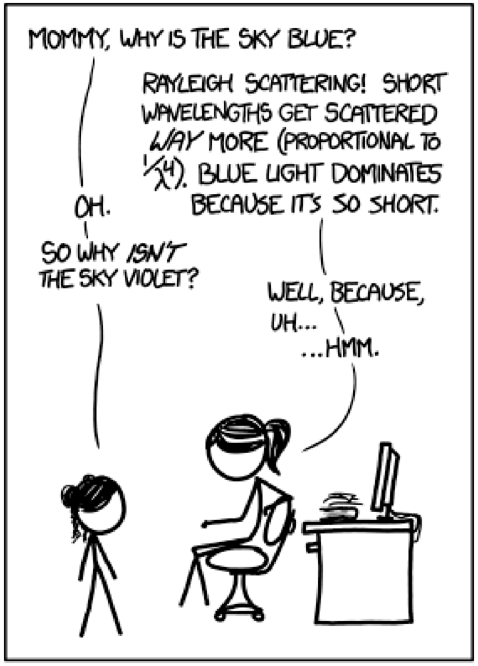
This has nothing to do with our class, but I like this question. First correct answer in the chat in the next 5 minutes gets an extra quiz point (that's worth about 0.04% of your grade!)



MY HOBBY: TEACHING TRICKY QUESTIONS TO THE CHILDREN OF MY SCIENTIST FRIENDS.

ECE 10C
Fall 2020
Slide Set 3
Instructor: Galan Moody
TA: Kamyar Parto

Tuesday
- Undriven RLC circuits

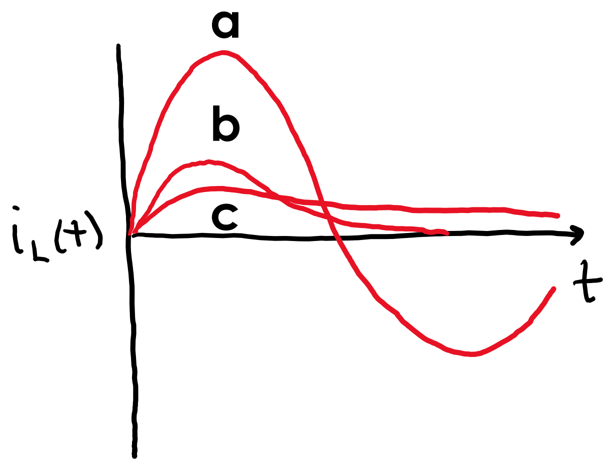
Today
- Review
- RLC circuit step response

Important Items:

- **HW #2 due FRIDAY 5pm**
- **Lab #2 due MONDAY now**
- **In 2 weeks**
  - **Lab # 3 due**
  - **HW # 3 due**
  - **Midterm**

Last Week's Quiz

[Q1] [3 points] In the diagram below for the current through an inductor in a series RLC circuit, pick the answer that corresponds to under-damped, over-damped, and critically damped (in that order):



a) underdamped
b) critically damped
c) overdamped

[Q2] [2 points] Without damping ($\alpha = 0$), the resonance frequency of an RLC circuit is $\omega_0 = 1/\sqrt{LC}$. For $\alpha \neq 0$ and smaller than ω_0, choose the correct answer:

~~**(a)** ω_0 is unaffected~~

(b) ω_0 is smaller

~~**(c)** ω_0 is larger~~

$$s_{1,2} = -\alpha \pm j\omega_d \qquad \omega_d \equiv \sqrt{\omega_0^2 - \alpha^2}$$

Typo: should have written "resonance frequency" instead of ω_0

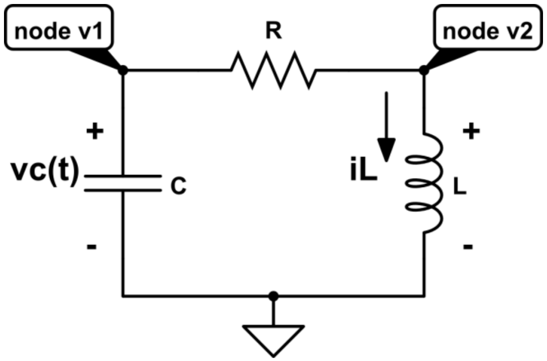
Quiz Time!

Informal course assessment: It's challenging to get a pulse on the class in a virtual setting, so I'd like your input to see how things are going.

Responses are anonymous (I promise!) Don't feel obligated to respond if you'd rather not. Everyone here will get full credit for today's quiz.

Please answer honestly: I want to use your feedback to improve the course and lectures for the rest of the quarter, where possible. If you have additional feedback on my instruction, the lectures, content, etc., don't hesitate to email me. I really value your input to help improve!

Review: Series RLC



- Node 1 KCL: $$-C\frac{dv_1(t)}{dt} - \frac{v_1(t) - v_2(t)}{R} = 0$$

- Node 2 KCL: $$\frac{v_1(t) - v_2(t)}{R} - \frac{1}{L}\int_{-\infty}^{t} v_2(t')dt' = 0$$

$$\boxed{\frac{d^2v_1(t)}{dt^2} + \frac{R}{L}\frac{dv_1(t)}{dt} + \frac{1}{LC}v_1(t) = 0}$$

Review: Series RLC

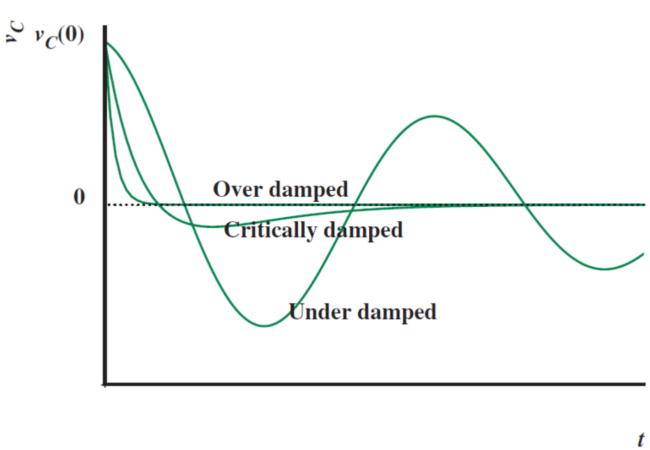
$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

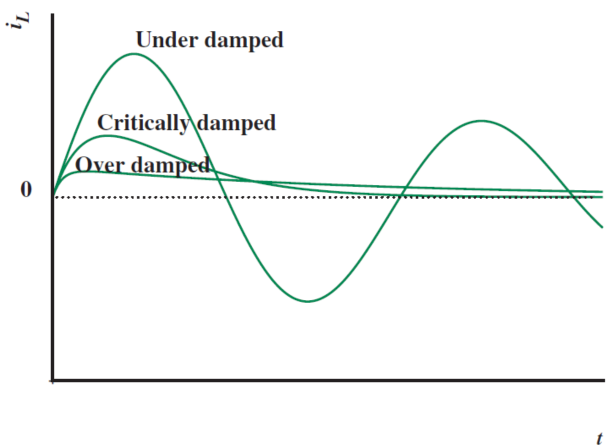
$$y(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \alpha = \frac{R}{2L}$$

***unless $s_1 = s_2$**

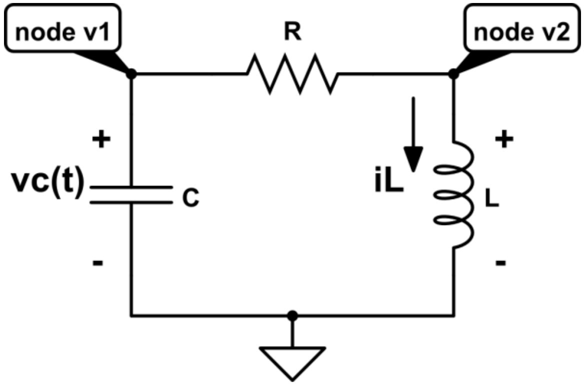
The dynamic behavior of an RLC circuit is different for three cases:

$\boldsymbol{\alpha < \omega_0} \rightarrow$ **under-damped dynamics**
$\boldsymbol{\alpha = \omega_0} \rightarrow$ **critically damped dynamics**
$\boldsymbol{\alpha > \omega_0} \rightarrow$ **over-damped dynamics**





Review: Series RLC



$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$y(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \alpha = \frac{R}{2L}$$

For 2nd-order (RLC) circuits, we need to find:

$$i_L(0^-) = i_L(0^+) \qquad \frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C}$$

$$v_c(0^-) = v_c(0^+) \qquad \frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L}$$

1. Once we find $v_C(0)$ and $i_L(0)$, then we can use capacitor element law law to determine $dv_C(0)/dt$ using $i_C(0)$
2. We can use the inductor element law to determine $di_L(0)/dt$ using $v_L(0)$

Review: Parallel RLC

- We can also write the characteristic equation as:

$$s^2 + 2\alpha s + \omega_o^2 = 0$$

- Where we have defined:

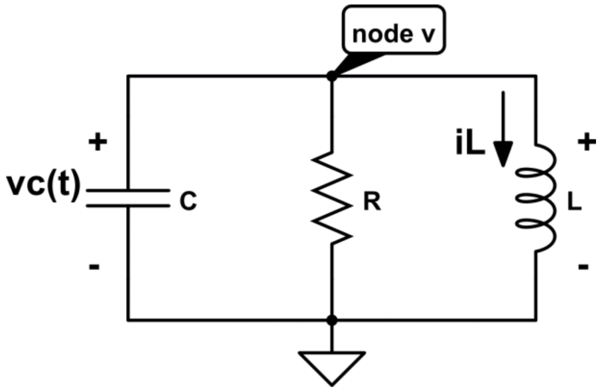
$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

- The roots of the characteristic equation are:

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$y(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$



Review: Let's Put it All Together

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\alpha = \frac{R}{2L}$$

(series RLC)

$$\alpha = \frac{1}{2RC}$$

(parallel RLC)

$\alpha < \omega_0 \rightarrow$ under-damped dynamics

$$y(t) = e^{-\alpha t}(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t})$$

$$s_{1,2} = -\alpha \pm j\omega_d$$

$$\omega_d \equiv \sqrt{\omega_0^2 - \alpha^2}$$

$$Q \equiv \frac{\omega_0}{2\alpha}$$

Amplitude decays to ~4% after Q oscillations

$\alpha = \omega_0 \rightarrow$ critically damped dynamics

$$y(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t}$$

$$s_1 = s_2 = -\alpha$$

$s_{1,2}$ are both real #s

$\alpha > \omega_0 \rightarrow$ over-damped dynamics

$$y(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

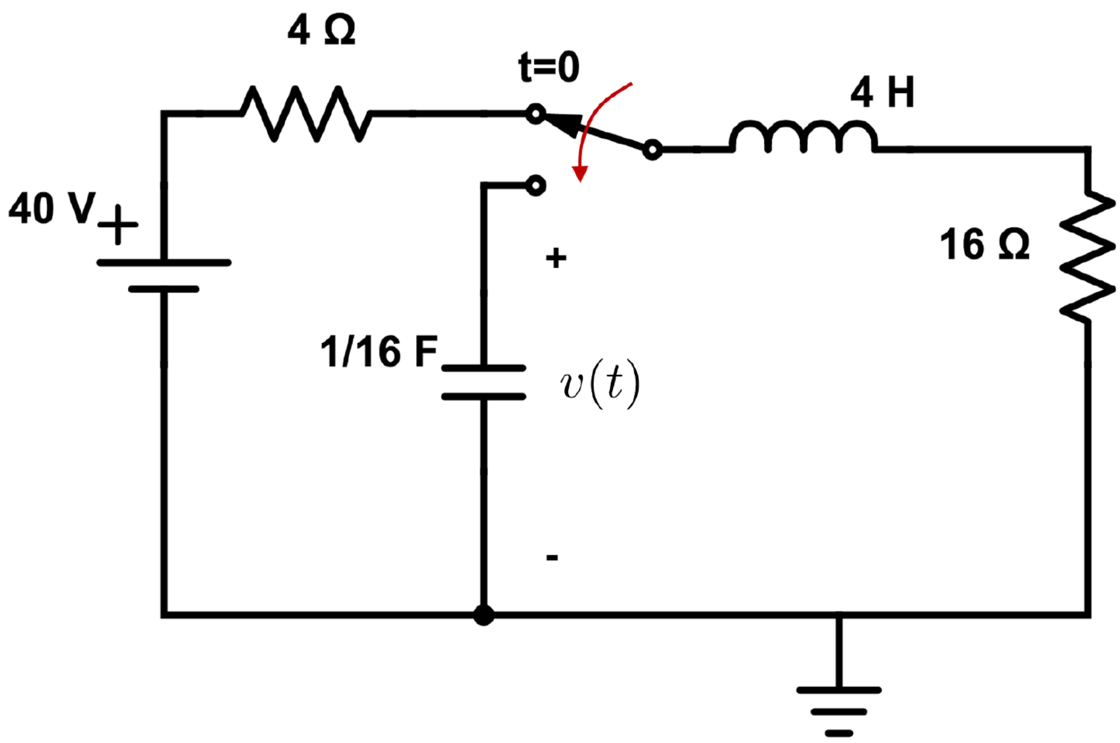
$s_{1,2}$ are both real #s

In-Class Exercise

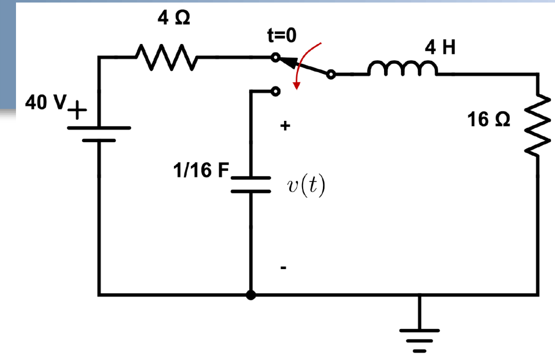
The switch has been at position 1 for all $t < 0$. At time $t = 0$, the switch flips to position 2. Assume v(0) = 0.

Find i(0).

Determine v(t) for t > 0.



In-Class Exercise



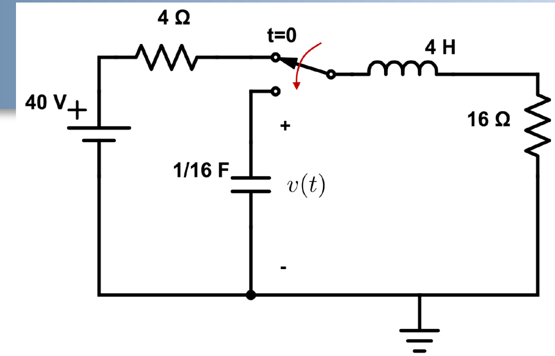
The switch has been at position 1 for all t < 0. At time t = 0, the switch flips to position 2. Assume v(0) = 0.

Find i(0).Determine v(t) for t > 0.

$i_L(0\text{-})$: $\frac{40V}{20\Omega} = 2A$

$\alpha = \frac{R}{2L} = 2$; $\omega_0 = \frac{1}{\sqrt{LC}} = 2$

In-Class Exercise



The switch has been at position 1 for all $t < 0$. At time $t = 0$, the switch flips to position 2. Assume $v(0) = 0$.

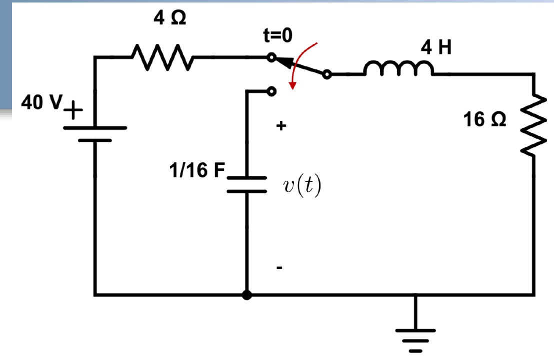
Find i(0).Determine v(t) for t > 0.

$i_L(0-)$: $\frac{40V}{20\Omega} = 2A$

$\alpha = \frac{R}{2L} = 2$; $\omega_0 = \frac{1}{\sqrt{LC}} = 2$

critically damped!

In-Class Exercise



The switch has been at position 1 for all $t < 0$. At time $t = 0$, the switch flips to position 2. Assume v(0) = 0.

Find i(0).Determine v(t) for t > 0.

$i_L(0-)$: $\frac{40V}{20\Omega} = 2A$

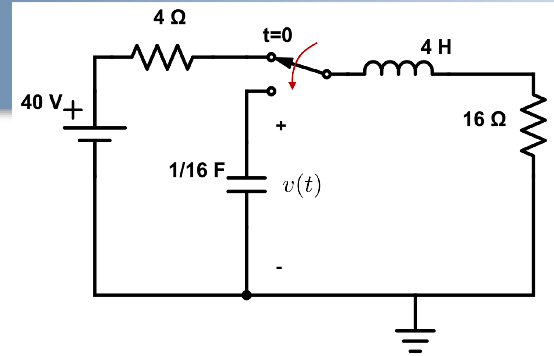
$\alpha = \frac{R}{2L} = 2$; $\omega_0 = \frac{1}{\sqrt{LC}} = 2$

critically damped!

$V_c(t) = A_1 e^{-2t} + A_2 t e^{-2t}$

with v(0) = 0, then: $A_1 = 0$.

In-Class Exercise



The switch has been at position 1 for all $t < 0$. At time $t = 0$, the switch flips to position 2. Assume v(0) = 0.

Find i(0).Determine v(t) for t > 0.

$i_L(0-)$: $\frac{40V}{20\Omega} = 2A$

$\alpha = \frac{R}{2L} = 2$; $\omega_0 = \frac{1}{\sqrt{LC}} = 2$

critically damped!

$V_c(t) = A_1 e^{-2t} + A_2 t e^{-2t}$

with v(0) = 0, then: $A_1 = 0.$

$V_c(t) = A_2 t e^{-2t}$

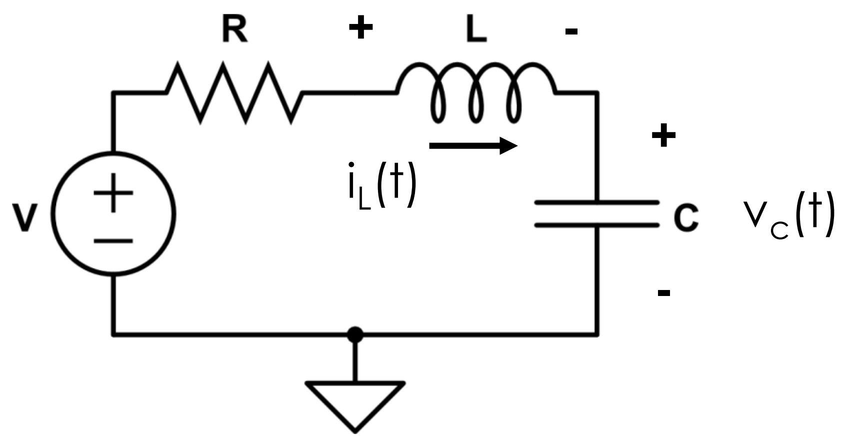
What is A_2?

$\rightarrow A_2 = \frac{dV_c(0)}{dt} = \frac{i_c(0)}{C} = \frac{-i_L(0)}{C} = \frac{-2A}{C}$

What's Next

- We'll move to driven RLC circuits (*i.e.* step source)
  - "Intuitive" method for analyzing circuits
- Next week:
  - RLC circuits with a sinusoidal input signal
  - A new method to analyze these circuits (impedance method) – it'll simplify things
- Following week: Review + Midterm
- Then:
  - Frequency domain analysis, plotting circuit frequency response, and filter design
  - Real-world examples related to signal processing, communications, quantum circuits and devices

Driven LC Circuit: 2nd-Order Transients



- Using the node method, we get:

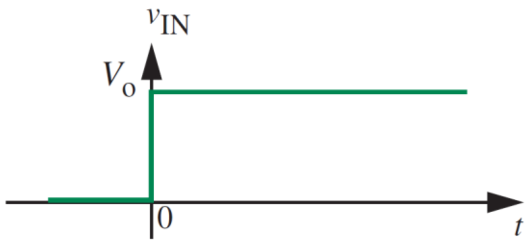
$$\frac{d^2v_C(t)}{dt^2} + \frac{R}{L}\frac{dv_C(t)}{dt} + \frac{1}{LC}v_C(t) = \frac{1}{LC}v_{IN}(t)$$

- **Let's assume the circuit is <u>under-damped</u> and <u>at-rest</u>:**

$$v_{CH}(t) = e^{-\alpha t}(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}) \qquad v_C(0) = i_L(0) = 0$$

Let's find the step response to $v_{IN}(t) = V_0 u(t)$ → need particular response

Driven LC Circuit: 2nd-Order Transients

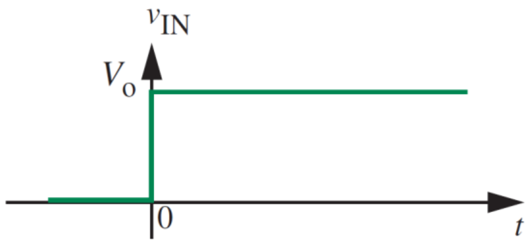


- Find the particular solution that satisfies:

$$\frac{d^2 v_C(t)}{dt^2} + \frac{R}{L}\frac{dv_C(t)}{dt} + \frac{1}{LC}v_C(t) = \frac{1}{LC}V_0$$

$$v_{CP}(t) =$$

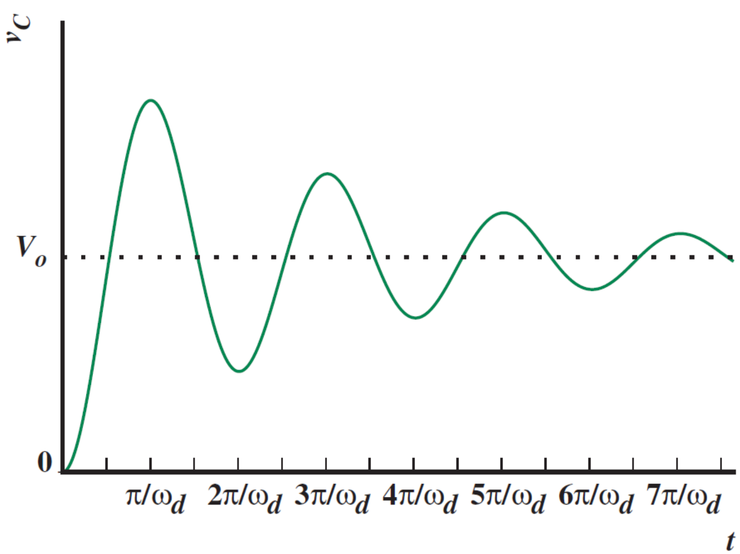
Driven LC Circuit: 2nd-Order Transients

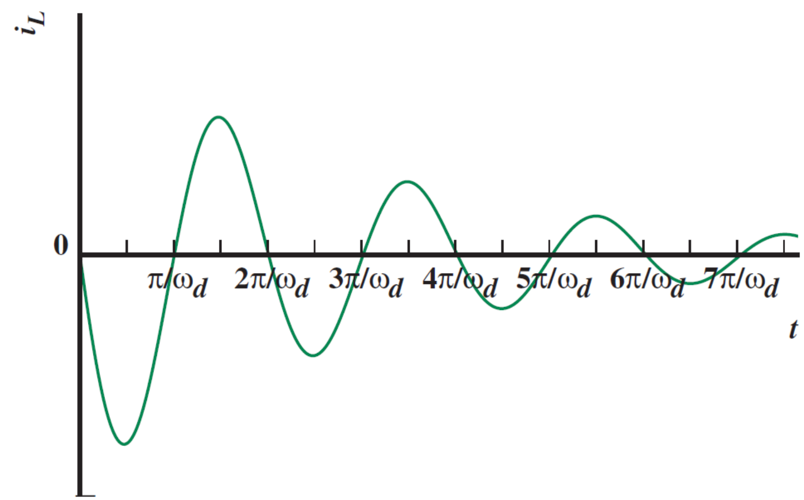


- Find the particular solution that satisfies:

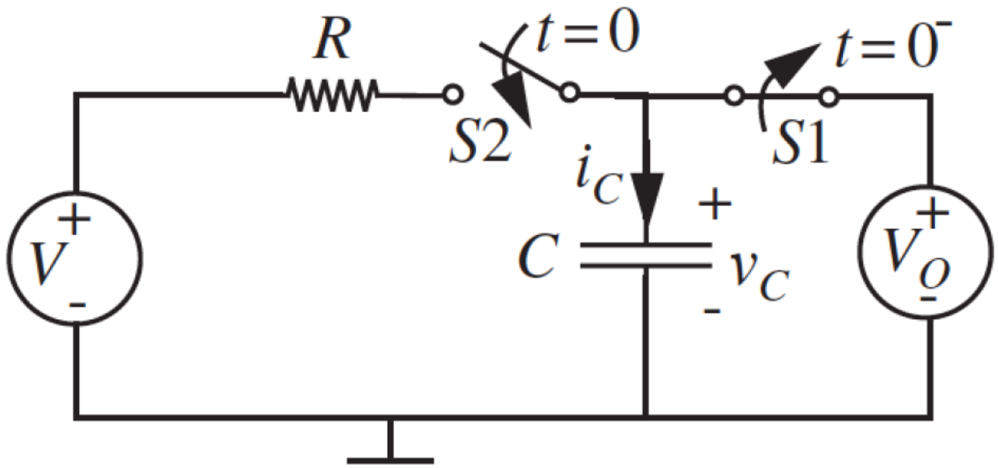
$$\frac{d^2 v_C(t)}{dt^2} + \frac{R}{L}\frac{dv_C(t)}{dt} + \frac{1}{LC}v_C(t) = \frac{1}{LC}V_0$$

$v_{CP}(t) = K = V_0$ **total solution:** $v_C(t) = v_{CH}(t) + v_{CP}$





Intuitive Analysis – RC Step Response

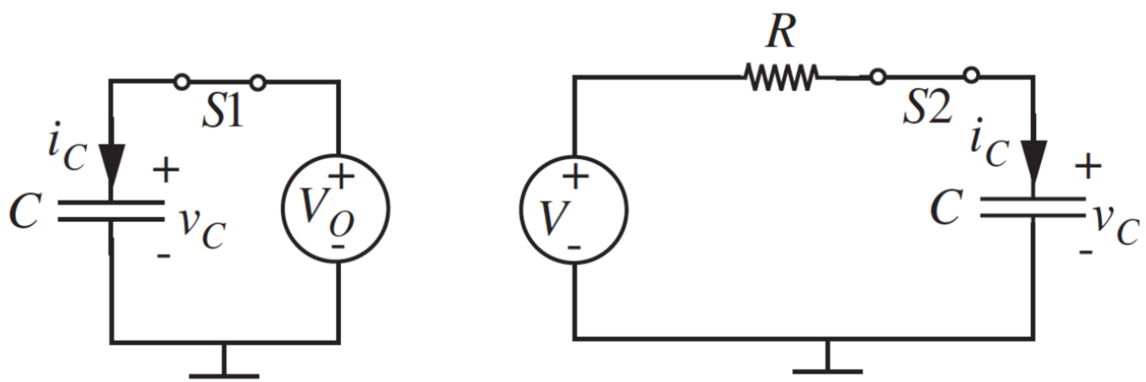


Steps for Intuitive Analysis

1. Initial interval $(t \leq 0+)$
2. Final interval $(t \gg \infty)$
3. Transition interval $(t > 0+)$ → need to have an understanding of how the circuit will behave

Intuitive Analysis – RC Step Response

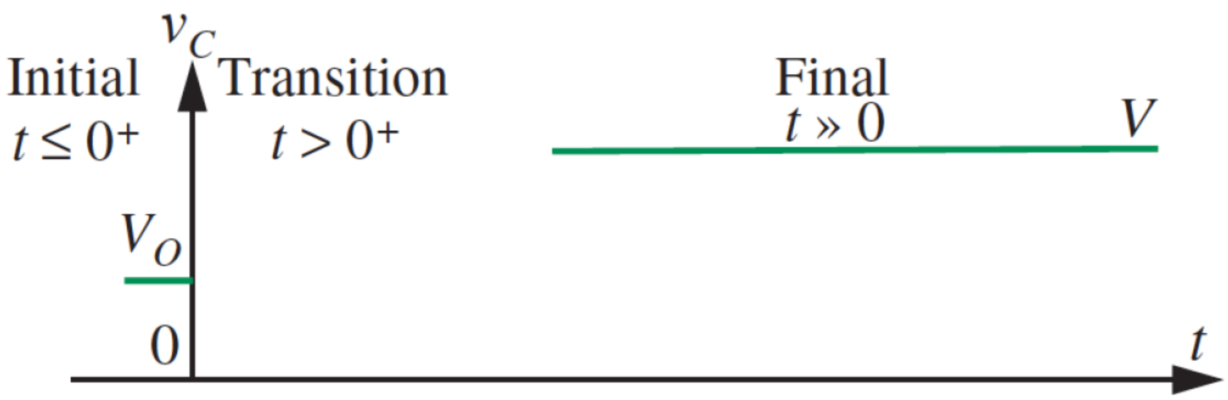
$t \leq 0-$ $\qquad$ $t \rightarrow \infty$



Steps for Intuitive Analysis

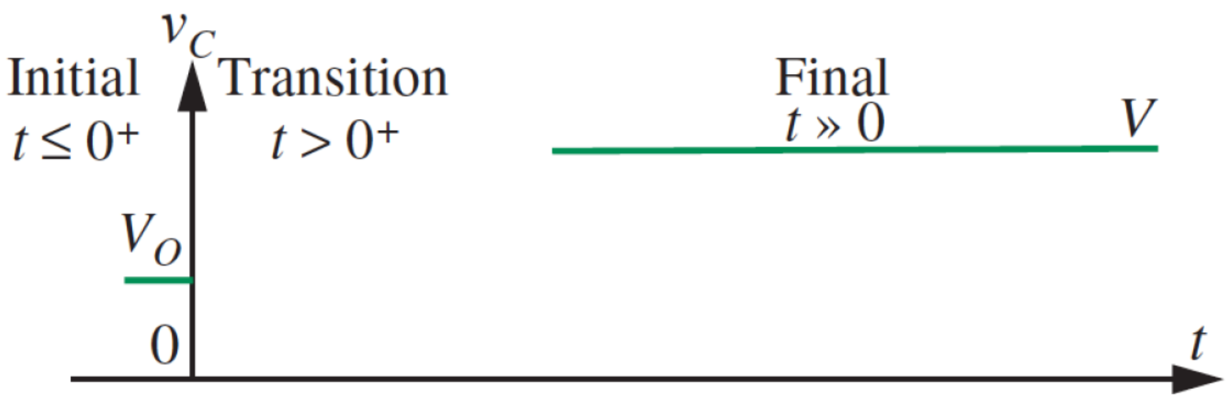
1. Initial interval ($t \leq 0+$)
2. Final interval ($t \gg \infty$)
3. Transition interval ($t > 0+$) → need to have an understanding of how the circuit will behave

Intuitive Analysis – RC Step Response

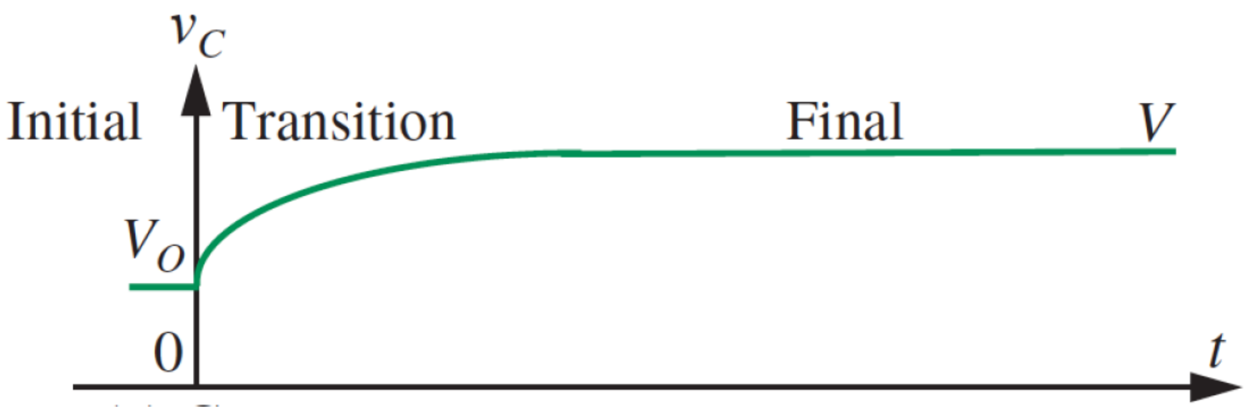


We know that for an RC circuit the transient follows an exponential form of either rising $(1 - e^{-t/RC})$ or falling $(e^{-t/RC})$ with time constant *RC*

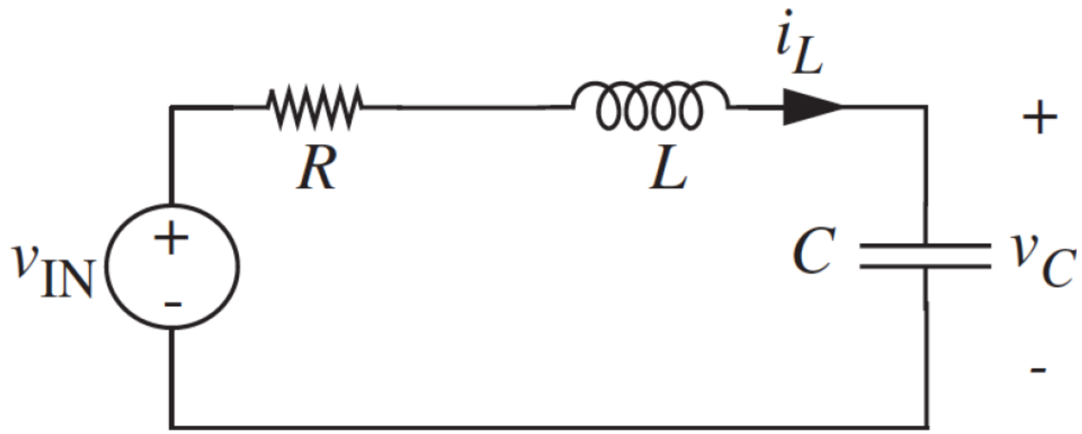
Intuitive Analysis – RC Step Response



We know that for an RC circuit the transient follows an exponential form of either rising $(1 - e^{-t/RC})$ or falling $(e^{-t/RC})$ with time constant *RC*



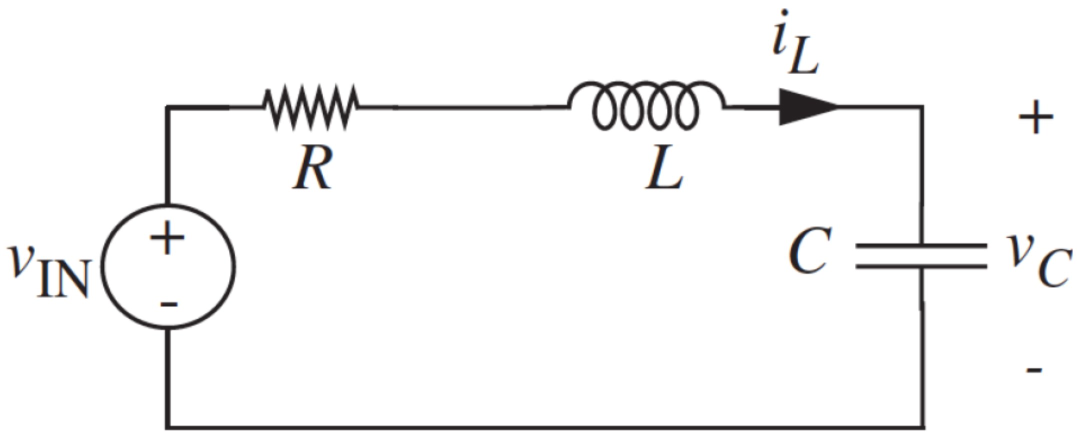
Intuitive Analysis – Driven Series RLC



Steps for Intuitive Analysis

1. Initial interval ($t \leq 0+$)
2. Final interval ($t \gg \infty$)
3. Find the initial trajectory of the transient
4. Find the frequency of oscillation (if any)
5. Find the approximate length of time over which the oscillations last (if any)

Intuitive Analysis – Driven Series RLC



- Suppose that:

$$L = 100\ \mu H$$
$$C = 100\ \mu F$$
$$R = 0.2\ \Omega$$

- With initial conditions:

$$v_C(0) = 0.5V$$
$$i_L(0) = -0.5A$$

- And an input step response:

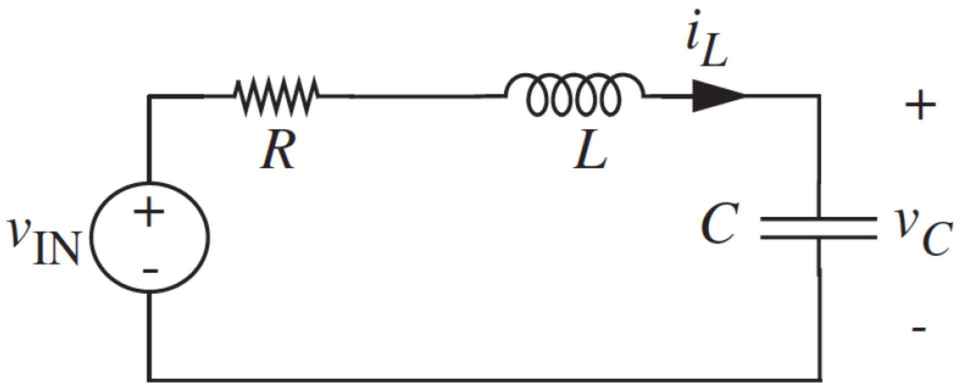
$$v_{IN} = 1V$$

Intuitive Analysis – Driven Series RLC

Initial Interval

No instantaneous jump in v_c and i_L

$$v_c(0+) = 0.5\ \mathrm{V}$$
$$i_L(0+) = -0.5\ \mathrm{A}$$



$$v_C(0) = 0.5V$$
$$i_L(0) = -0.5A$$

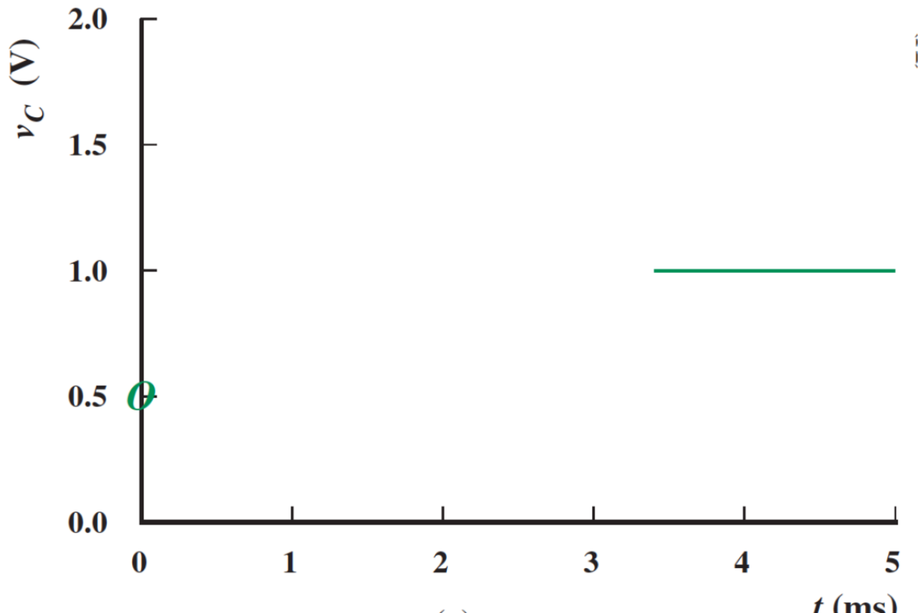
$$v_{IN} = 1V$$

Final Interval

C = open circuit in steady state

$$v_c(\infty) = 1\ \mathrm{V}$$
$$i_L(\infty) = 0\ \mathrm{A}$$

Intuitive Analysis – Driven Series RLC



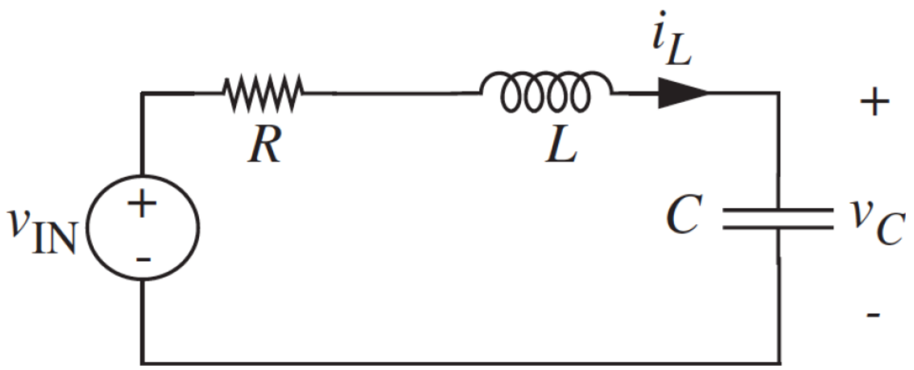
(a)

Circuit Parameters to Determine Transient Behavior

$$\alpha = \frac{R}{2L} = 10^3 \ 1/\text{s} \qquad \omega_0 = \sqrt{\frac{1}{LC}} = 10^4 \text{ rad/s} \qquad \boldsymbol{\alpha < \omega_0}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \approx 9950 \text{ rad/s} \qquad Q = \frac{\omega_0}{2\alpha} = 5$$

Transition Analysis

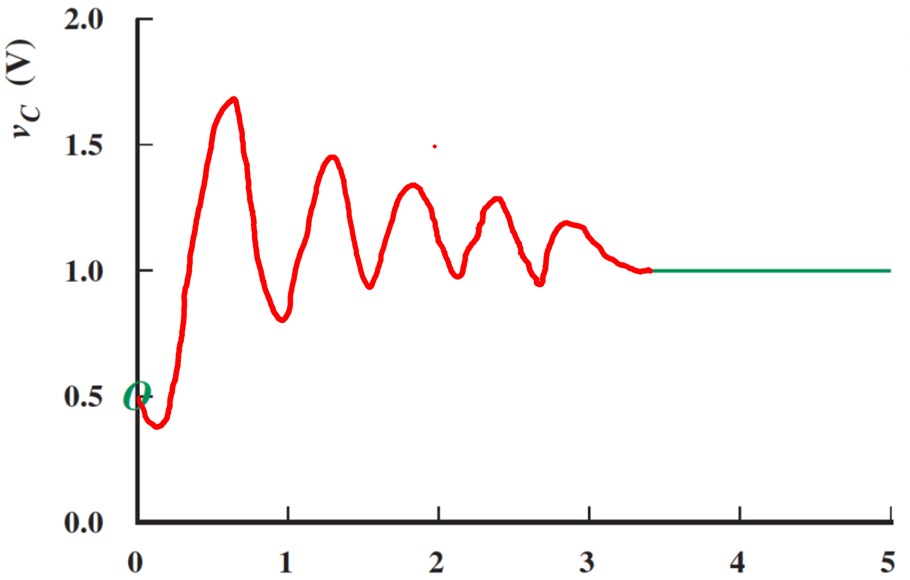


Remember: $i_L(0)$ = -0.5 A, thus capacitor will discharge initially, so $v_c(t)$ will *decrease*

Steps for Intuitive Analysis

1. Initial interval $(t \leq 0+)$
2. Final interval $(t \gg \infty)$
3. **Find the initial trajectory of the transient**
4. Find the frequency of oscillation (if any)
5. Find the approximate length of time over which the oscillations last (if any)

Transition Analysis



(a)

What We Know

Oscillation period = $\frac{2\pi}{9950} \approx 0.6$ ms and oscillations last ~ Q = 5 cycles

We won't work on finding the amplitude (too complex)

Summary

You can find the transient response of any quantity:

1. Find the ODE using the node method
2. Characteristic equation $s^2 + 2\alpha s + \omega_o^2 = 0 \rightarrow$ Roots + Response
3. Based on $v(0)$ and $dv(0)/dt \rightarrow$ Find constants

Steps for Intuitive Analysis

1. Initial interval $(t \leq 0+)$
2. Final interval $(t \gg \infty)$
3. Find the initial trajectory of the transient
4. Find the frequency of oscillation (if any)
5. Find the approximate duration for the oscillations last (if any)

Remember: HW #2 due Friday, 5pm.
Read up for next week: Sinusoidal sources with RLC, impedance method
HW #3 will be posted Tuesday, due on 11/5 (day of midterm).