MAST20005/MAST90058: Week 7 Problems

Some useful information for many of the problems is shown at end of this problem sheet.

- 1. A ball is drawn from one of two bowls. Bowl A contains 100 red balls and 200 white balls; Bowl B contains 200 red balls and 100 white balls. Let p denote the probability of drawing a red ball from the chosen bowl. Then p is unknown as we don't know which bowl is being used. We shall test the simple null hypothesis, $H_0: p = 1/3$, against the simple alternative, $H_1: p = 2/3$. We draw three balls at random with replacement from the selected bowl. Let X be the number of red balls drawn. Let the critical region be $x \in \{2,3\}$. What are the probabilities α and β respectively of Type I and Type II errors?
- 2. Let $Y \sim \text{Bi}(100, p)$. To test $H_0: p = 0.08$ against $H_1: p < 0.08$, we reject $H_0: f < 0.08$.
 - (a) Determine the significance level α of the test.
 - (b) Find the probability of a Type II error if, in fact, p = 0.04.
- 3. If a newborn baby has a birth weight that is less than 2500 grams we say the baby has a low birth weight. The proportion of babies with low birth weight is an indicator of nutrition for the mothers. In the USA, approximately 7% of babies have a low birth weight. Let p be the proportion of babies born in the Sudan with low birth weight. Test the null hypothesis H_0 : p = 0.07 against the alternative H_1 : p > 0.07. If p = 23 babies out of a random sample of p = 209 babies had low birth weight, what is your conclusion at the following significance levels:
 - (a) $\alpha = 0.05$?
 - (b) $\alpha = 0.01$?
 - (c) Find the p-value of this test. (Recall the p-value is the probability of the observed value or something more extreme when the null hypothesis is true).
- 4. Let p_m and p_f be the respective proportions of male and female white crowned sparrows that return to their hatching site. Give the endpoints for a 95% confidence interval for $p_m p_f$, given that 124 out of 894 males and 70 out of 700 females returned. Does this agree with the conclusion of the test of H_0 : $p_m = p_f$ against H_1 : $p_m \neq p_f$ with $\alpha = 0.05$?
- 5. Vitamin B_6 is one of the vitamins in a multivitamin pill manufactured by a pharmaceutical company. The pills are produced with a mean of 50 milligrams of vitamin B_6 per pill. The company believes there is a deterioration of 1 milligram per month, so that after 3 months they expect that $\mu = 47$. A consumer group suspects that $\mu < 47$ after 3 months.
 - (a) Define a critical region to test H_0 : $\mu = 47$ against H_1 : $\mu < 47$ with a significance level of $\alpha = 0.05$ based on a random sample of size n = 20.
 - (b) If the 20 pills resulted in a mean of $\bar{x} = 46.94$ and a standard deviation of s = 0.15, what is your conclusion?
 - (c) Give limits for the p-value of this test.
- 6. Let X represent the height of professional female volleyball players. Assume that $X \sim N(\mu, \sigma^2)$ approximately. Suppose it is known that $\mu = 1.9$ metres worldwide. Do Australian female players differ from this? We explore this using a random sample of size n = 9.
 - (a) Define the null hypothesis.

- (b) Define the alternative hypothesis.
- (c) Define a critical region for which $\alpha = 0.05$.
- (d) Calculate the value of the test statistic if $\bar{x} = 2.05$ and s = 0.17.
- (e) What is your conclusion?
- (f) Give limits for the p-value of this test.
- 7. In May, the weights of 2-kilogram boxes of laundry detergent had a mean of 2.13 kilograms with a standard deviation of 0.095. The goal was to decrease the standard deviation. The company decided to adjust the filling machines and then test H_0 : $\sigma = 0.095$ against H_1 : $\sigma < 0.095$. In June, a random sample of size n = 20 gave $\bar{x} = 2.10$ and s = 0.065.
 - (a) At an $\alpha = 0.05$ significance level, was the company successful?
 - (b) What is an approximate p-value for this test?
- 8. The World Health Organisation collects air quality data around the world, which includes a measurement of suspended particles in μ g/m³. Let X and Y equal the concentration of suspended particles in the city centres of Melbourne and Sydney, respectively. Using n = 13 observations of X and m = 16 observations of Y, we shall test $H_0: \mu_X = \mu_Y$ against $H_1: \mu_X \neq \mu_Y$ using a significance level of 5%.
 - (a) Define the test statistic and the critical region assuming the variances are equal.
 - (b) If $\bar{x} = 72.9$, $s_X = 25.6$, $\bar{y} = 81.7$ and $s_Y = 28.3$, calculate the value of the test statistic and state your conclusion.
 - (c) Give limits for the p-value of this test.
 - (d) Test whether the assumption of equal variances is valid. Let $\alpha = 0.05$.

Some potentially helpful R output:

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> dbinom(0:3, 3, 1/3)
[1] 0.29629630 0.44444444 0.22222222 0.03703704
> dbinom(0:3, 3, 2/3)
[1] 0.03703704 0.22222222 0.44444444 0.29629630
> pnorm(c(-0.737, -0.553, 1.276, 1.531, 2.269))
[1] 0.2305612 0.2901317 0.8990222 0.9371153 0.9883658
> p1 \leftarrow c(0.75, 0.9, 0.95, 0.975, 0.99)
> qnorm(p1)
[1] 0.6744898 1.2815516 1.6448536 1.9599640 2.3263479
> qt(p1, 27)
[1] 0.683685 1.313703 1.703288 2.051831 2.472660
> qt(p1, 20)
[1] 0.6869545 1.3253407 1.7247182 2.0859634 2.5279770
> qt(p1, 19)
[1] 0.6876215 1.3277282 1.7291328 2.0930241 2.5394832
> qt(p1, 8)
[1] 0.7063866 1.3968153 1.8595480 2.3060041 2.8964594
> p2 <- c(0.025, 0.05, 0.95, 0.975)
> qchisq(p2, 19)
[1] 8.906516 10.117013 30.143527 32.852327
> qf(p2, 12, 15)
[1] 0.3147424 0.3821387 2.4753130 2.9632824
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