ECE 10C
Fall 2020
Slide Set 12
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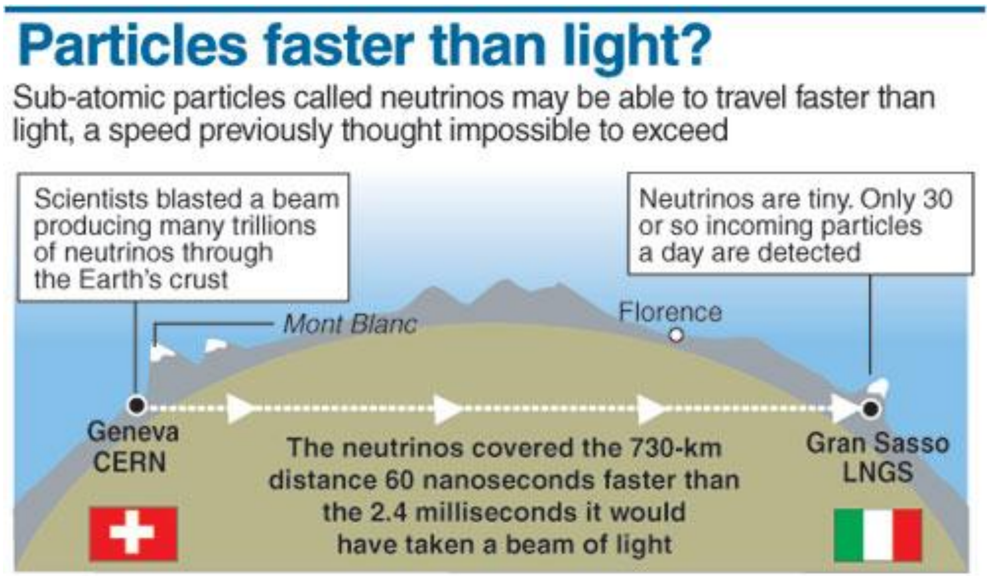
TA: Kamyar Parto



1st correct answer in chat = +2 quiz points:
How is this possible? Is Einstein wrong? If so, why? If not, why?

Previously
- 2nd-order Parallel RLC

Toda
- 2nd-order Series RLC
- Examples

Important Items:

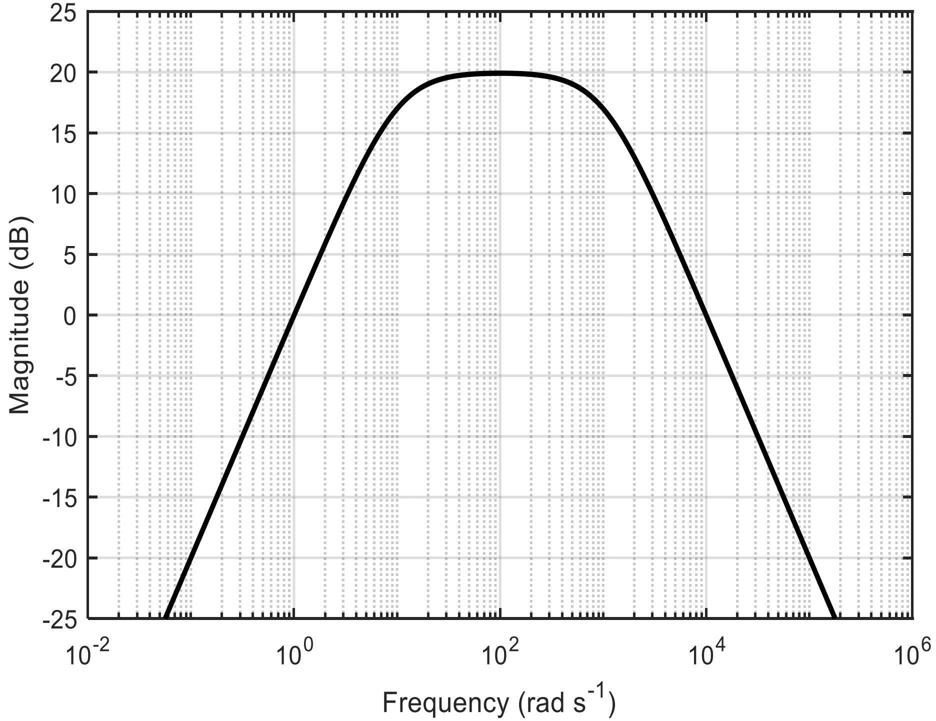
- HW #5 due Today, 12/3
- Lab #5 due Tomorrow
- ESCI questionnaire
- HW #6 posted, due 12/10
- Lab #6 posted, due 12/11

Quiz Last Time

Q1: $\boxed{\text{True}}$ or False: An RC circuit with the output voltage measured across the capacitor is an example of a low-pass filter.

Q2: True or $\boxed{\text{False:}}$ An RL circuit with the output voltage measured across the inductor is an example of a low-pass filter.

Q3: Pick the correct transfer function for the gain showed in the plot below:



a) $\dfrac{(1+j\omega)}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{100}\right)}$

b) $\left(1+\dfrac{j\omega}{10}\right)\left(1+\dfrac{j\omega}{100}\right)$

c) $\boxed{\dfrac{j\omega}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{1000}\right)}}$

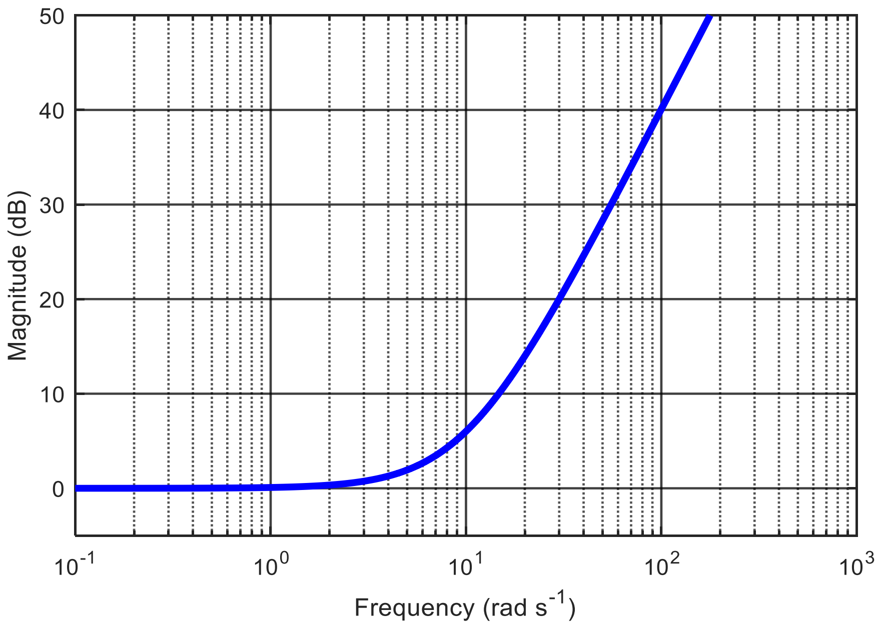
d) $\dfrac{j\omega}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{100}\right)}$

Quiz Time!

Q1: True or False: If the quality factor of an RLC circuit, Q= $\omega/2\alpha > 0.5$, then the circuit response (*e.g.* voltage across capacitor) will oscillate.

Q2: True or False: An underdamped RLC circuit oscillates with a resonant frequency $\omega_0 = 1/\sqrt{RLC}$ and has a transfer function that looks the same as a low-pass filter.

Q3: Pick the correct transfer function for the gain showed in the plot below:



a) $(j\omega)^2$

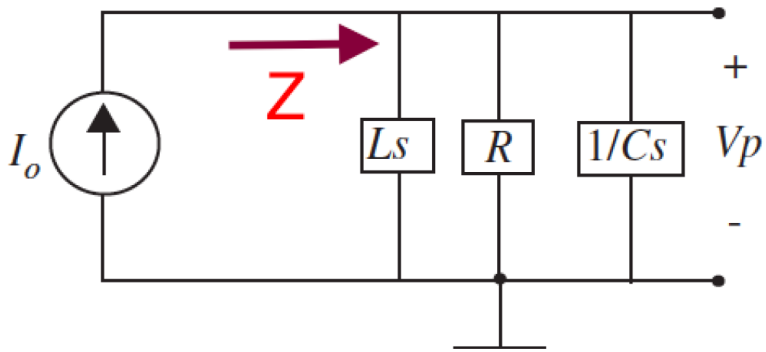
b) $(j\omega)^4$

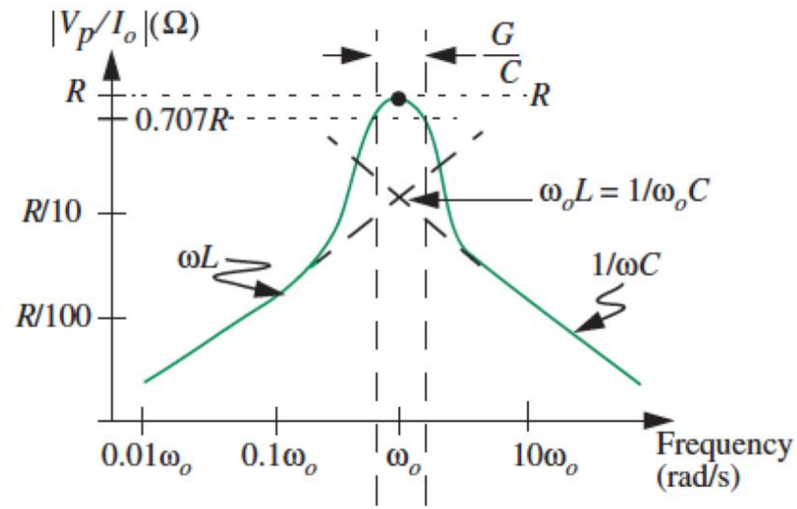
c) $(1 + j\omega/10)^2$

d) $(1 + j\omega/10)^4$

Review: Parallel RLC Filters

$$H(j\omega) = \frac{V_c}{I} = Z = \frac{1}{\frac{1}{R} + j\left(\omega_\circ C - \frac{1}{\omega_\circ L}\right)}$$





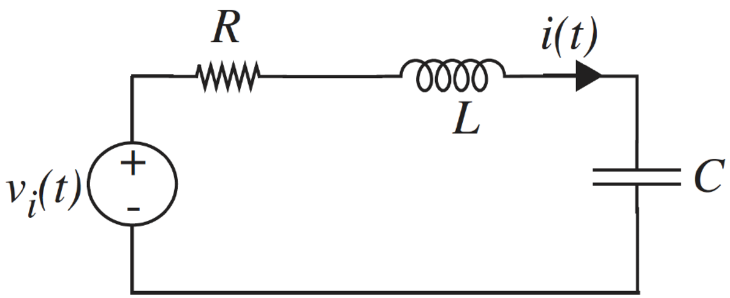
Value of $H(j\omega)$ at $\omega_\circ = \frac{1}{\sqrt{LC}}$?

$$\omega_\circ C - \frac{1}{\omega_\circ L} = 0 \rightarrow \quad H(j\omega_\circ) = R$$

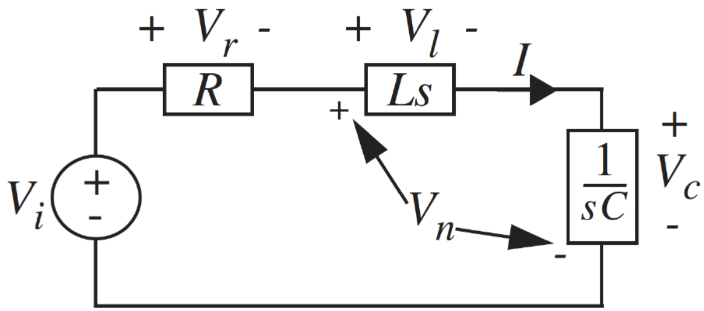
Cut-off frequencies and Bandwidth

$\omega_{cut} = \pm\frac{G}{2C} + \sqrt{(\frac{G}{2C})^2 + \frac{1}{LC}} \rightarrow$ Bandwidth $= \frac{G}{C} = \frac{1}{RC} = 2\alpha = \frac{\omega_\circ}{Q}$

Next: Series RLC Circuit



(a) Circuit



(b) Impedance Model

What we will learn:

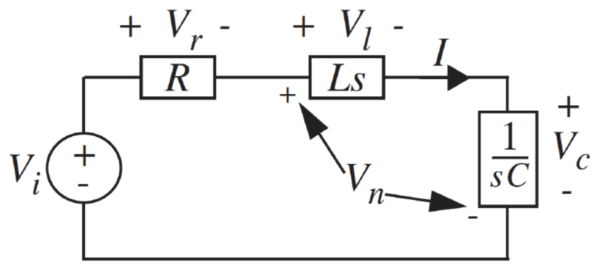
- Depending on where the output is taken, the series and parallel RLC resonant circuits can be used as filters of various types.
- The higher the Q factor of the circuit, the higher the selectivity.

General Steps for 2nd-Order Resonant Filter Analysis

1. Sketch the approximate frequency response, using Bode plots or asymptotes. Be careful to clarify units on the plots;
2. Determine the filter type from approximate sketch (lowpass, highpass, bandpass, bandstop);
3. If resonant bandpass or bandstop $\rightarrow$ Calculate the exact gain at resonance frequency and add in the "peak" in sketch (this might not be the exact value of the peak);
4. Determine filter bandwidth.

Let's apply these steps to the series RLC circuit with different outputs

Complex Amplitude of the Current

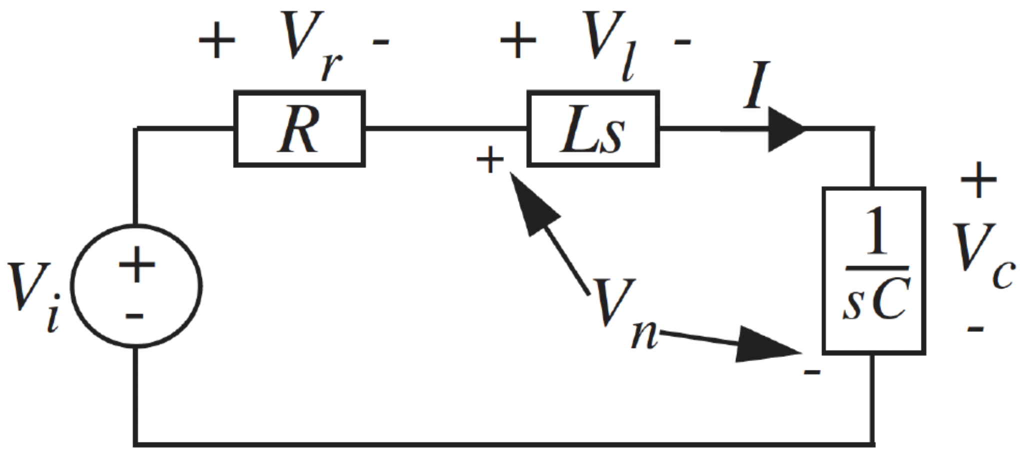


$$I = \frac{V_i}{R + Ls + 1/Cs} = \frac{(s/L)V_i}{s^2 + sR/L + 1/LC} = \frac{(s/L)V_i}{s^2 + 2\alpha s + \omega_\circ^2}$$

$$\omega_\circ = \sqrt{1/LC}, \quad \alpha = \frac{R}{2L}$$

We will assume the second order term has complex roots, e.g.,
$R = 1\Omega, L = 1\mu H, C = 1\mu F \rightarrow \omega_\circ = 10^6 rad/s, \alpha = 5 \times 10^5 s, Q = 1$

Output #1: Resistor Voltage



$$V_r = IR = \frac{\frac{sR}{L} V_i}{s^2 + 2\alpha s + \omega_\circ^2}$$

$$H_r(s) = \frac{V_r}{V_i} = \frac{\frac{sR}{L}}{s^2 + 2\alpha s + \omega_\circ^2} = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_\circ^2}$$

Let's Follow Our General Set of Rules

$$H(s) = \frac{V_r}{V_i} = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_\circ^2}$$

1. Sketch the approximate frequency response, using Bode plots or asymptotes. Be careful to clarify units on the plots;
2. Determine the filter type from approximate sketch (lowpass, highpass, bandpass, bandstop);
3. Calculate the exact gain at resonance frequency and add in the "peak" in sketch;

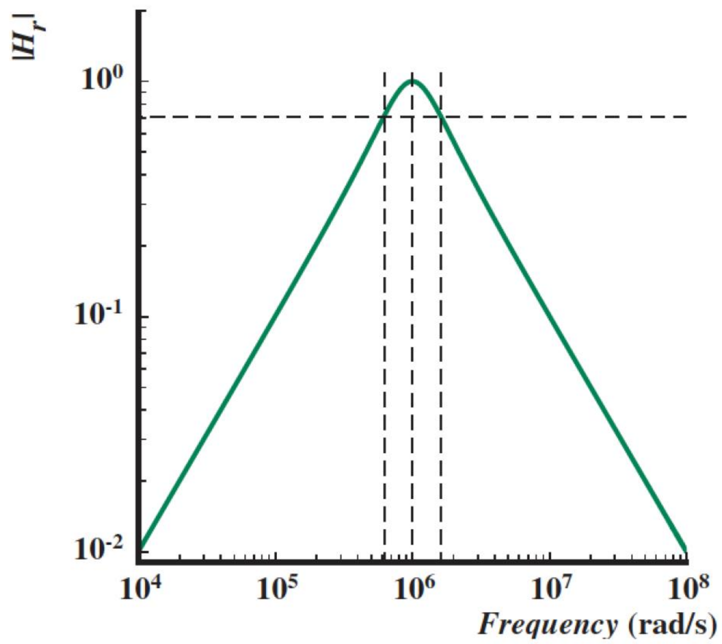
$$|H(j\omega_\circ)| = 1 \quad (0\ dB)$$

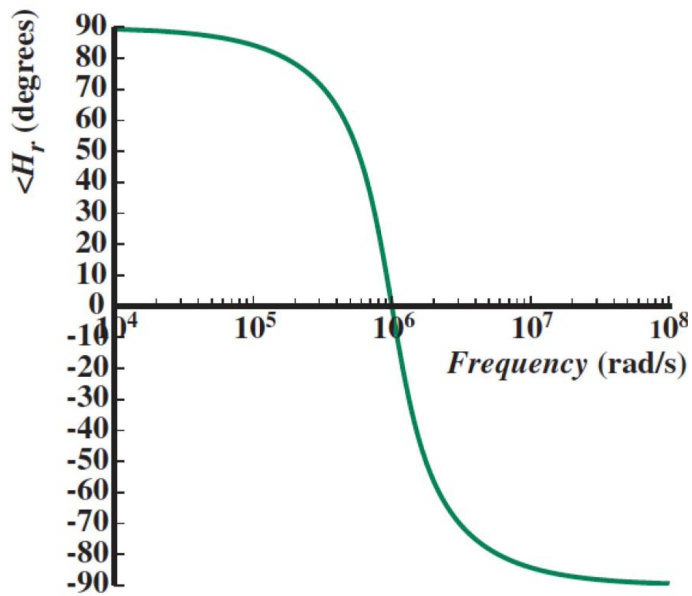
4. Calculate $Q = \frac{\omega_\circ}{2\alpha}$ and filter bandwidth (by calculating cut off freqs).

$$Q = \frac{\omega_\circ L}{R}, \quad \text{Bandwidth} = 2\alpha = 10^6\text{rad}$$

Output #1: Resistor Voltage

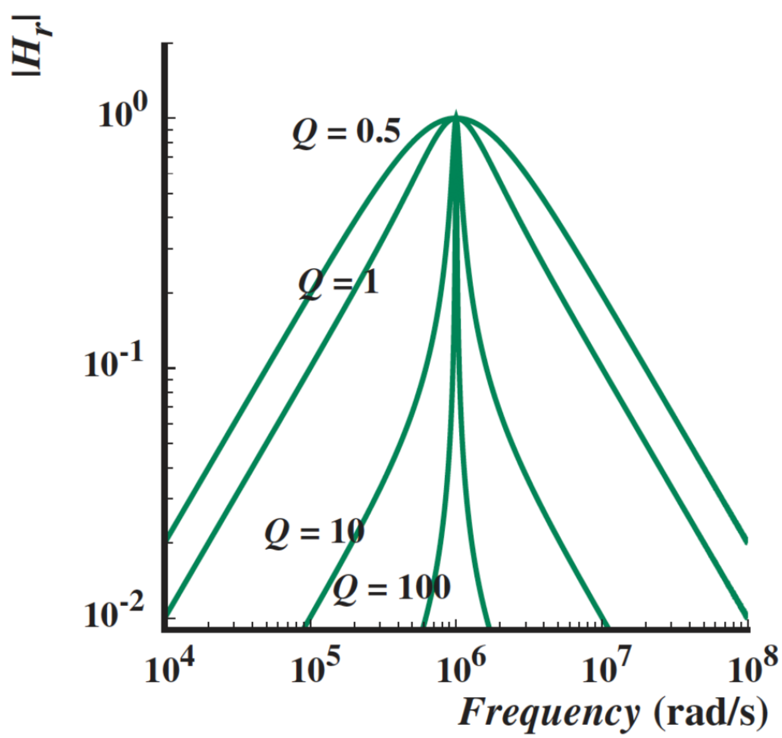
For plots:
$R = 1\Omega, L = 1\mu H, C = 1\mu F \rightarrow \omega_\circ = 10^6 rad/s, \alpha = 5 \times 10^5 s, Q = 1$

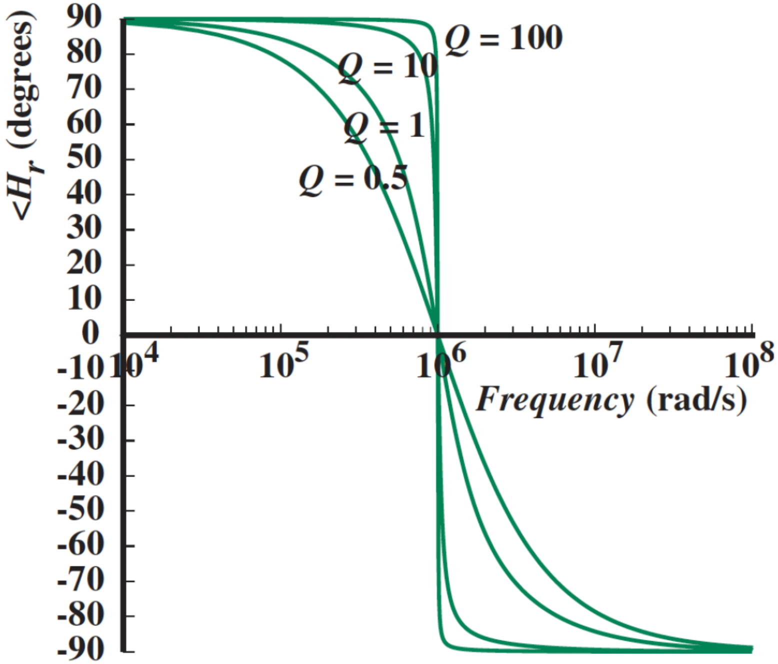




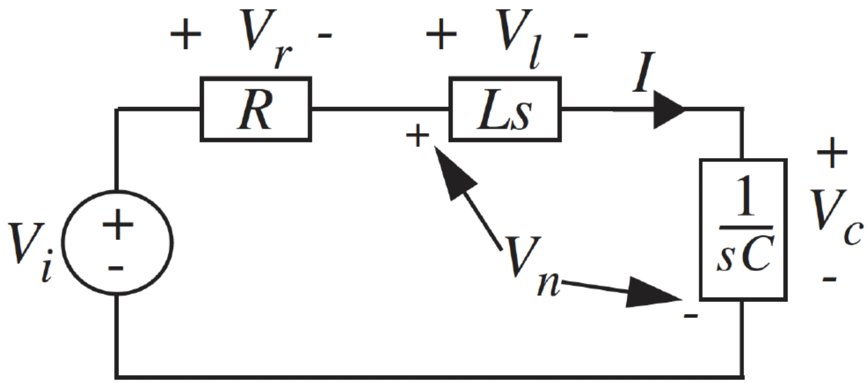
Output #1: Resistor Voltage

Notice that the maximum gain is always equal to 1





Output #2: Capacitor Voltage



$$V_c = \frac{I}{sC} = \frac{\frac{1}{LC} V_i}{s^2 + 2\alpha s + \omega_\circ^2}$$

$$H_c(s) = \frac{V_c}{V_i} = \frac{\frac{1}{LC}}{s^2 + 2\alpha s + \omega_\circ^2} = \frac{\omega_\circ^2}{s^2 + 2\alpha s + \omega_\circ^2}$$

Let's Follow Our General Set of Rules

$$H(s) = \frac{V_r}{V_i} = \frac{\omega_\circ^2}{s^2 + 2\alpha s + \omega_\circ^2}$$

1. Sketch the approximate frequency response, using Bode plots or asymptotes. Be careful to clarify units on the plots;
2. Determine the filter type from approximate sketch (lowpass, highpass, bandpass, bandstop);
3. If bandpass or bandstop $\rightarrow$ Calculate the exact gain at resonance frequency and add in the "peak" in sketch;

$$|H(j\omega_\circ)| = \frac{\omega_\circ^2}{2\alpha\omega_\circ} = Q \quad (20\log Q \ dB)$$

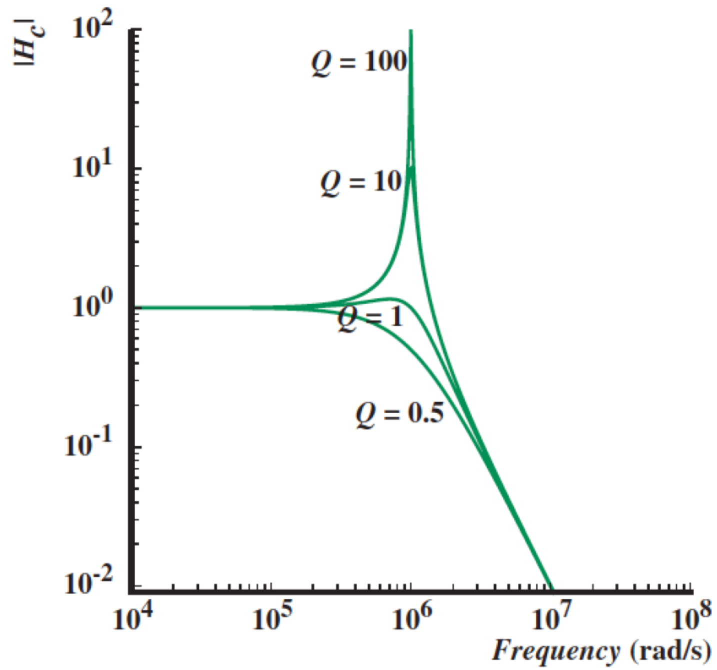
4. Calculate filter bandwidth (by calculating cut off freqs).

$$\text{Bandwidth} \approx \omega_\circ$$

5. Exact BW? Solve $|H(j\omega_c)| = \frac{1}{\sqrt{2}}|H(j0)| = \frac{1}{\sqrt{2}}$

The Gain at Resonance Frequency

Let's look at the gain:

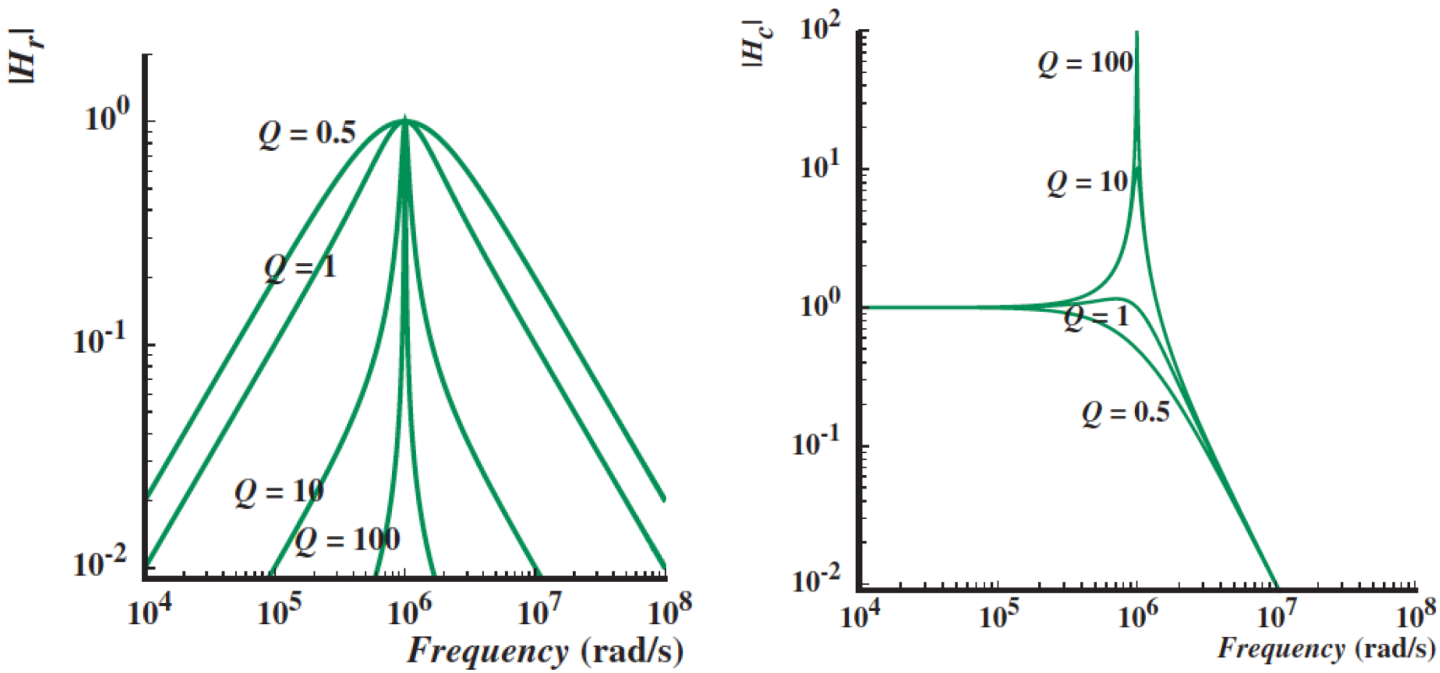


High Q circuits are not useful low-pass filters (as they produce voltages that are significantly higher than the input when $\omega \approx \omega_\circ$)

Need a good low-pass filter? Choose $Q = 1$

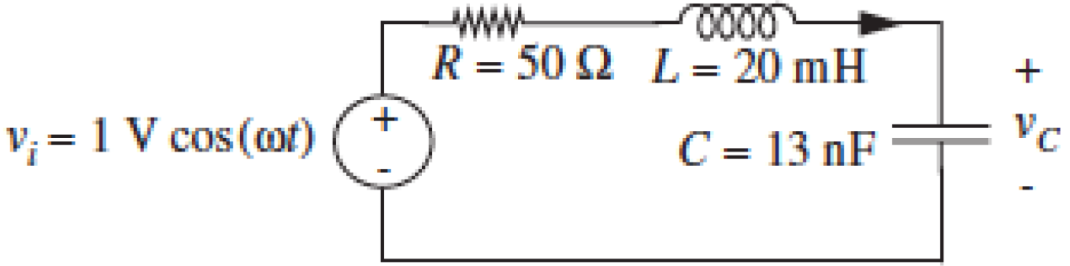
The Gain at Resonance Frequency

Compare the peak gain between the two cases (V_r as output vs. V_c as output)



$$v_i(t) = V_i \cos(\omega t) \longrightarrow \boxed{H(j\omega)} \longrightarrow v_o(t) = |H(j\omega)| V_i \cos(\omega t + \angle H(j\omega))$$

Example: Need To Be Careful About Resonance!



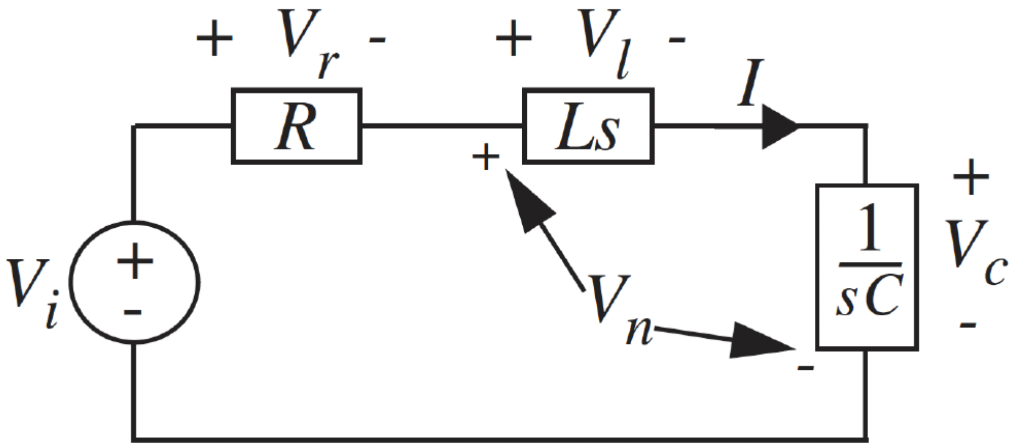
Let's find the magnitude of v_c at $\omega = \omega_\circ = \frac{1}{\sqrt{LC}} = 62.017$ krad/s

$$V_c(j\omega) = \frac{I}{j\omega C} = V_i \frac{1}{R + j(L\omega - 1/C\omega)} \frac{1}{j\omega C}$$

$$|V_c(j\omega_\circ)| = \frac{1V}{R}\frac{1}{\omega_\circ C} = Q \times 1V = 24.80V$$

At the resonance frequency:
The amplitude of v_C in response to a 1V input is 24.8069 V

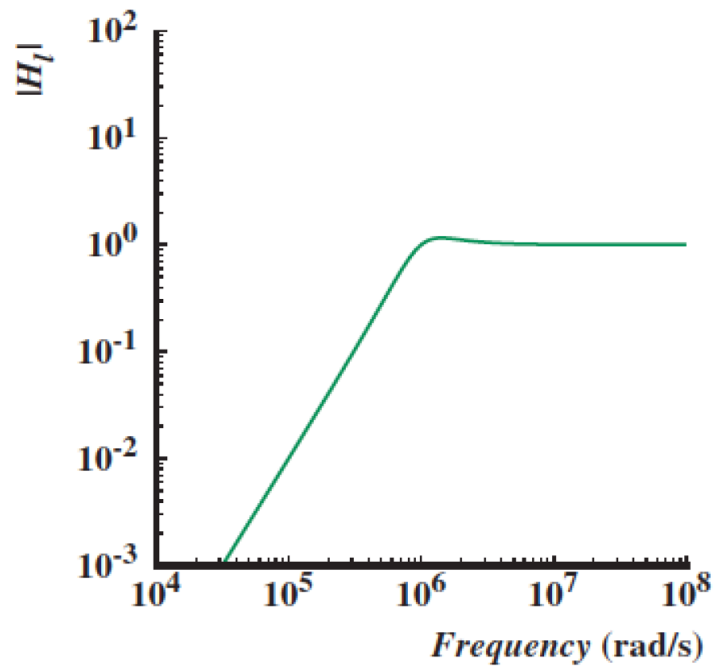
Output #3: Inductor Voltage

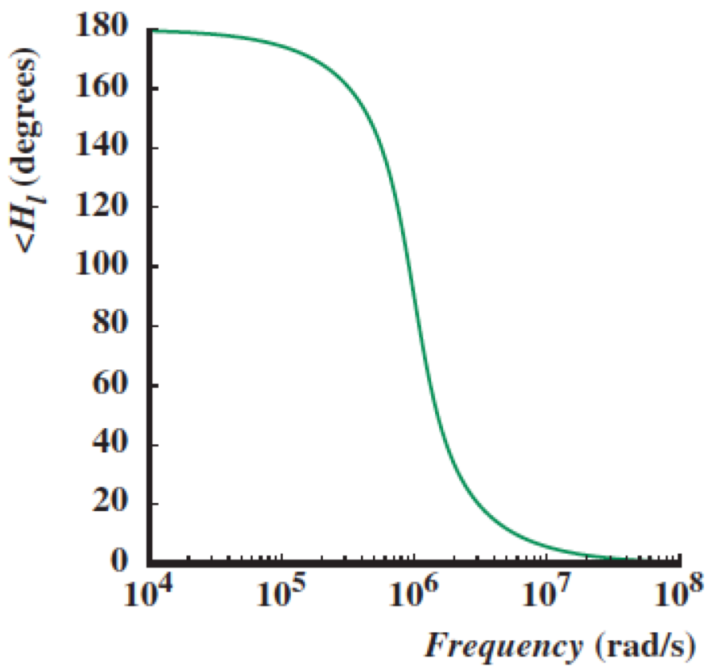


$$V_L = IsL = \frac{s^2 V_i}{s^2 + 2\alpha s + \omega_\circ^2}$$

$$H_L(s) = \frac{V_L}{V_i} == \frac{s^2}{s^2 + 2\alpha s + \omega_\circ^2}$$

Output #3: Inductor Voltage

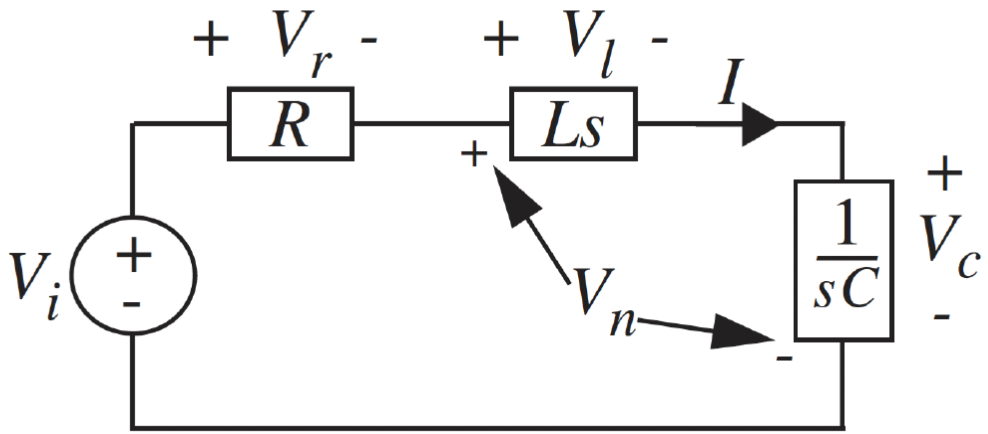




$$H_L(s) = \frac{V_L}{V_i} = \frac{s^2}{s^2 + 2\alpha s + \omega_\circ^2}$$

Need a good high pass filter? Select $Q = 1$

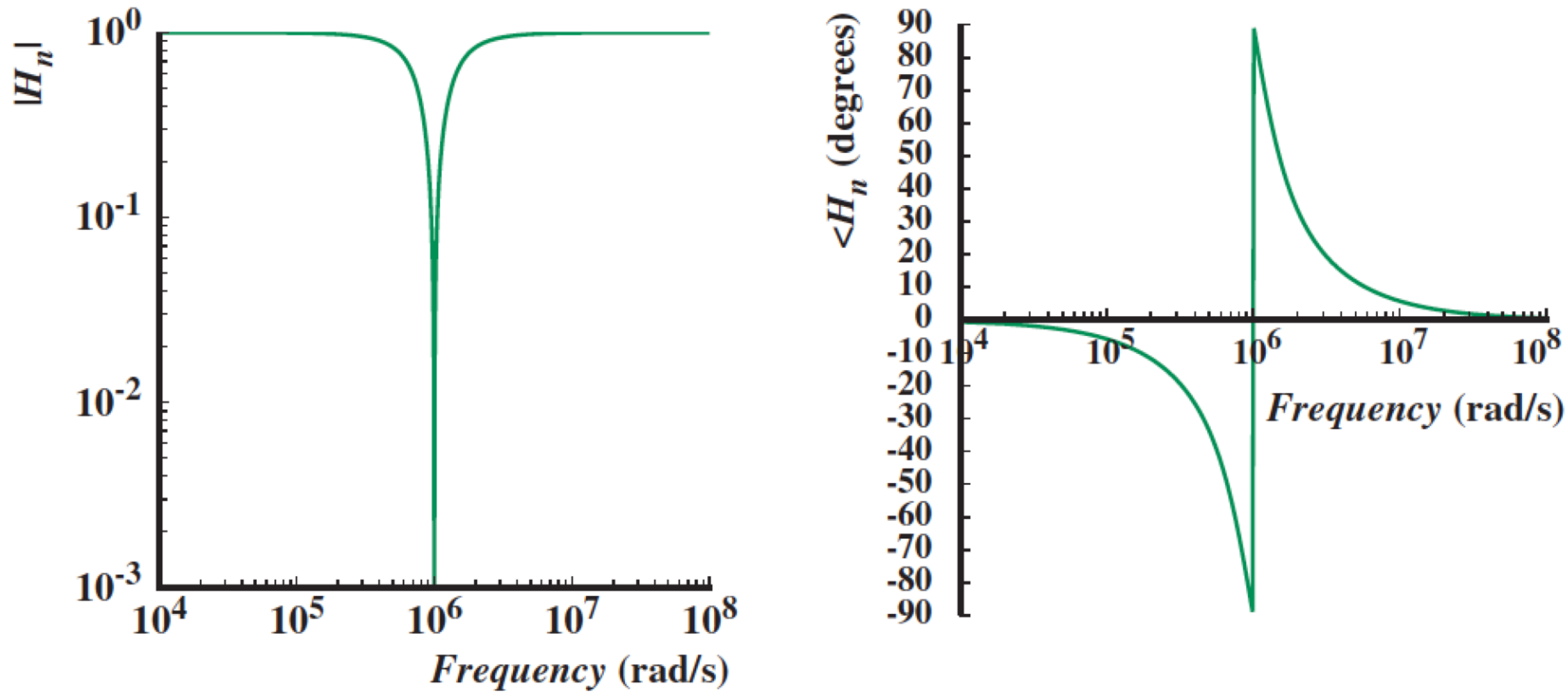
Output #4: Inductor & Capacitor Voltage



$$V_n = I(sL + \frac{1}{sC}) = \frac{(s^2 + \frac{1}{LC})V_i}{s^2 + 2\alpha s + \omega_\circ^2}$$

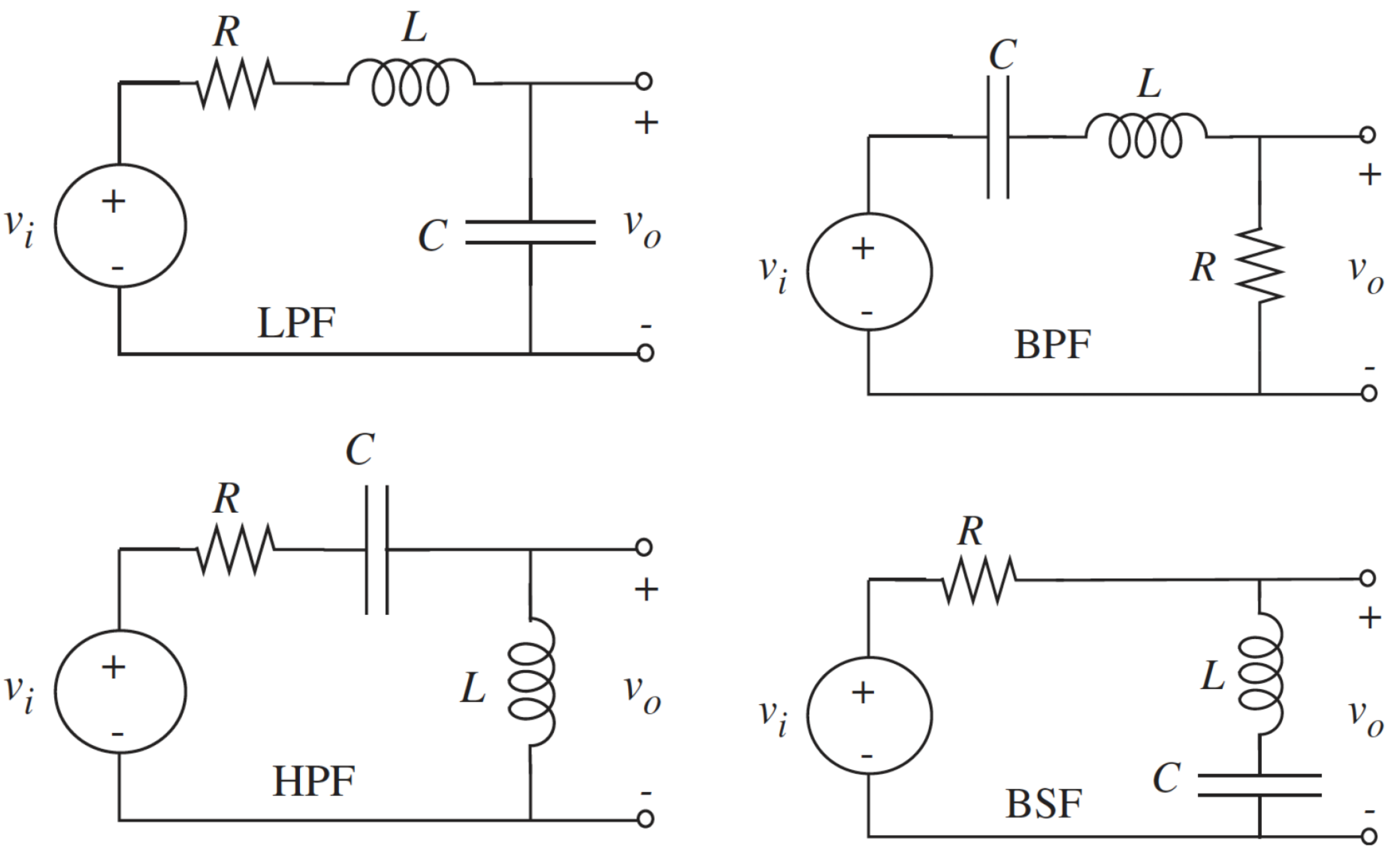
$$H_n(s) = \frac{V_n}{V_i} = \frac{(s^2 + \frac{1}{LC})}{s^2 + 2\alpha s + \omega_\circ^2}$$

Output #4: Inductor & Capacitor Voltage



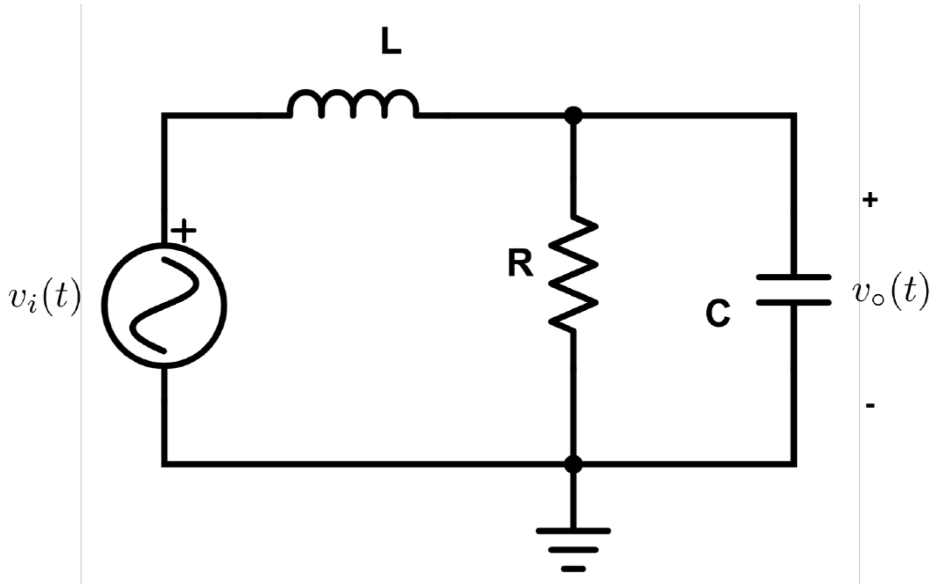
$$H_n(s) = \frac{V_n}{V_i} = \frac{(s^2 + \omega_\circ^2)}{s^2 + 2\alpha s + \omega_\circ^2}$$

Recap: All Different Filter Types Using Same Circuit



We can repeat the same steps for a general resonant filter
(not parallel or series)

Example #1



$$H(j\omega) = \frac{V_o}{V_i} = \frac{R||\frac{1}{Cj\omega}}{R||\frac{1}{Cj\omega} + j\omega L} = \frac{R}{s^2 RLC + sL + R}, \quad s = j\omega$$

Assume the second order term has complex roots

Example #1

Step 1: What does the frequency response look like? (asymptotes or Bode)

Step 2: What type of filter is this?

Step 3: Calculate the peak (no need to do this for lowpass or highpass)

Step 4: Calculate the bandwidth $\rightarrow$ approximate or exact

$$|H(j\omega)| = \frac{R}{\sqrt{(R - \omega^2 RLC)^2 + \omega^2 L^2}}$$

At cut-off ω_c:

$$|H(j\omega_c)| = \frac{R}{\sqrt{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}} = \frac{1}{\sqrt{2}}(\text{DC gain}) = \frac{1}{\sqrt{2}}$$

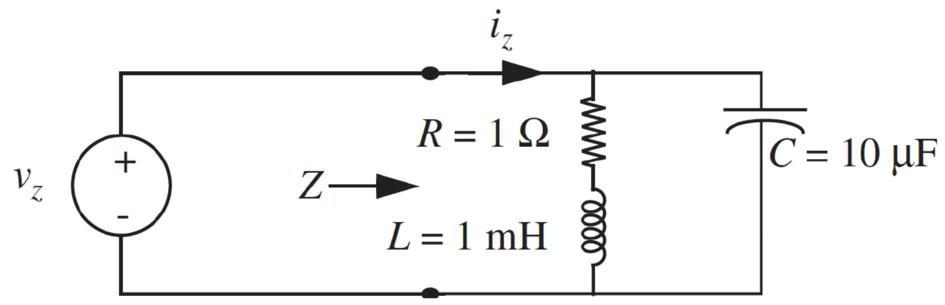
$$2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2 \quad [\text{quadratic in } \omega_c^2]$$

$$\text{Bandwidth} = \omega_c$$

Example #2

Study the response of the following circuit given:

$$L = 1mH,\ C = 10\mu F, R = 1\Omega$$



Desired system function (Z denotes input impedance):

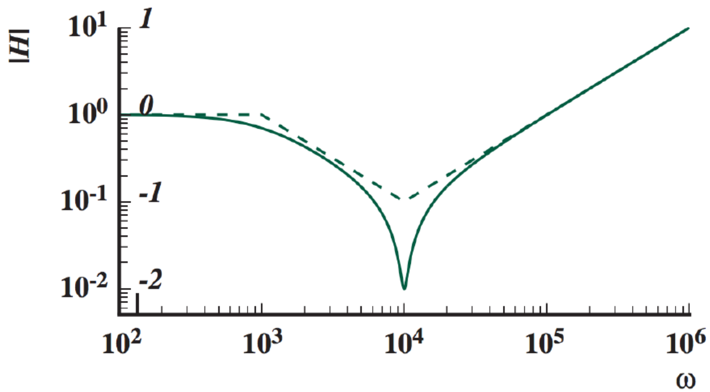
$$H(s) = \frac{I_z}{V_z} = \frac{1}{Z} = \frac{s^2 + s\frac{R}{L} + \frac{1}{LC}}{\frac{s}{C} + \frac{R}{LC}}$$

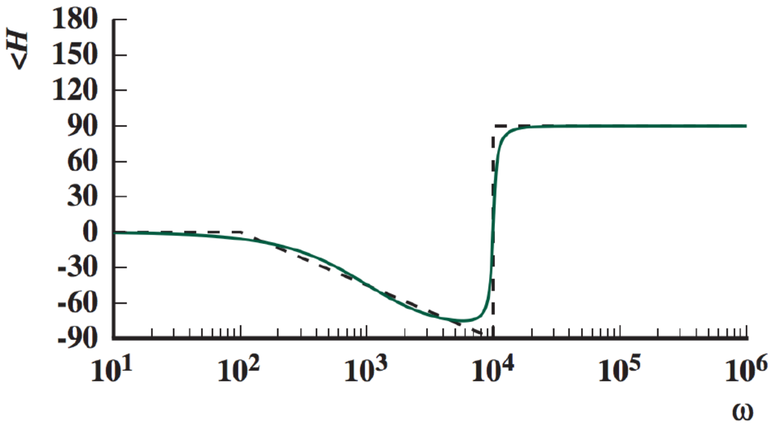
Let's check whether the second order term has complex roots:

$$Q = \frac{2\alpha}{\omega_\circ} = 10 > 0.5 \rightarrow \text{resonant}$$

Example #2

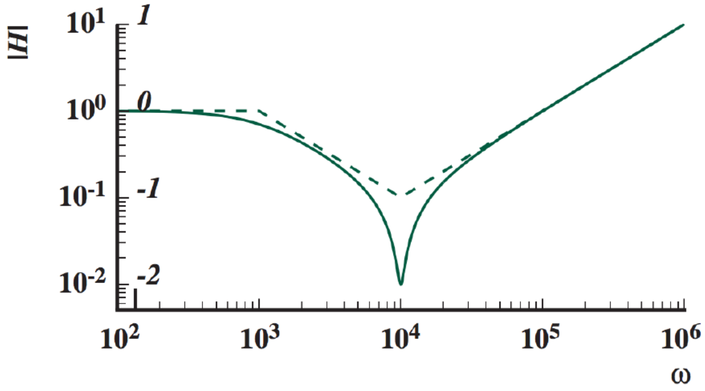
Step 1: What does the frequency response look like?
(Derived using asymptotes in example 14.5 or Bode plot in section 14.4)





Example #2

Step 2: What type of filter is this?



Step 3: Calculate the peak (no need to do this for lowpass or highpass)

$$|H(j\omega_\circ)| \approx 0.01$$

Summary

- RLC filters are very versatile, with many different configurations for designing low pass, high pass, band pass, and band stop filters.

- Remember the filter analysis/design steps!

1. Sketch the approximate frequency response, using Bode plots or asymptotes. Be careful to clarify units on the plots;
2. Determine the filter type from approximate sketch (lowpass, highpass, bandpass, bandstop);
3. If resonant bandpass or bandstop $\rightarrow$ Calculate the exact gain at resonance frequency and add in the "peak" in sketch (this might not be the exact value of the peak);
4. Determine filter bandwidth.

Let's apply these steps to the series RLC circuit with different outputs

- Last HW and Lab (#6) due next week. Finals week after.

- Next week: Review on Tuesday, practice problems/examples Thursday