Stochastic Optimization by Computer Simulation

Graduate coursework, Fall 2019

Instructor: Professor Felisa Vázquez-Abad.
Office Hours: by appointment.

Course Description

Optimization is the "science for finding the best". Today, there is a sound body of models and methods to find the best decision or choices. These methods are widely used in airlines, hospitals, banks, computer engineering, manufacturing and scheduling, among other sectors. We will specialize our study to *stochastic* optimization, which incorporates randomness (uncertainty). Among the problems that we will discuss are the following. Statistical learning using data, adversarial games to reach economic equilibria, optimal path allocation for transportation and telecommunications, optimal inventory ordering policies, optimal production strategies and target tracking (used in supervised learning).

Algorithms can be compared in terms of speed of execution and accuracy in the approximation. Central to our course is the analysis of convergence (in a probabilistic sense) of these computer methods. We will illustrate difficult concepts with actual problems for students to code and see the methods at work.

We will study *continuous* optimization, dealing with continuous decision variables (how much to invest, how much hydroelectric energy to produce, etc), for which we will provide a summary of known convergence results. In a final stage, we will discuss some methods for *discrete* optimization (how many buses should a transportation company buy, DNA sequencing, etc) and encourage reading current research papers.

The subject of stochastic optimization integrates sophisticated knowledge in probability theory, functional analysis, dynamical systems and computer simulation. Our pedagogical formula focuses on individual needs and goals, and we will emphasize understanding through hands-on experience with examples and computer exercises. We will provide a full set of lecture notes with the mathematical detail.

Syllabus

- W1: Overview of deterministic constrained optimization
- W2: The iterative method as an Ordinary Differential Equation
- W3: Review of Probability Theory
- W4: Stochastic approximation methodology, exogenous noise model
- W4 (Tutorial): Review of Markov Chains
- W5–W6: Stochastic approximation methodology, endogenous noise model
- W7: Asymptotic efficiency of numerical algorithms
- W8-W9: Statistical estimation of gradients, sensitivity analysis
- W10: Overview of discrete optimisation
- W11: Student Presentations of research projects
- W12–14: Student project development, report writing and final exams

Learning Outcomes

Besides acquiring fundamental knowledge in the area of stochastic processes and optimization, students who succeed the course will be capable of

- undertake important consulting projects in the public and private sector,
- have an understanding of the different optimization techniques, and be able to identify appropriate models for practical problems,
- have an understanding of the various gradient estimation methods available, and
- be able to report the results using statistical confidence intervals.

Prerequisites

A good background in probability and statistics, and basic knowledge of matrix algebra and calculus. Basic knowledge of dynamical systems and mathematical analysis are desirable, as well as some experience with computer simulation. This course will be suitable for students specializing in Computer Science, Mathematics and Statistics, Electrical and Computer Engineering, Chemistry, Physics, Economics and Bioinformatics.

Lectures

The course consists of an equivalent of 12 two-hour lectures with assigned reading. There will be student presentations of research projects. A full set of Lecture Notes will be provided with exercise suggestions.

Assessment

Assignment 1 (week 4): 10%. Assignment 2 (week 7): 10%. Assignment 3 (week 10): 10%. Group project presentations (week 11): 10%. Group project report (week 12): 30%. Exam (in the exam period) : 30%.

Threshold: in order to pass the subject, a minimum of 55 in the exam is required.

Historical Remarks

This course has evolved over the years to its present form. Parts of the material were developed earlier on from 1996 to 2004, as part of a course in Stochastic Modelling and Simulation that I taught at the Computer Science Department at the University of Montreal, with students in various joint programs in Computer Science, Management, Industrial Engineering, Finance, and Mathematics. I taught a preliminary version of this subject to graduate and postdoctoral students in Electrical Engineering at the University of Melbourne in 1999, and then as an Honours course for students in Mathematics and Statistics from 2004 to 2008 and as a Doctoral course for Engineering students in the University of Vienna in 2009. The lecture notes are co-authored with Prof Bernd Heidergott (of the Vrije Universiteit) as a full book, incorporating our recent research on the subject of gradient estimation. Since then we have delivered the course in parallel at the City University New York (CUNY) and the Vrije Universiteit (Amsterdam) as a graduate course. Students have expressed in their evaluations that they have enjoyed and appreciated the learning process just as much as the acquired knowledge, particularly through the research projects. The topic of stochastic modeling and simulation is my main research area. I have worked on it for many years and some of my publications are the result of refinement of student projects, which they have co-authored. I love the subject and I try to communicate my enthusiasm to younger generations.