \in O(b^d)$
Iterative Deepening Search	$(d)b + (d-1)b^2 + \cdots + 3b^{d-2} + 2b^{d-1} + 1b^d \in O(b^d)$

Example: $b = 10, d = 5$

Breadth-First Search	$10 + 100 + 1,000 + 10,000 + 100,000 = 111,110$
Iterative Deepening Search	$50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$

$\rightarrow$ IDS combines the advantages of breadth-first and depth-first search. It is the preferred blind search method in large state spaces with unknown solution depth.

Heuristic Search Algorithms: Systematic

→ Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Systematic heuristic search algorithms:

- Greedy best-first search.
  → One of 3 most popular algorithms in satisficing planning.
- Weighted A^*.
  → One of 3 most popular algorithms in satisficing planning.
- A^*.
  → Most popular algorithm in optimal planning. (Rarely ever used for satisficing planning.)
- IDA^*, depth-first branch-and-bound search, breadth-first heuristic search, . . .

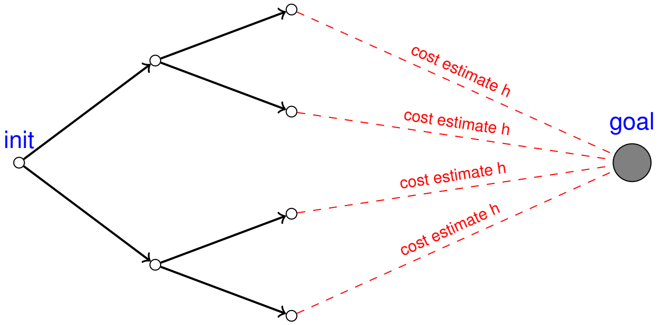
Heuristic Search Algorithms: Local

→ Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Local heuristic search algorithms:

- Hill-climbing.
- Enforced hill-climbing.
  → One of 3 most popular algorithms in satisficing planning.
- Beam search, tabu search, genetic algorithms, simulated annealing, . . .

Heuristic Search: Basic Idea



→ Heuristic function h estimates the cost of an optimal path to the goal; search gives a preference to explore states with small h.

Heuristic Functions

Heuristic searches require a heuristic function to estimate remaining cost:

Definition (Heuristic Function). *Let Π be a planning task with state space Θ_Π. A heuristic function, short heuristic, for Π is a function $h : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$. Its value $h(s)$ for a state s is referred to as the state's heuristic value, or h-value.*

Definition (Remaining Cost, h^*). *Let Π be a planning task with state space Θ_Π. For a state $s \in S$, the state's remaining cost is the cost of an optimal plan for s, or ∞ if there exists no plan for s. The perfect heuristic for Π, written h^*, assigns every $s \in S$ its remaining cost as the heuristic value.*

Heuristic Functions: Discussion

What does it mean to "estimate remaining cost"?

- For many heuristic search algorithms, h does not need to have any properties for the algorithm to "work" (= be correct and complete).

  $\rightarrow$ h is *any* function from states to numbers ...
- Search performance depends crucially on "how well h reflects h^*"!!

  $\rightarrow$ This is informally called the informedness or quality of h.
- For some search algorithms, like A^*, we can *prove* relationships between formal quality properties of h and search efficiency (mainly the number of expanded nodes).
- For other search algorithms, "it works well in practice" is often as good an analysis as one gets.

$\rightarrow$ We will analyze in detail approximations to one particularly important heuristic function in planning: h^+.

Heuristic Functions: Discussion, ctd.

"Search performance depends crucially on the informedness of h . . . "

Any other property of h that search performance crucially depends on?

". . . and on the computational overhead of computing h!!"

Extreme cases:

- $h = h^*$: Perfectly informed; computing it = solving the planning task in the first place.
- $h = 0$: No information at all; can be "computed" in constant time.

→ Successful heuristic search requires a good trade-off between h's informedness and the computational overhead of computing it.

→ This really is what research is all about! Devise methods that yield good estimates at reasonable computational costs.

Properties of Heuristic Functions

Definition (Safe/Goal-Aware/Admissible/Consistent). *Let Π be a planning task with state space $\Theta_\Pi = (S, L, c, T, I, S^G)$, and let h be a heuristic for Π. The heuristic is called:*

- *safe if $h^*(s) = \infty$ for all $s \in S$ with $h(s) = \infty$;*
- *goal-aware if $h(s) = 0$ for all goal states $s \in S^G$;*
- *admissible if $h(s) \leq h^*(s)$ for all $s \in s$;*
- *consistent if $h(s) \leq h(s') + c(a)$ for all transitions $s \xrightarrow{a} s'$.*

$\rightarrow$ **Relationships?**

Proposition. *Let Π be a planning task with state space $\Theta_\Pi = (S, L, c, T, I, S^G)$, and let h be a heuristic for Π. If h is consistent and goal-aware, then h is admissible. If h is admissible, then h is goal-aware. If h is admissible, then h is safe. No other implications of this form hold.*

Proof. $\rightarrow$ **Exercise, perhaps**.

Greedy Best-First Search

Greedy Best-First Search (with duplicate detection)

open $:=$ **new** priority queue ordered by ascending $h(\mathit{state}(\sigma))$
open.insert(make-root-node(init()))
closed $:= \emptyset$
while not *open*.empty():
    $\sigma :=$ *open*.pop-min() /* get best state */
    if $\mathit{state}(\sigma) \notin \mathit{closed}$: /* check duplicates */
        $\mathit{closed} := \mathit{closed} \cup \{\mathit{state}(\sigma)\}$ /* close state */
        if is-goal(state(σ)): **return** extract-solution(σ)
        for each $(a, s') \in$ succ($\mathit{state}(\sigma)$): /* expand state */
            $\sigma' :=$ make-node(σ, a, s')
            if $h(\mathit{state}(\sigma')) < \infty$: *open*.insert$(\sigma')$
return unsolvable

Greedy Best-First Search: Remarks

Properties:

- Complete? Yes, for safe heuristics. (and duplicate detection to avoid cycles)
- Optimal? No.[1]
- Invariant under all strictly monotonic transformations of h (e.g., scaling with a positive constant or adding a constant).

Implementation:

- Priority queue: e.g., a min heap.
- "Check Duplicates": Could already do in "expand state"; done here after "get best state" *only* to more clearly point out relation to A^*.

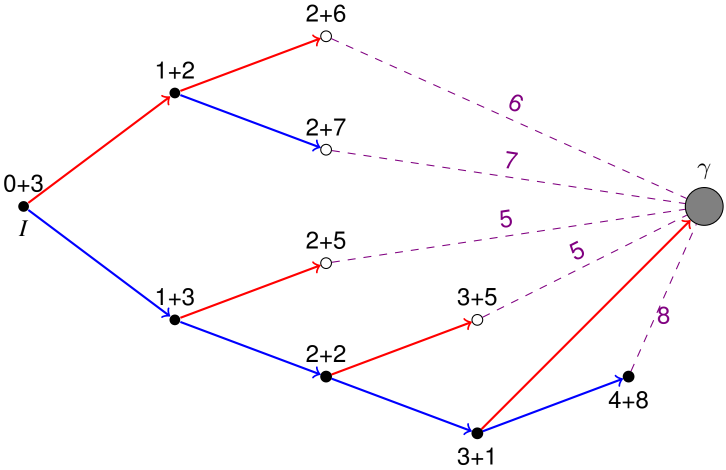
[1] Even for perfect heuristics! E.g., say the start state has two transitions to goal states, one of which costs a million bucks while the other one is free. Nothing keeps Greedy Best-First Search from choosing the bad one.

A^*

A^* (with duplicate detection and re-opening)

open := **new** priority queue ordered by ascending $g(\mathit{state}(\sigma)) + h(\mathit{state}(\sigma))$
open.insert(make-root-node(init()))
closed := $\emptyset$
best-g := $\emptyset$/* maps states to numbers */
while not *open*.empty():
    σ := *open*.pop-min()
    if *state*(σ) $\notin$ *closed* **or** $g(\sigma) <$ *best-g*(*state*(σ)):
      /* re-open if better g; note that all σ' with same state but worse g
        are *behind* σ in *open*, and will be skipped when their turn comes */
      closed := *closed* $\cup$ {*state*(σ)}
      best-g(*state*(σ)) := $g(\sigma)$
      if is-goal(state(σ)): **return** extract-solution(σ)
      for each $(a, s') \in$ succ(*state*(σ)):
        σ' := make-node(σ, a, s')
        if $h(\mathit{state}(\sigma')) < \infty$: *open*.insert($\sigma'$)
return unsolvable

A^*: Example



A^*: Terminology

- f-value of a state: defined by $f(s) := g(s) + h(s)$.
- Generated nodes: Nodes inserted into *open* at some point.
- Expanded nodes: Nodes σ popped from *open* for which the test against *closed* and *distance* succeeds.
- Re-expanded nodes: Expanded nodes for which *state*$(\sigma) \in$ *closed* upon expansion (also called re-opened nodes).

A^*: Remarks

Properties:

- Complete? Yes, for safe heuristics. (Even without duplicate detection.)
- Optimal? Yes, for admissible heuristics. (Even without duplicate detection.)

Implementation:

- Popular method: break ties ($f(s) = f(s')$) by smaller h-value.
- If h is admissible and consistent, then A^* never re-opens a state. So if we know that this is the case, then we can simplify the algorithm.
- Common, hard to spot bug: check duplicates at the wrong point. (Russel & Norvig are way too imprecise about this.)
- Our implementation is optimized for readability not for efficiency!

Quizz@SpeakUp

Question!

If we set $h(n) := 0$ for all n, what does $\mathbf{A}^*$ become?

(A): Breadth-first search.
(B): Depth-first search.
(C): Uniform-cost search.
(D): Depth-limited search.

$\rightarrow$ (C): Same expansion order. (Details in book-keeping of open/closed states may differ.)

Weighted A^*

Weighted A^* (with duplicate detection and re-opening)

open $:=$ **new** priority queue ordered by ascending $g(\textit{state}(\sigma)) + W * h(\textit{state}(\sigma))$
open.insert(make-root-node(init()))
closed $:= \emptyset$
best-g $:= \emptyset$
while not *open*.empty():
  $\sigma :=$ *open*.pop-min()
  if $\textit{state}(\sigma) \notin \textit{closed}$ **or** $g(\sigma) < \textit{best-g}(\textit{state}(\sigma))$:
    closed $:=$ *closed* $\cup \{\textit{state}(\sigma)\}$
    $\textit{best-g}(\textit{state}(\sigma)) := g(\sigma)$
    if is-goal(state(σ)): **return** extract-solution(σ)
    for each $(a, s') \in$ succ$(\textit{state}(\sigma))$:
      $\sigma' :=$ make-node(σ, a, s')
      if $h(\textit{state}(\sigma')) < \infty$: *open*.insert$(\sigma')$
return unsolvable

Weighted A^*: Remarks

The weight $W \in \mathbb{R}_0^+$ is an **algorithm parameter:**

- For $W = 0$, weighted A^* behaves like uniform-cost search.
- For $W = 1$, weighted A^* behaves like A^*.
- For $W \to \infty$, weighted A^* behaves like greedy best-first search.

Properties:

- For $W > 1$, weighted A^* is bounded suboptimal: if h is admissible, then the solutions returned are at most a factor W more costly than the optimal ones.

Hill-Climbing

Hill-Climbing

$\sigma :=$ make-root-node(init())
forever:
  if is-goal(state(σ)):
    return extract-solution(σ)
  $\Sigma' :=$ { make-node(σ, a, s') | $(a, s') \in$ succ(state(σ)) }
  $\sigma :=$ an element of Σ' minimizing h /* (random tie breaking) */

Remarks:

- Makes sense only if $h(s) > 0$ for $s \notin S^G$.
- Is this complete or optimal? No.
- Can easily get stuck in local minima where immediate improvements of $h(\sigma)$ are not possible.
- Many variations: tie-breaking strategies, restarts, . . .

Enforced Hill-Climbing

Enforced Hill-Climbing: Procedure *improve*

def *improve*(σ_0):
  queue := **new** fifo queue
  queue.push-back(σ_0)
  closed $:= \emptyset$
  while not *queue*.empty():
    $\sigma =$ *queue*.pop-front()
    if *state*$(\sigma) \notin$ *closed*:
      closed := *closed* $\cup$ {*state*(σ)}
      if $h($*state*$(\sigma)) < h($*state*$(\sigma_0))$: **return** σ
      for each $(a, s') \in$ succ(*state*(σ)):
        $\sigma' :=$ make-node(σ, a, s')
        queue.push-back(σ')
  fail

$\rightsquigarrow$ Breadth-first search for state with strictly smaller h-value.

Enforced Hill-Climbing, ctd.

Enforced Hill-Climbing

$\sigma :=$ make-root-node(init())
while not is-goal(state(σ)):
  $\sigma :=$ improve(σ)
return extract-solution(σ)

Remarks:

- Makes sense only if $h(s) > 0$ for $s \notin S^G$.
- Is this optimal? No.
- Is this complete? In general, no. Under particular circumstances, yes. Assume that h is goal-aware.
  $\rightarrow$ Procedure *improve* fails: no state with strictly smaller h-value reachable from s, thus (with assumption) goal not reachable from s.
  $\rightarrow$ This can, for example, not happen if the state space is undirected, i.e., if for all transitions $s \rightarrow s'$ in Θ_Π there is a transition $s' \rightarrow s$.

Properties of Search Algorithms

	DFS	BrFS	ID	A*	HC	IDA*
Complete	No	Yes	Yes	Yes	No	Yes
Optimal	No	Yes*	Yes	Yes	No	Yes
Time	∞	b^d	b^d	b^d	∞	b^d
Space	$b \cdot d$	b^d	$b \cdot d$	b^d	b	$b \cdot d$

- Parameters: d is solution depth; b is branching factor
- Breadth First Search (BrFS) optimal when costs are uniform
- A*/IDA* optimal when h is **admissible**; $h \leq h^*$

Quiz@Speakup

Question!

If we set $h(n) := 0$ for all n, what does A^* become?

(A): Breadth-first search. (B): Depth-first search.
(C): Uniform-cost search. (D): Depth-limited search.

$\rightarrow$ (C): Same expansion order. (Details in book-keeping of open/closed states may differ.)

Question!

If we set $h(n) := 0$ for all n, what can greedy best-first search become?

(A): Breadth-first search. (B): Depth-first search.
(C): Uniform-cost search. (D): A), B) and C)

$\rightarrow$ h implies no ordering of nodes at all, so this fully depends on how we break ties in the open list. (A): FIFO, (B): LIFO, (C): Order on g. (Details in book-keeping of open/closed states may differ.)

Quiz@Speakup, ctd.

Question!

Is informed search always better than blind search?

(A): Yes. (B): No.

$\rightarrow$ In greedy best-first search, the heuristic may yield larger search spaces than uniform-cost search. E.g., in path planning, say you want to go from Melbourne to Sydney, but $h(\text{Perth}) < h(\text{Canberra})$.

$\rightarrow$ In A^* with an admissible heuristic and duplicate checking, we cannot do worse than uniform-cost search: $h(s) > 0$ can only reduce the number of states we must consider to prove optimality.

$\rightarrow$ Also, in the above example, A^* doesn't expand Perth with *any* admissible heuristic, because $g(\text{Perth}) > g(\text{Sydney})$!

$\rightarrow$ "Trusting the heuristic" has its dangers! Sometimes g helps to reduce search.

Summary

Distinguish: World states, search states, search nodes.

- World state: Situation in the world modelled by the planning task.
- Search state: Subproblem remaining to be solved.
  - In progression, world states and search states are identical.
  - In regression, search states are sub-goals describing sets of world states.
- Search node: Search state + info on "how we got there".

Search algorithms mainly differ in order of node expansion:

- Blind vs. heuristic (or informed) search.
- Systematic vs. local search.

Summary (ctd.)

- Search strategies differ (amongst others) in the order in which they expand search nodes, and in the way they use duplicate elimination. Criteria for evaluating them are completeness, optimality, time complexity, and space complexity.
- Breadth-first search is optimal but uses exponential space; depth-first search uses linear space but is not optimal. Iterative deepening search combines the virtues of both.

Summary (ctd.)

Heuristic Functions: Estimators for remaining cost.

- Usually: The more informed, the better performance.
- Desiderata: Safe, goal-aware, admissible, consistent.
- The ideal: Perfect heuristic h^*.

Heuristic Search Algorithms:

- Most common algorithms for satisficing planning:
  - Greedy best-first search.
  - Weighted A^*.
  - Enforced hill-climbing.
- Most common algorithm for optimal planning:
  - A^*.

Reading

- *Artificial Intelligence: A Modern Approach (Third Edition)* , Chapter 3 “Solving Problems by Searching” and the first half of Chapter 4 “Beyond Classical Search”.

  Content: An overview of various search algorithms, including blind searches as well as greedy best-first search and A^*.
- Search Tutorial in the context of path-finding `http://www.redblobgames.com/pathfinding/a-star/introduction.html`