Introduction to Computer Graphics

Assignment 1

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Derivation of Cylinder - Ray intersection and normal

We consider the plane defined by a point M in the ray, the center C of the cylinder and the unity vector defining its axis $\boldsymbol{a}$. In this plane we have the adjacent schematic with M belonging to the cylinder if and only if $PM = R$.

We derive that the squared length perpendicular to the axis PC^2 is defined as

$$PC^2 = (\overrightarrow{CM} \cdot \boldsymbol{a})^2$$

We define the ray equation as $M(t) = t \cdot \boldsymbol{dir} + O, \boldsymbol{oc} = \overrightarrow{OC}$ and therefore $\overrightarrow{CM} = t \cdot \boldsymbol{dir} + \boldsymbol{oc}$.

Because MPC is a right triangle we can derive

$$PC^2 + MP^2 = MC^2$$

The implicit intersection equation is then

$$\begin{aligned} M \in Cylinder &\iff MP^2 = R^2 \\ &\iff MC^2 - PC^2 - R^2 = 0 \\ &\iff \|t \cdot \boldsymbol{dir} + \boldsymbol{oc}\|^2 - ((t \cdot \boldsymbol{dir} + \boldsymbol{oc}) \cdot \boldsymbol{a})^2 - R^2 = 0 \end{aligned}$$

Which leads us to this final 2^{nd} degree polynomial equation

$$\begin{aligned} &(\boldsymbol{dir} \cdot \boldsymbol{dir} - (\boldsymbol{dir} \cdot \boldsymbol{a})^2) \cdot t^2 \\ &+2(\boldsymbol{dir} \cdot \boldsymbol{oc} - (\boldsymbol{dir} \cdot \boldsymbol{a}) \cdot (\boldsymbol{oc} \cdot \boldsymbol{a})) \cdot t \\ &+\boldsymbol{oc} \cdot \boldsymbol{oc} - (\boldsymbol{oc} \cdot \boldsymbol{a})^2 - R^2 = 0 \end{aligned}$$

After having found t as a solution to the previous equation, we can find the intersection point $M(t)$ by using the dedicated ray method. We then check if solution is in the finite cylinder with the condition $CP \leq H/2$.

$$CP \leq H/2 \iff \left|\overrightarrow{CM} \cdot \boldsymbol{a}\right| \leq H/2 \iff |(M(t) - C) \cdot \boldsymbol{a}| \leq H/2$$

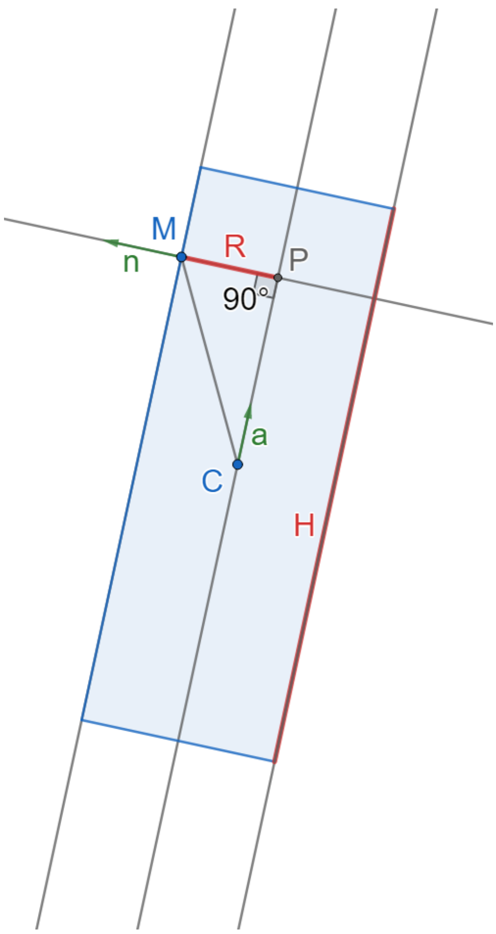

Figure 1: Cylinder schematic

At the end the intersection normal $\boldsymbol{n}$ can be calculated as in figure 1 as a normalization of $\overrightarrow{PM}$

$$\boldsymbol{n} = \overrightarrow{PM}/R = (\overrightarrow{CM} - \overrightarrow{CP})/R = (\overrightarrow{CM} - (\overrightarrow{CM} \cdot \boldsymbol{a}) \cdot \boldsymbol{a})/R) = (M(t) - C - ((M(t) - C) \cdot \boldsymbol{a}) \cdot \boldsymbol{a})/R$$