ECE 10C
Fall 2020
Slide Set 9
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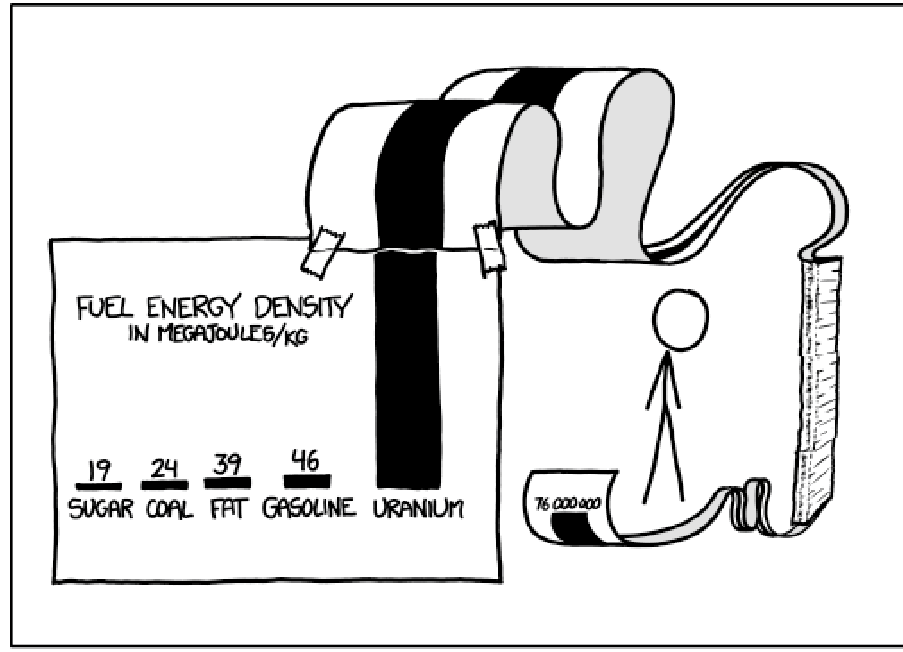


SCIENCE TIP: LOG SCALES ARE FOR QUITTERS WHO CAN'T FIND ENOUGH PAPER TO MAKE THEIR POINT *PROPERLY.*

Last Week
- Op Amps
- Transfer Function

This Week
- **Bode Plot**
- Filter Design
- Fast Fourier Transform

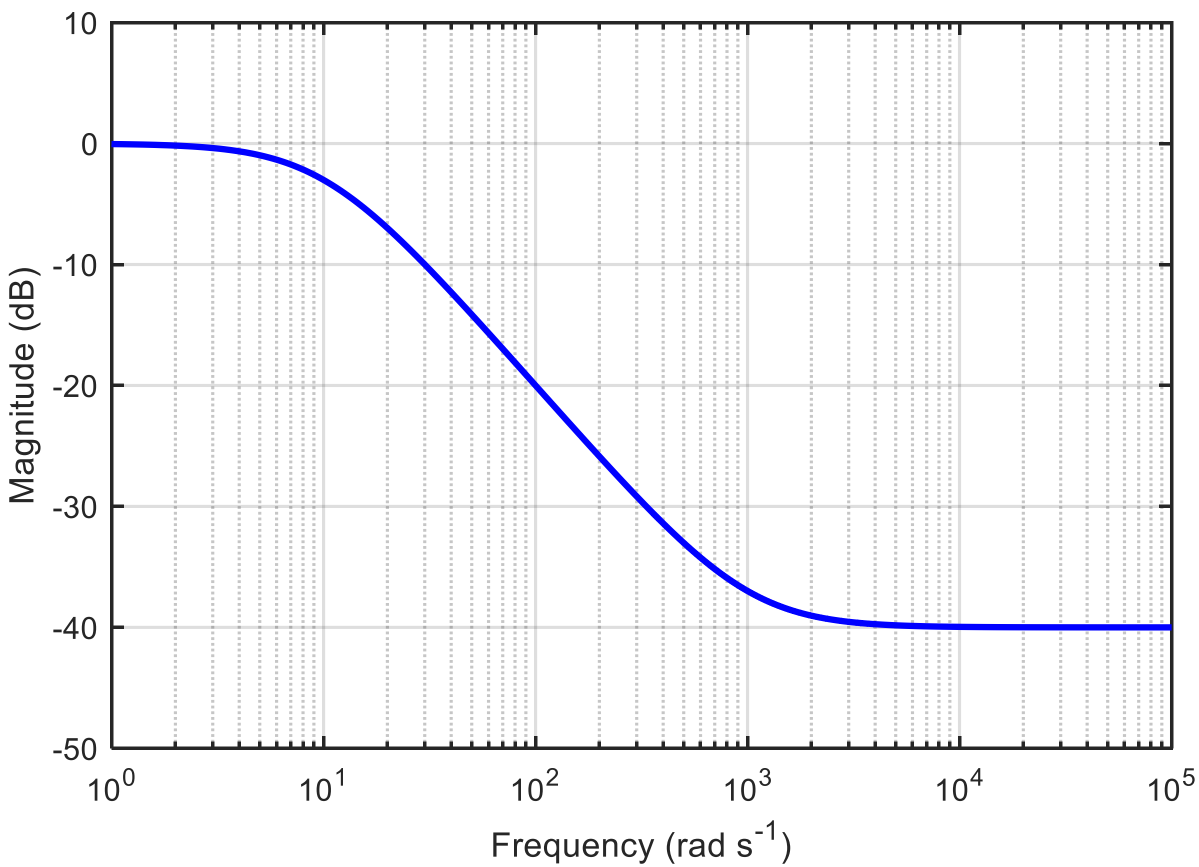
Important Items:

- HW #4 due Thurs, 11/19
- Lab #4 due 11/20

Quiz Time!

Q1: True or False: The transfer function is the ratio of the complex amplitude of a circuit's input voltage to the circuit's output voltage.

Q2: The figure below shows the magnitude of a transfer function versus frequency on a log-log plot. Pick the correct expression that matches the figure.



a) $$|H(j\omega)| = \frac{1}{1 + \frac{\omega}{10^1}}$$

b) $$|H(j\omega)| = 1 + \frac{\omega}{10^1}$$

c) $$|H(j\omega)| = \frac{1 + \frac{\omega}{10^3}}{1 + \frac{\omega}{10^1}}$$

d) $$|H(j\omega)| = \frac{1 + \frac{\omega}{10^1}}{1 + \frac{\omega}{10^3}}$$

Summary

typically V_0 or I_0

transfer function = $H(j\omega) = \dfrac{\textit{complex amplitude of the output}}{\textit{complex amplitude of the input}}$

$$v_i(t) = V_i \cos(\omega t) \longrightarrow \boxed{H(j\omega)} \longrightarrow v_o(t) = |H(j\omega)| V_i \cos\left(\omega t + \angle H(j\omega)\right)$$

When asked to plot the frequency response of a system, you need to draw the approximate sketches of the magnitude and phase as follows:

Plotting the Frequency Response

- Plot $|H(j\omega)|$: the log magnitude plotted against log frequency
- Plot $\angle H(j\omega)$: the angle in linear scale plotted against log frequency

Intuitive Sketching, Magnitude $|H(j,\omega)|$

- First, find the asymptotes of the magnitude plot:

$$|H(j\omega)| = \left| \frac{\frac{R}{L}}{\frac{R}{L} + j\omega} \right|$$

- At low frequencies ($\omega \to 0$):

$$|H(j\omega)| \approx 1$$

  - Hence the magnitude appears as a horizontal line at low frequencies

- At high frequencies $\omega \gg R/L$, ω dominates in the denominator

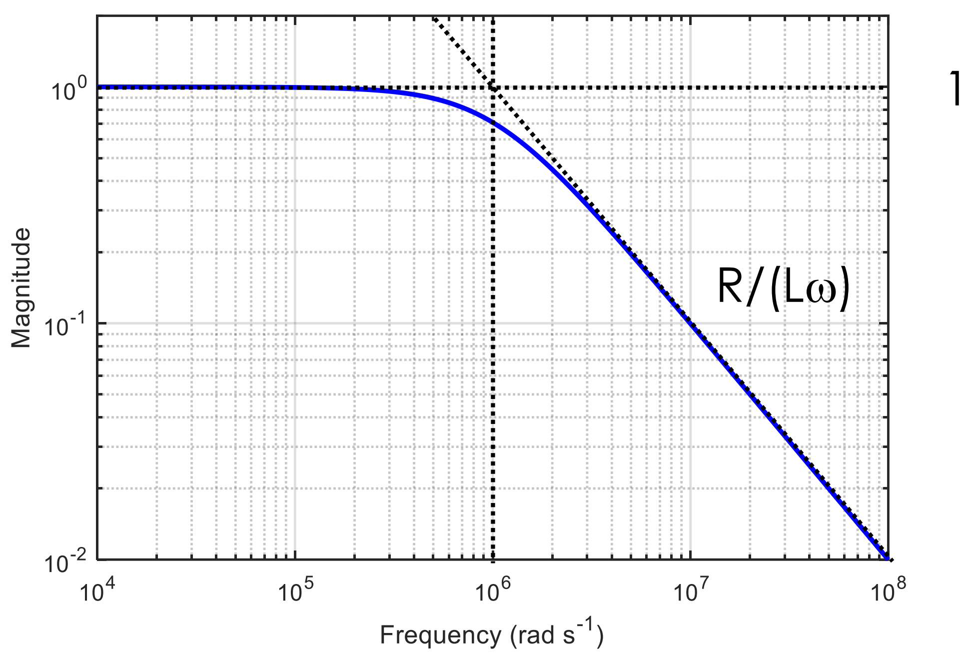
$$|H(j\omega)| \approx \frac{R/L}{\omega} \to \log|H(j\omega)| \approx \log R/L - \log \omega$$

  - On a log scale, a line with slope = -1 passing through (R/L, 1)

Intuitive Sketching, Magnitude $|H(j,\omega)|$

- At low frequencies ($\omega \to 0$): $|H(j\omega)| = 1$
- At high frequencies $\omega \to \infty$, ω dominates in the denominator

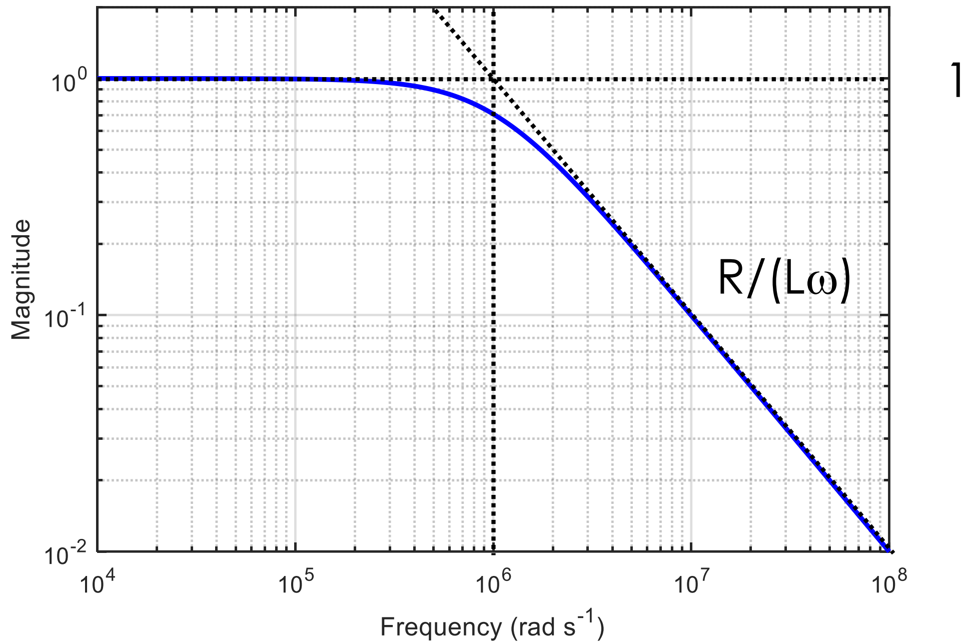
$$|H(j\omega)| \approx \frac{R/L}{\omega} \to \log|H(j\omega)| \approx \log R/L - \log \omega$$



Intuitive Sketching, Magnitude $|H(j,\omega)|$

- The two asymptotes intersect at: $\omega = \dfrac{R}{L}$
- This frequency is called the "corner" or "cutoff" frequency

> At the cutoff frequency, the real and imaginary components of $H(j,\omega)$ are equal, and $|H(j,\omega)| = \frac{1}{\sqrt{2}} = 0.707$



$$|H(j\omega)| \approx \frac{R/L}{\omega} \rightarrow \log|H(j\omega)| \approx \log R/L - \log\omega$$

Alternative: The Decibel Scale

The decibel (dB) is a logarithmic unit used to express the (dimensionless) ratio of two values of a physical quantity.

$$|H(s)| = \left|\frac{V_0}{V_i}\right|$$

Instead of plotting $log_{10}|H(s)|$, in Bode plots, we normally plot:

$$G_{dB} = 10\ log_{10}|H(s)|^2 = 10\ log_{10}\left|\frac{V_0^2}{V_i^2}\right| \text{ dB}$$

Why do we use the Decibel (dB) scale in this form?

- Power is proportional to the voltage squared

Example: $G_{dB} = 10\ log_{10}\left|\frac{1000\ W}{1\ W}\right| = 30\ dB$

- Each factor of 10 dB = order of magnitude

$$G_{dB} = 20\ log_{10}\ |H(s)| = 20\ log_{10}\left|\frac{V_0}{V_i}\right| \text{ dB}$$

Alternative: The Decibel Scale

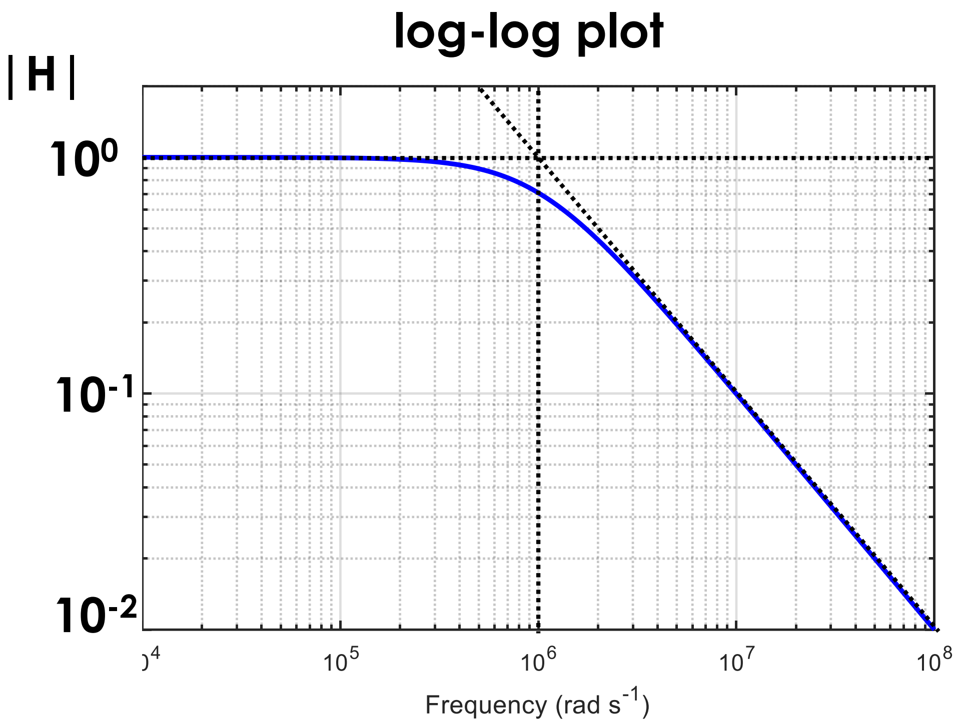
$$G_{dB} = 20\ log_{10}\ |H(s)| = 20\ log_{10}\left|\frac{V_0}{V_i}\right| \text{ dB}$$

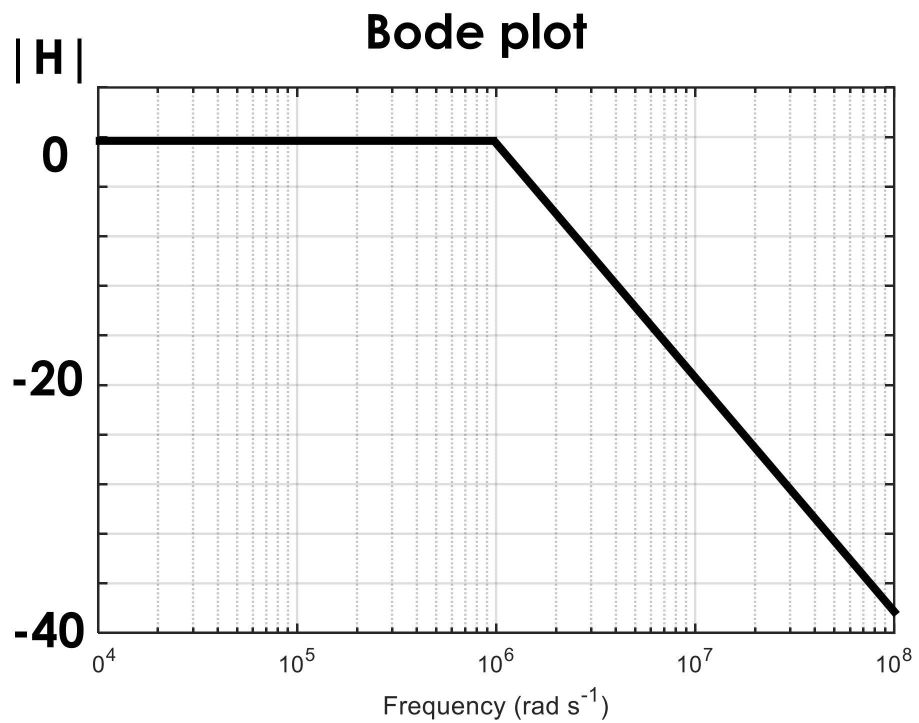
Some useful #s:

V	V²	
Root-Power Ratio	**Power Ratio**	**Decibels (dB)**
√P	**P**	**10*log(P)**
31.6228	1000	30.00
10.0000	100	20.00
3.1623	10	10.00
1.4142	2	3.01
1.0000	1	0.00
0.7071	0.5	-3.01
0.3162	0.1	-10.00
0.1000	0.01	-20.00
0.0316	0.001	-30.00

The Bode Plot

Graph of the frequency response of a system on a Decibel scale using asymptotic approximations and straight-line segments





Provides an easier way to break down a complex response into individual components, sum them, and plot an approximate response

The Bode Plot

Using asymptotes, we now can **approximately plot**
$H(s) \in \{1/(s+a), (s+a), s/(s+a), (s+a)/s\}$

How about more general frequency response forms?

$$H(s) = \frac{K_\circ s^l (s+a_1)(s+a_2)\dots(s^2+2\alpha_1 s+\omega_1^2)\dots}{(s+a_3)(s+a_4)\dots(s^2+2\alpha_2 s+\omega_2^2)\dots}$$

$$\begin{aligned}
log|H(s)| = {} & \log K_o + \\
& \log|s| + \log|s| + \cdots \ (l \text{ terms}) + \\
& \log|s+a_1| + \log|s+a_2| + \cdots - \log|s+a_3| - \log|s+a_4| + \cdots \\
& \log|s^2+2\alpha_1 s+\omega_1^2| + \cdots - \log|s^2+2\alpha_2 s+\omega_2^2| + \cdots
\end{aligned}$$

$$\begin{aligned}
\angle H(s) = {} & \angle K_o + \\
& \angle s + \angle s + \cdots \ (l \text{ terms}) \\
& \angle(s+a_1) + \angle(s+a_2) + \qquad + a_3) - \angle(s+a_4) - \cdots \\
& + \angle(s^2+2\alpha_1 s+\omega_1^2) + \cdots - \angle(s^2+2\alpha_2 s+\omega_2^2) - \cdots
\end{aligned}$$

The Bode Plot

$$G_{dB} = 20\, log_{10}\, |H(s)|\ \text{dB}$$

Bode plot terms		
Term	Magnitude	Phase
Constant: K	$\log(\|K\|)$	0 if $k > 0$, $\pm 180^\circ$ if $k < 0$
s	line with slope $20dB$ passing through 0 at $\omega = 1$	$+90^\circ$
$\frac{1}{s}$	line with slope $-20dB$ passing through 0 at $\omega = 1$	-90°
$\frac{1}{1+\frac{s}{\omega_\circ}}$	Draw low frequency asymptote (constant = 0 dB) and high frequency asymptote (line with slope $-20dB$). Connect lines at $\omega_\circ$	Draw low frequency asymptote at 0°. Draw high frequency asymptote at -90°. Connect with a straight line from 0.1 $\omega_\circ$ to 10 $\omega_\circ$
$1 + \frac{s}{\omega_\circ}$	like above but high frequency asympotote has slope $20dB$	like above but high frequency asymptote at $+90^\circ$

The Bode Plot: Rules

- Determine the transfer function: $$H(s)=\frac{K(s+z_1)}{s(s+p_1)}$$
- Rewrite it by factoring numerator and denominator into "standard" form.
  - The "z's" are called zeros
  - The "p's" are called poles

$$H(s)=\frac{Kz_1(s/z_1+1)}{sp_1(s/p_1+1)}$$

- Replace ***s*** with ***jω***. Find the magnitude of the transfer function.
- Take $\log_{10}$ and multiply by 20.

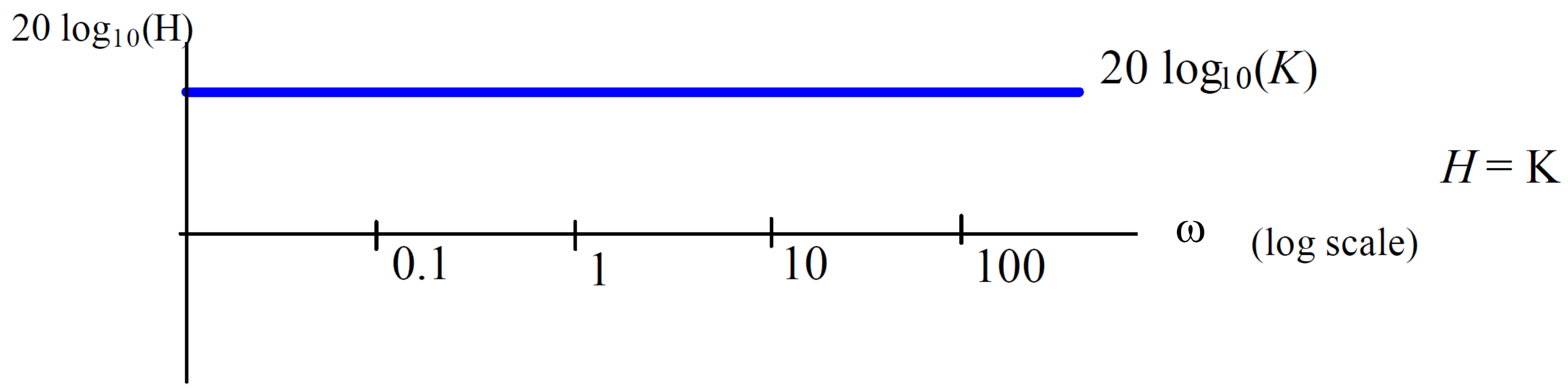
$$20\log_{10}(H(jw)) = 20\log_{10}\left(\frac{Kz_1(jw/z_1+1)}{jwp_1(jw/p_1+1)}\right)=$$

$$20\log_{10}|K|+20\log_{10}|z_1|+20\log_{10}\left|(jw/z_1+1)\right|-20\log_{10}|p_1|-20\log_{10}|jw|-20\log_{10}\left|(jw/z_1+1)\right|$$

- Each of these terms is fairly straightforward to show on a log scale. For the Bode plot, graph each one individually, and then connect the straight-line segments. With a little practice, we can do this quickly. Let's look at each term.

<u>Effect of Constant Terms:</u>

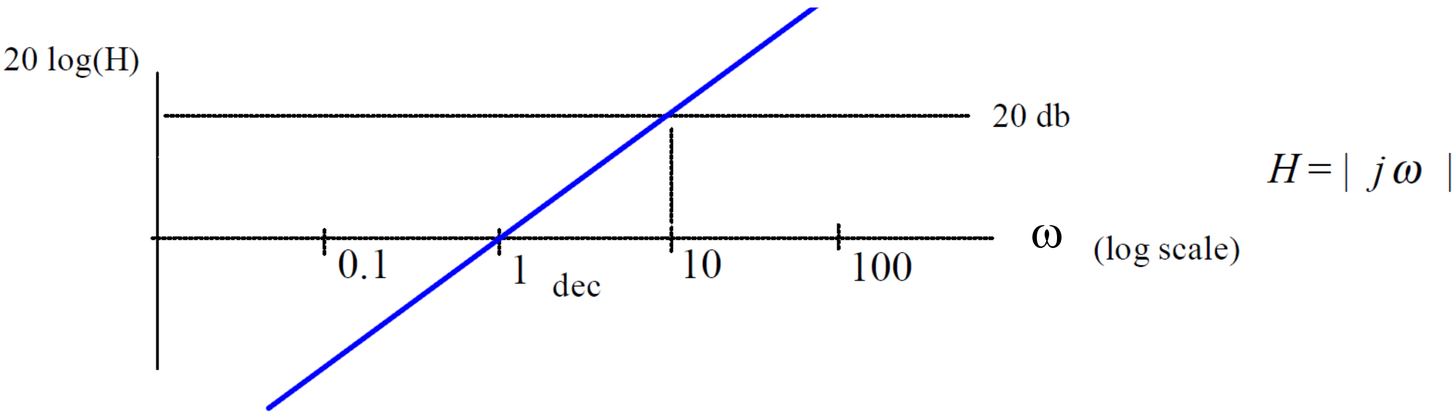
Constant terms such as ***K*** contribute a straight horizontal line of magnitude 20 $\log_{10}$(K)



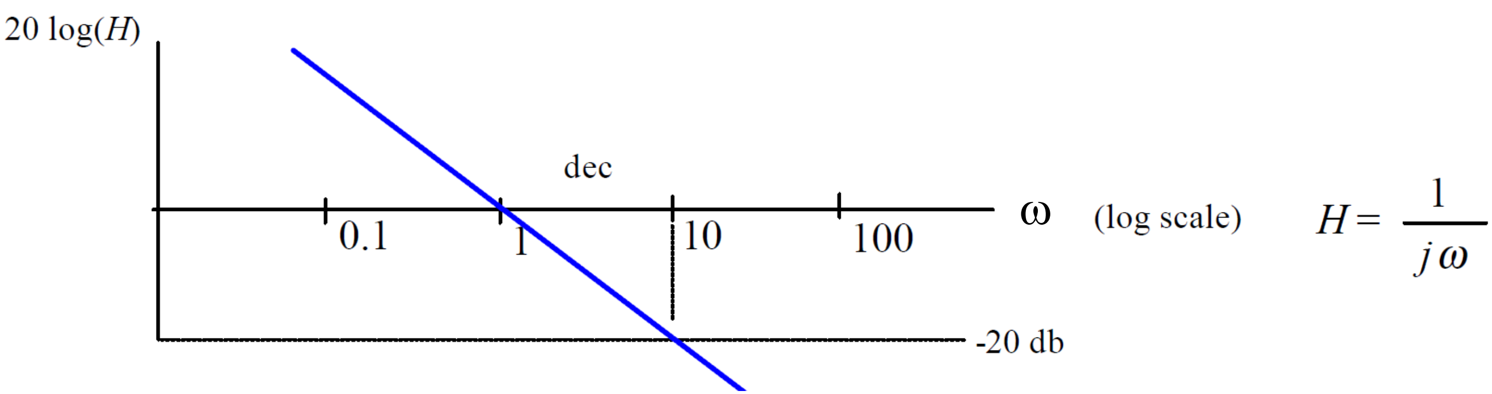
The Bode Plot: Rules

Effect of Individual Zeros and Poles at the Origin:

A **zero** at the origin occurs when there is an ***s*** or ***jω*** multiplying the numerator. Each occurrence of this causes a positively sloped line passing through ω = 1 with a rise of 20 dB/decade



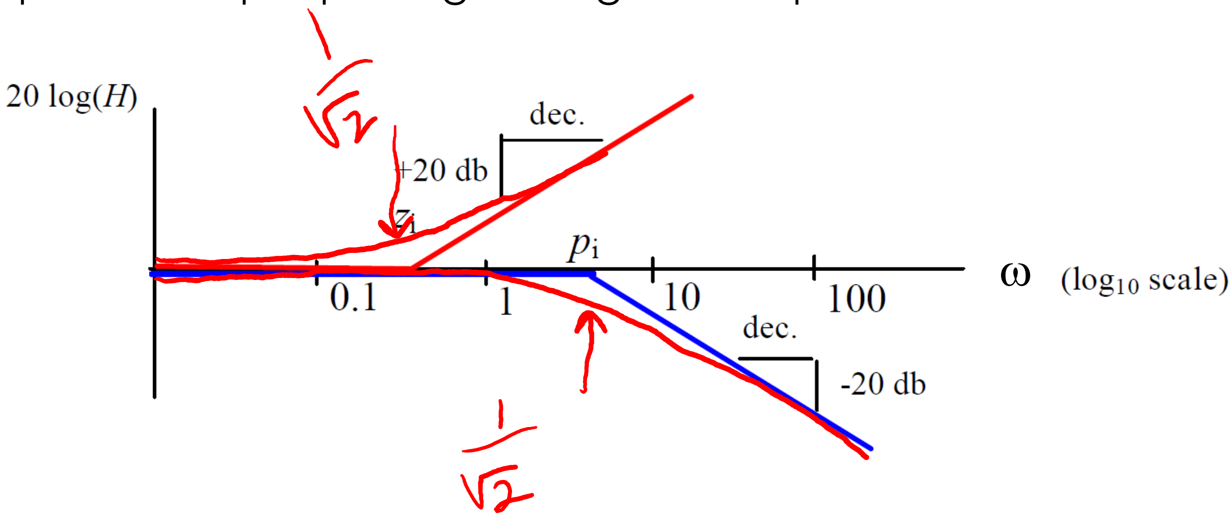
A **pole** at the origin occurs when there is an ***s*** or ***jω*** multiplying the denominator. Each occurrence of this causes a negatively sloped line passing through ω = 1 with a drop of 20 dB/decade



The Bode Plot: Rules

Effect of Zeros and Poles Not at the Origin

Zeros and poles **not at the origin** occurs when there are (**1+*j*ω/z$_i$**) and (**1+*j*ω/p$_i$**) terms. The values of zi and pi are called critical/cutoff/break/corner frequencies. Below their cutoff frequencies, these terms don't contribute to the log magnitude plot. Above their cutoff frequencies, they represent a 20 dB/decade ramp function. Zeros give positive slope, poles give negative slope.

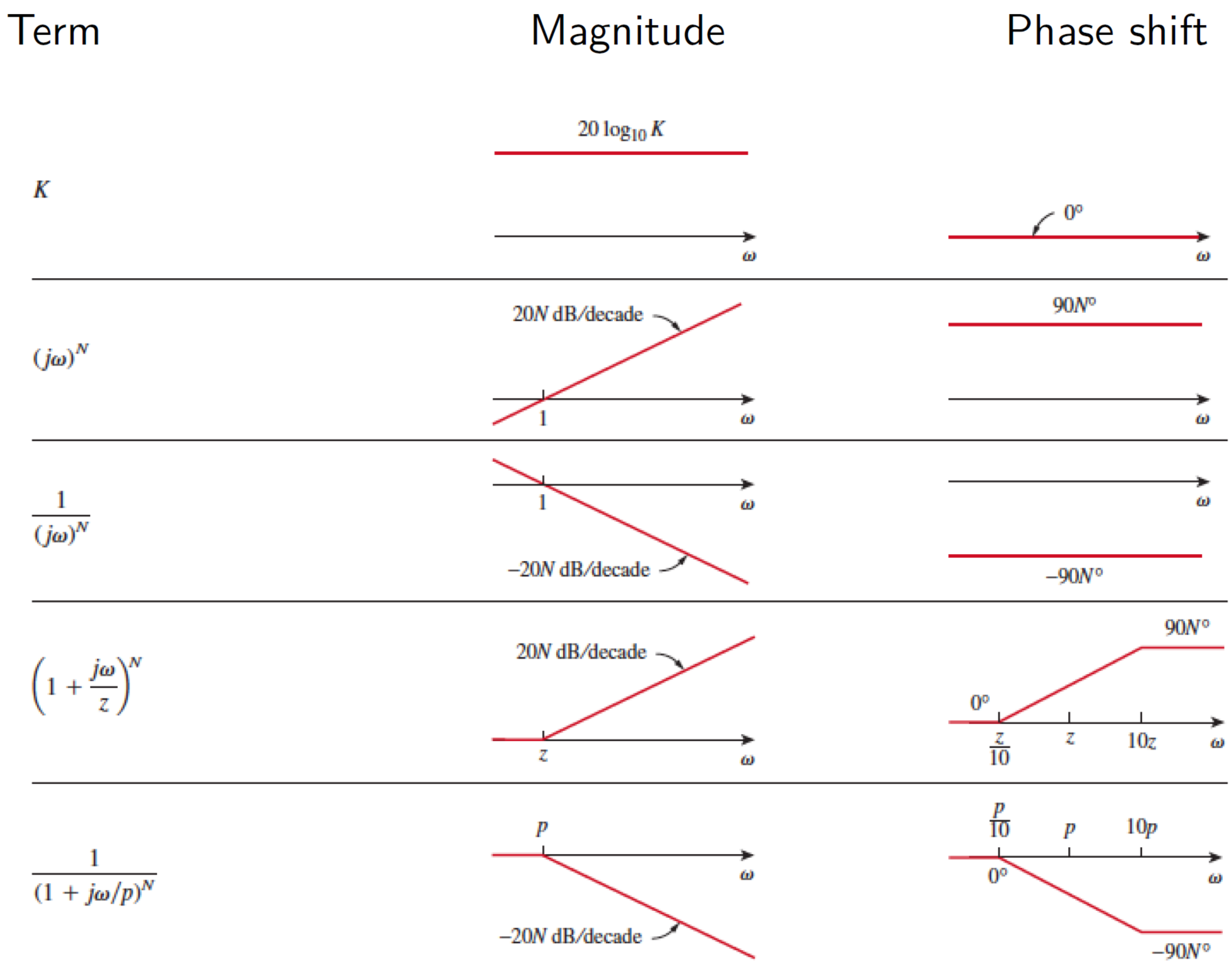


$$H = \frac{\left|1 + \dfrac{j\omega}{z_i}\right|}{\left|1 + \dfrac{j\omega}{p_i}\right|}$$

On the Bode plot, graph each individual term, then use superposition and add up all terms like you would as if you were dealing with a linear scale graph

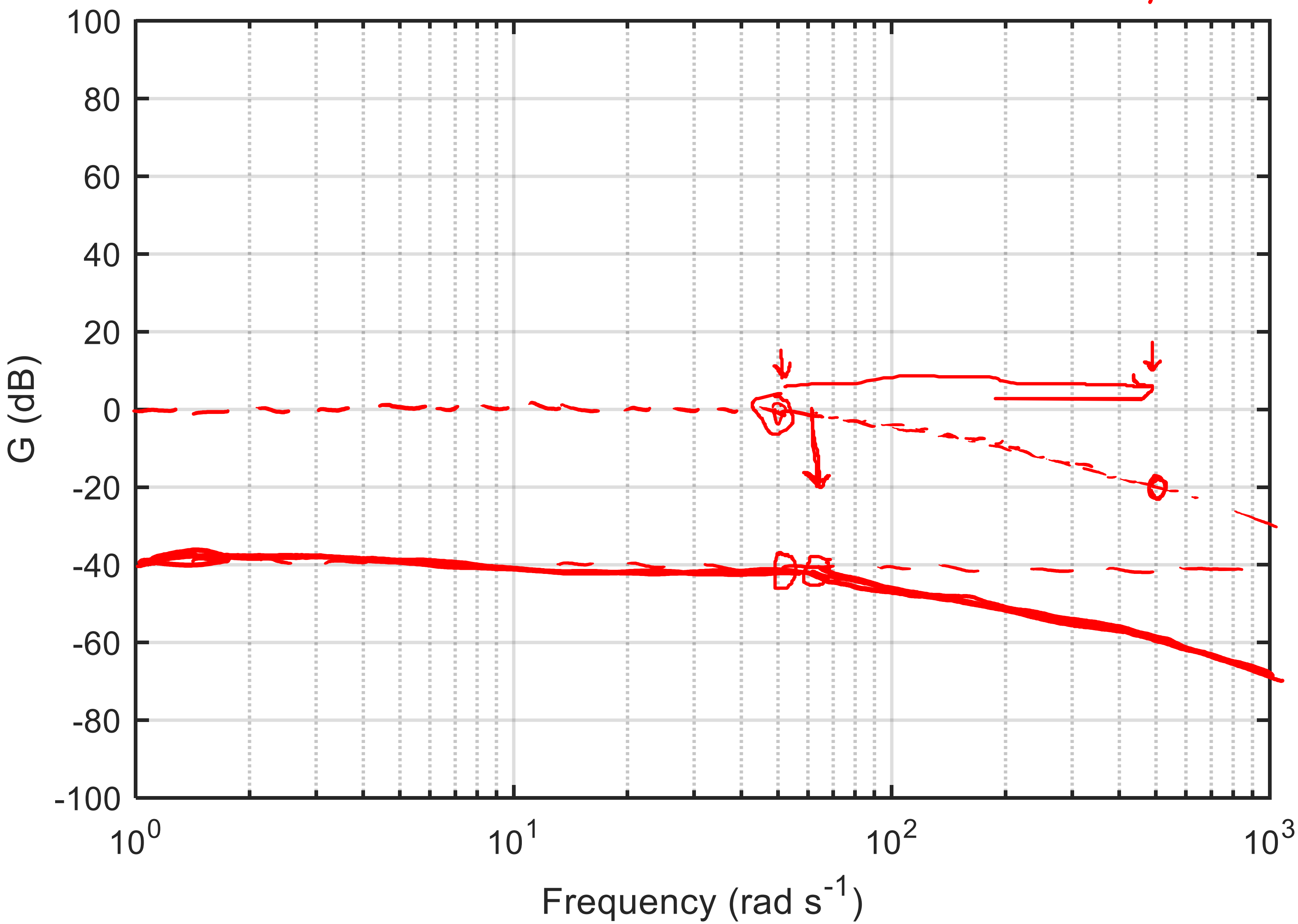
The Bode Plot

$$G_{dB} = 20\ log_{10}\ |H(s)|\ \text{dB}$$



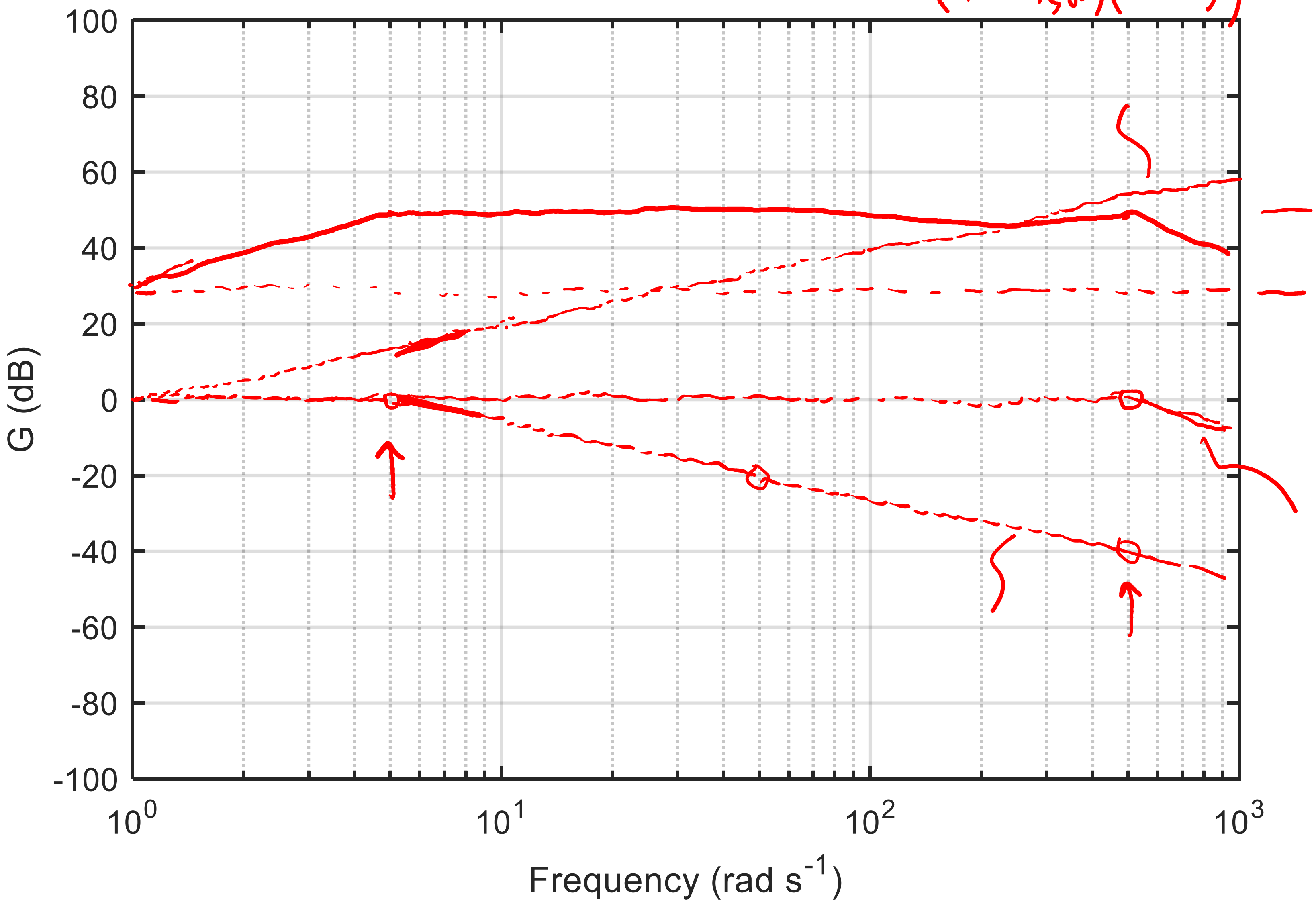
Practice Problem 1

$$H = \frac{1}{(2j\omega + 100)} = \frac{1}{100} \cdot \frac{1}{1 + s/50}$$



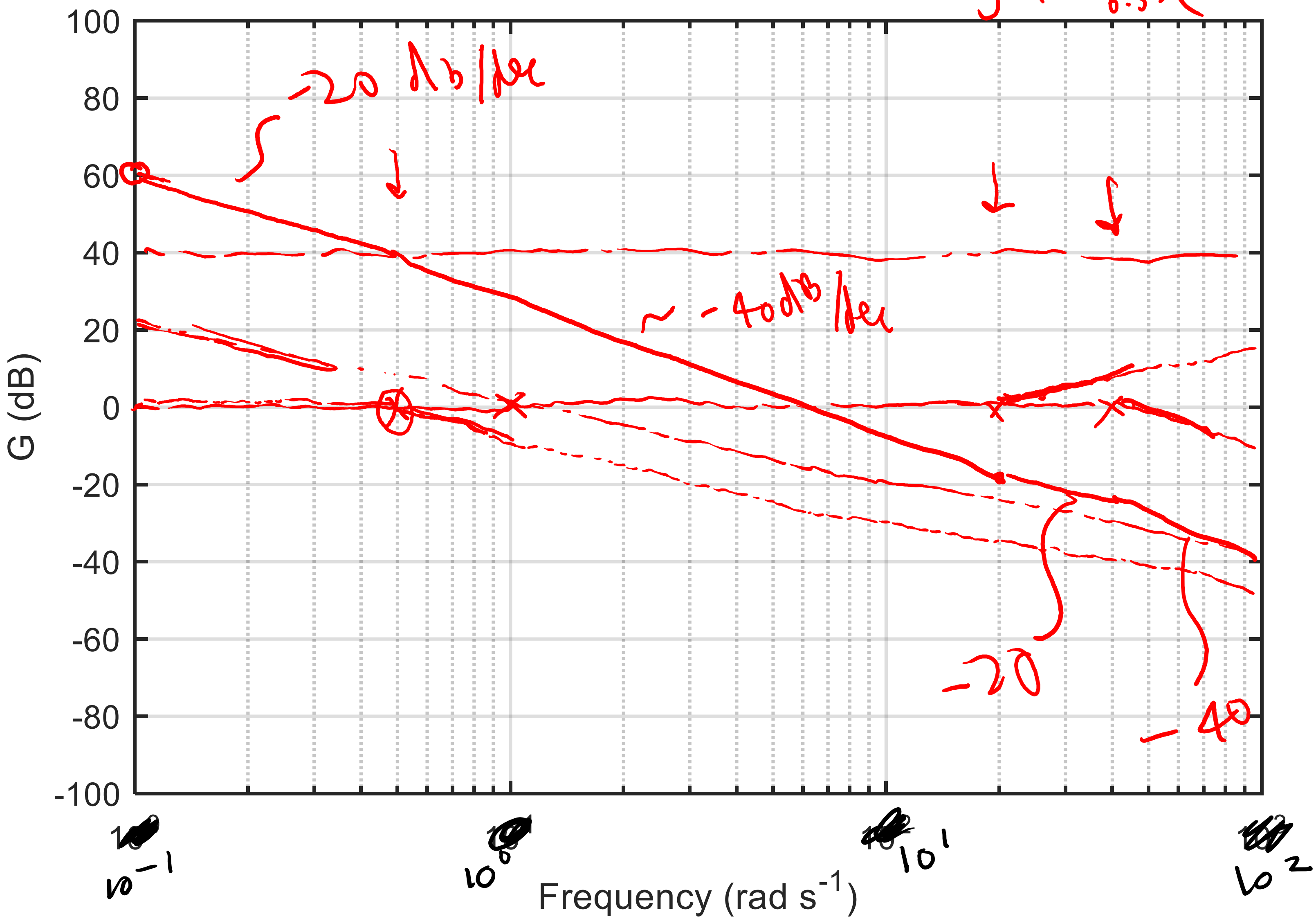
Practice Problem 2

$$H = \frac{5E4j\omega}{(j\omega + 500)(j\omega + 5)} = \frac{20\, j\omega}{(1 + j\omega/500)(1 + j\omega/5)}$$



Practice Problem 3

$$H = \frac{200(j\omega + 20)}{j\omega(2j\omega + 1)(j\omega + 40)} = \frac{100(1 + j\omega/20)}{j\omega\left(1 + \frac{j\omega}{0.5}\right)(1 + j\omega/40)}$$



Summary: Bode Plot

- Determine the transfer function: $H(s) = \dfrac{K(s+z_1)}{s(s+p_1)}$
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$$H(s) = \frac{Kz_1(s/z_1 + 1)}{sp_1(s/p_1 + 1)}$$

- Replace ***s*** with ***jω***. Find the magnitude of the transfer function.
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$$20\log_{10}(H(jw)) = 20\log_{10}\left(\frac{Kz_1(jw/z_1 + 1)}{jwp_1(jw/p_1 + 1)}\right) =$$

$$20\log_{10}|K| + 20\log_{10}|z_1| + 20\log_{10}\left|(jw/z_1 + 1)\right| - 20\log_{10}|p_1| - 20\log_{10}|jw| - 20\log_{10}\left|(jw/z_1 + 1)\right|$$

- Graph each term individually. Then sum up to get total frequency response on a log scale.

- **THURSDAY: Filters, Square Wave Decomposition**