Happy Halloween!

ECE 10C
Fall 2020
Slide Set 5
Instructor: Galan Moody
TA: Kamyar Parto

Tuesday
- Transient analysis
- Impedance method

Today
- Impedance method
- Examples

Important Items:

- **HW #3 due Friday, 11/6**
- **Lab #3 due 11/6**
- **Midterm on Thurs, 11/5**
- **HW 2 average: 84%**

Midterm on Thursday, November 5

Will email it to everyone at 4 pm PDT

When finished upload to GauchoSpace, timestamped by 6:15 pm PDT

I'll log on to our zoom link to answer any questions from 5 – 6:15 pm PDT

Open book/notes (no internet, no collaboration with classmates)

4-5 questions – will be able to complete within the 75 minute class time

Quiz on Tuesday

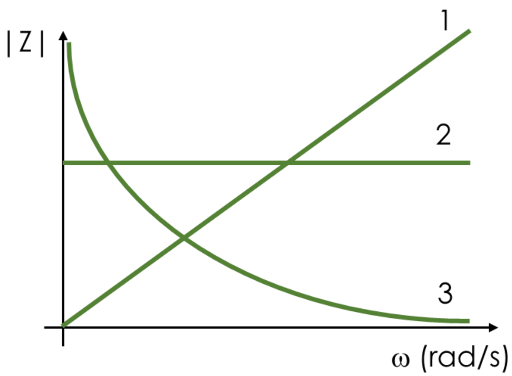
everyone received full credit

Q1 [2 points]. What is the magnitude and phase of the complex number $Z = 3e^{j0} + 4e^{j\pi/2}$?

(a) $|Z| = 4,\ \varphi = \tan^{-1}(4/3)$
(b) $|Z| = 5,\ \varphi = \tan^{-1}(3/4)$
(c) $|Z| = 5,\ \varphi = \tan^{-1}(4/3)$

$$Z = 3 + 4j$$

Q2 [3 points]. In the figure below, the three curves represent the frequency dependence of the impedance for a capacitor, an inductor, and a resistor. Label which curve corresponds to which element. [**HINT #1:** the impedance Z of an element can be thought of as its "resistance", which allows us to apply a generalization of Ohm's law V = I×Z to each element.] [**HINT #2:** how do the capacitor, inductor, and resistor behave at DC, *i.e.* as $\omega \to 0$ rad/s?]



(a) 1 = R, 2 = L, 3 = C
(b) RCL
(c) LRC
(d) LCR
(e) CRL
(f) CLR

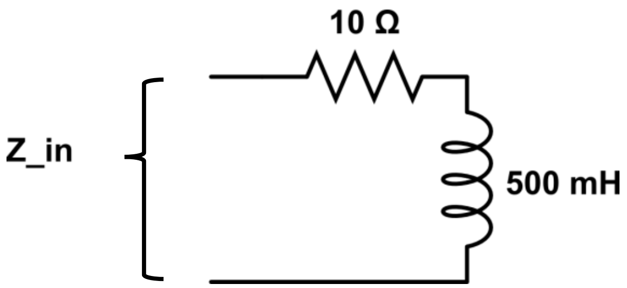
$$Z_R = R$$

$$Z_C = \frac{1}{sC} = \frac{1}{j\omega C}$$

$$Z_L = sL = j\omega L$$

Quiz Time!

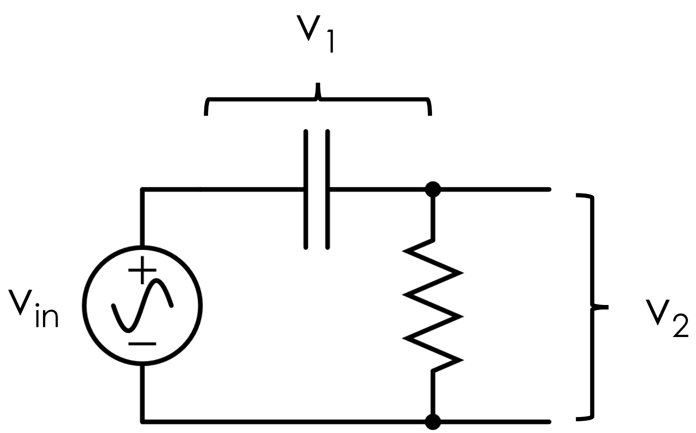
Q1 [2 pts] What is the impedance of the circuit below at angular frequency w = 10 rad/s?



(a) $10 + 0.5j\ \Omega$
(b) $0.5 + 10j\ \Omega$
(c) $10 + 5j\ \Omega$
(d) $5 + 10j\ \Omega$
(e) None of the above

Q2 [1 pt] True or False? You can combine and simplify complex impedances in parallel or in series in a similar way to resistors, i.e. $Z = Z_1 + Z_2 + Z_3$ (series) and $1/Z = 1/Z_1 + 1/Z_2 + 1/Z_3$ (parallel).

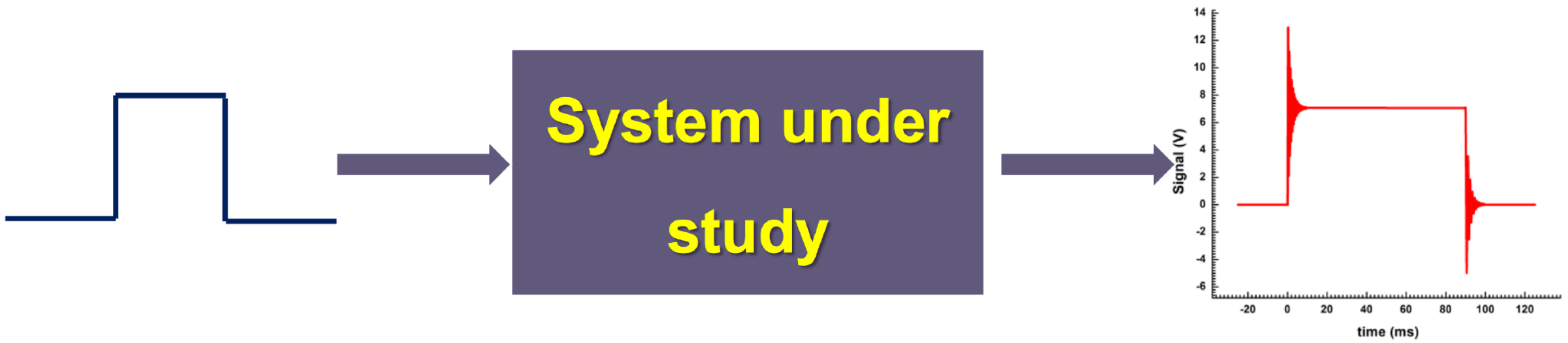
Q3 [2 pts] When measuring the voltages v_1 and v_2, the circuit acts as a (respectively):



(a) High pass filter / Low pass filter
(b) Low pass filter / High pass filter
(c) High pass filter / High pass filter
(d) Low pass filter / Low pass filter
(e) None of the above

Transient Analysis

We care about the exact response of a system at all times *t*:



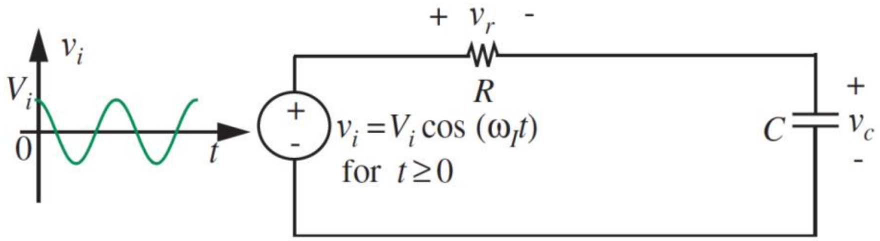
input signal

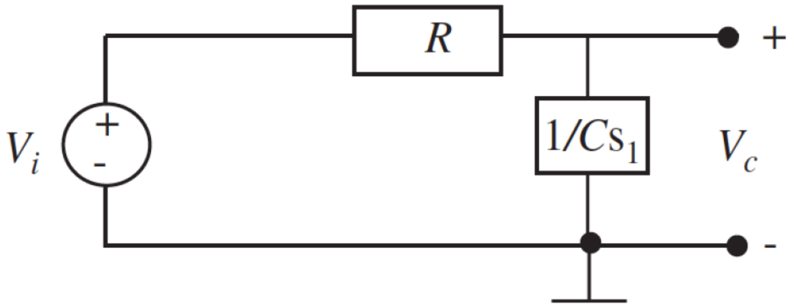
output signal

- We can use what we've learned with 1st and 2nd order ODEs to calculate the system's response.
- A different method: impedances. We treat a complex circuit as a bunch of elements that appear similar to resistors.

A New Circuit with “Boxes”

- We can redraw the circuit, replacing resistors with *R* boxes, capacitors with 1/Cs boxes, and cosine sources by their amplitudes
- We can then analyze the circuits using a generalized version of Ohm’s law for each element
- Once we figure out what the complex amplitudes are, then we can plug these back into our generic time-domain expressions. Makes the analysis MUCH easier than working through the ODEs





The Concept of Impedance

Impedance: A generalization of resistance for sinusoidal steady state inputs. Associated with each element is a characteristic impedance that affects the flow of current/voltage drop across the element, analogous to a resistor.

We can now analyze complex circuits with R's, C's, and L's using our standard tools, like KVL, KCL, Node method, and treat each element in the circuit using Ohm's law with it's characteristic impedance.

Ohm's law is easier to use for evaluating circuits. No more ODE's! (mostly)

Generalization of Ohm's Law for Sinusoidal Steady State

Resistor: $V = IR \rightarrow Z_R = R$

Capacitor: $V = I\frac{1}{sC} \rightarrow Z_C = \frac{1}{sC} = \frac{1}{j\omega C}$

Inductor: $V = sLI \rightarrow Z_L = sL = j\omega L$

Complex Amplitude of a Source

Remember the trick we used with the source

Replace the source of $v_i(t) = V\cos(\omega t)$ with the form:

$$\tilde{v}_i(t) = Ve^{j\omega t}$$

We can see that: $v_i(t) = Re\{\tilde{v}_i(t)\}$

The complex amplitude is: $V = |V|e^{-j\angle V}$

$$v_i(t) = Re\{\tilde{v}_i(t)\} = |V|\cos(\omega t + \angle V)$$

General form for v_i, where the input has an amplitude, frequency and phase

- Essentially, what we're doing is analyzing the frequency response of the complex amplitudes, which tells us what we need to know about how the circuits will respond to the sinusoidal input

Example

Replace the sinusoidal sources by their complex amplitudes:

$$\sin(\varphi) = \cos(\varphi - 90°)$$

$$-\cos(\varphi) = \cos(\varphi + 180°)$$

$$-\sin(\varphi) = \sin(\varphi + 180°)$$

$$i(t) = 6\cos(50t - 40°) \longrightarrow I = 6\angle{-40°}$$

$$v(t) = -4\sin(30t + 50°)$$

Example

Replace the sinusoidal sources by their complex amplitudes:

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$$i(t) = 6\cos(50t - 40°) \longrightarrow I = 6\angle -40°$$

$$v(t) = -4\sin(30t + 50°) \longrightarrow V = 4\angle 140°$$

The relationship between v*(t)* and the complex amplitude $V = |V|e^{-j\angle V}$ is:

$$v(t) = Re\{V(j\omega)e^{j\omega t}\} = |V|\cos(\omega t + \angle V)$$

IMPORTANT! Remember that $\omega = 2\pi f$ is the angular frequency. Some questions you come across may give you the frequency f or the angular frequency ω. If you're given (or asked for) f, make sure to convert to (from) ω via $\omega = 2\pi f$. **ALSO**: one period is equal to $T = \frac{1}{f} = \frac{2\pi}{\omega}$

Steps for the Impedance Method

1. First, replace the (sinusoidal) sources by the complex amplitudes

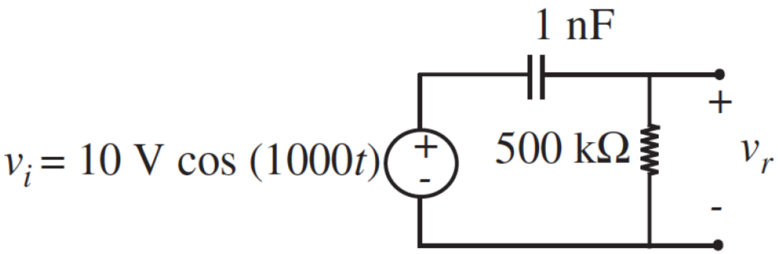
2. Then, replace the circuit elements by their impedances. The resulting diagram is called **the impedance model of the network**.

3. Now, determine the complex amplitudes of the voltages and currents in the circuits (e.g. $V_c(j\omega)$). You can use any standard linear circuit analysis technique you like—Node method, KVL, KCL, Thevenin, intuitive method based on series/parallel simplifications, etc.

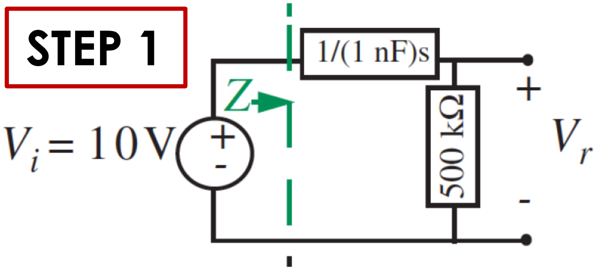
4. Finally, we can obtain the time variables from the complex amplitudes and plug into the general expression for the dynamics. This isn't usually necessary though. As an example, the time domain response corresponding to the node variable V_c is given by:

$$v_c(t) = Re\{V_c(j\omega)e^{j\omega t}\} = |V_c|\cos(\omega t + \angle V_c)$$

Example: RC Circuit



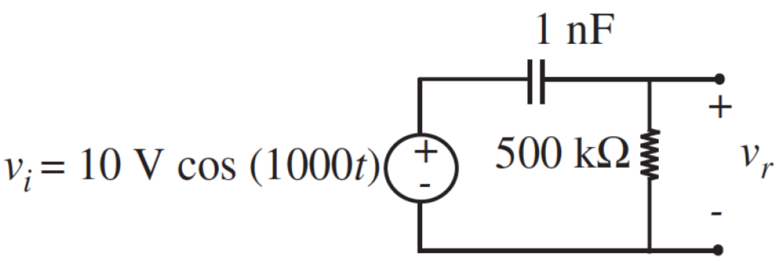
STEP 1



STEP 2 The impedances are:

$$Z_R = 500 * 10^3 \Omega, \quad Z_C = \frac{1}{Cj\omega} = \frac{1}{1 \times 10^{-9} j1000} \Omega$$

Example: RC Circuit



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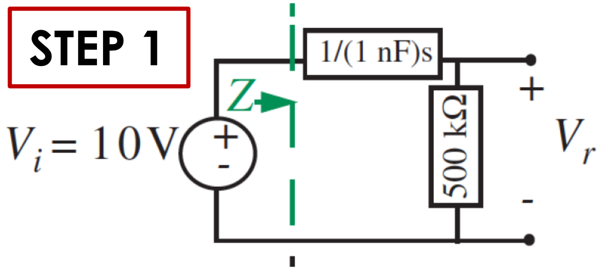

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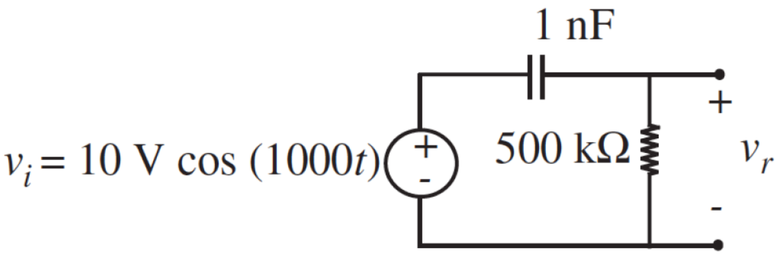
$$Z_R = 500 * 10^3\Omega, \quad Z_C = \frac{1}{Cj\omega} = \frac{1}{1 \times 10^{-9}j1000}\Omega$$

STEP 3 Using the voltage divider relation for V_r

$$V_r = \frac{Z_R}{Z_R + Z_C}V_i = \frac{500 * 10^3}{500 * 10^3 + \frac{1}{1\times 10^{-9}j1000}}10 = \frac{5}{0.5 - j} = 4.47e^{j1.1}$$

Example: RC Circuit



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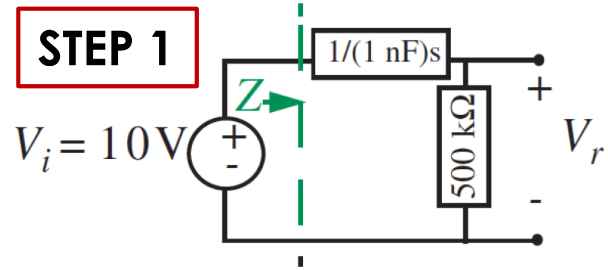


STEP 2 The impedances are:

$$Z_R = 500 * 10^3 \Omega, \quad Z_C = \frac{1}{Cj\omega} = \frac{1}{1 \times 10^{-9} j1000} \Omega$$

$\phi = \tan^{-1}\left(\frac{Y}{X}\right)$

$z = X + jY$

$|z| = \sqrt{X^2 + Y^2}$

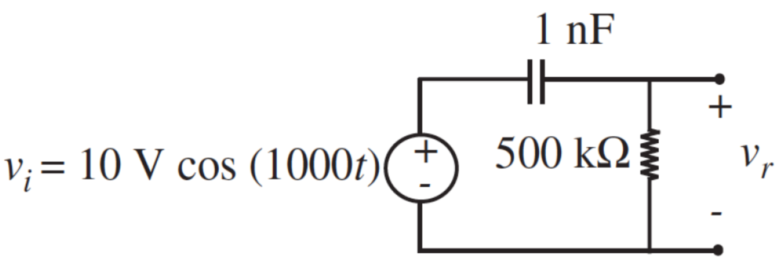
STEP 3 Using the voltage divider relation for V_r

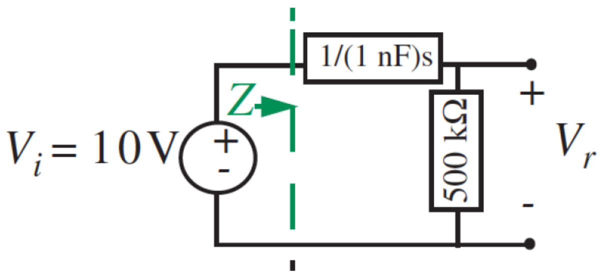
$$V_r = \frac{Z_R}{Z_R + Z_C} V_i = \frac{500 * 10^3}{500 * 10^3 + \frac{1}{1 \times 10^{-9} j1000}} 10 = \frac{5}{0.5 - j} = 4.47 e^{j1.1}$$

Complex amplitude of the voltage $V_r = 4.47 e^{j1.1}$

Magnitude $|V_r| = 4.47$ and Phase $\angle V_r = 1.1$ rad

Example: RC Circuit



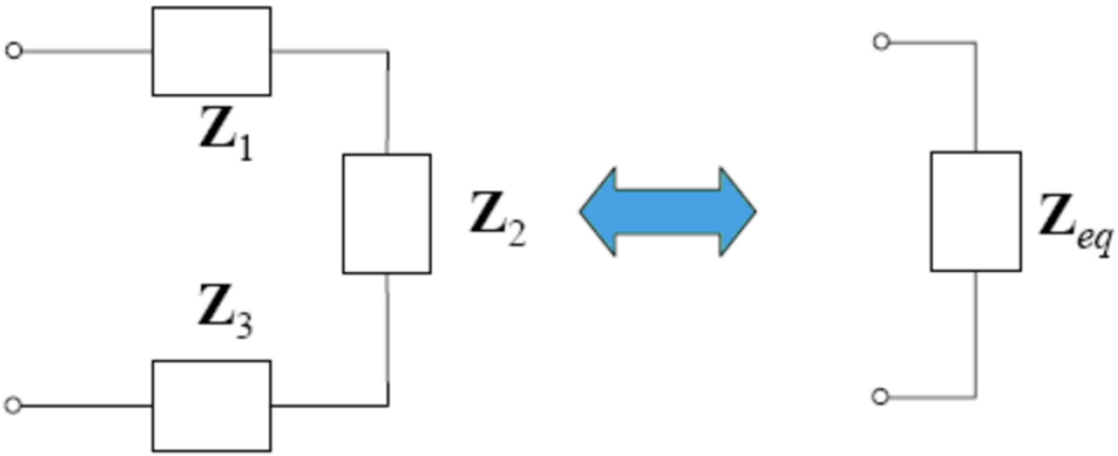


$$V_r = 4.47e^{j1.1}$$

STEP 4 So in the time domain, $v_r(t)$ is given by:

$$v_r = \Re\left[V_r e^{j\omega t}\right] = \Re\left[4.47e^{j1.1}e^{j1000t}\right] = 4.47\cos(1000t + 1.1)V$$

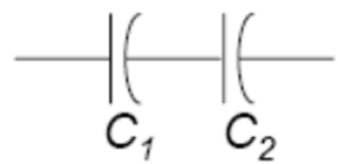
Impedance Combinations: Series



$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3$$

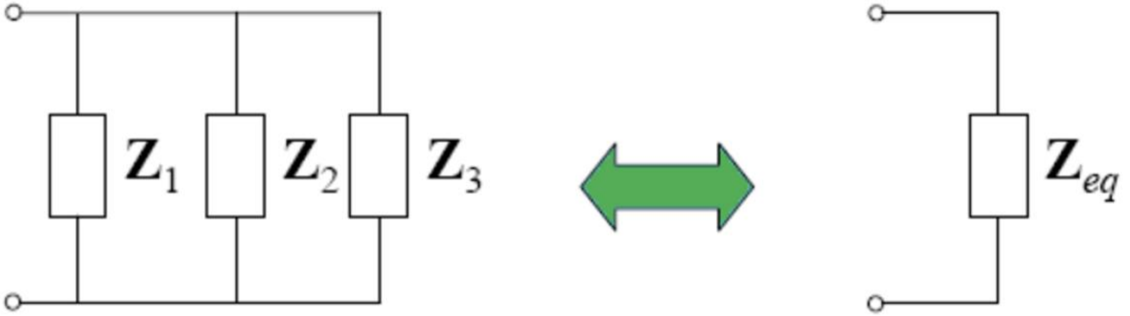
For example:

$$\mathbf{Z}_{eq} = j\omega(L_1 + L_2)$$



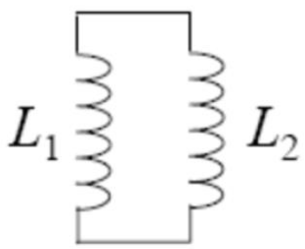
$$\mathbf{Z}_{eq} = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}$$

Impedance Combinations: Parallel

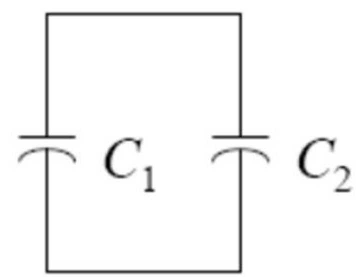


$$1/\mathbf{Z}_{eq} = 1/\mathbf{Z}_1 + 1/\mathbf{Z}_2 + 1/\mathbf{Z}_3$$

For example:

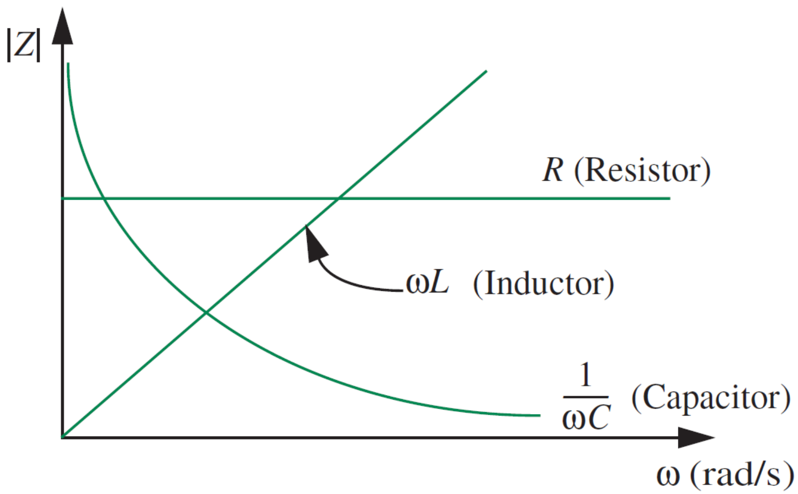


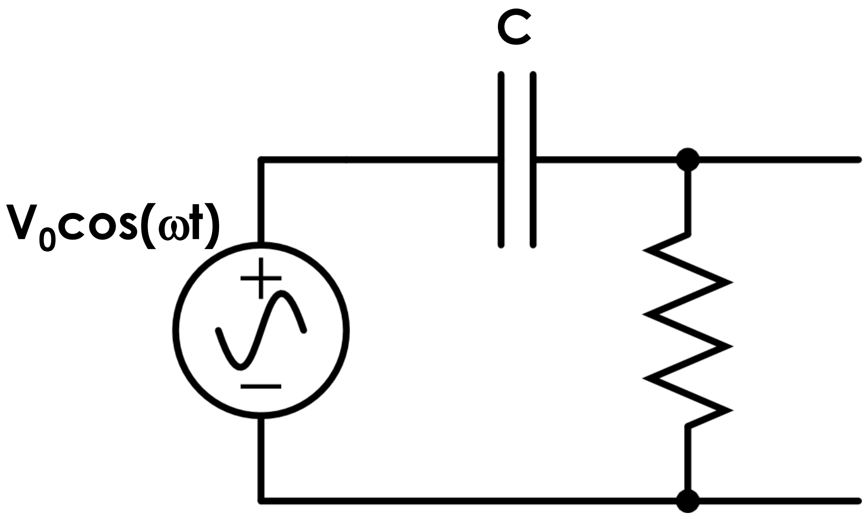
$$\mathbf{Z}_{eq} = j\omega\frac{L_1 L_2}{(L_1 + L_2)}$$



$$\mathbf{Z}_{eq} = \frac{1}{j\omega(C_1 + C_2)}$$

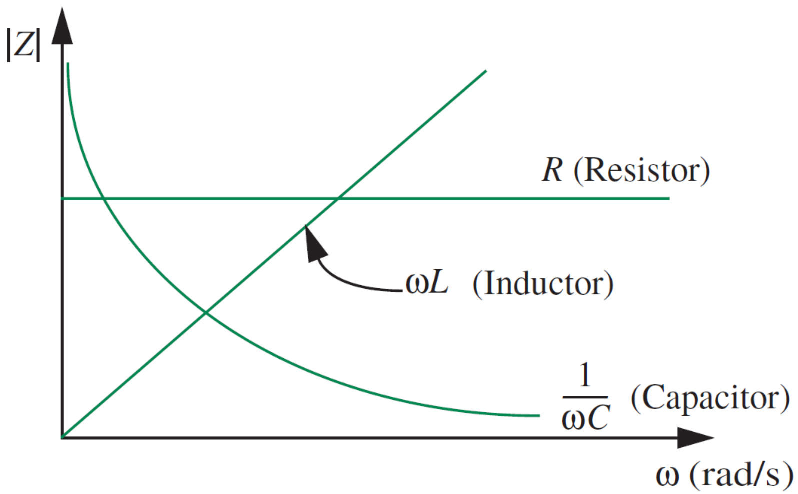
Let's Examine a Capacitor's Impedance

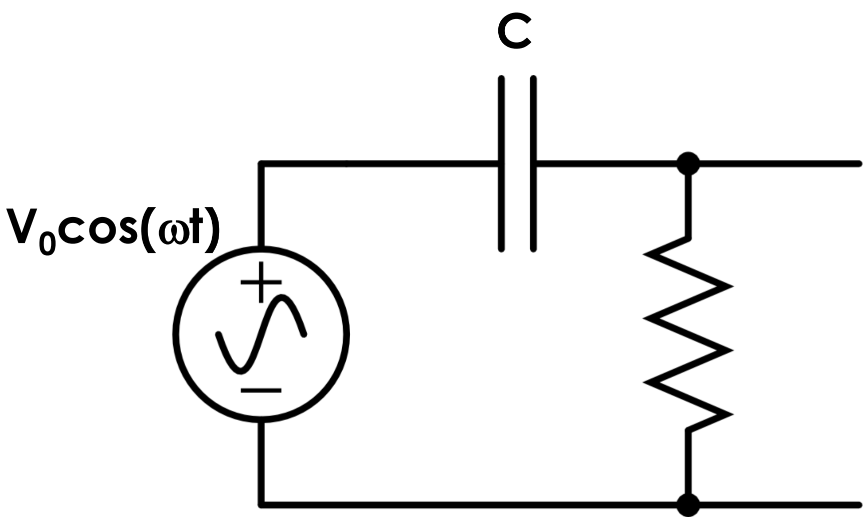




Low frequencies $\omega \sim 0$:

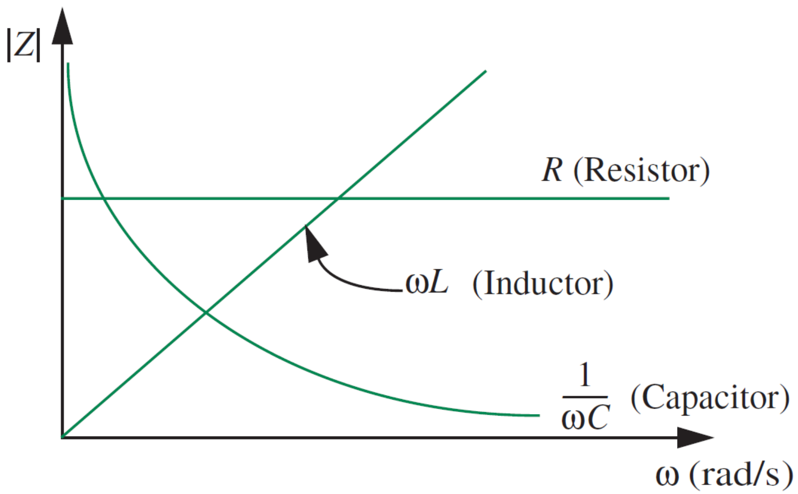
Let's Examine a Capacitor's Impedance

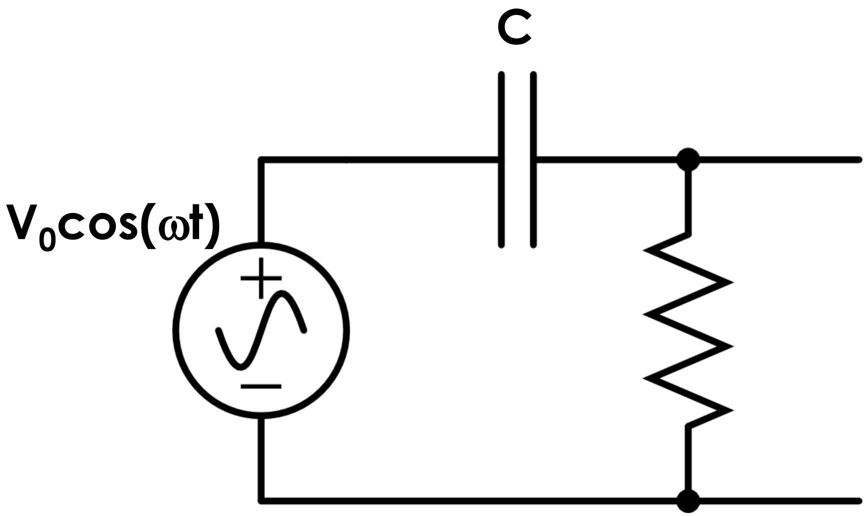




Low frequencies $\omega \sim 0$: Looks like a DC source. The capacitor has **large** impedance, which means that the voltage V_0 is dropped across the capacitor. Thus the voltage across the resistor is **zero.**

Let's Examine a Capacitor's Impedance





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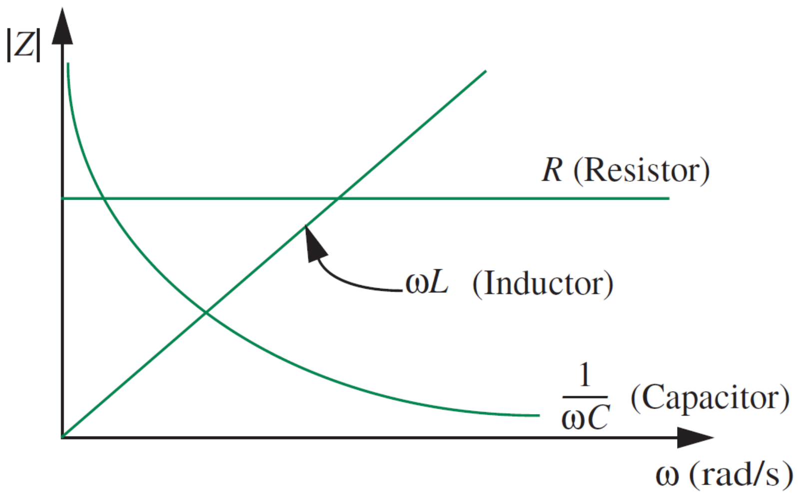
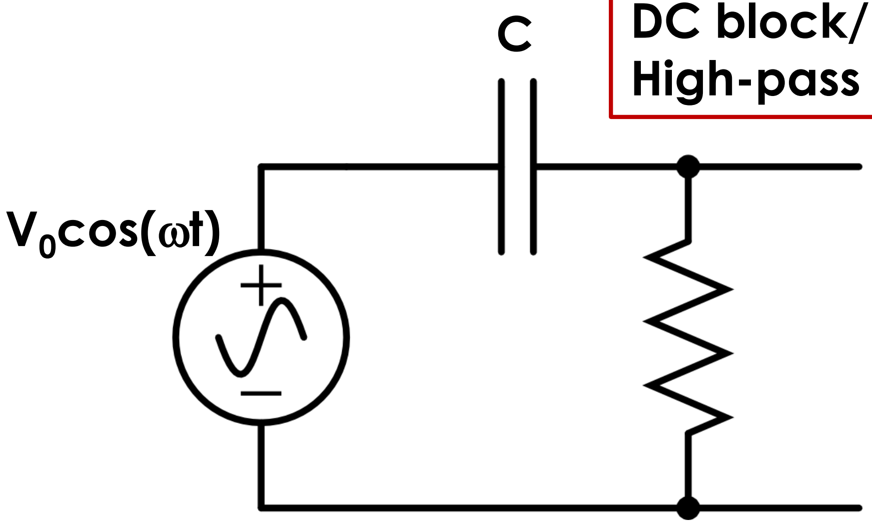
High frequencies $\omega \to \infty$:

Let's Examine a Capacitor's Impedance



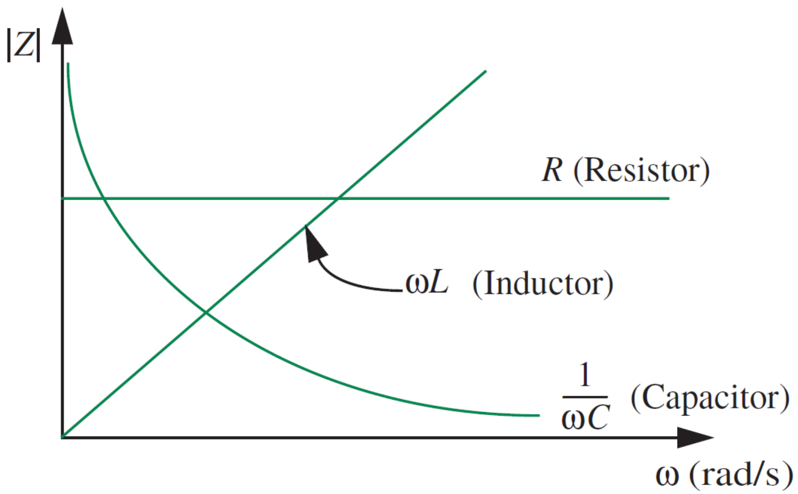


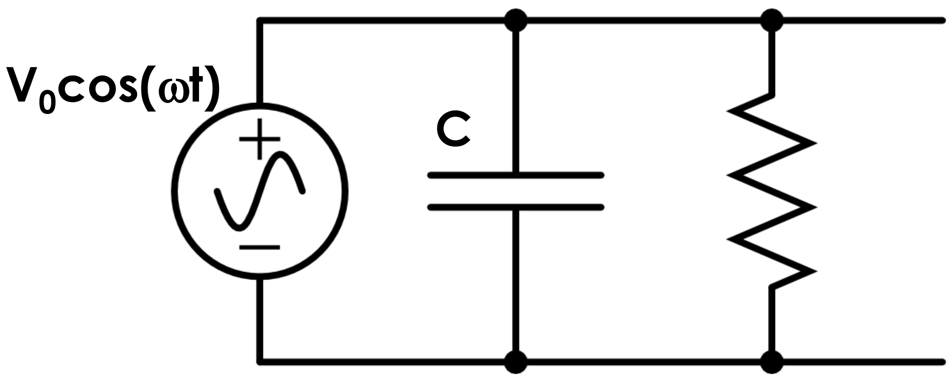
DC block/
High-pass filter

Low frequencies $\omega \sim 0$: Looks like a DC source. The capacitor has **large** impedance, which means that the voltage V_0 is dropped across the capacitor. Thus the voltage across the resistor is **zero.**

High frequencies $\omega \rightarrow \infty$: The source frequency is too fast for the electrons on the capacitor plates to respond. Capacitor has **small** impedance, no voltage dropped across C, and resistor voltage is $\mathbf{V_0}$.

Let's Examine a Capacitor's Impedance





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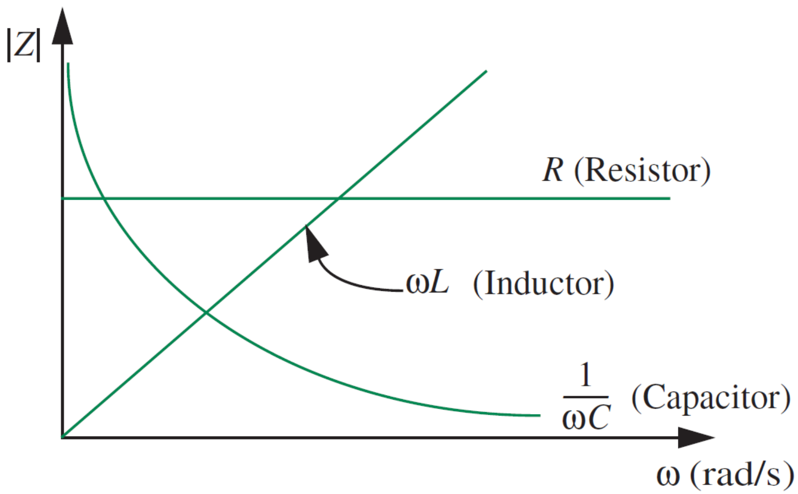
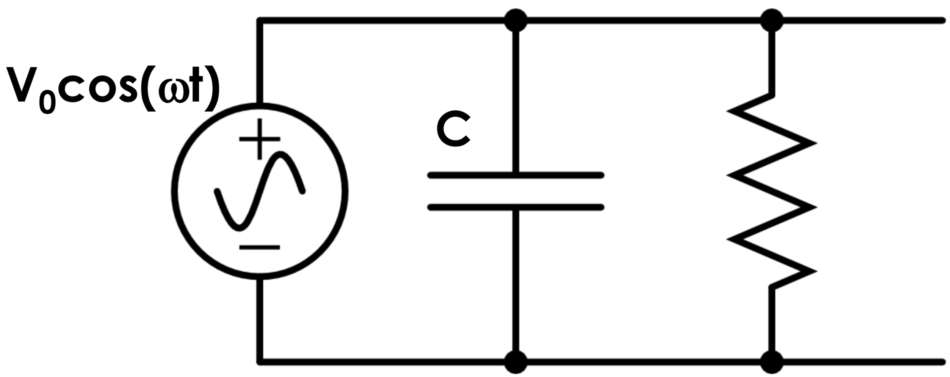


Low-pass filter

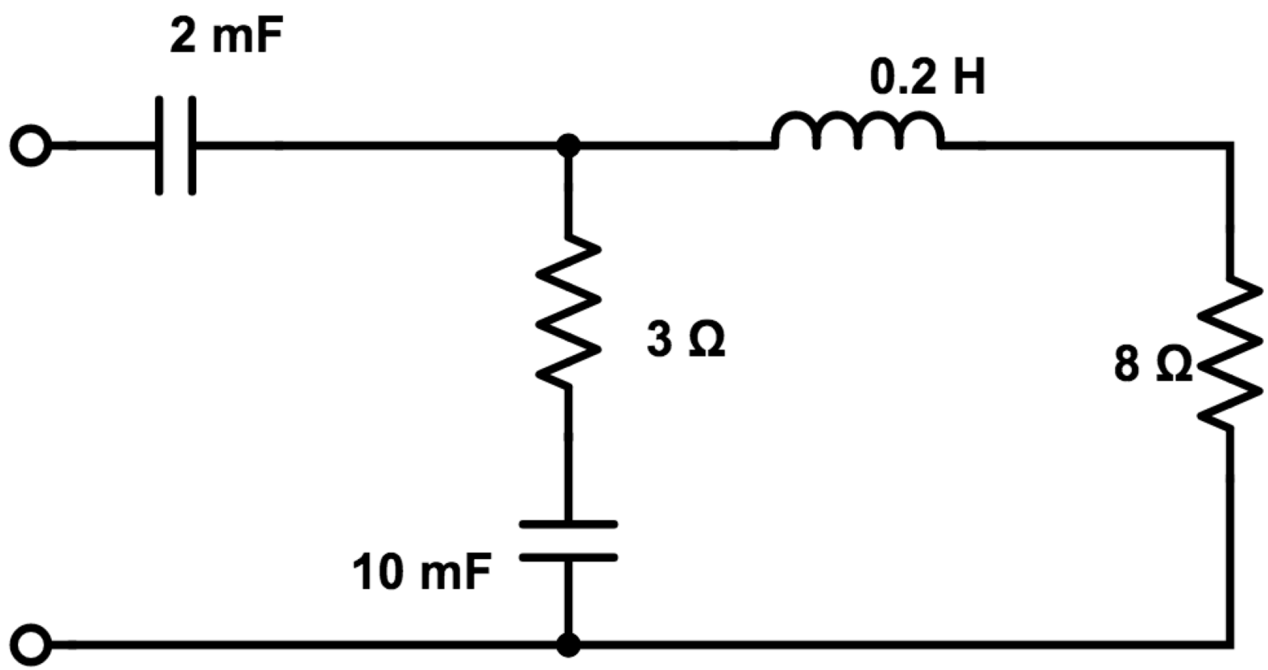


Low frequencies $\omega \sim 0$: Looks like a DC source. The capacitor has **large** impedance, which means that the voltage V_0 is dropped across the capacitor. Thus the voltage across the resistor is also $\mathbf{V_0}$.

High frequencies $\omega \to \infty$: The source frequency is too fast for the electrons on the capacitor plates to respond. Capacitor has **small** impedance, no voltage dropped across C, and resistor voltage is **zero**

In-Class Exercise #1

Find the input impedance of the circuit (at frequency ω = 50 rad/s):



$Z_R = R \qquad Z_C = \frac{1}{j\omega C} \qquad Z_L = j\omega L$

In-Class Exercise #1

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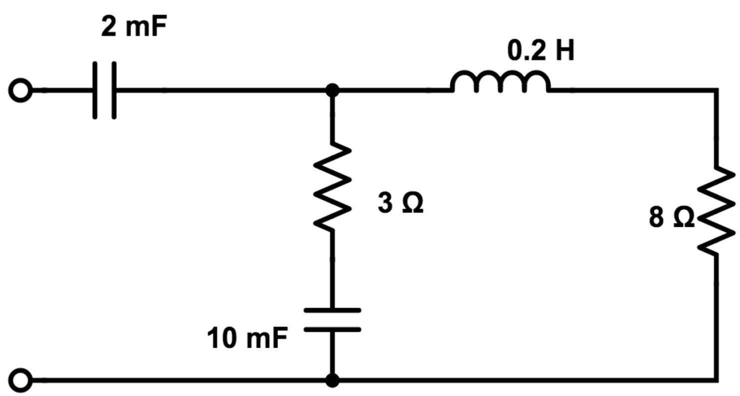
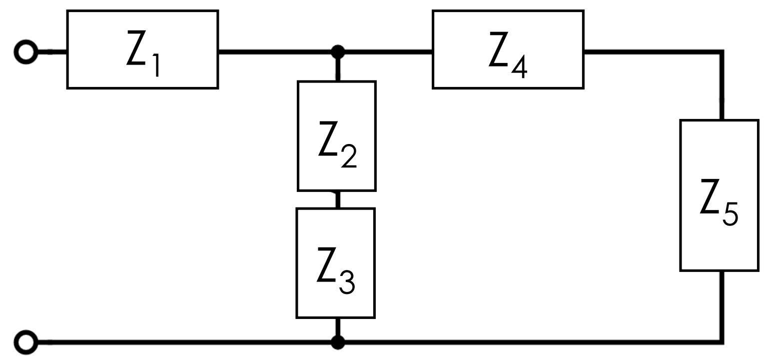
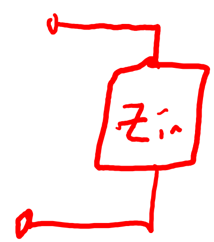




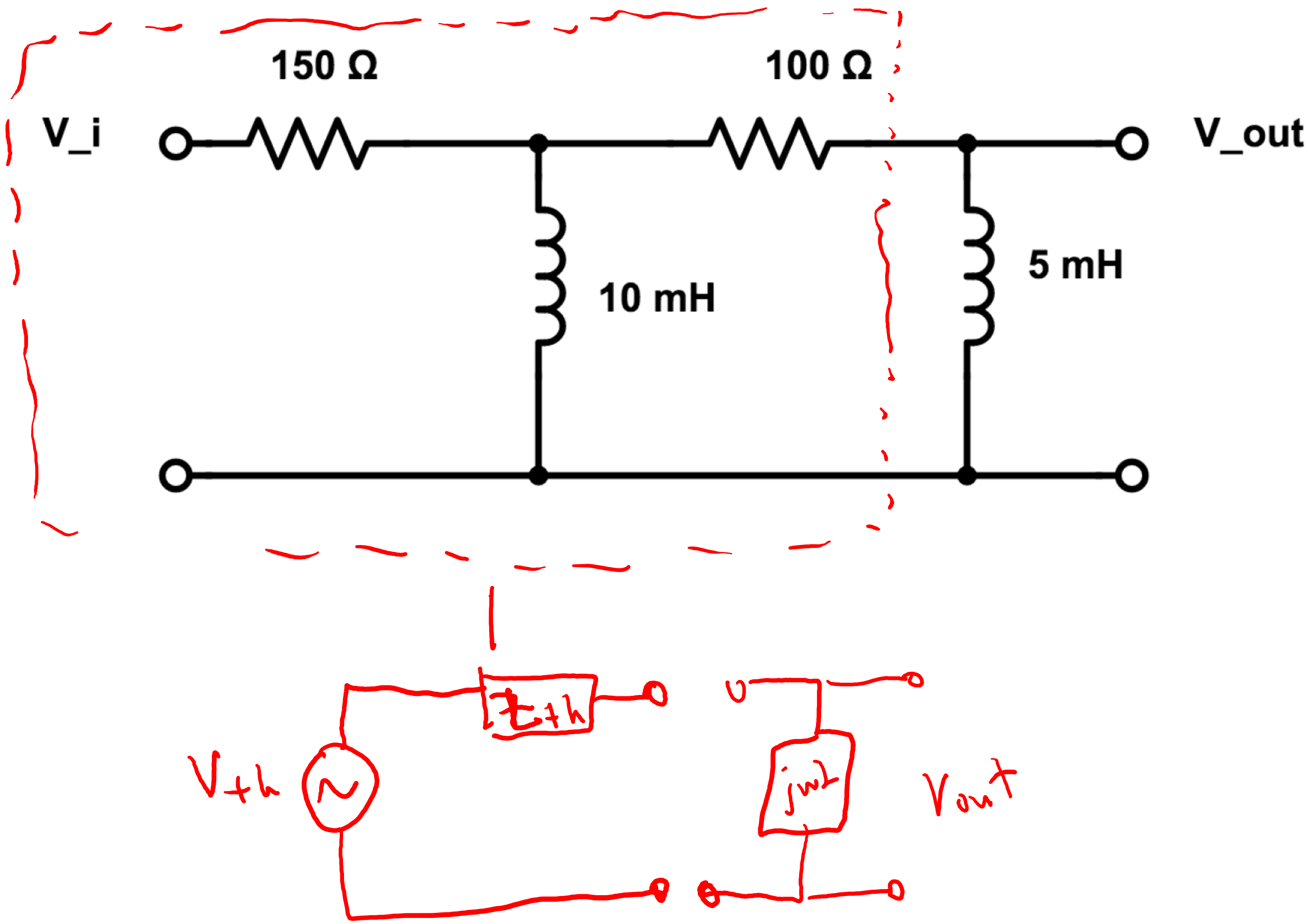
$$Z_1 + (Z_2 + Z_3) \,//\, (Z_4 + Z_5) \qquad Z_C = \frac{1}{j\omega C} \qquad Z_L = j\omega L$$

$$Z_{in} = 2.68 - 9.95j$$



In-Class Exercise #2

Find the phase shift produced by the following circuit at ω = 2 kHz:



In-Class Exercise #2

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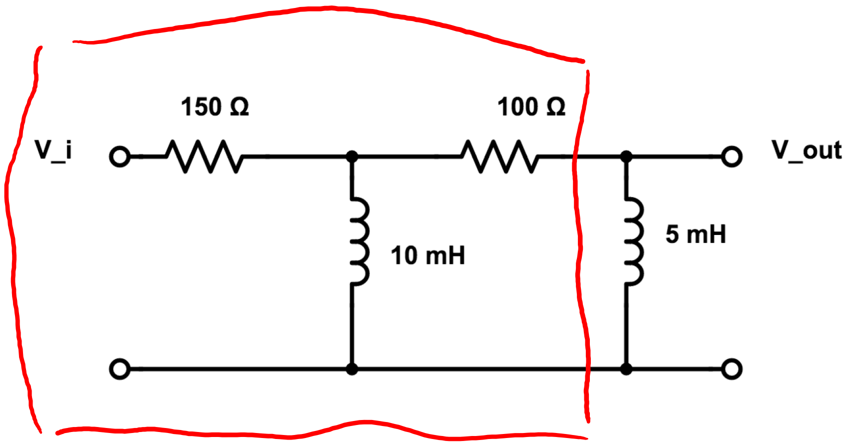
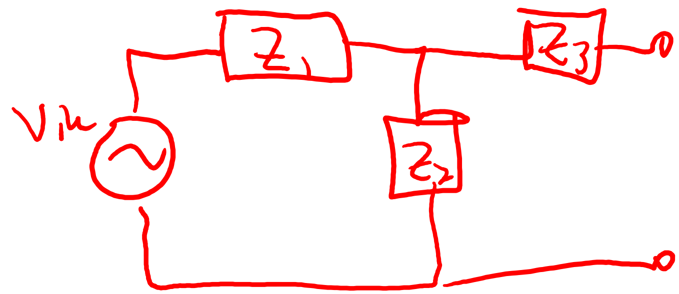
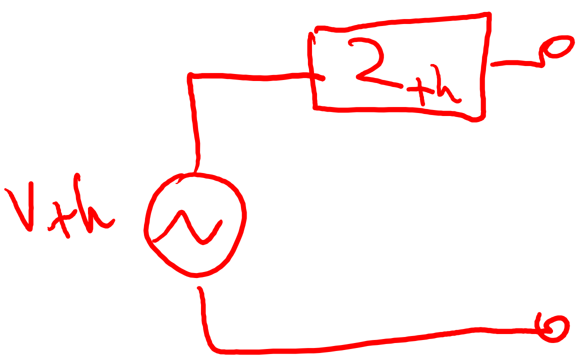
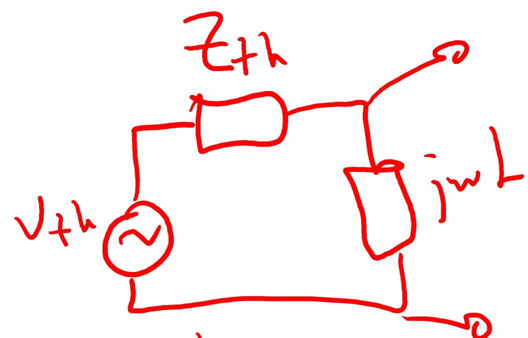




$$Z_{th} = Z_1 // Z_2 + Z_3$$

$$V_{th} = \frac{Z_2}{Z_1 + Z_2} V_{in}$$





$$V_{out} = \left(\frac{j\omega L}{Z_{th} + j\omega L}\right) V_{th}$$

$$X + jY \rightarrow \phi = \tan^{-1}\left(\frac{Y}{X}\right)$$

In-Class Exercise #3

The following two current sources are in parallel in a circuit. Write a single equivalent current source that combines them:

$$i_1(t) = 4\cos(\omega t + 30^\circ)A, \quad i_2(t) = 5\sin(\omega t - 20^\circ)$$

In-Class Exercise #3

The following two current sources are in parallel in a circuit. Write a single equivalent current source that combines them:

$\cos(\phi) = \sin(\phi - 90^\circ)$

$$i_1(t) = 4\cos(\omega t + 30^\circ)A, \quad i_2(t) = 5\sin(\omega t - 20^\circ)$$

You can replace sinusoidal sources by their complex amplitude and phase when using the impedance method.

$\dot{i}_t = \dot{i}_1 + \dot{i}_2$

$\dot{i}_1 = |\dot{i}_1| \angle \dot{i}_1$

$\dot{i}_1 = 4 \angle 30^\circ$

$\dot{i}_2 = |\dot{i}_2| \angle \dot{i}_2$

$\dot{i}_2 = 5 \angle -110^\circ$

$\dot{i}_t = \dot{i}_1 + \dot{i}_2 = 4e^{j30^\circ} + 5e^{-j110^\circ}$

$\dot{i}_t = 3.22 \angle -57^\circ$

$i_t(t) = 3.22\cos(\omega t - 57^\circ)$

In-Class Exercise #3

What is we had these two sources instead?

$$i_1(t) = 4\cos(10t + 30^\circ)A, \quad i_2(t) = 5\sin(1000t - 20^\circ)$$

Can you combine then like you just did using the impedance method?

In-Class Exercise #3

What is we had these two sources instead?

$$i_1(t) = 4\cos(10t + 30^\circ)A, \quad i_2(t) = 5\sin(1000t - 20^\circ)$$

Can you combine then like you just did using the impedance method?

No! But, superposition theorem does apply to sinusoidal steady state circuits the same way it applies to DC circuits. So you can calculate the response to one source at one frequency, and then repeat for each source at a different frequency. Then sum up the final responses.