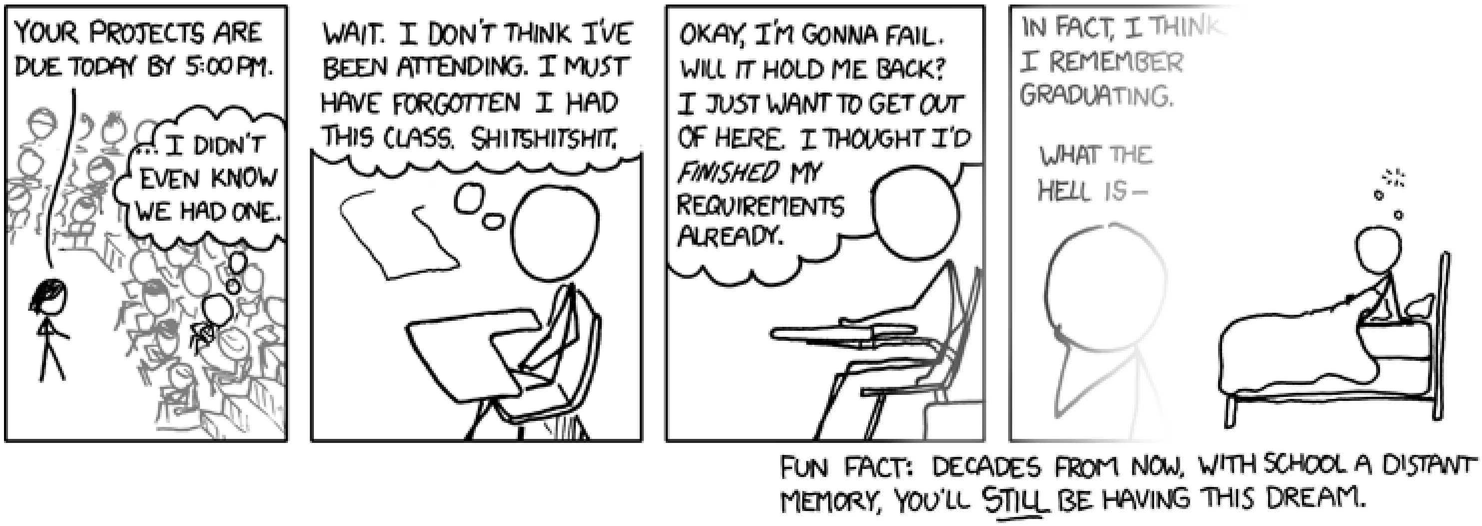


ECE 10C
Fall 2020
Slide Set 6

Instructor: Galan Moody
TA: Kamyar Parto

Important Items:

- HW #3 due Friday, 11/6
- Lab #3 due 11/6
- Midterm on Thurs, 11/5

Poll

Q1: What is your familiarity with using operational amplifiers (Op Amps)?

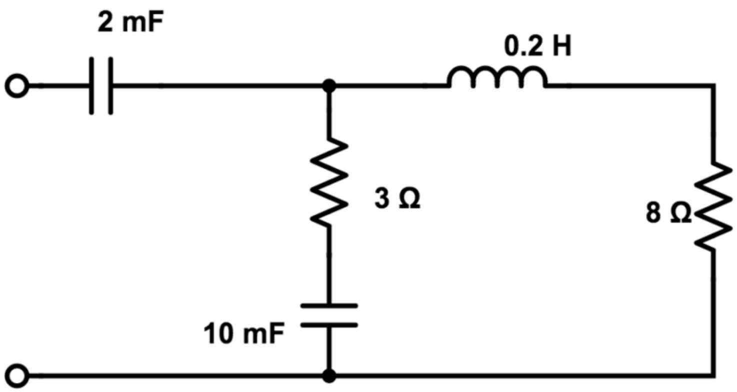
Q2: Extra Study Session Times Tomorrow

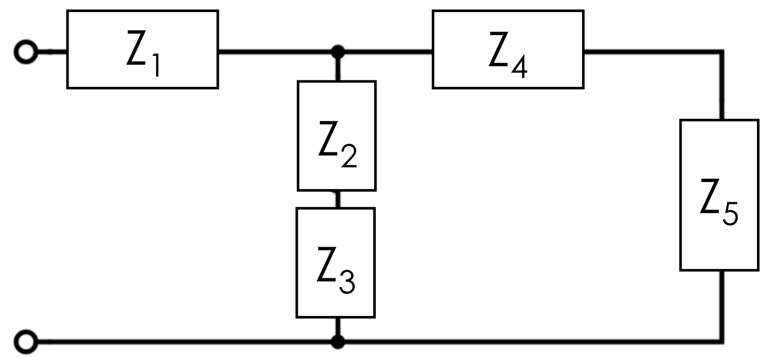
Midterm

- Will post practice problems solutions tomorrow
- Open notes/slides/lectures/HWs/practice midterm from this quarter
- Covers all HW/quiz content
- Combination of ~4-5 quiz-like questions, 3-4 HW-like questions

From Thursday

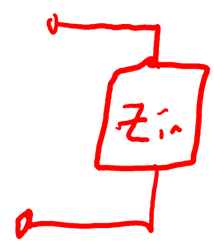
Find the input impedance of the circuit (at frequency ω = 50 rad/s):





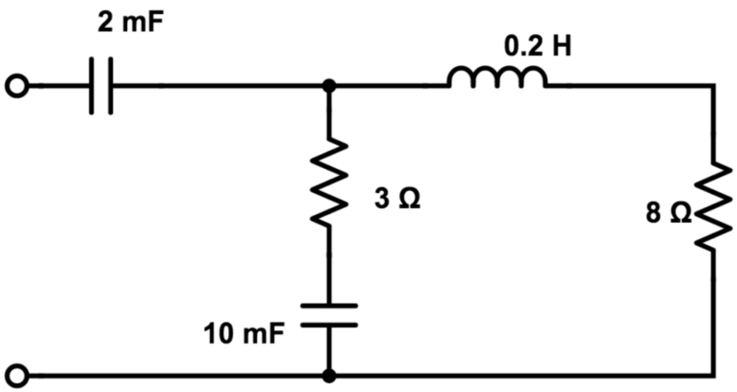
$Z_1 + (Z_2+Z_3) // (Z_4+Z_5)$ $\quad Z_C = \frac{1}{j\omega C} \quad Z_L = j\omega L$

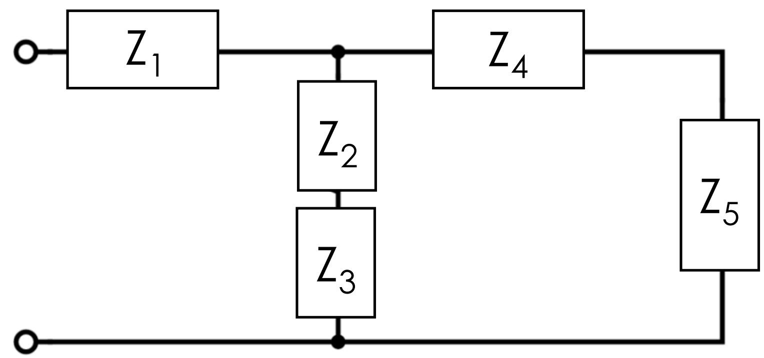
$Z_{in} = 2.68 - 9.95j$



From Thursday

Find the input impedance of the circuit (at frequency ω = 50 rad/s):

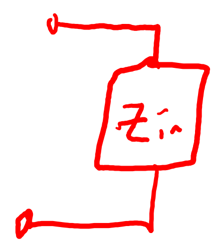




$Z_1 + (Z_2 + Z_3) // (Z_4 + Z_5)$

$Z_C = \frac{1}{j\omega C}$ $Z_L = j\omega L$

$\cancel{Z_{in} = 2.68 - 9.95j}$



Z_{in} = 3.22 – 11.07j

Summary: Capacitor and Inductor 1st Order ODEs

$$\mathrm{y(t)} = \text{final value} + (\text{initial value } - \text{final value})e^{-t/t_c}$$

$$v_c(\mathrm{t}) = v_c(\infty) + (v_c\,(0)\ - v_c(\infty))e^{-t/RC}$$

$$i_L(\mathrm{t}) = i_L(\infty) + (i_L(0)\ - i_L(\infty))e^{-Rt/L}$$

****applies to branch variables such as C or L current, resistor V, etc.****

Capacitor

- Behaves as instantaneous **short** circuit if initially at rest for a long time
- Behaves as **open** circuit at long times when driven by DC voltage source

Inductor

- Behaves as instantaneous **open** circuit if initially at rest for a long time
- Behaves as **short** circuit at long times when driven by DC current source

Power and Energy Relation in a Two-Terminal Element

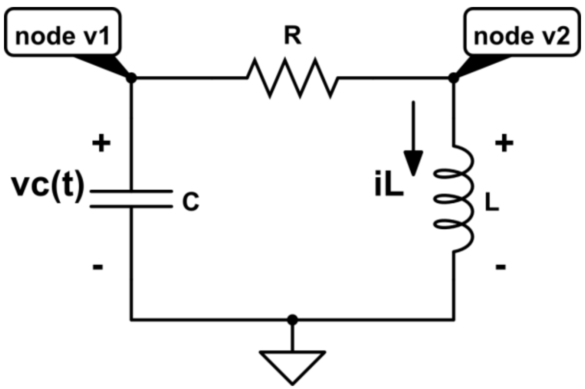
Power:

$$P(t) = i(t)v(t) = \frac{v(t)^2}{R}$$

Energy:

$$w(t) = \int_0^T p(t)dt$$

Review: 2nd Order ODEs, Series RLC



$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$v_1(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \alpha = \frac{R}{2L}$$

**if critically damped, $A_1 e^{s_1 t} + A_2 \boldsymbol{t} e^{s_2 t}$

For 2nd-order (RLC) circuits, we need to find:

$$i_L(0^-) = i_L(0^+) \qquad \frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C}$$

$$v_c(0^-) = v_c(0^+) \qquad \frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L}$$

1. Once we find $v_C(0)$ and $i_L(0)$, then we can use capacitor element law to determine $dv_C(0)/dt$ using $i_C(0)$
2. We can use the inductor element law to determine $di_L(0)/dt$ using $v_L(0)$

Review: 2nd Order ODEs, Parallel RLC

- We can also write the characteristic equation as:

$$s^2 + 2\alpha s + \omega_o^2 = 0$$

- Where we have defined:

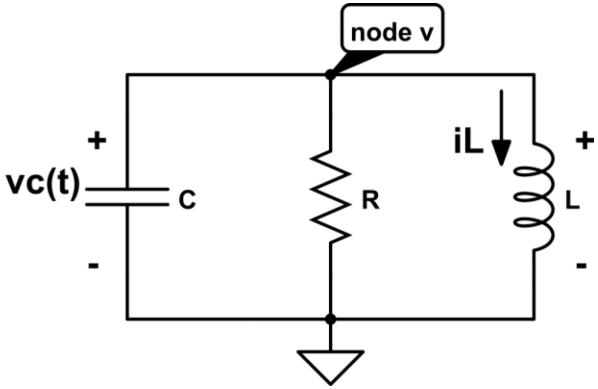
$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

- The roots of the characteristic equation are:

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

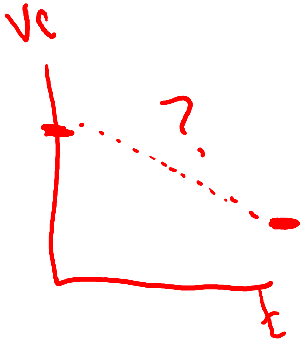
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

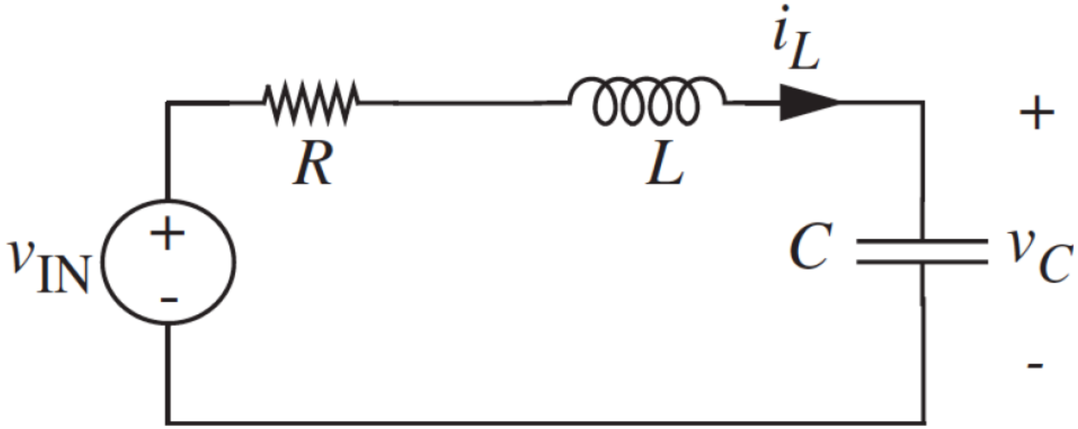
$$v_1(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$



**if critically damped, $A_1 e^{s_1 t} + A_2 \boldsymbol{t} e^{s_2 t}$

Intuitive Analysis – Driven Series RLC





Steps for Intuitive Analysis

1. Initial interval ($t \leq 0+$)
2. Final interval ($t \gg \infty$)
3. Find the initial trajectory of the transient
4. Find the frequency of oscillation (if any)
5. Find the approximate length of time over which the oscillations last (if any)

Steps for Analyzing these Circuits So Far:

If you're asked to find equation for, e.g. $v_c(t)$:

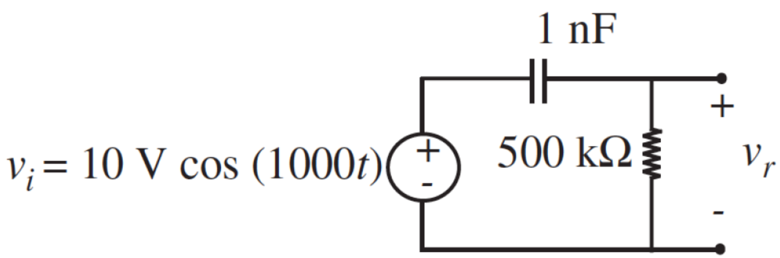
1. Find the ODE using the node method
2. Characteristic equation $s^2 + 2\alpha s + \omega_o^2 = 0 \rightarrow$ Roots + Response
3. General form*: $v_1(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (or $A_1 e^{s_1 t} + A_2 t e^{s_2 t}$) **[homogeneous]**
4. Find $v(0+)$ and $dv(0+)/dt$ based on inspecting circuit. Relate back to v(t) to find constants A_1 and A_2
5. Determine instantaneous power, energy dissipated

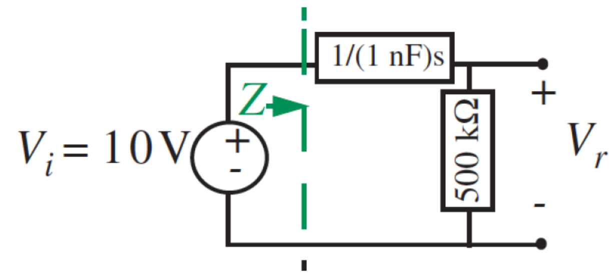
For driven circuit, try intuitive analysis:

1. Initial interval $(t \leq 0+)$ [**FIND**: $v_C(0+)$ & $dv_C(0+)/dt$; same for i_L; KVL + KCL]
2. Final interval $(t \gg \infty)$ [**FIND**: $v_C(\infty)$; $i_L(\infty)$]
3. Inspect circuit to determine quantities of interest:
    1. α, ω_0, ω_d
    2. Damping condition
    3. Q, period = $2\pi/\omega_d$ (if applicable)
    4. Total solution = homogeneous + particular [$v_c(\infty)$ for step input]

The Concept of Impedance

Impedance: A generalization of resistance for sinusoidal steady state inputs. Associated with each element is a characteristic impedance that affects the flow of current/voltage drop across the element, analogous to a resistor.





$$v_0(t) = |V_0|\cos(\omega t + \angle V_0) = Re\{|V_0|e^{j\angle V_0}\cancel{e^{j\omega t}}\}$$

Generalization of Ohm's Law for Sinusoidal Steady State

Resistor: $V = IR \rightarrow Z_R = R$

Capacitor: $V = I\frac{1}{sC} \rightarrow Z_C = \frac{1}{sC} = \frac{1}{j\omega C}$

Inductor: $V = sLI \rightarrow Z_L = sL = j\omega L$

Impedance Method

$$v_0(t) = |V_0| \cos(\omega t + \angle V_0) = Re\{V_0(j\omega)e^{j\omega t}\}$$

1. First, **replace** the (sinusoidal) sources by the complex amplitudes
2. Then, replace the circuit elements by their impedances. The resulting diagram is called **the impedance model of the network.**
3. Now, **determine the complex amplitudes** of the voltages and currents in the circuits (e.g. $V_c(j\omega)$). You can use any standard linear circuit analysis technique you like—Node method, KVL, KCL, Thevenin, intuitive method based on series/parallel simplifications, etc.
4. Finally, we can obtain the time variables from the complex amplitudes and **plug into the general expression for the dynamics**. This isn't usually necessary though. As an example, the time domain response corresponding to the node variable V_c is given by: