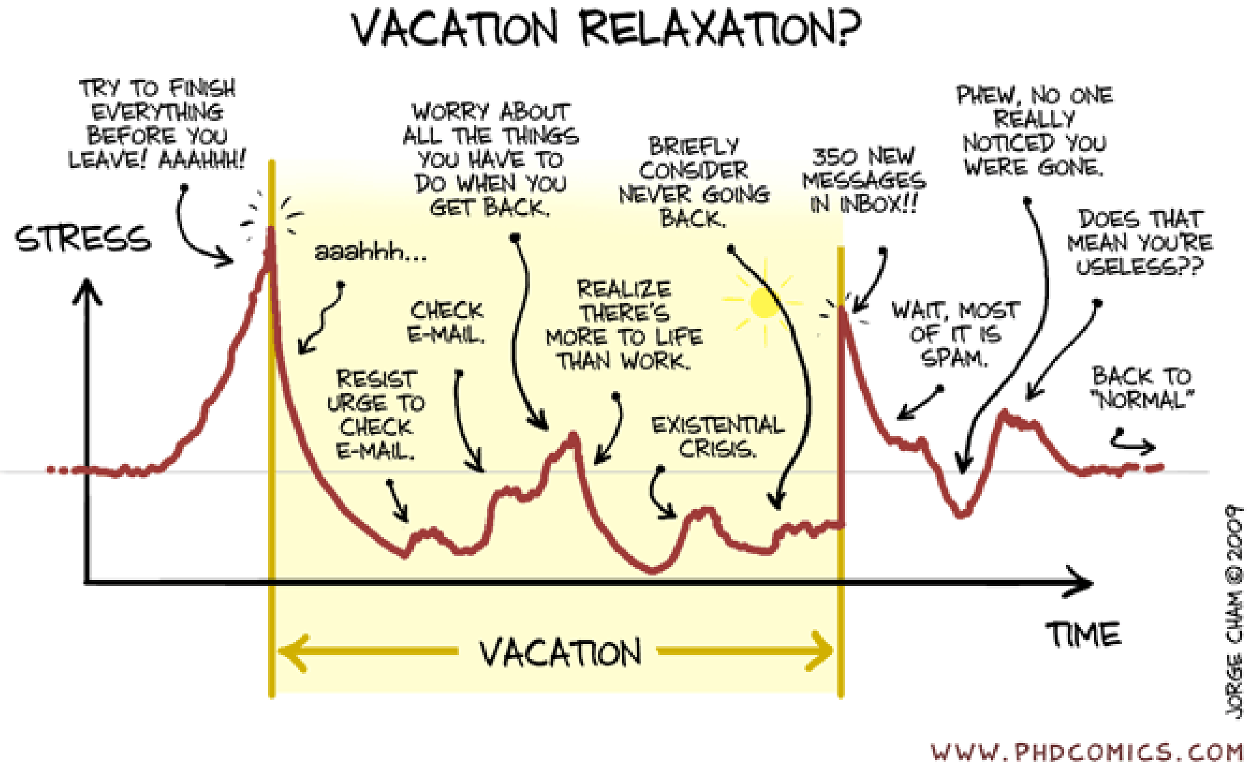
ECE 10C
Fall 2020
SlideSet 10



Tuesday
- Bode Plot

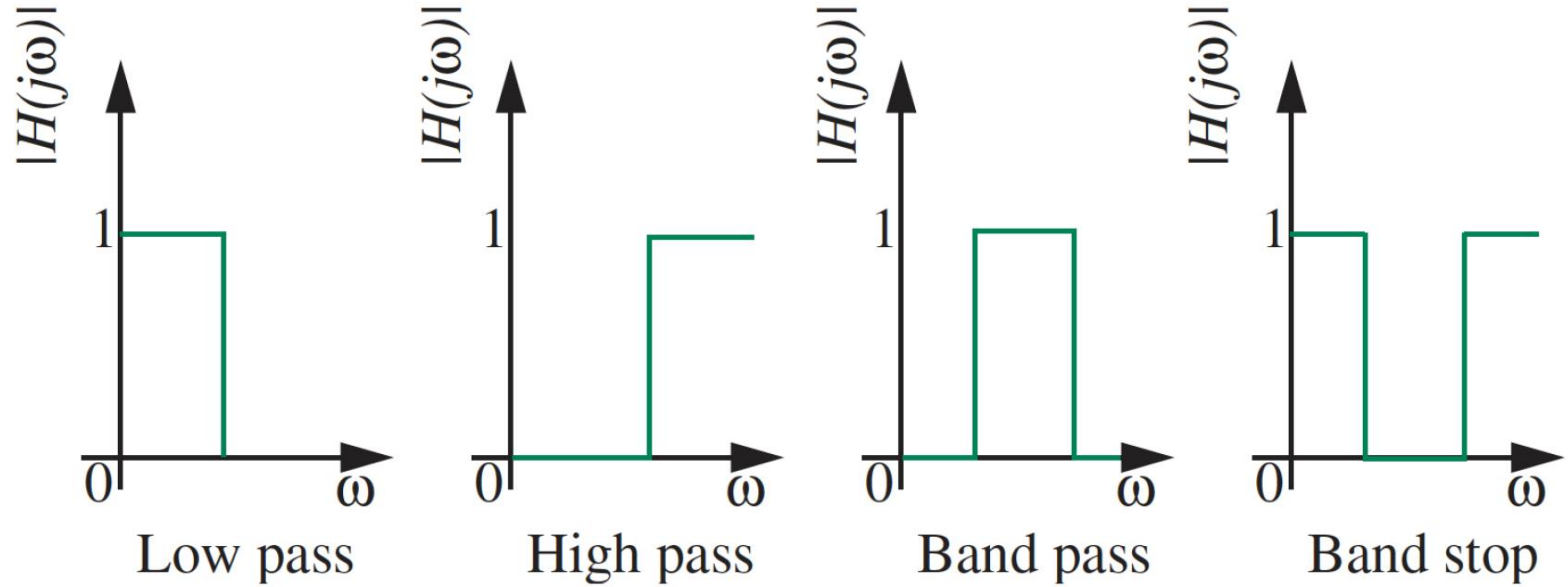
Today
- 1^{st} Order Filters
- Square Waves
- Fourier Transform

<u>Important Items:</u>

- Review next Tuesday
- No class next Thursday
- HW #5 due Thurs, 12/3
- Lab #5 due Friday, 12/4

Filters: Frequency Selective Behavior

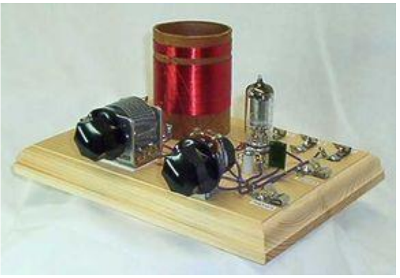
- We can use circuits to process signals according to their frequency
- You've been experimenting with 1st order circuits as filters in the labs
- Some generic "ideal" filters:



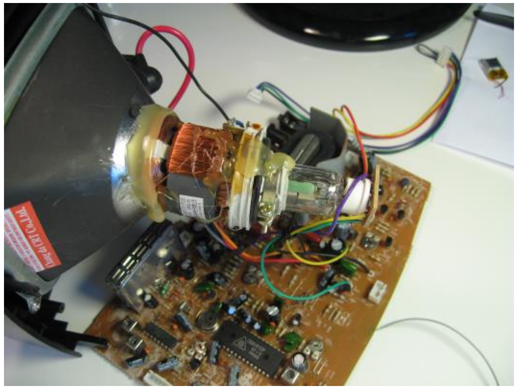
- Actual filters can only ***approximate*** these idea filter behaviors

Filters: Frequency Selective Behavior

- Radios



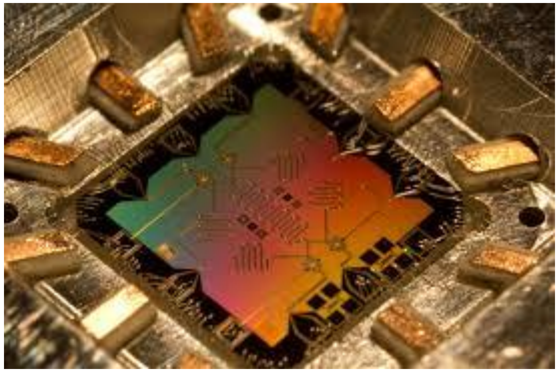
- TVs



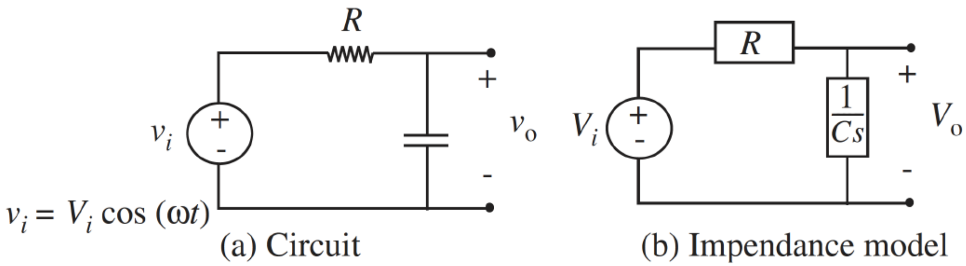
- Audio Amplifiers

- Communication Systems

- Quantum Computing



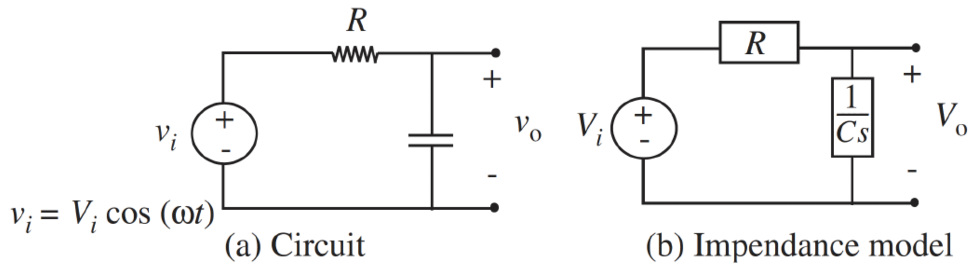
Filters: The Oldest Analog Filter — the RC Circuit



$$H(s) = \frac{1/sC}{R + 1/sC} = \frac{1}{sRC + 1}$$

<-- Pole with ω_c = 1/RC

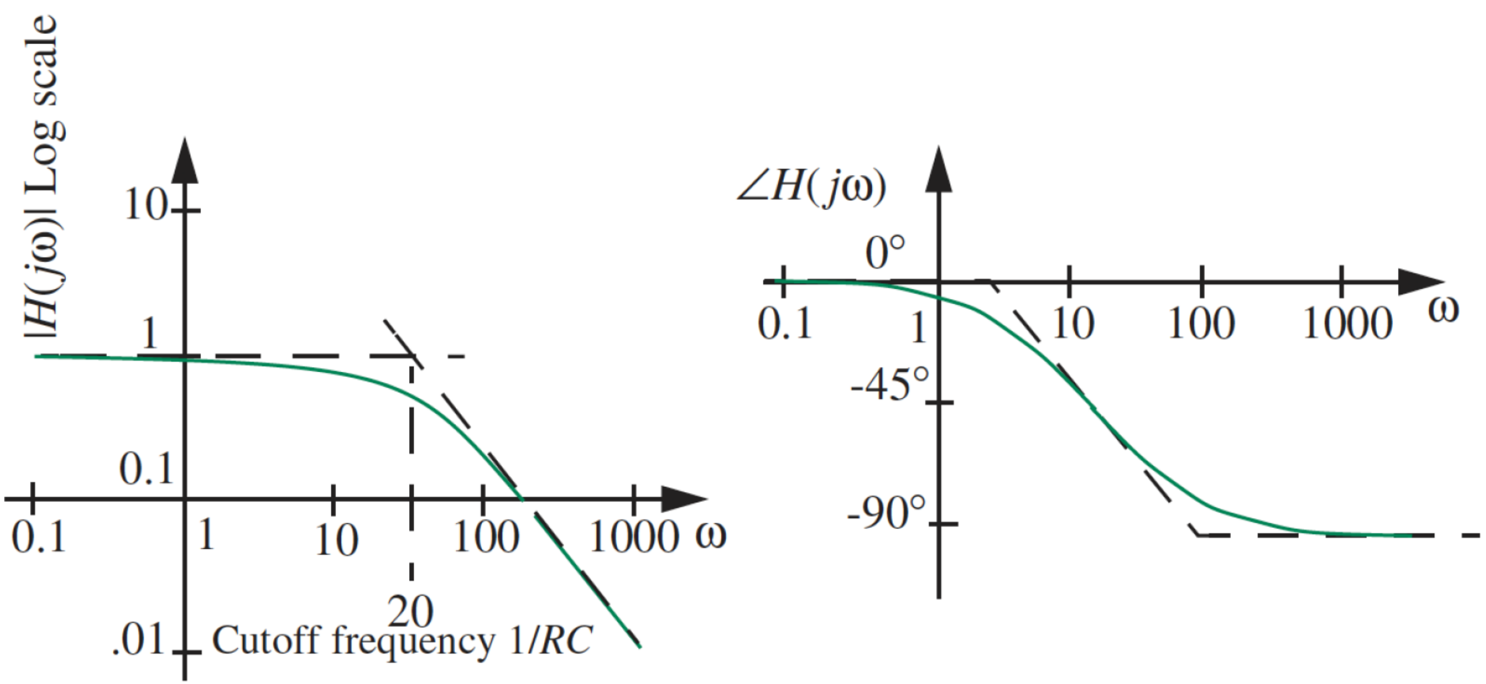
Filters: The Oldest Analog Filter — the RC Circuit



$$H(s) = \frac{1/sC}{R + 1/sC} = \frac{1}{sRC + 1}$$

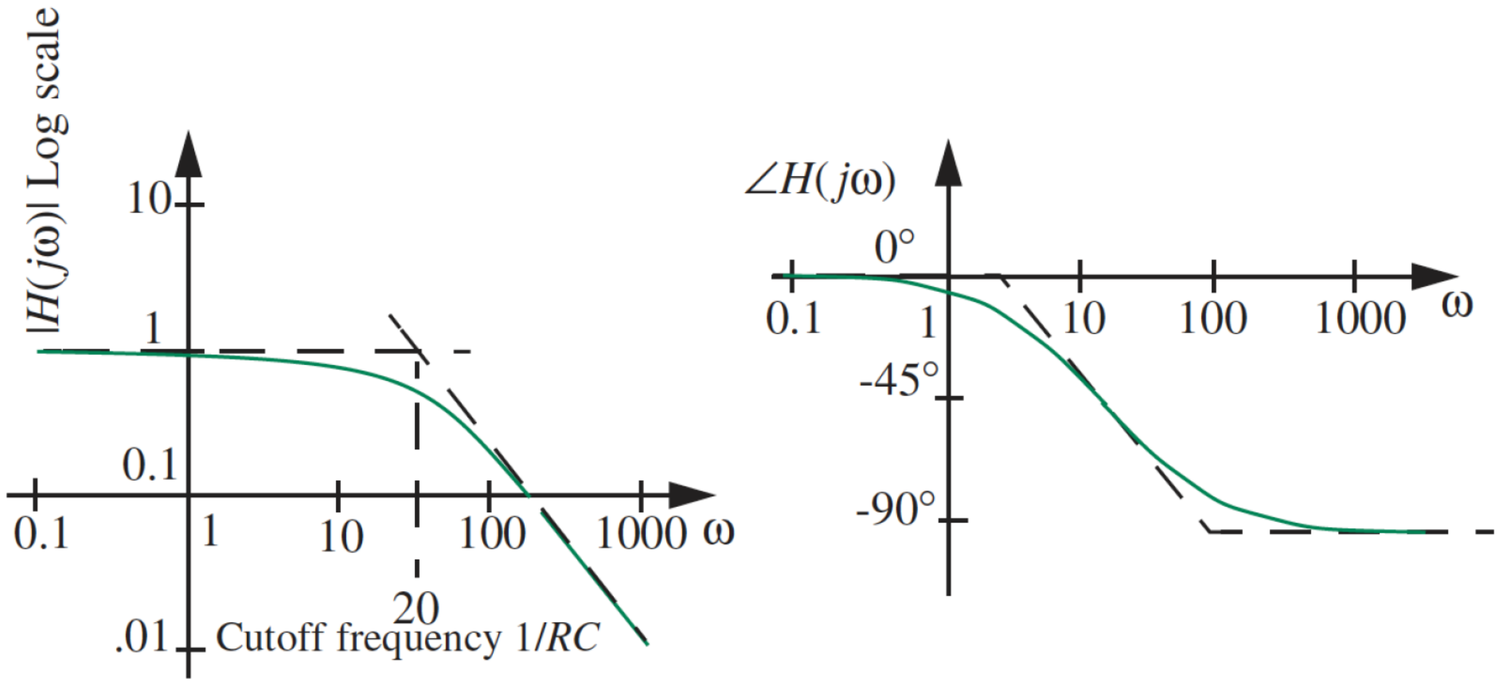
<-- Pole with ω_c = 1/RC

Let's look at this when $RC = 1/20$:



Filters: The Oldest Analog Filter — the RC Circuit

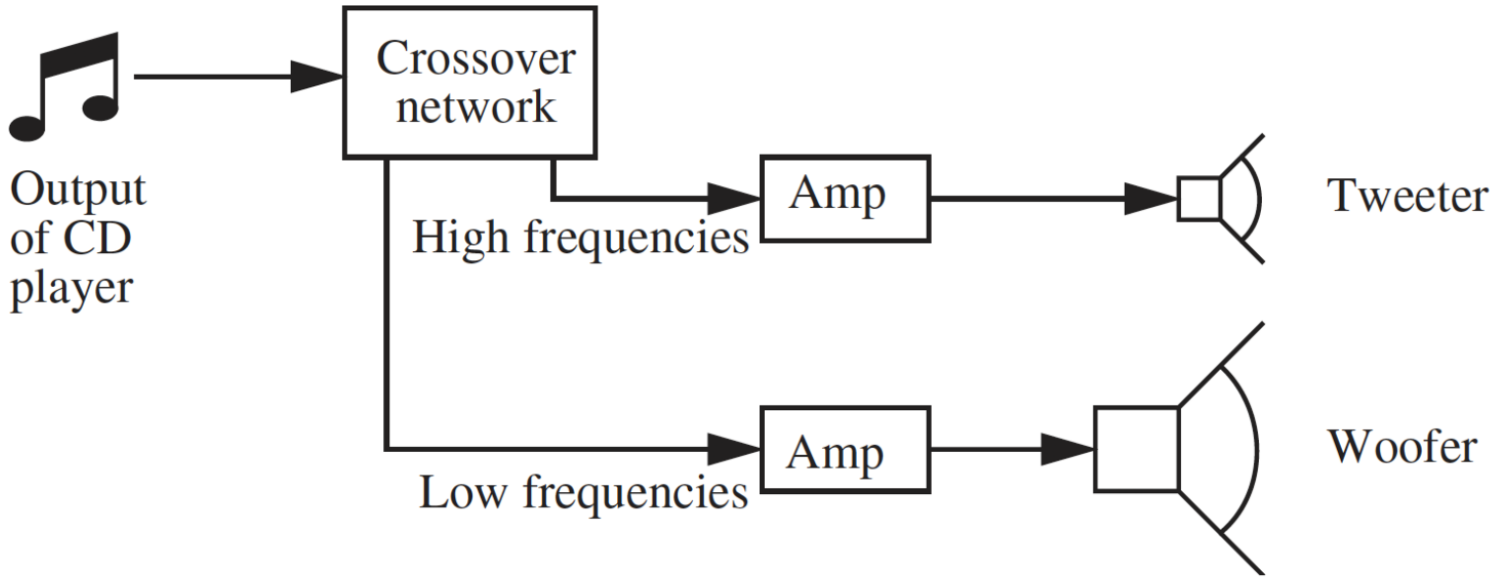
The shape of the magnitude plot is indicative of a low-pass filter:



- RC circuit, output taken as voltage across C = low-pass filter
- **Homework #5: Other variants of RC and RL**

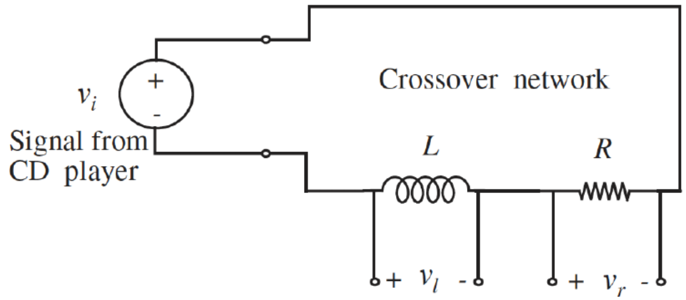
Example: Cross-Over Amplifier System

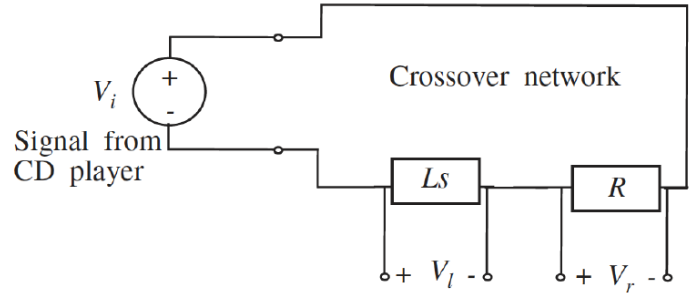
Let's say we want a stereo amplifier system that splits high and low frequencies and amplifies them:



A cross-over network acts as a filter to send the right signals to your speakers and tweeters

Example: Cross-Over Amplifier System





The transfer function to the inductor voltage is

$$H_L(s) = \frac{V_L(s)}{V_{\text{i}}(s)} = \frac{Ls}{R + Ls}$$

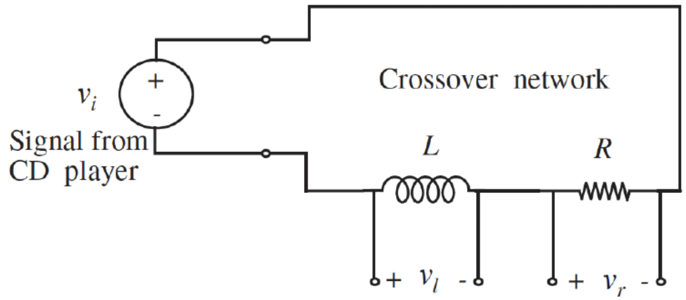
Similarly, the transfer function, to the resistor voltage is

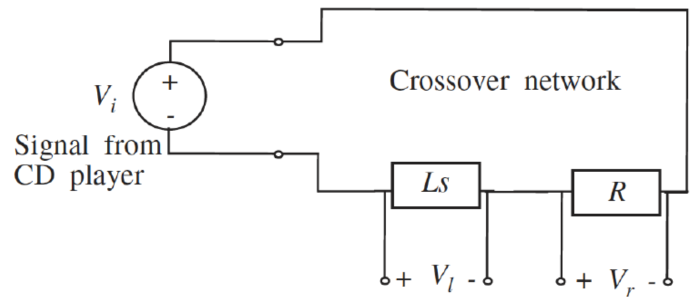
$$H_R(s) = \frac{V_R(s)}{V_{\text{i}}(s)} = \frac{R}{R + Ls}$$

$$|H_L(j\omega)| = \left|\frac{V_L(\omega)}{V_{\text{i}}(\omega)}\right| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}, \quad \angle H_L(j\omega) = \tan^{-1}\left(\frac{R}{\omega L}\right)$$

$$|H_R(j\omega)| = \left|\frac{V_R(\omega)}{V_{\text{i}}(\omega)}\right| = \frac{R}{\sqrt{R^2 + (\omega L)^2}}, \quad \angle H_R(j\omega) = \tan^{-1}\left(-\frac{\omega L}{R}\right)$$

Example: Cross-Over Amplifier System





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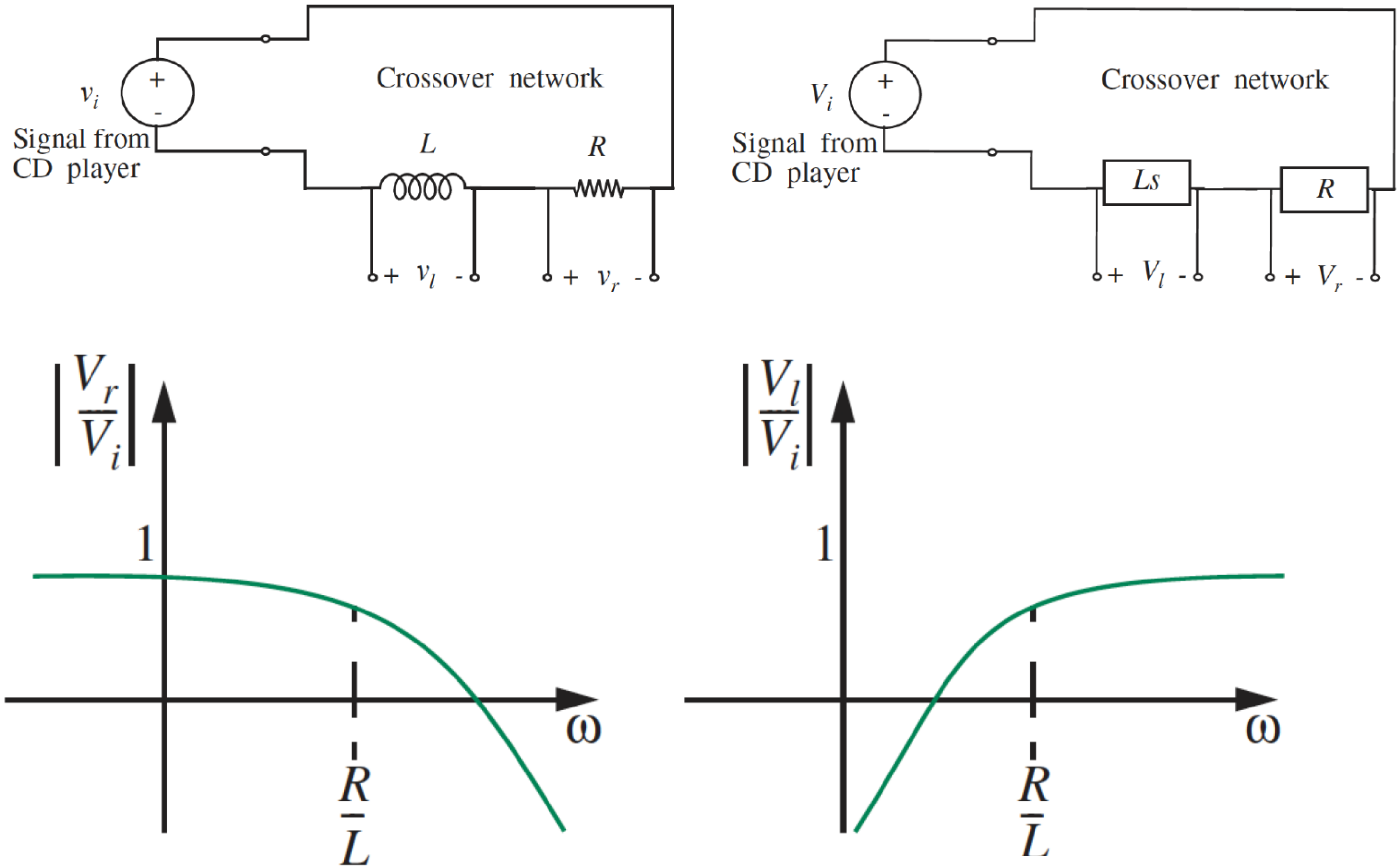
What are the cutoff frequencies?

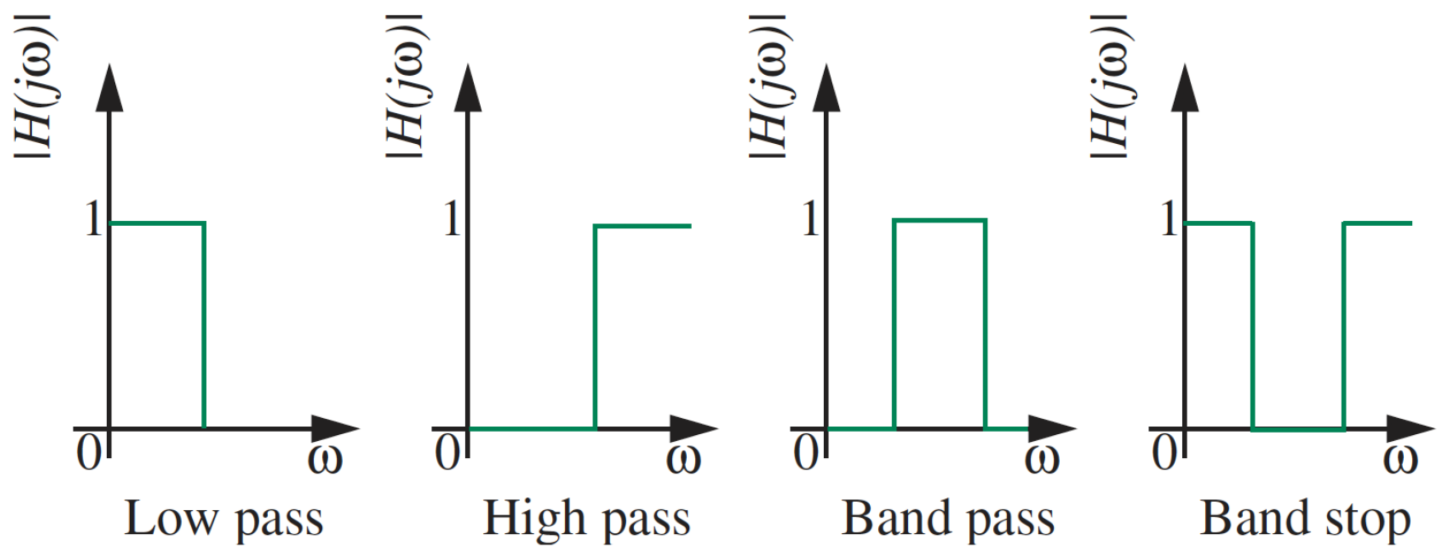
Example: Cross-Over Amplifier System



Series RL/RC: Can make these high pass or low pass depending on where we measure the output voltage

How to Build Filters

Remember our "ideal filters":

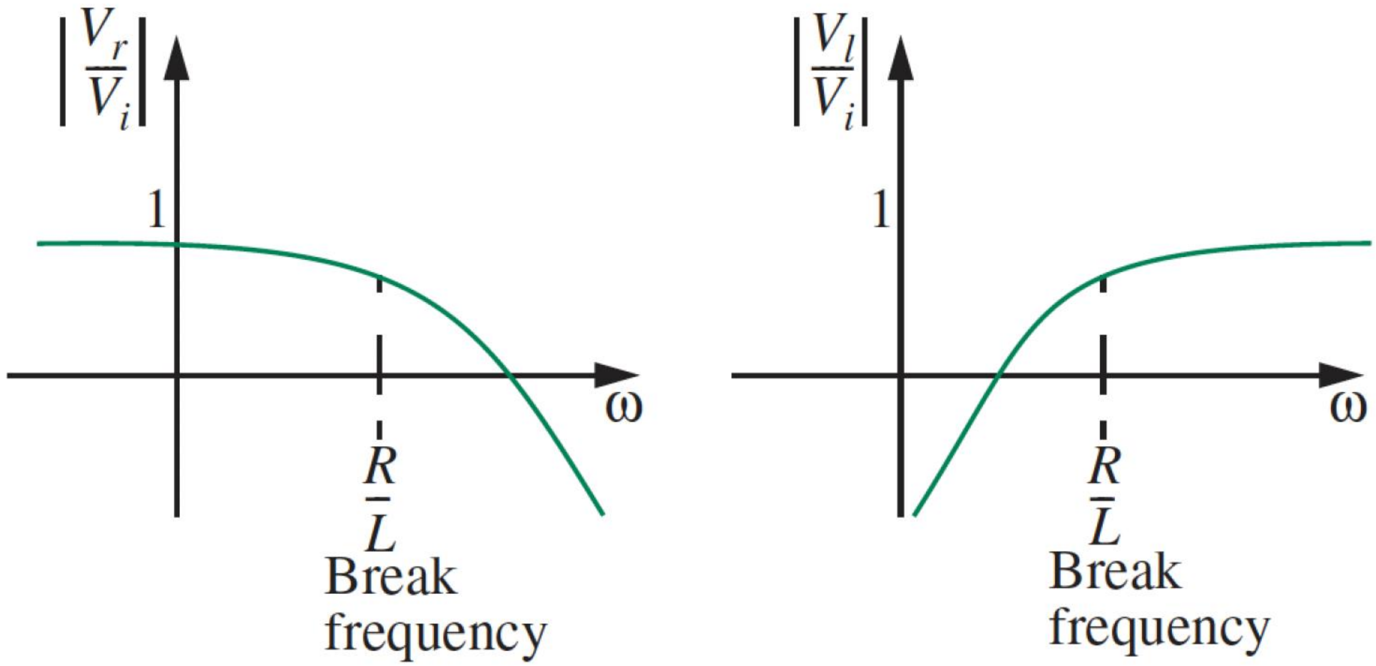


- We've looked at RC and RL circuits in this lecture and in HW #5. These are useful for low-pass and high-pass filters

- Week after Thanksgiving: We'll see how to make band-pass and band-stop filters using RLC circuits

- **First, let's look at an example about why we might care about filtering signal**

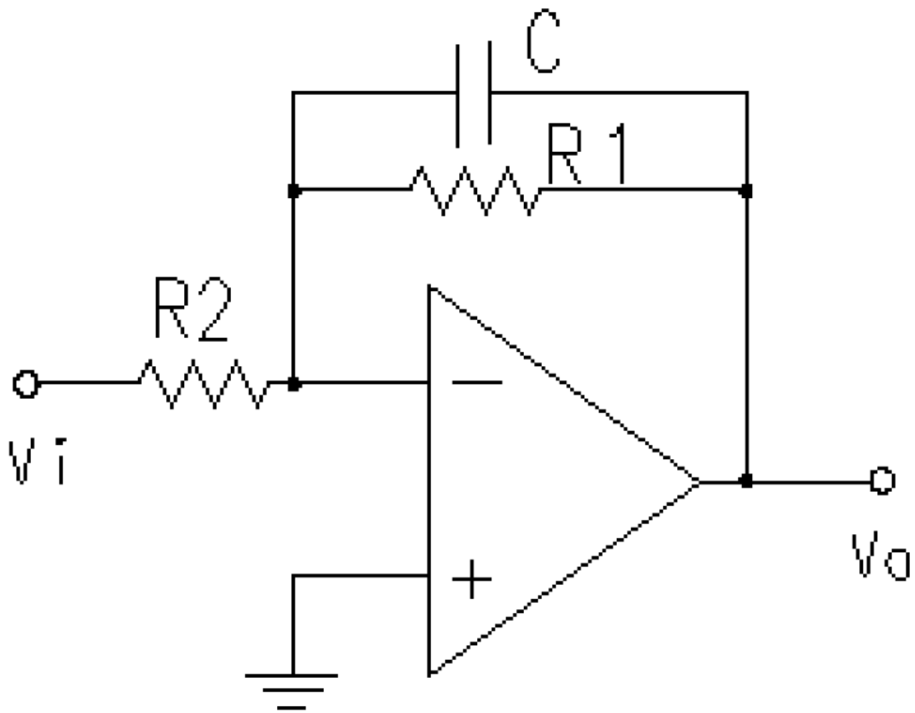
First Question: What if You Need More Gain?



add an op-amp

First Question: What if You Need More Gain?

Remember the inverting amplifier:



$$H(j\omega) = \frac{V_o}{V_i} = -\frac{(R_1 || \frac{1}{Cj\omega})}{R_2} = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_f R_f}$$

Second Question: Why Care about Sinusoidal Inputs?

Second Question: Why Care about Sinusoidal Inputs?

Main reason: Decomposition of signals

Sines and cosines form a complete linear basis for reconstructing any realistic signal

What this means:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{b=1}^{\infty} b_n \sin(n\omega_0 t)$$

We can write any realistic signal using the above expression with the appropriate terms, frequencies, and coefficients

Example: Let's see how sinusoids can be used to reconstruct a square wave

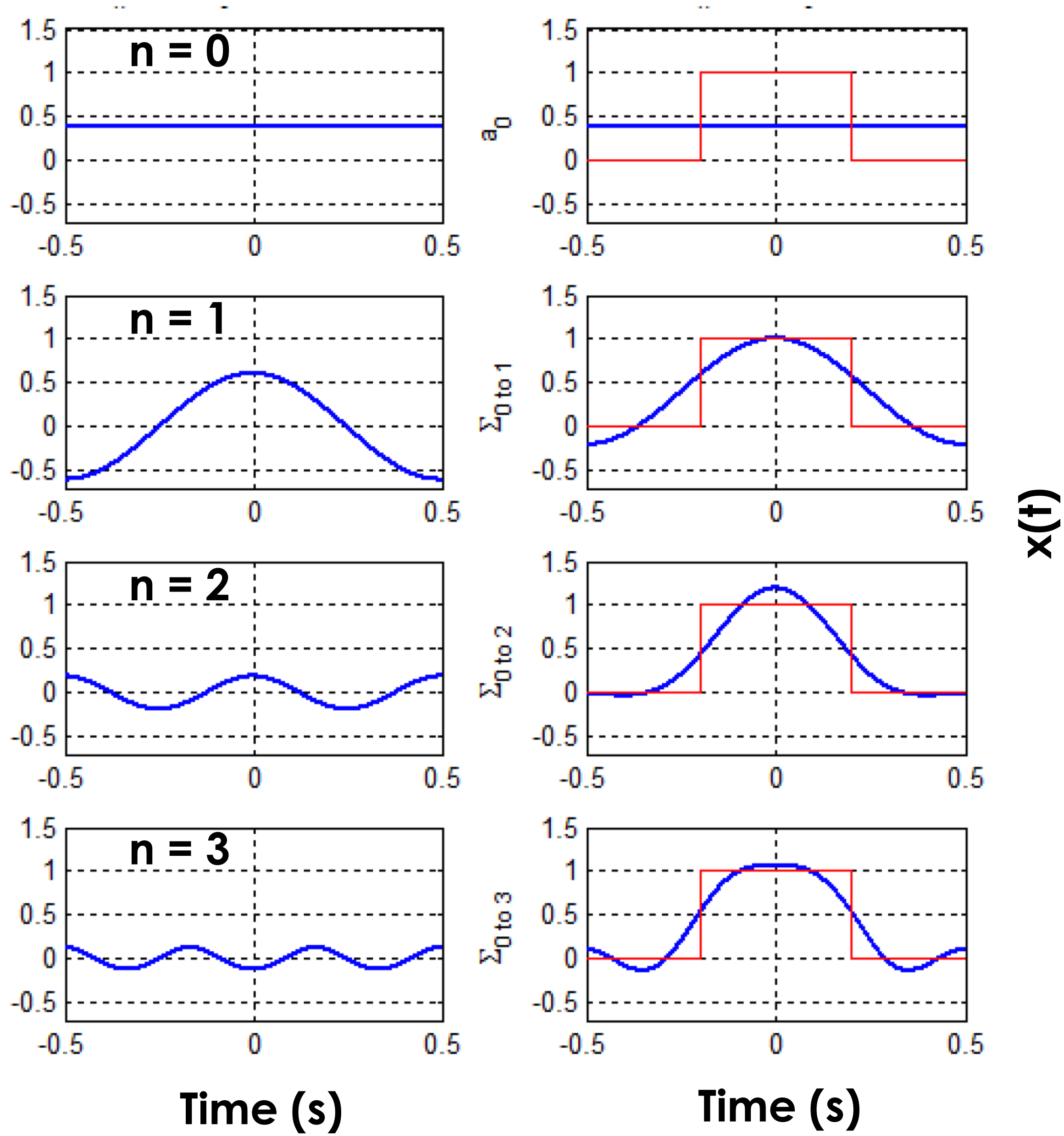
Frequency Components of a Square Wave

$$x(t) = a_0 +$$

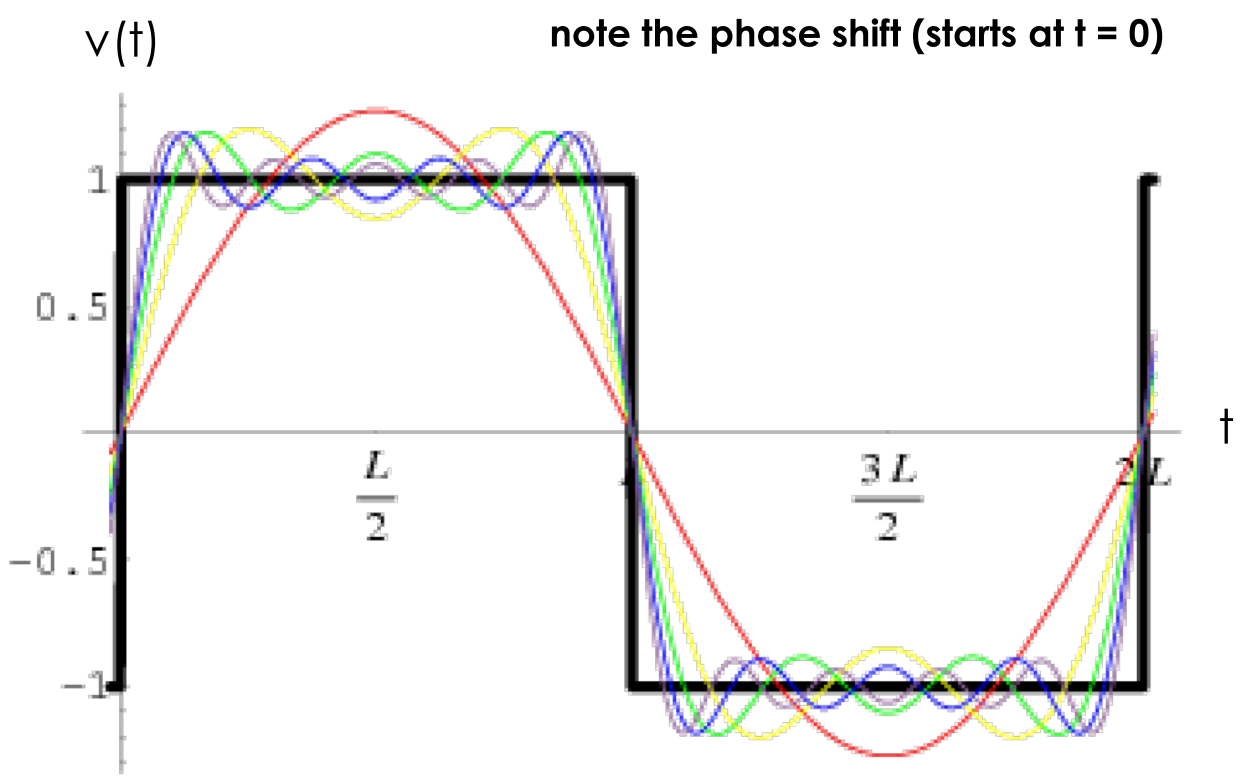
$$\sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) +$$

$$\sum_{b=1}^{\infty} b_n \sin(n\omega_0 t)$$

b_n's = 0
ω = 2π x 1 rad/s



Frequency Components of a Square Wave



$$v_i(t) = \frac{4}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{(n\pi t)}{L}\right) \qquad \text{(harmonics of } \frac{\pi}{L}\text{)}$$

Frequency Components of a Square Wave

$$v_i(t) = \frac{4}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin\left(\frac{(n\pi t)}{L}\right) \qquad \text{(harmonics of } \frac{\pi}{L}\text{)}$$

We know how to determine the output response to sinusoidal signals:

$$v_i(t) = V_i\cos(\omega t) \longrightarrow \boxed{H(j\omega)} \longrightarrow v_o(t) = |H(j\omega)|V_i\cos\left(\omega t + \angle H(j\omega)\right)$$

$$v_o(t) = \frac{4}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\left|H\left(j\frac{(n\pi)}{L}\right)\right|\sin\left(\frac{(n\pi t)}{L} + \angle H\left(j\frac{(n\pi)}{L}\right)\right)$$

Recap

Single sinusoid going through a system/filter:

$$v_i(t) = V_i \cos(\omega t) \longrightarrow \boxed{H(j\omega)} \longrightarrow v_o(t) = |H(j\omega)| V_i \cos\left(\omega t + \angle H(j\omega)\right)$$

A general periodic signal is a collection of sinusoidal components:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{b=1}^{\infty} b_n \sin(n\omega_0 t)$$

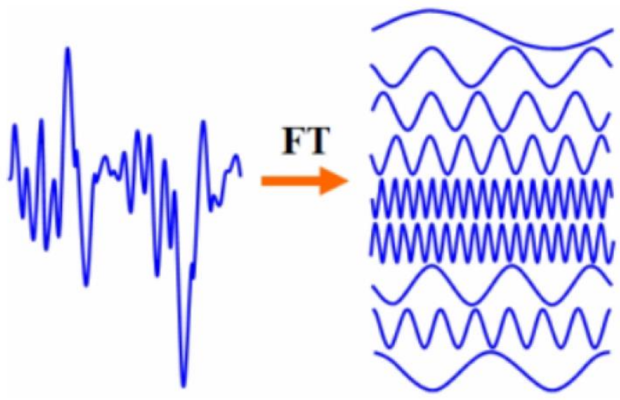
$$\begin{aligned} y(t) = {} & a_0 |H(j0)| \cos(\angle H(j0)) + \sum_{n=1}^{\infty} a_n |H(jn\omega_0)| \cos(n\omega_0 t + \angle H(jn\omega_0)) \\ & + \sum_{b=1}^{\infty} b_n |H(jn\omega_0)| \sin(n\omega_0 t + \angle H(jn\omega_0)) \end{aligned}$$

So far, we've discussed an analog sinusoidal input signal to a circuit, and what the frequency response looks like.

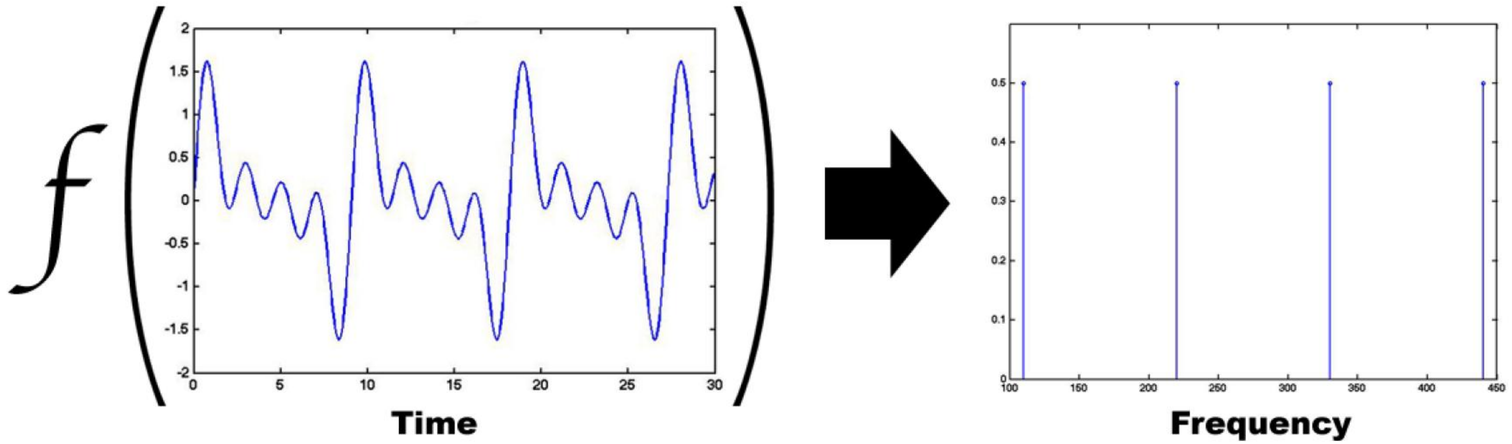
How do you find the frequency components of an electrical signal that you've recorded digitally?

Fast Fourier Transform

A continunous (analog) signal can be decomposed into a summation of continuous sinusoidal functions via the Fourier transform/ Fourier series

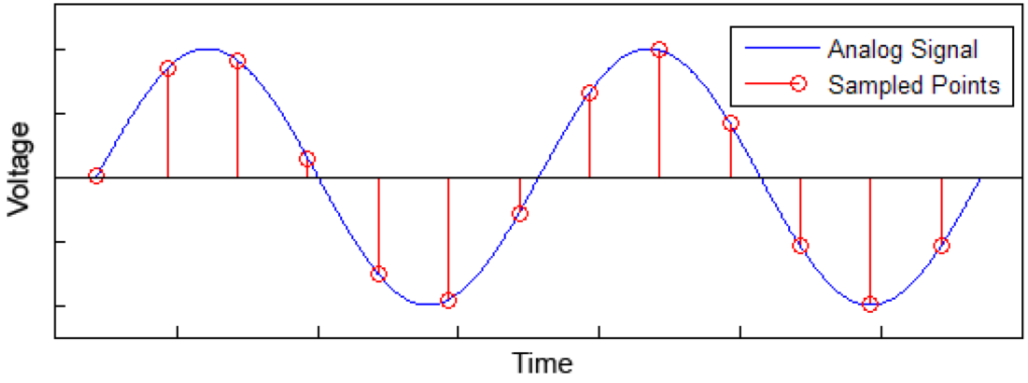


A good way of plotting the sinusoidal harmonics that make up the signal is using the "frequency domain" representation of the signal



Fast Fourier Transform

In a computer, we only store dicrete signals that are samples of the analog signal

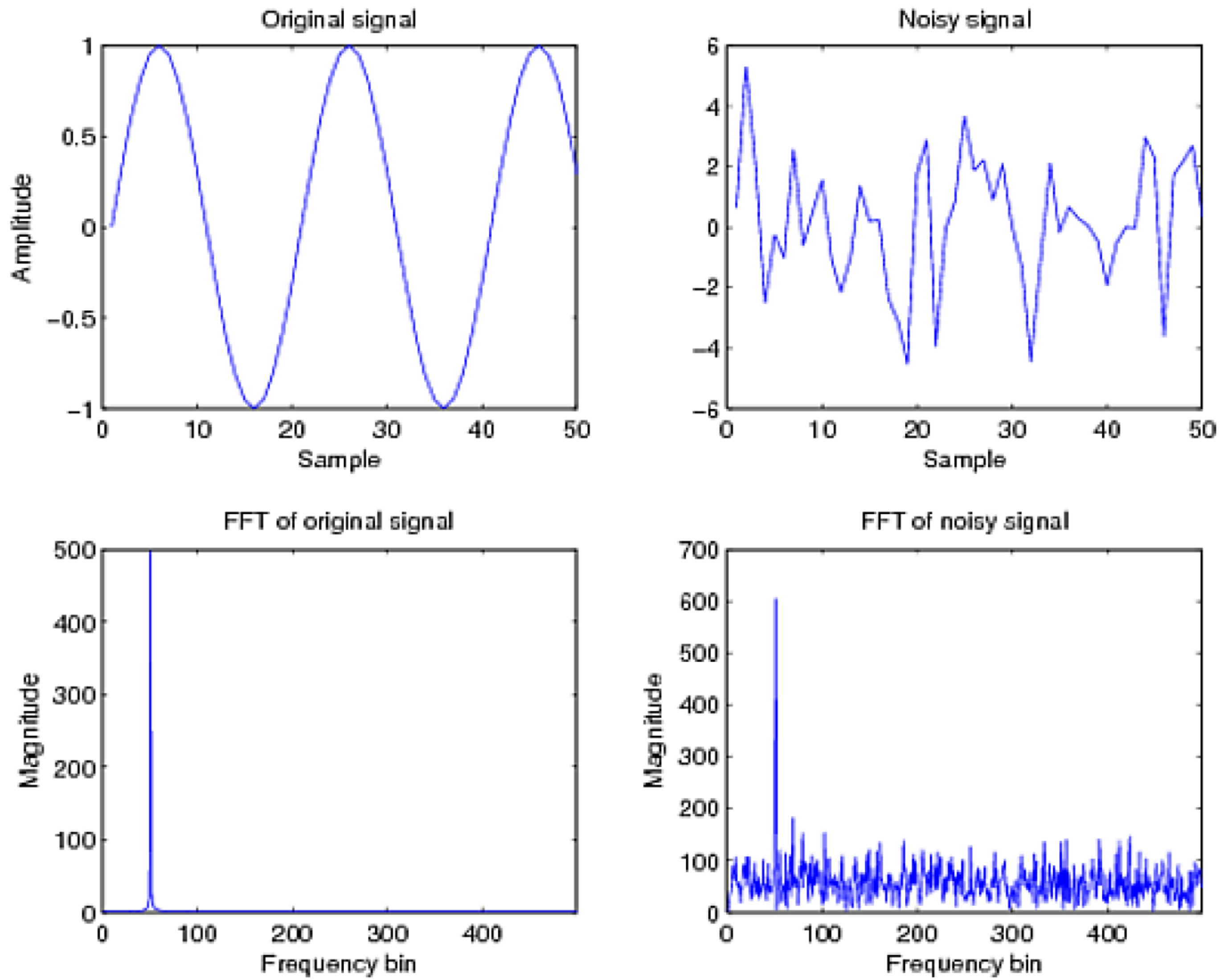


FFT is a fast algorithm to find the frequency components of such discrete signals:

$$Y = \mathrm{fft}(X)$$

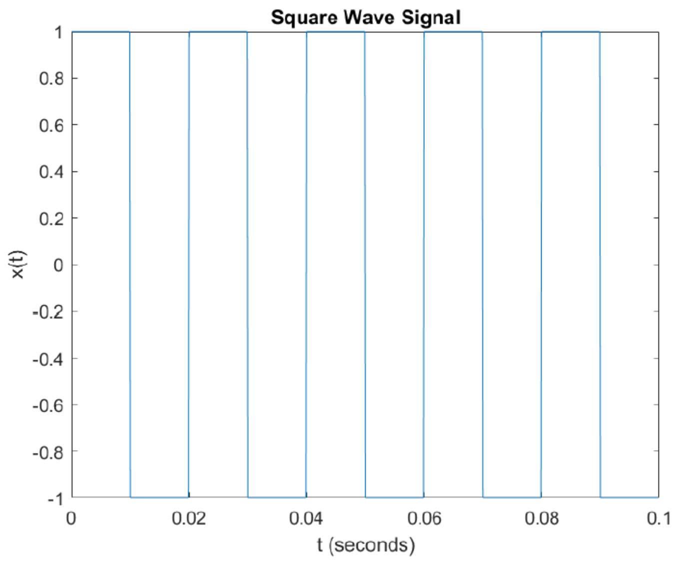
(X can be samples of one period of the signal if the signal is periodic)

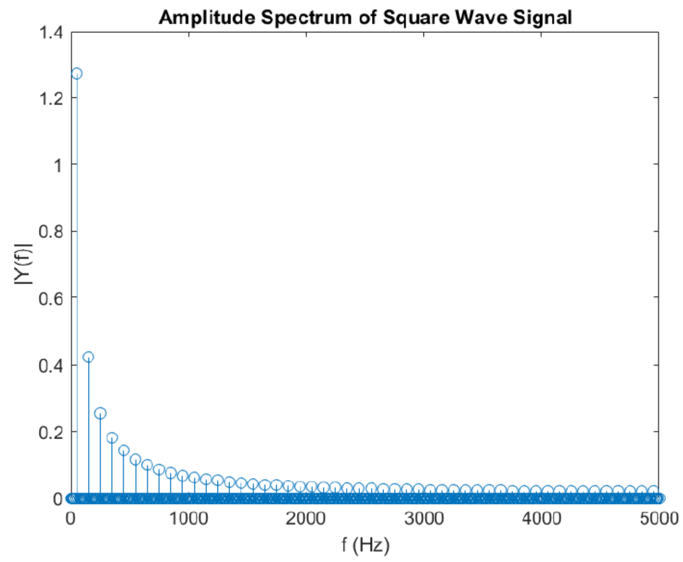
Fast Fourier Transform



The “frequency bins” need to be mapped to Hz based on “sampling rate” of the signal

Fast Fourier Transform





You need to sample the signal fast enough (at a high enough rate) to reproduce it digitally with high fidelity
(Nyquist criterion: sampling rate > 2 x highest signal frequency)

1. **Take FFT of input signal to get the sines/cosines.**
2. **Analyze each component individually through the circuit or system using the transfer function and impedance model.**
3. **Another way to think about it: Knowing the frequencies in the signal via the FFT, design a circuit based on this knowledge**

Power and Energy in Impedance

- Power and energy are critical issues in the design of circuits
- The size of the battery required by a device so it will function for a desired amount of time is related to the energy efficiency
- We know how to do power and energy analysis in the time domain

Power and Energy Relation in a Two-Terminal Element

Power: $P(t) = i(t)v(t)$

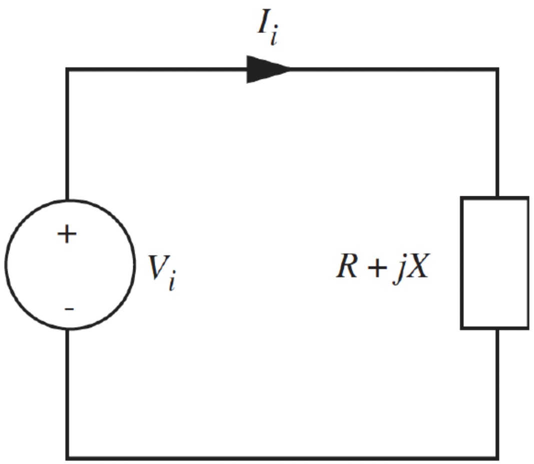
where *i(t)* is defined to be positive if it enters at the positive terminal.

Energy: $w(t) = \int_0^T p(t)dt$

- In the special case of sinusoidal inputs, we develop power and energy calculation methods using impedances

Power and Energy in Impedance

$$v_i(t) = |V_i|\cos(\omega t + \phi)$$



Hence the voltage and current complex amplitudes are:

$$\begin{aligned} V_i &= |V_i|e^{j\phi} \\ I_i &= \frac{V_i}{Z} = \frac{|V_i|e^{j\phi}}{R + jX} \\ &= \frac{|V_i|e^{j(\phi-\theta)}}{\sqrt{R^2 + X^2}} \\ &= |I_i|e^{j(\phi-\theta)} \end{aligned}$$

where

$$\theta = \tan^{-1}\frac{X}{R}$$

Power and Energy in Impedance

- First let us look at the instantaneous power:

$$p(t) = v_i(t)i_i(t) = [|V_i|\cos(\omega t + \phi)]\left[\frac{|V_i|}{\sqrt{R^2+X^2}}\cos(\omega t + \phi - \theta)\right]$$

$$= \frac{1}{2}\frac{|V_i|^2}{\sqrt{R^2+X^2}}[\cos(2\omega t + 2\phi - \theta) + \cos\theta]$$

The instantaneous power for sinusoidal drive has 2 components:

1. a sinusoidal component at twice the frequency of the input signal
2. a DC component

Average Power

$$p(t) = v_i(t) i_i(t) = \frac{1}{2} \frac{|V_i|^2}{\sqrt{R^2 + X^2}} [\cos(2\omega t + 2\phi - \theta) + \cos\theta]$$

Because the average value of a sinusoid is 0, the average power flowing into an arbitrary impedance is just the DC term:

$$\begin{aligned} \bar{p} = \frac{1}{2} \frac{|V_i|^2}{\sqrt{R^2 + X^2}} \cos\theta &= \frac{1}{2} |V_i||I_i| \cos\theta \\ &= \frac{1}{2} \Re[V_i I_i^*] \\ &= \frac{1}{2} \Re[V_i^* I_i] \end{aligned} \tag{1}$$

where where X^* denotes the complex conjugate of X:

$$X = |X| e^{j\theta} \ \rightarrow \ X^* = |X| e^{-j\theta}$$

Special Case 1: Purely Resistive

If the impedance $Z = R + jX$ is pure resistance $\rightarrow \;\; X = 0$

$$p(t) = \frac{V_i^2}{2R}(1 + \cos(2\omega t))$$

$$\bar{p} = \frac{V_i^2}{2R}$$

The average power delivered by a sinusoidal input to a resistance $=$ $\frac{1}{2}$ (the power delivered by a DC voltage source of the same amplitude)

Special Case 2: Purely Reactive

- If the impedance $Z = R + jX$ is pure reactance $\rightarrow \;\; R = 0$
- Inductance $\rightarrow X = \omega L, \theta = \pi/2$:

$$p(t) = \frac{V_i^2}{2X}\cos(2\omega t - \pi/2) = \frac{V_i^2}{2X}\sin(2\omega t)$$

- Capacitance $\rightarrow X = \omega L, \theta = -\pi/2$

$$p(t) = -\frac{V_i^2}{2X}\sin(2\omega t)$$

In both cases the average power $\bar{p}$ is zero

Reminders

- Next week: Review and practice problems on Tuesday. No class Thursday

- HW due Dec 3
- Lab due Dec 4

- Week after Thanksgiving: Resonant RLC filters