ECE 10C
Fall 2020
Slide Set 1
Instructor: Galan Moody
TA: Kamyar Parto

Last Week
- Introduction to course

This Week
- KVL/KCL, node methods
- Thevenin
- 1st-order RC/RL circuits

Important Items:

Homework #1 posted
Due Th, Oct 15 by 5 pm

Lab #1 posted
Due Fr, Oct 16

Piazza Forum for Class Discussions

Piazza is a Q&A platform designed to get you great answers from classmates and instructors fast.

1. **Edit questions and answers wiki-style.**
2. **Add a follow-up to comment or ask further questions.**
3. **Go anonymous.**
4. **Tag your posts.**
5. **Format code and equations.**
6. **View and download class details and resources.**

This Week

Review Chapters 9 and 10 of the textbook to get ready for the course, which really begins week 2

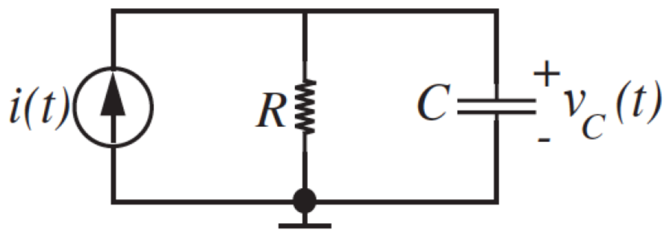
It's important to understand how to do the following:

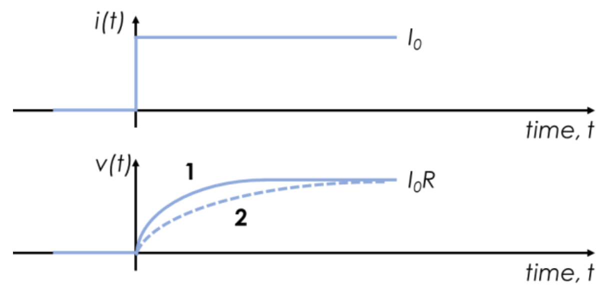
1. Use KVL, KCL and the node method to formulate the differential equations for analyzing a circuit with energy-storage elements
2. Write Thevenin equivalent circuits to simplify your analysis
3. For first-order RC and RL circuits: Find the homogeneous and particular solutions
4. Use the initial conditions to evaluate the constants in the homogeneous solution

Quiz Time!

Assume capacitors are uncharged at time t = 0.

Q1 [2 point]. For the parallel RC circuit shown on the left, with a constant-current step source shown on the top right, choose the answer that best describes the circuits for the capacitor voltage shown on the bottom right. Curves 1 and 2 correspond to circuits with R_1,C_1 and R_2,C_2, but are otherwise identical.

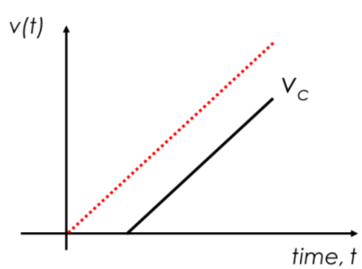




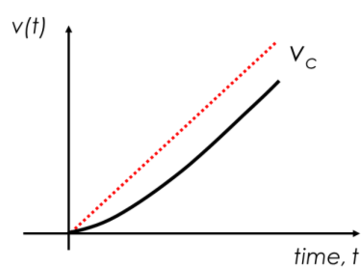
(a) $R_1C_1 = R_2C_2$ **(b)** $R_1 > R_2$ & $C_1 < C_2$ **(c)** $R_1C_1 > R_2C_2$ **(d)** $C_1 < C_2$

Q2 [3 points]. (a) For an RC series circuit with a linear voltage ramp starting at 0V, choose the correct capacitor voltage.

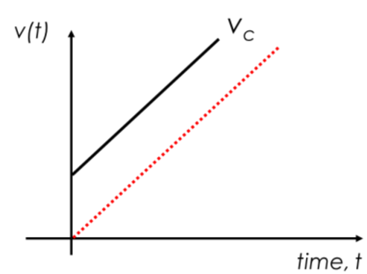
(a)



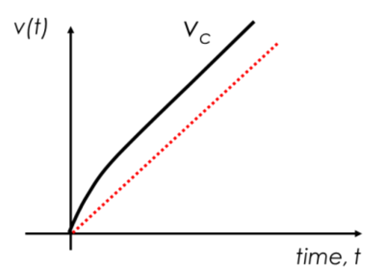
(b)



(c)

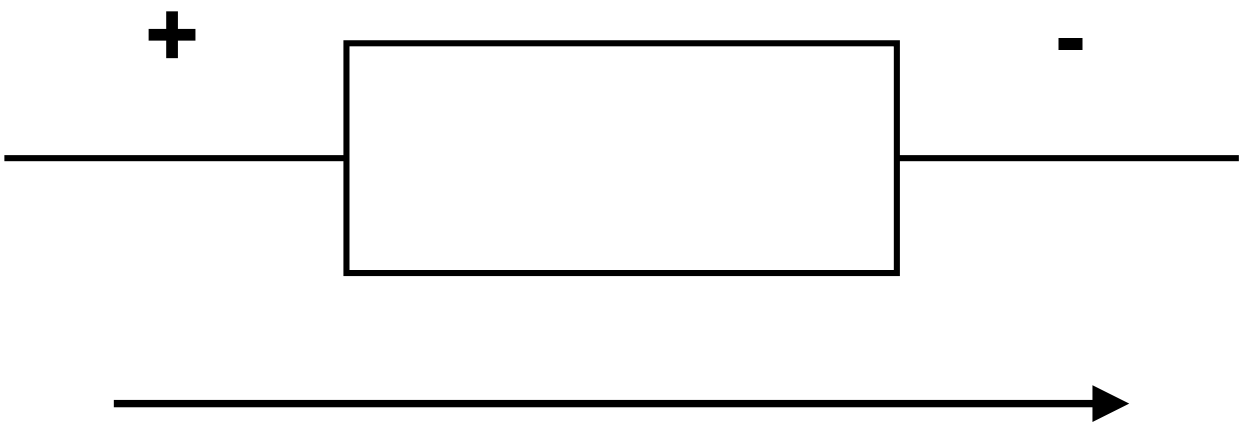


(d)



We Need to Pick a Convention

How we define the direction of current and voltage drop is up to us, as long as we are consistent

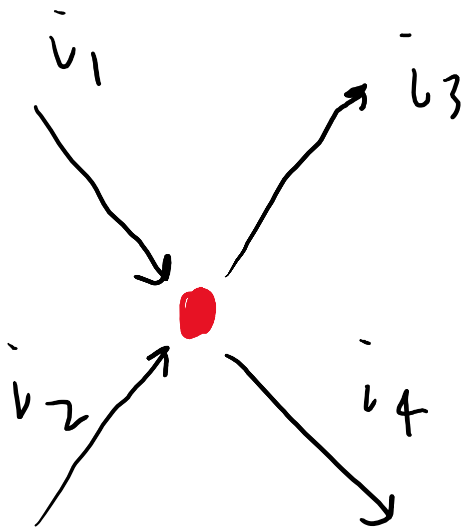


i, current

$$V_+ - V_- > 0$$

Review: KCL

Kirchoff's Current Law: Charge/current can't be created nor destroyed. Algebraic sum of current has to equal zero at a node

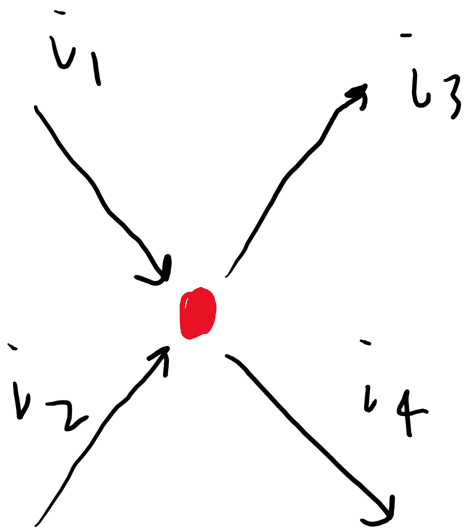


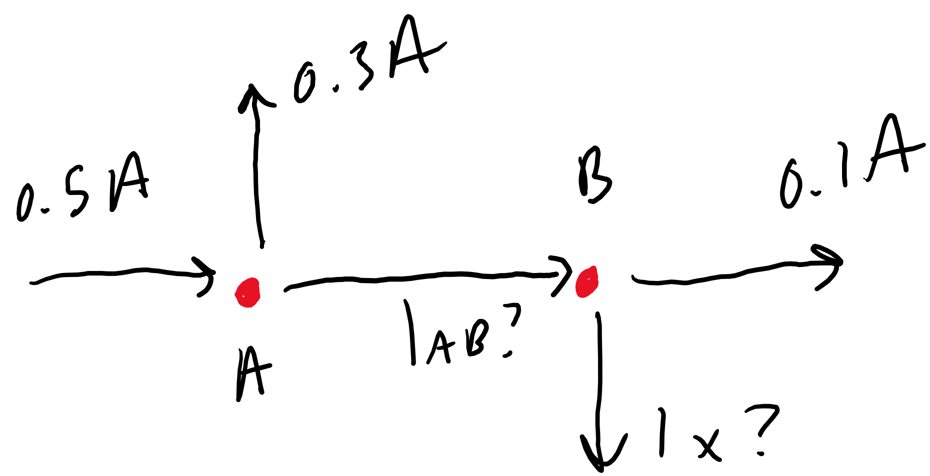
$$i_1 + i_2 - i_3 - i_4$$

$$= 0.$$

Review: KCL

Kirchoff's Current Law: Charge/current can't be created nor destroyed. Algebraic sum of current has to equal zero at a node





$$\bar{i}_1 + \bar{i}_2 - \bar{i}_3 - \bar{i}_4$$

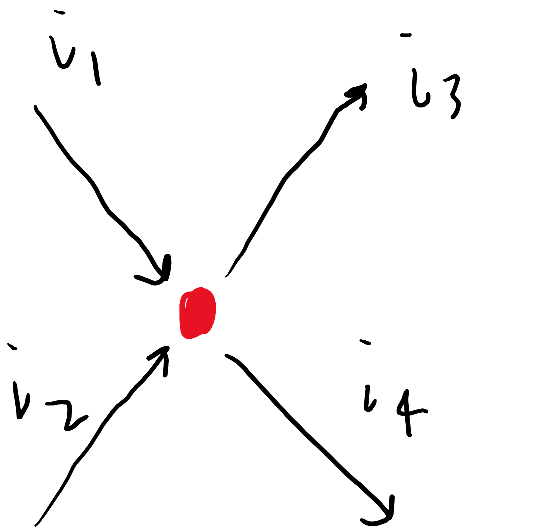
$$= 0.$$

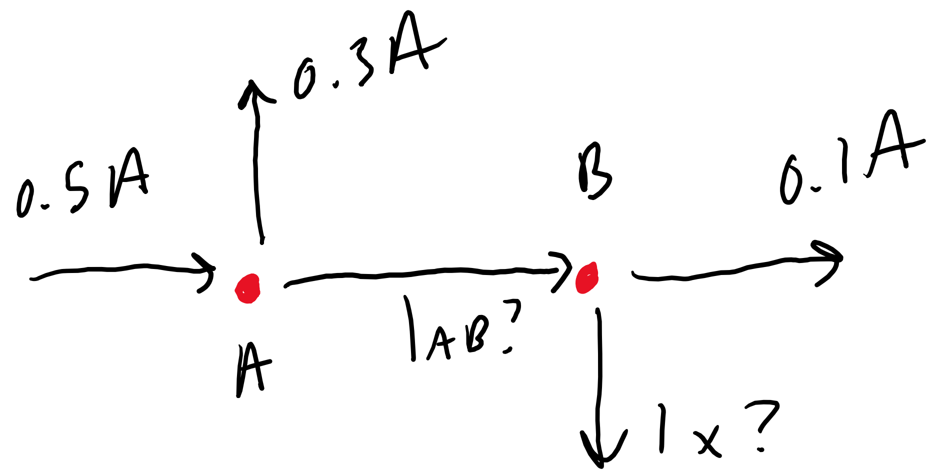
$$0.5A - 0.3A - I_{AB} = 0$$

$$I_{AB} = 0.2A$$

Review: KCL

Kirchoff's Current Law: Charge/current can't be created nor destroyed. Algebraic sum of current has to equal zero at a node





$$\bar{i}_1 + \bar{i}_2 - \bar{i}_3 - \bar{i}_4 = 0.$$

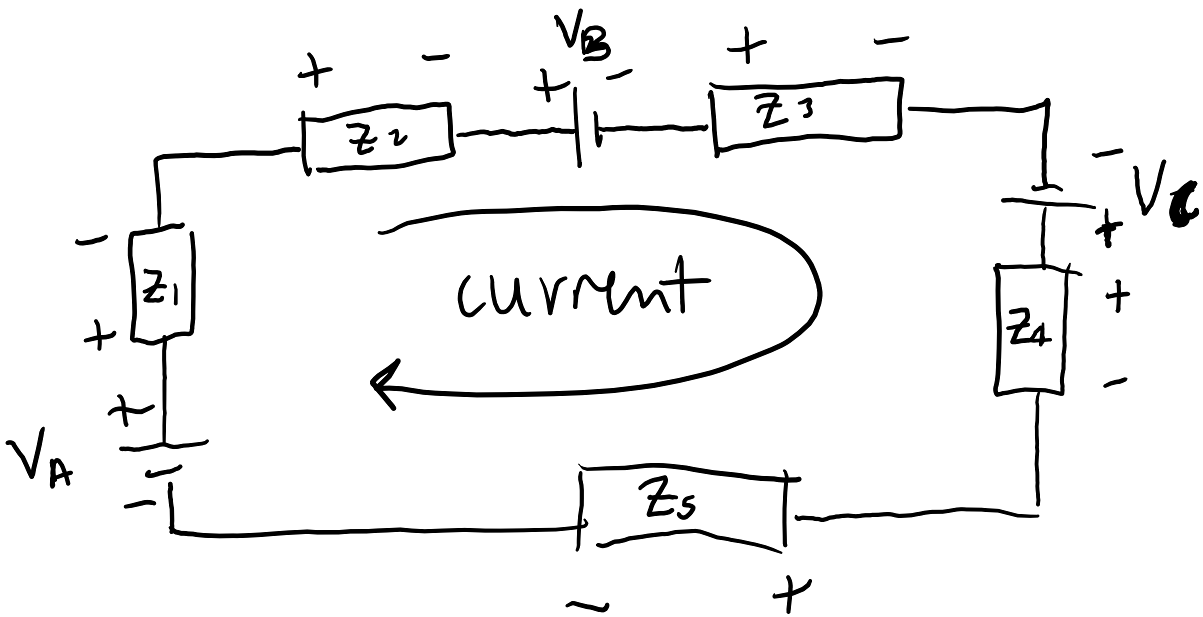
$$0.5A - 0.3A - I_{AB} = 0$$

$$I_{AB} = 0.2A$$

$$0.2A - 0.1A - I_x = 0; \quad I_x = 0.1A$$

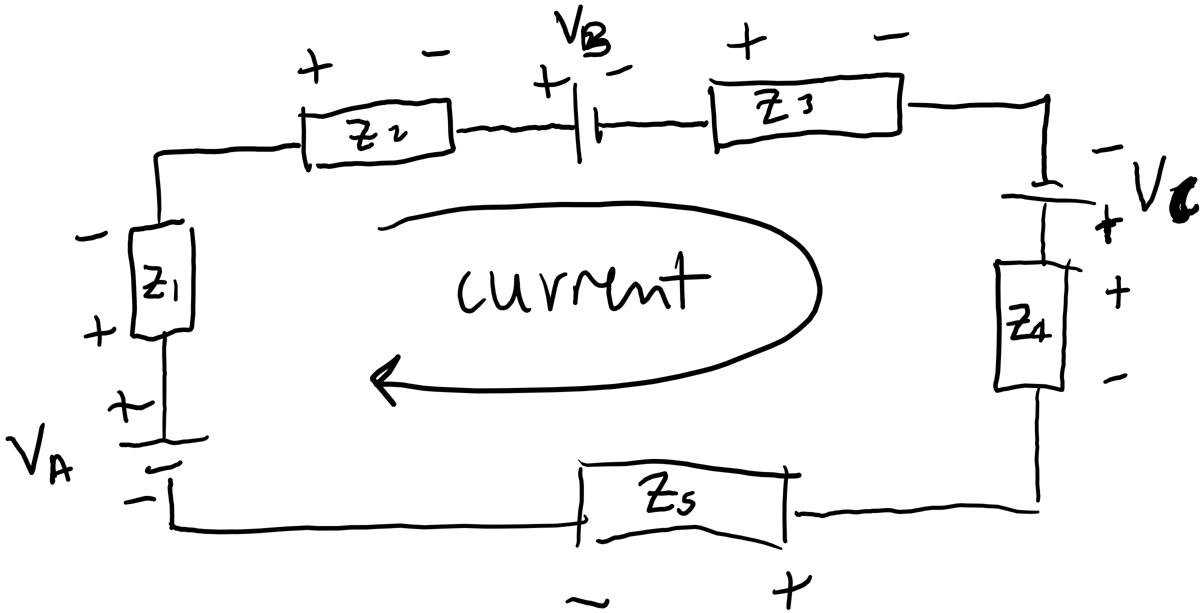
Review: KVL

Kirchoff's Voltage Law: Algebraic sum of voltage is equal zero around a loop



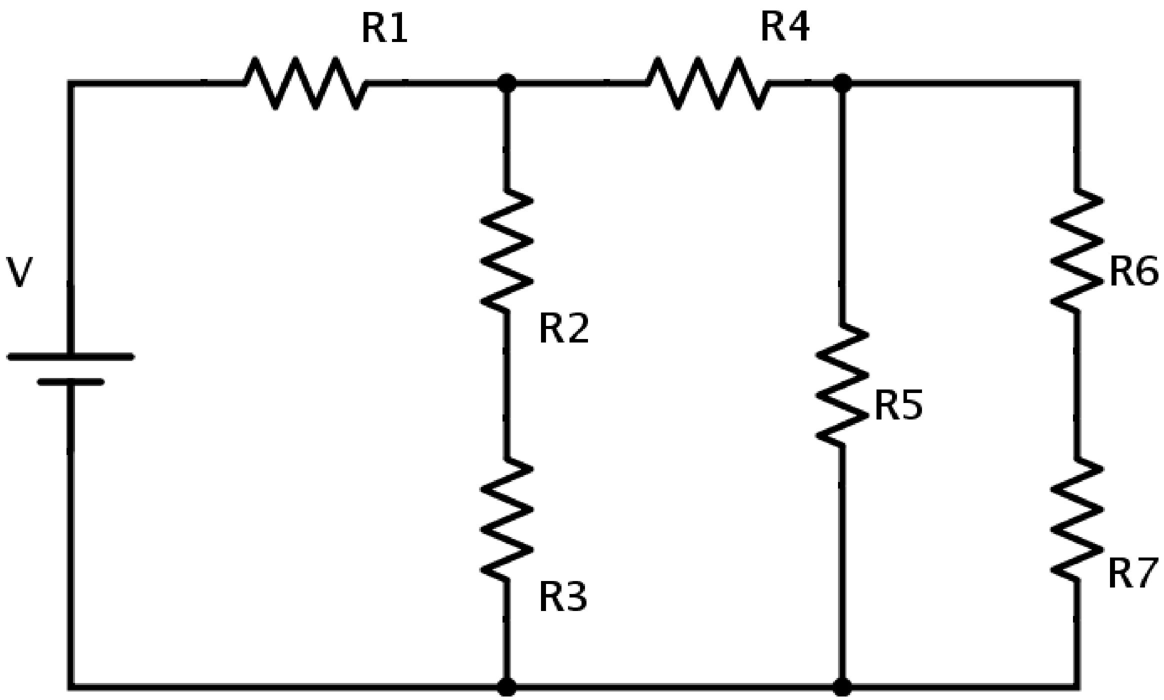
Review: KVL

Kirchoff's Voltage Law: Algebraic sum of voltage is equal zero around a loop



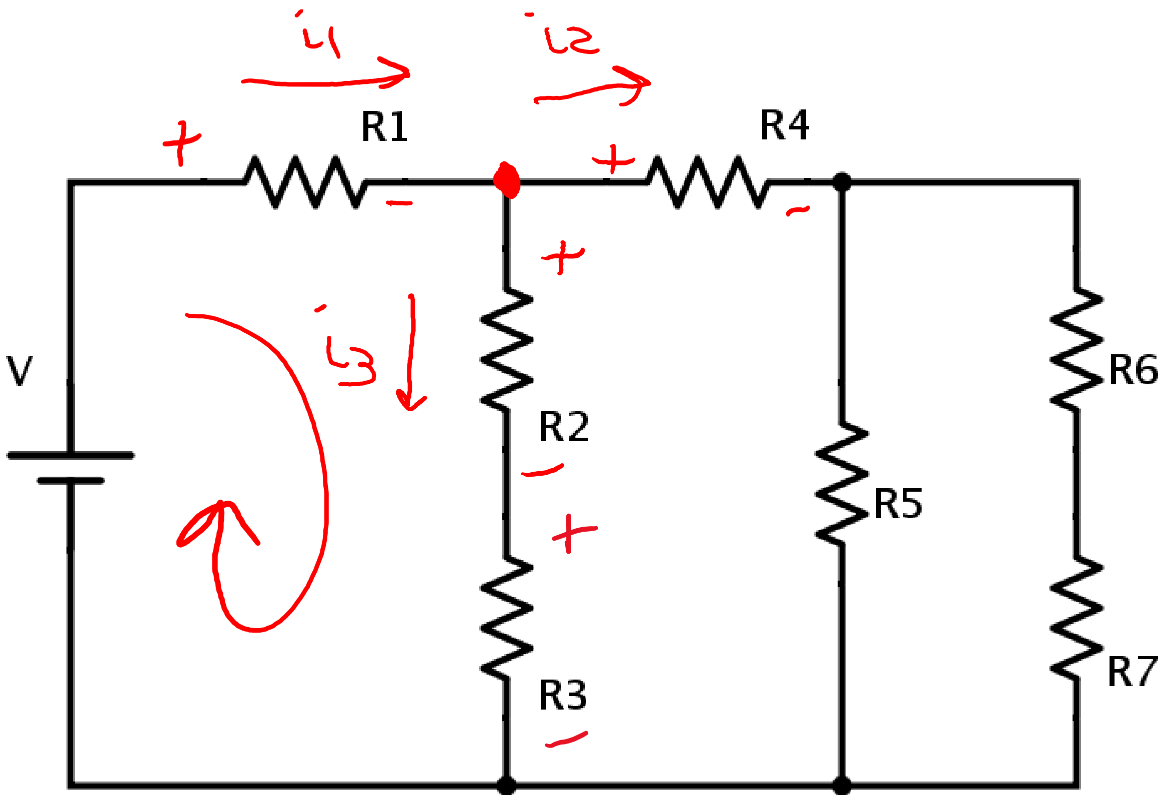
$$+V_A - V_1 - V_2 - V_B - V_3 + V_C - V_4 - V_5 = 0$$

Example of KVL/KCL



Example of KVL/KCL

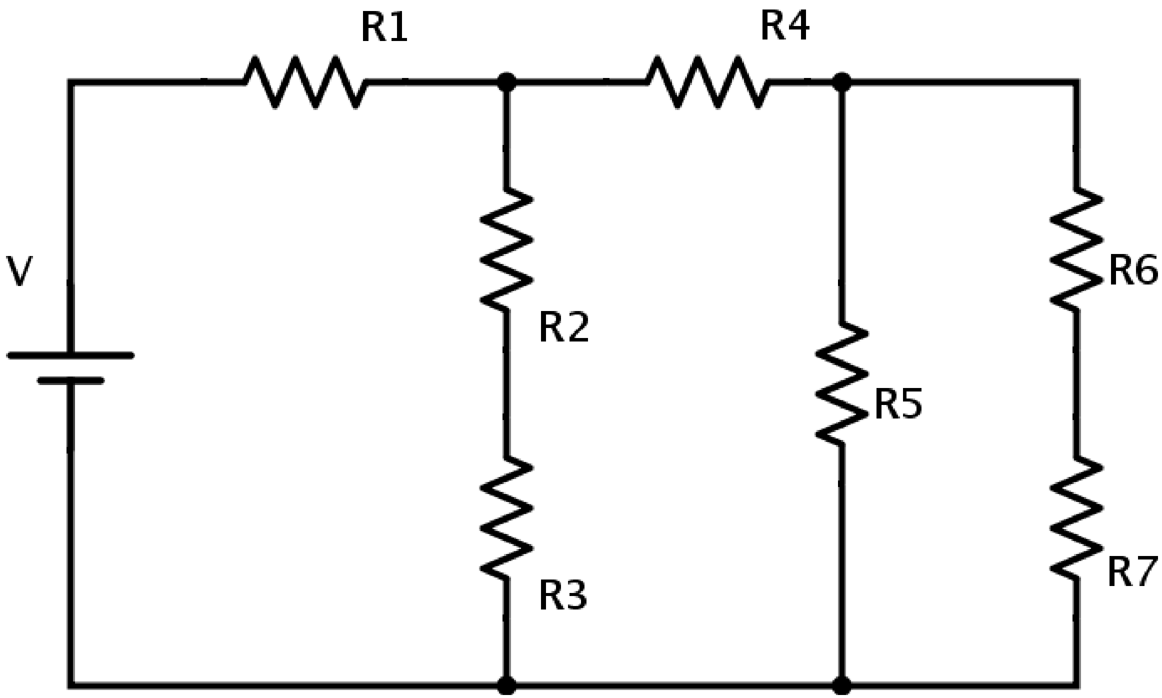
KCL $\sum i = \phi$ $\quad i_1 - i_2 - i_3 = 0$



KVL $\sum v = \phi$ $\quad V - V_1 - V_2 - V_3 = 0$

Review: Node Method

Combines KCL/KVL at a given node to write KCL in terms of what we know



Review: Node Method

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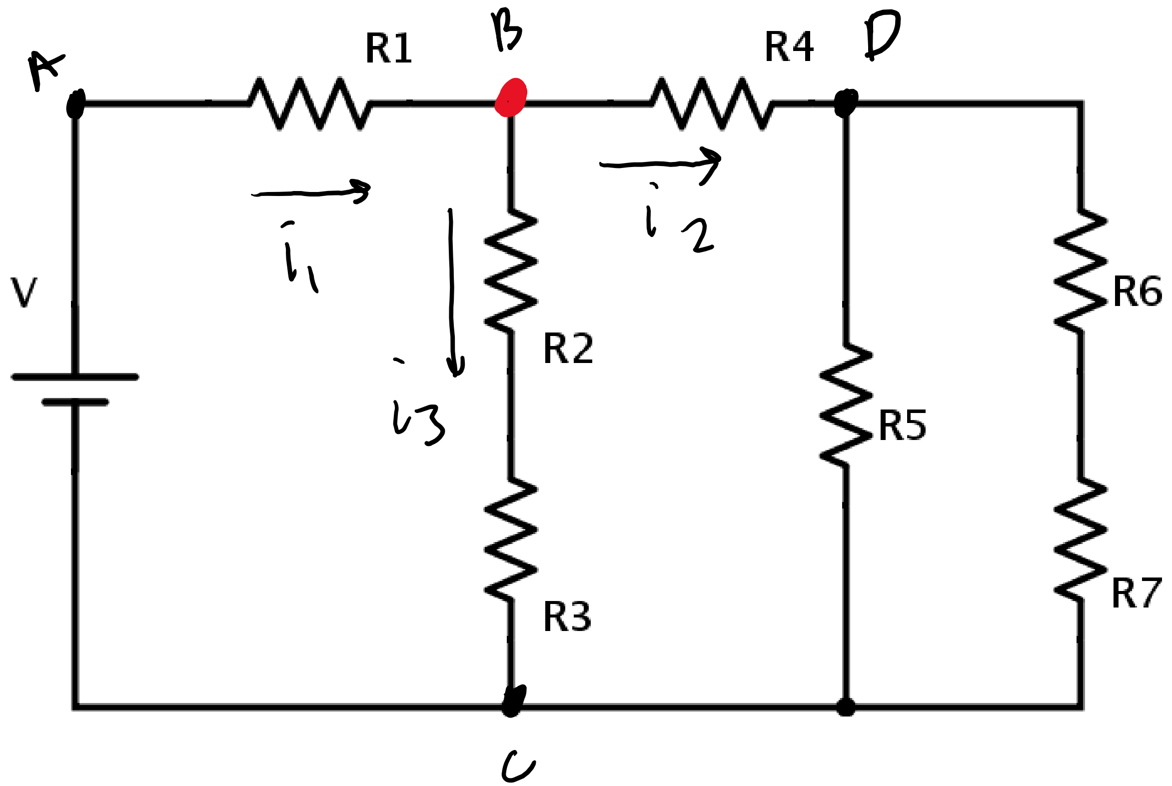
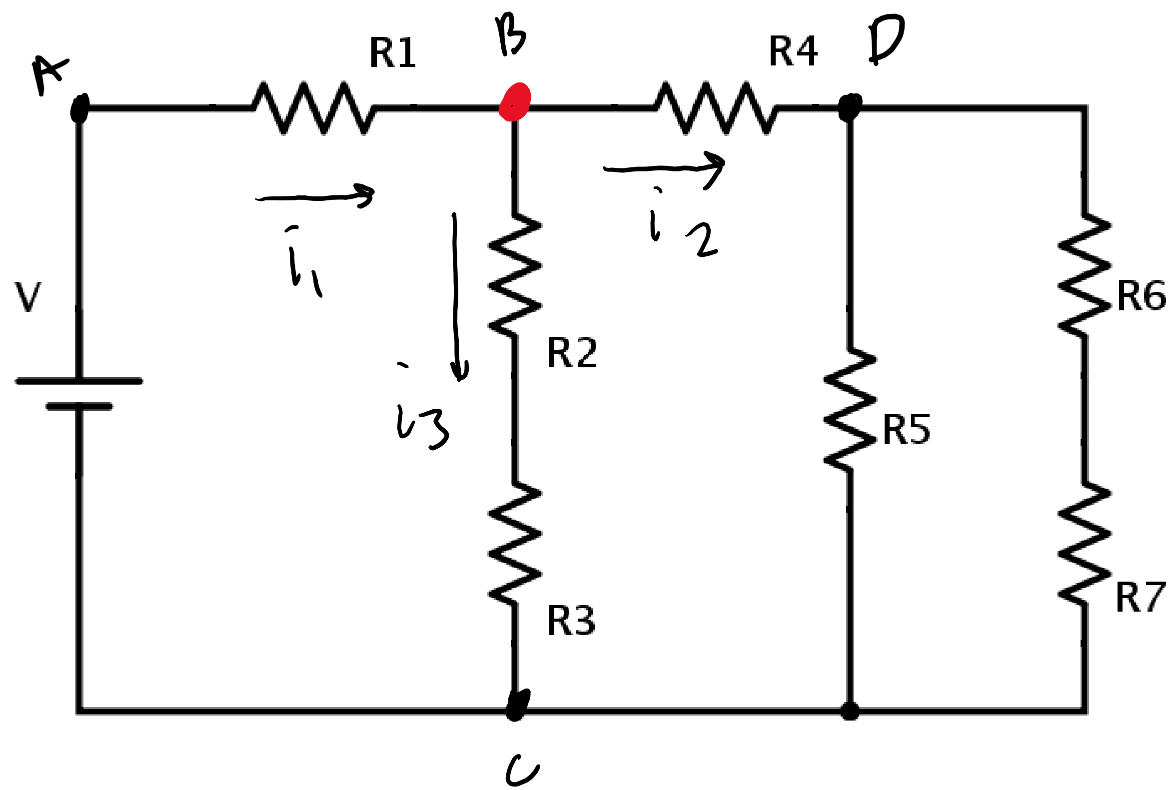


$$i_1 = i_2 + i_3$$

$$V = IR$$

Review: Node Method

Combines KCL/KVL at a given node to write KCL in terms of what we know



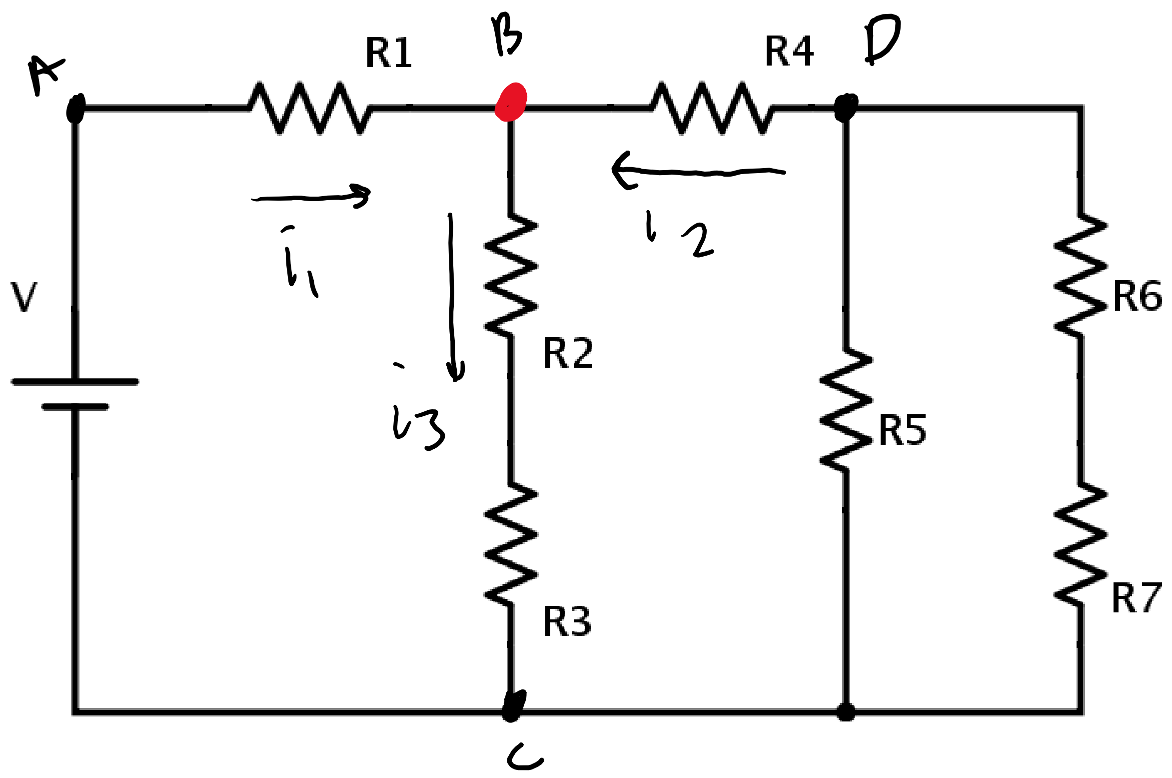
$i_1 = i_2 + i_3$

$V = IR$

$$\frac{V_A - V_B}{R_1} = \frac{V_B - V_D}{R_4} + \frac{V_B - V_C}{R_2 + R_3}$$

Review: Node Method

Combines KCL/KVL at a given node to write KCL in terms of what we know



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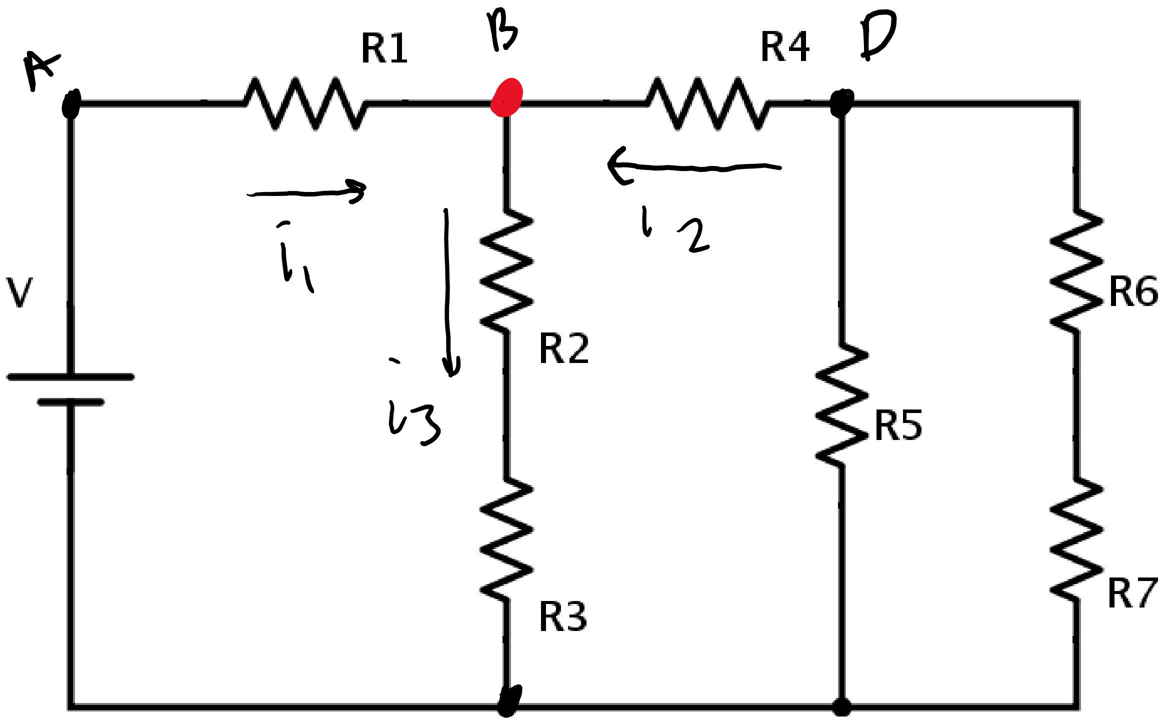
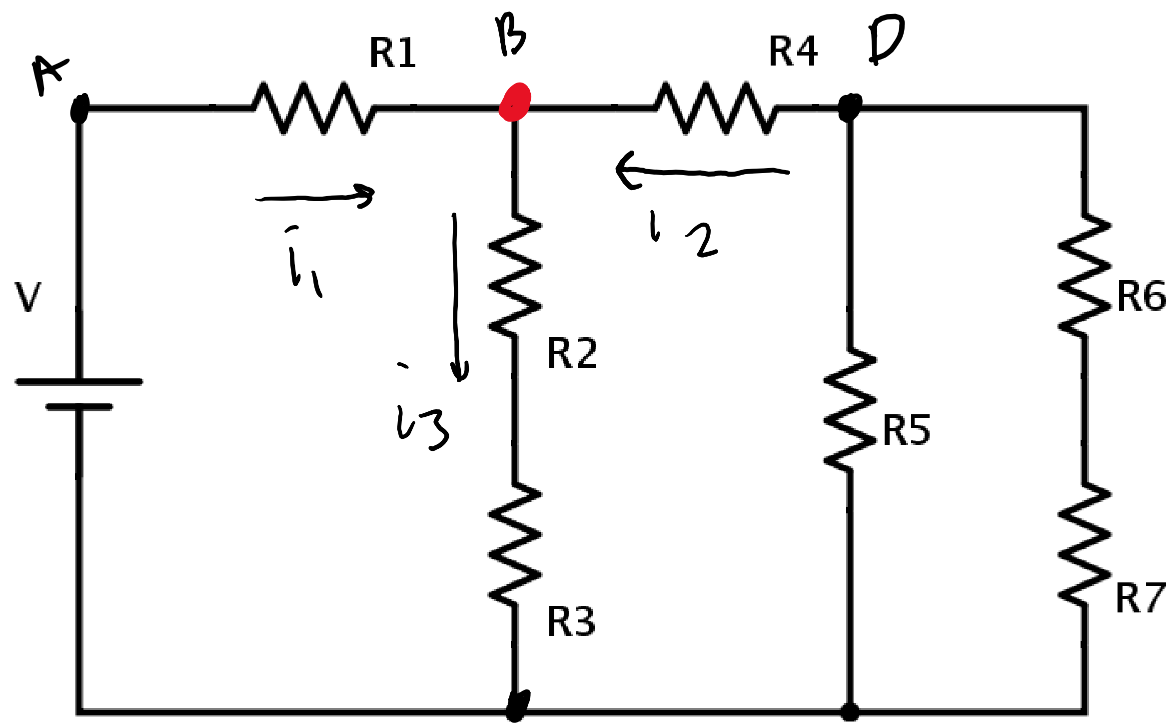


$$i_1 + i_2 = i_3$$

$$V = IR$$

Review: Node Method

Combines KCL/KVL at a given node to write KCL in terms of what we know

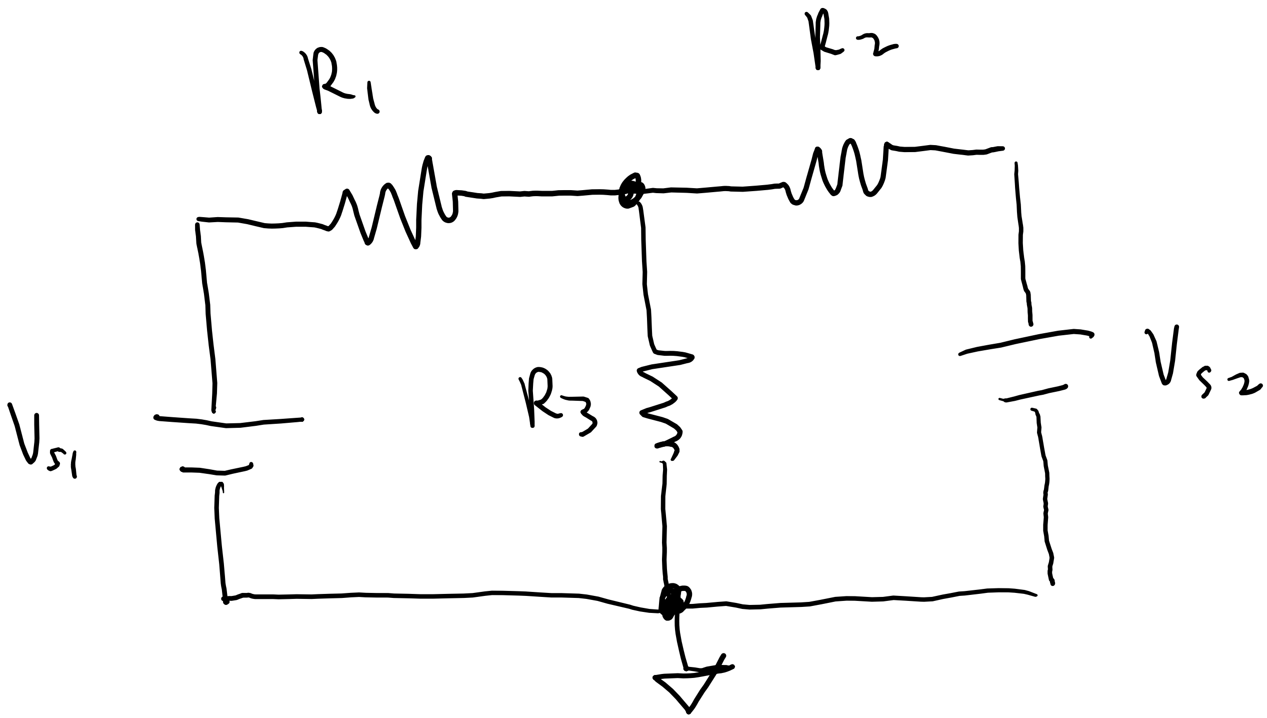


$$i_1 + i_2 = i_3$$

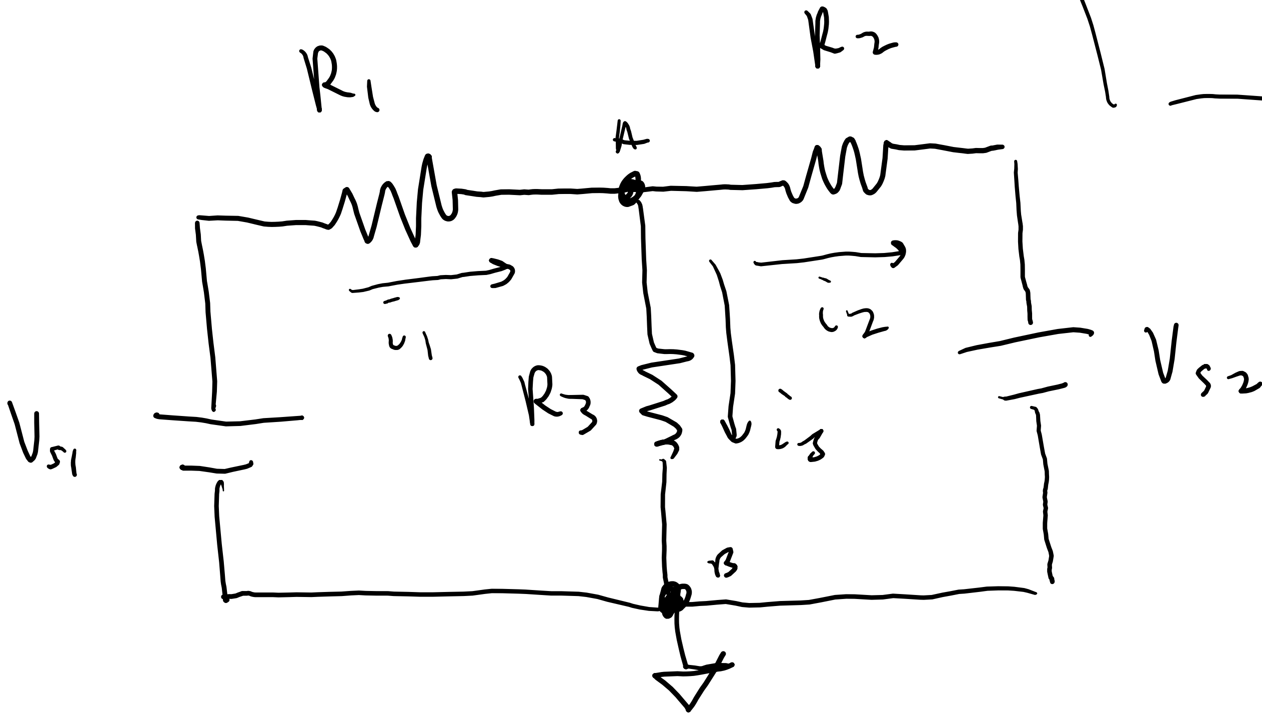
$$V = IR$$

$$\frac{V_A - V_B}{R_1} + \frac{V_D - V_B}{R_4} = \frac{V_B - V_C}{R_2 + R_3}$$

Example 1



Example 1



$$\boxed{i_1 = i_2 + i_3}$$

Example 1

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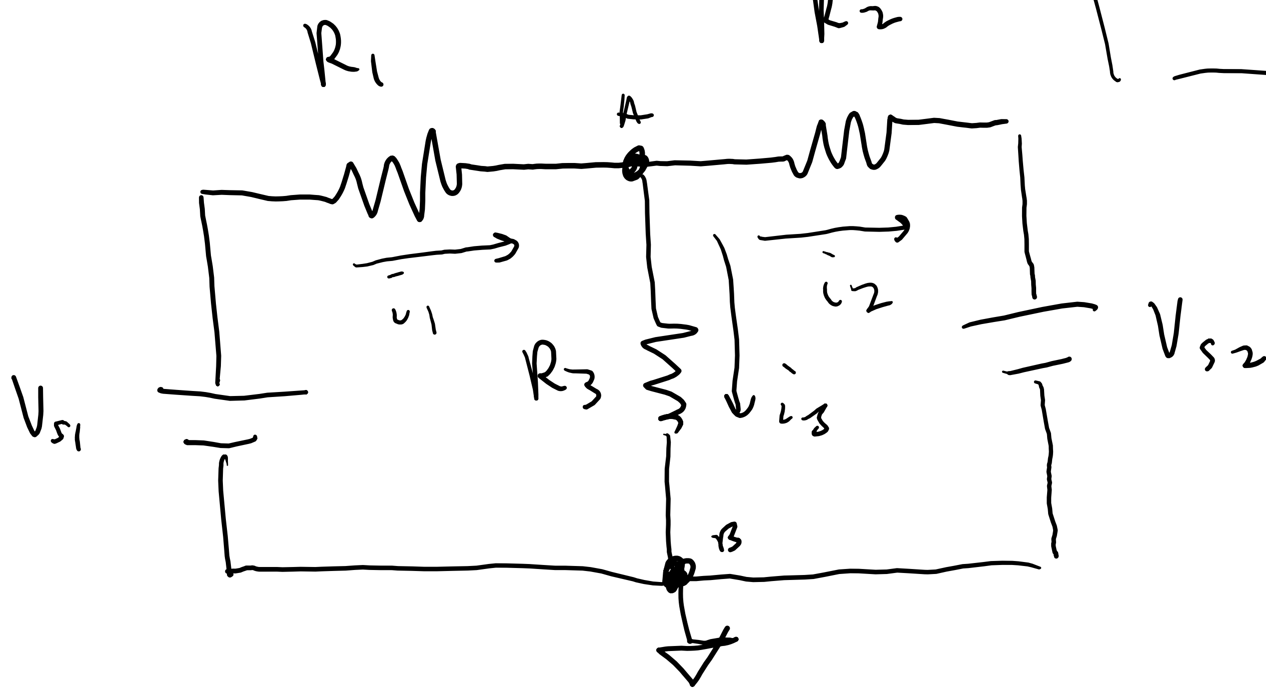
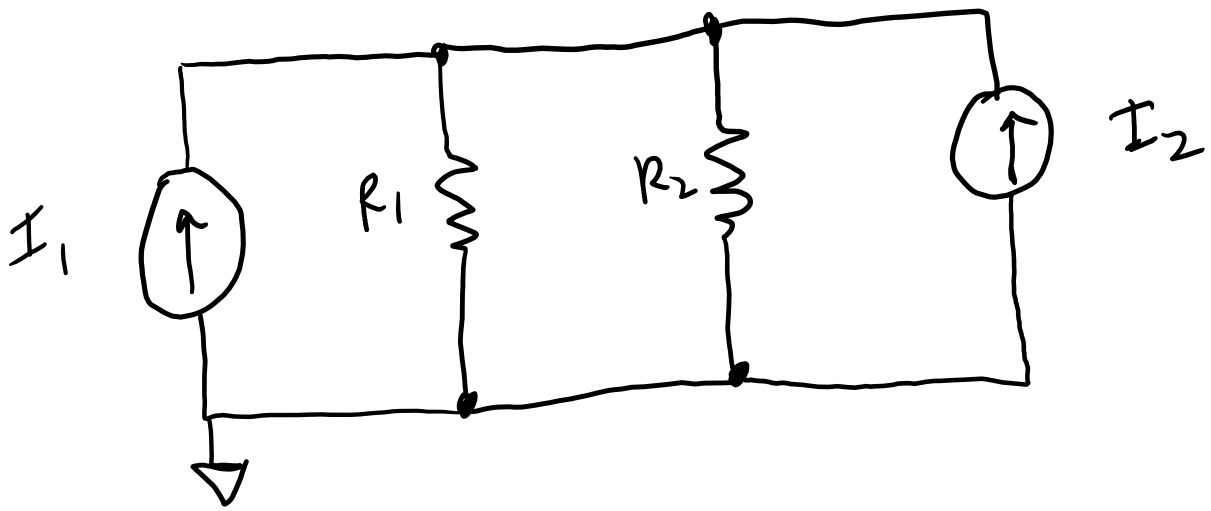
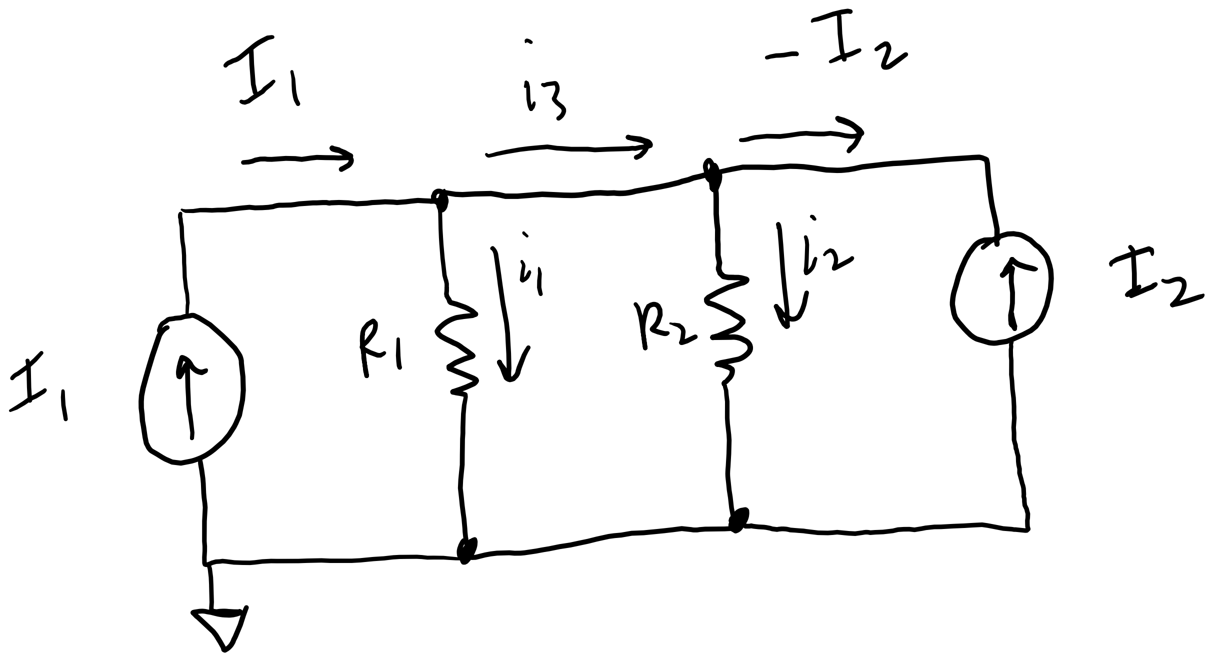


$$\frac{V_{s1} - V_A}{R_1} = \frac{V_A - V_{s2}}{R_2} + \frac{V_A - V_B}{R_3}$$

Example 2



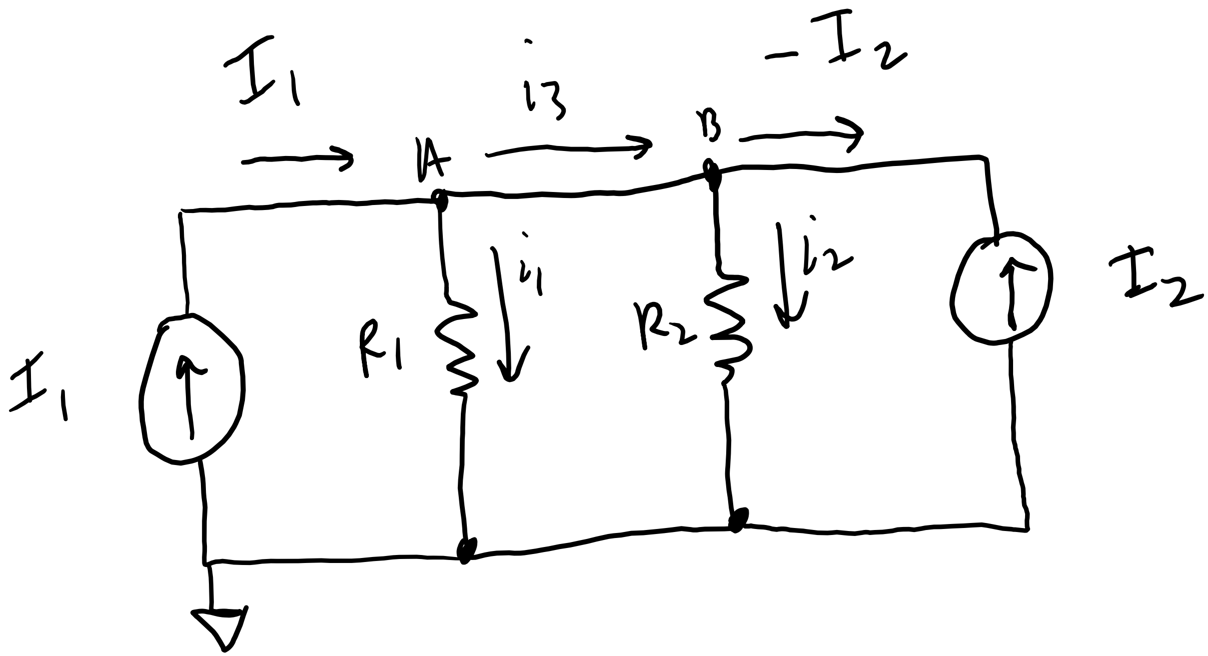
Example 2



$$I_1 = i_1 + i_3$$

$$i_3 = -I_2 + i_2$$

Example 2



$$I_1 = i_1 + i_3$$

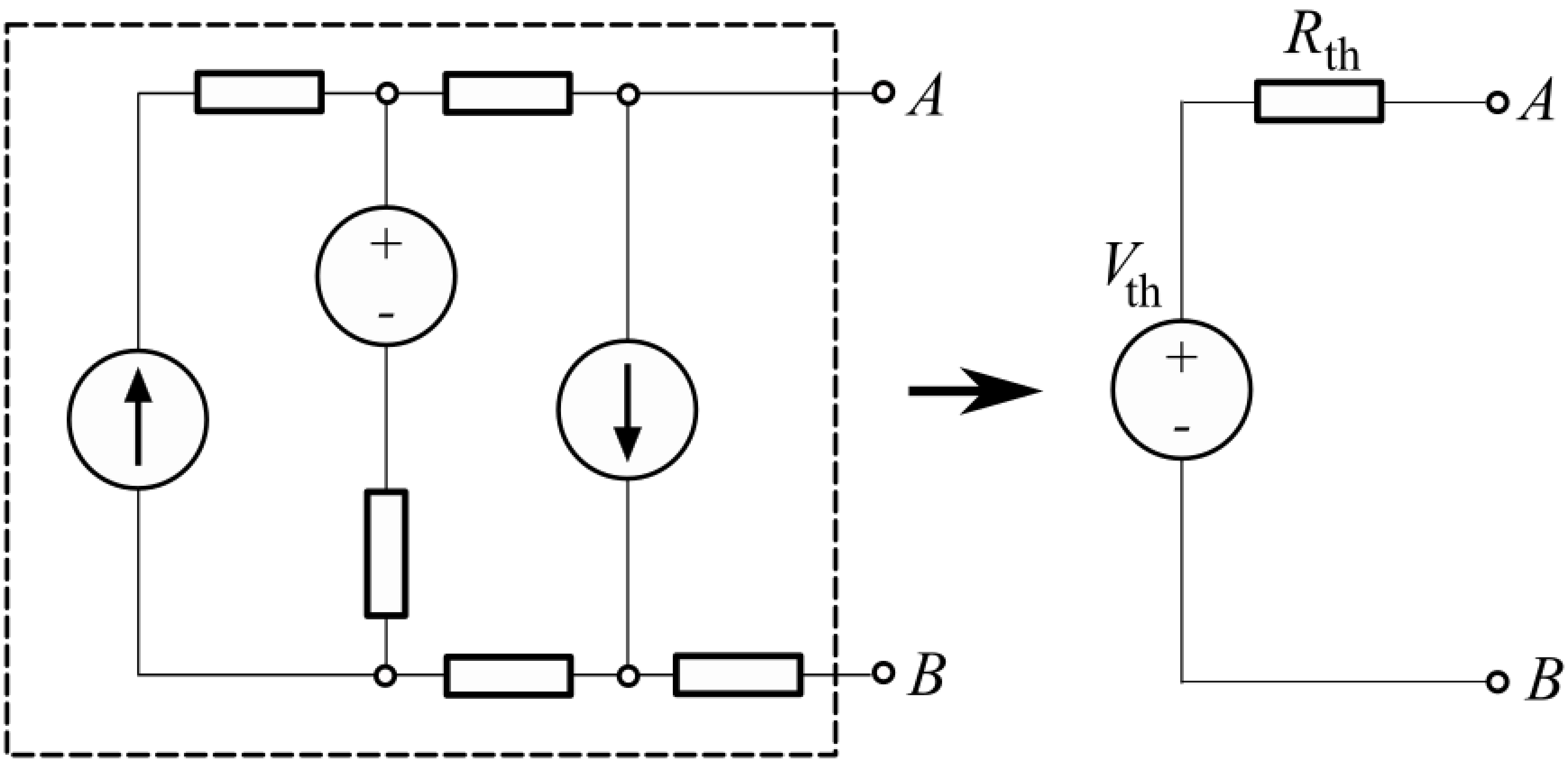
$$i_3 = -I_2 + i_2$$

$$I_1 = \frac{V_A}{R_1} + i_3$$

$$i_3 = -I_2 + \frac{V_B}{R_2}$$

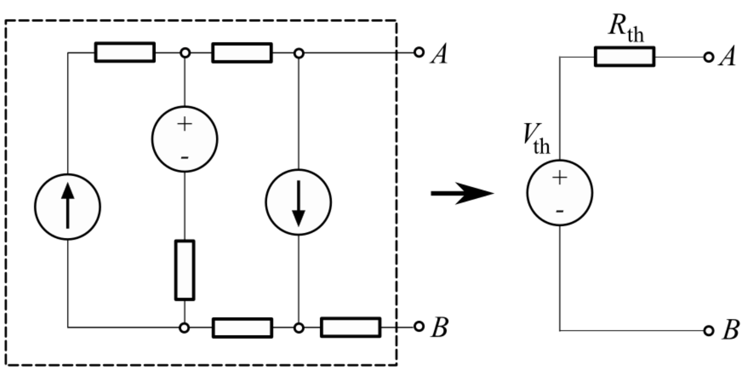
Review: Thevenin Equivalent

Thevenin's Theorem: Any ***linear*** electrical network containing only voltage sources, current sources, and resistances can be replaced at terminals A-B by an equivalent combination of a voltage source V_{th} in a series connection with a resistance R_{th}.

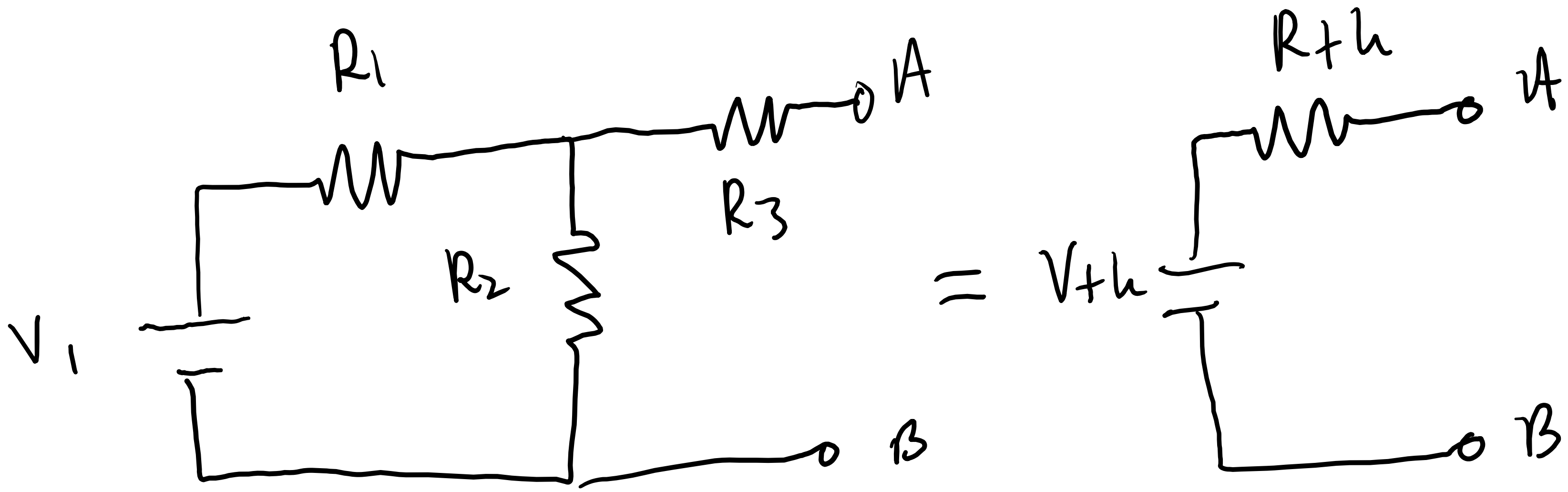


Thevenin Equivalent: Rules

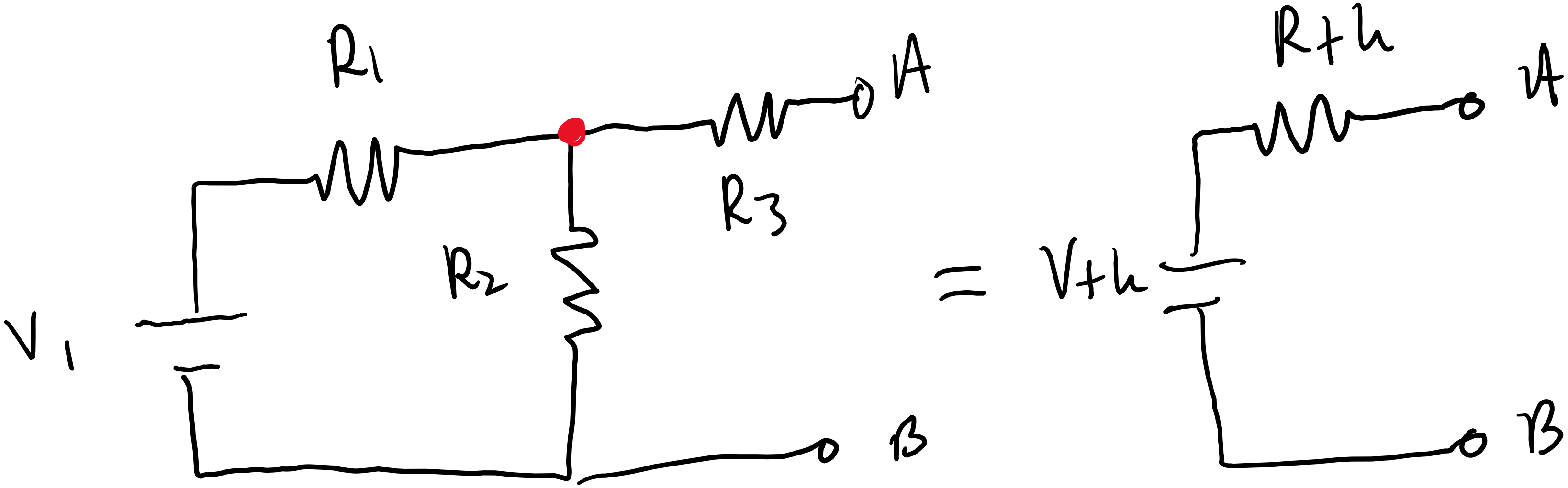
1. The equivalent voltage V_{th} is the voltage you'd get across terminals A-B
2. The equivalent resistance R_{th} is the resistance that the circuit would have across A-B if we replace voltage sources with short circuits and current sources with open circuits
3. If terminals A and B of the Thevenin equivalent circuit are connect to each other, the current flowing would be V_{th}/R_{th}



Thevenin Equivalent: Example 1

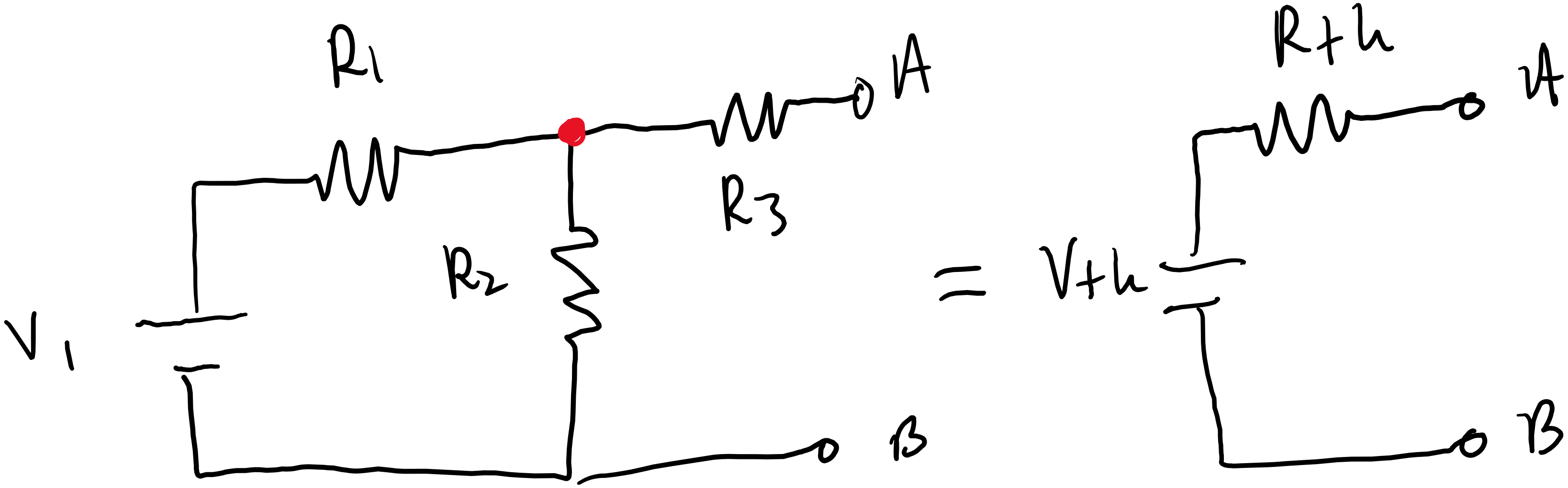


Thevenin Equivalent: Example 1



$$\frac{V_1 - V_A}{R_1} = \frac{V_A}{R_2} \longrightarrow \frac{V_1}{R_1} = \frac{V_A}{R_2} + \frac{V_A}{R_1}$$

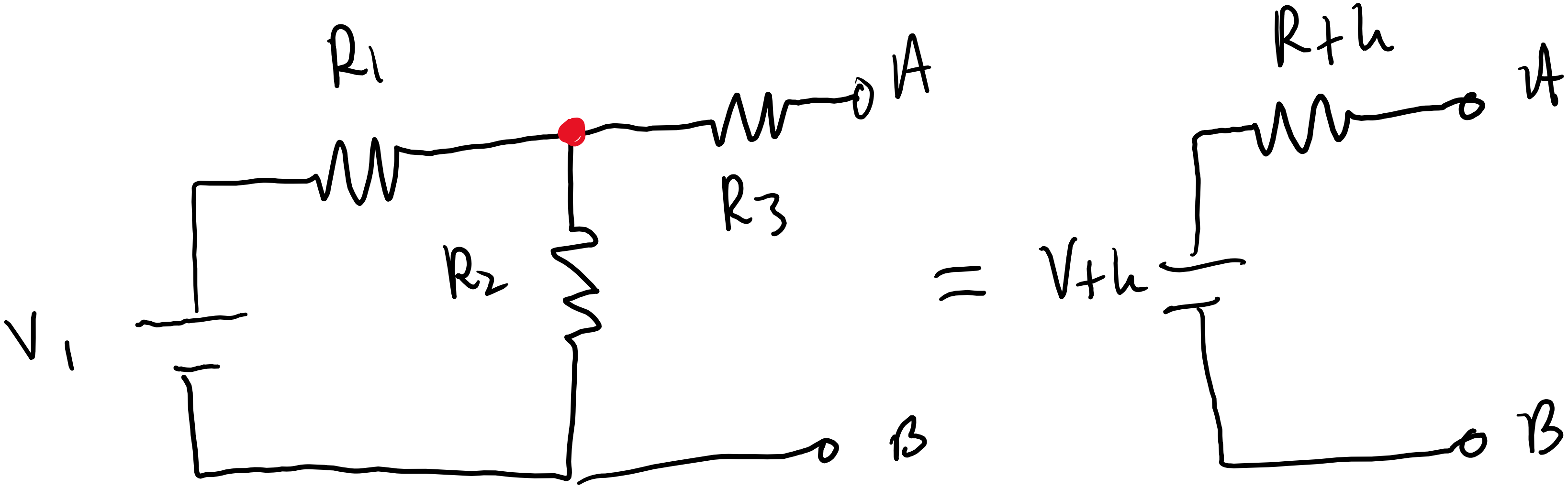
Thevenin Equivalent: Example 1



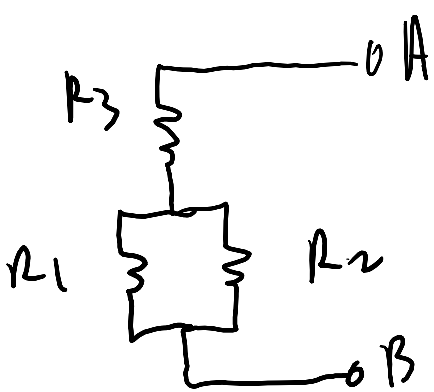
$$\frac{V_1 - V_A}{R_1} = \frac{V_A}{R_2} \longrightarrow \frac{V_1}{R_1} = \frac{V_A}{R_2} + \frac{V_A}{R_1}$$

$$\longrightarrow \frac{V_1}{R_1} \cdot \left(\frac{R_2 R_1}{R_1 + R_2}\right) = V_A \longrightarrow \underline{\underline{V_A = V_{th}}} = \frac{V_1 R_2}{R_1 + R_2}$$

Thevenin Equivalent: Example 1

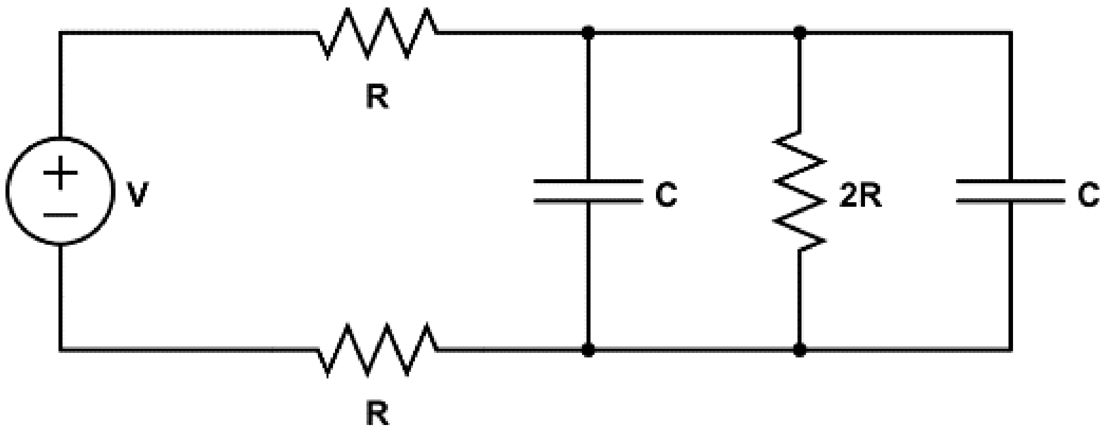


⇓ set V_1 = short

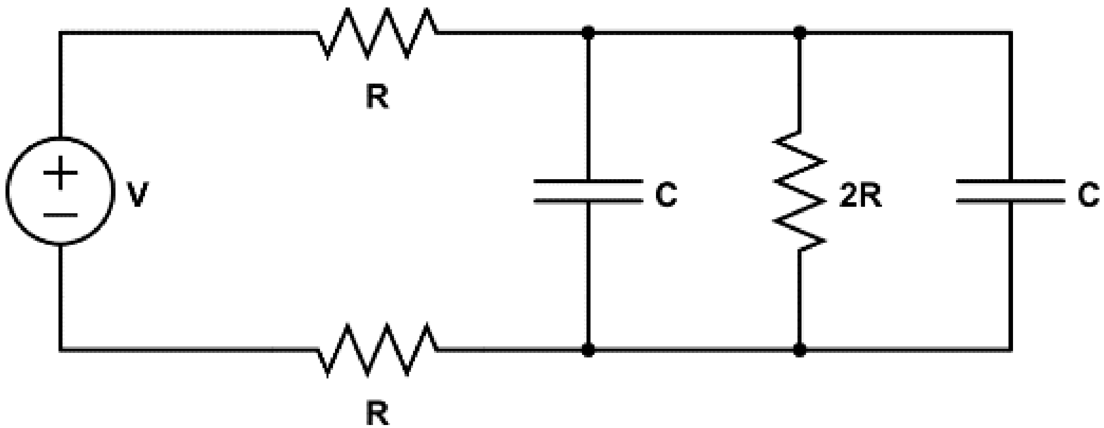


$$R_{th} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

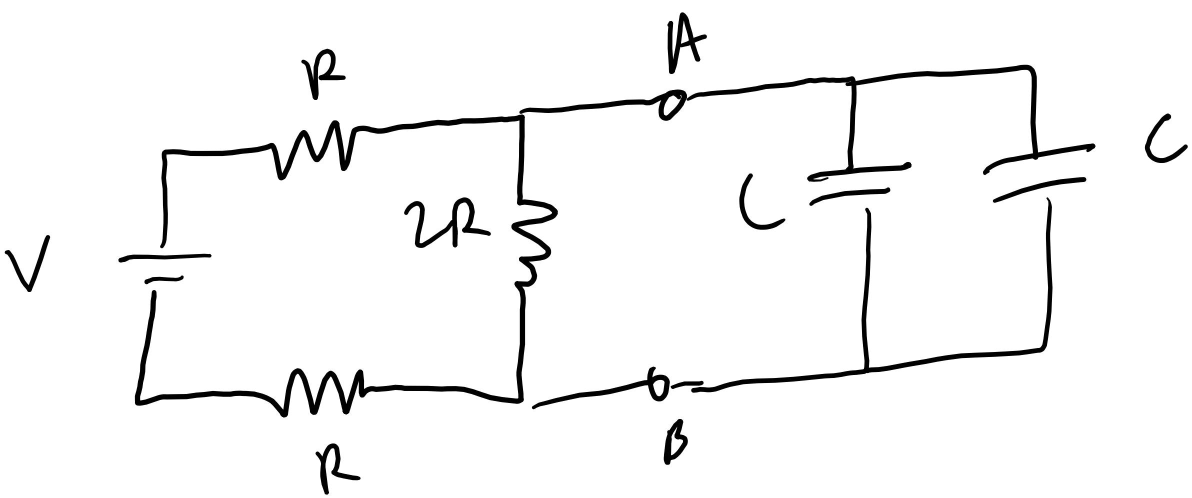
Thevenin Equivalent Example 2



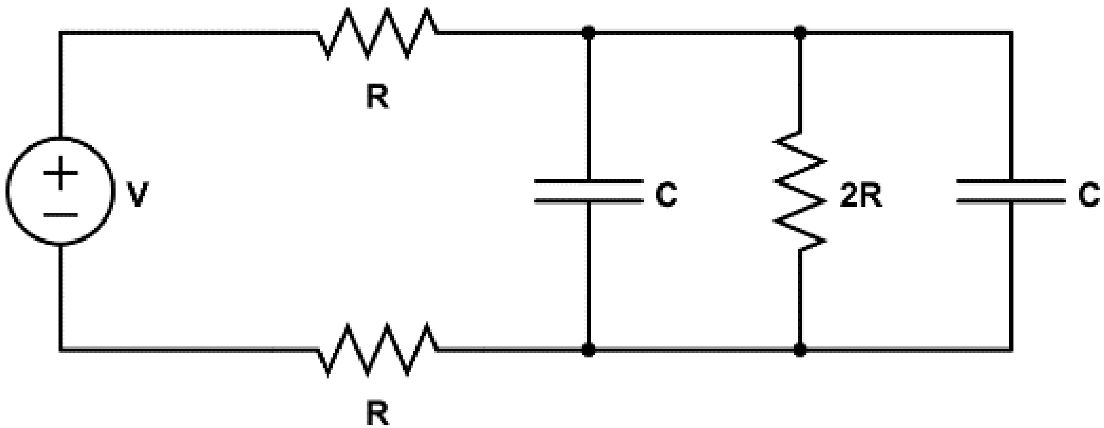
Thevenin Equivalent Example 2



Rewrite:



Thevenin Equivalent Example 2



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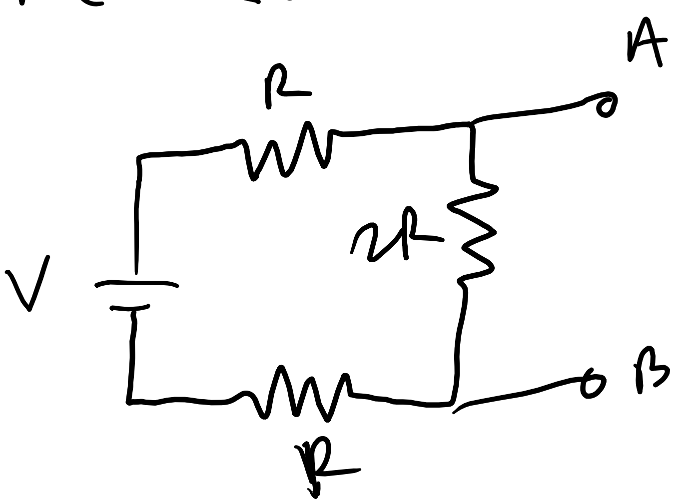
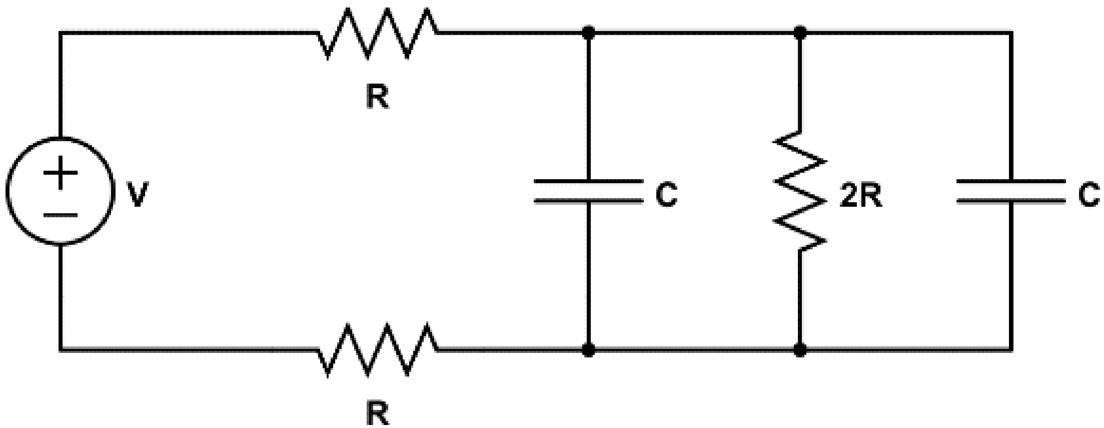


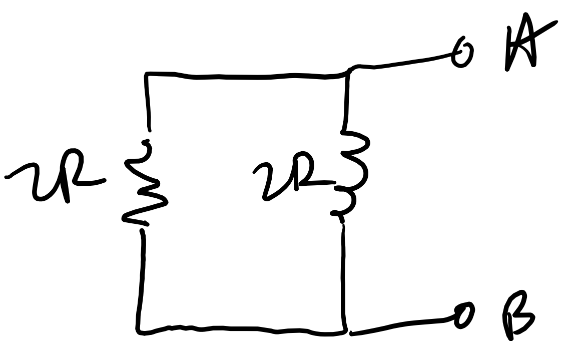
$$V = IR \rightarrow i = \frac{V}{4R}$$

$$V_{AB} = V_{th} = \frac{V}{4R} \cdot 2R = \frac{V}{2}$$

Thevenin Equivalent Example 2



Rewrite.



$$\left(\frac{1}{R_{th}} = \frac{1}{2R} + \frac{1}{2R} \right)$$

$$V = IR \rightarrow i = \frac{V}{4R}$$

$$V_{AB} = V_{th} = \frac{V}{4R} \cdot 2R = \frac{V}{2}$$

$$R_{th} = \frac{2R \cdot 2R}{2R + 2R} = \frac{4R^2}{4R} = R.$$

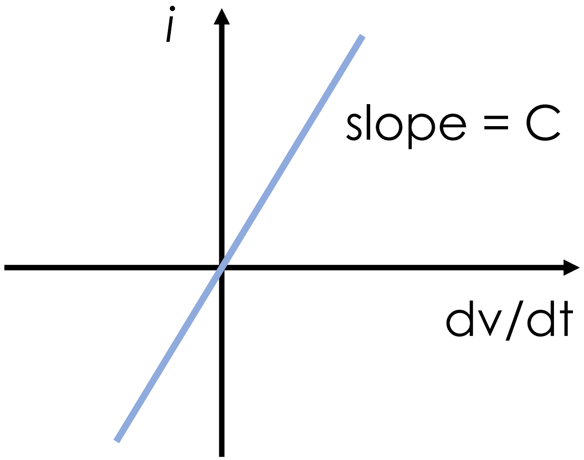
End of the KCL/KVL and Thevenin Review

Review: Storage Elements

Capacitor

Element law: $$i(t) = C\frac{dv(t)}{dt}$$

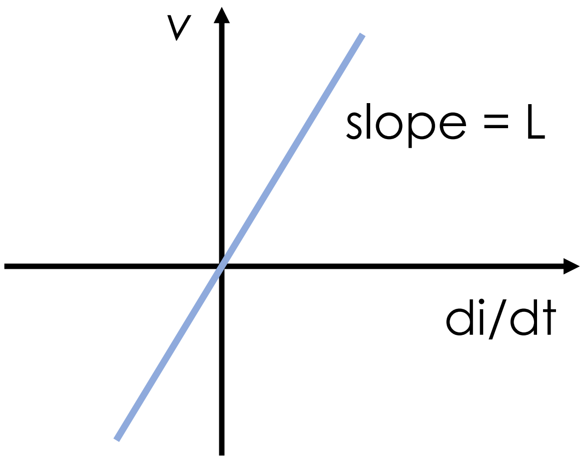
Stored energy: $$w_E(t) = \frac{Cv(t)^2}{2}$$



Inductor

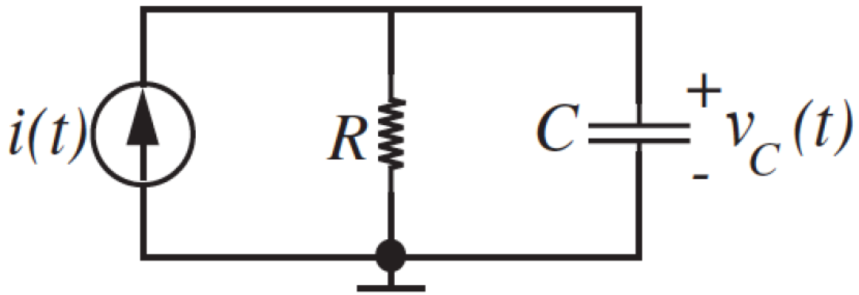
Element law: $$v(t) = L\frac{di(t)}{dt}$$

Stored energy: $$w_M(t) = \frac{Li(t)^2}{2}$$



RULE OF THUMB: $C \longleftrightarrow L$, $v(t) \longleftrightarrow i(t)$

Review: Simple RC Circuit – Step Response (I_0)

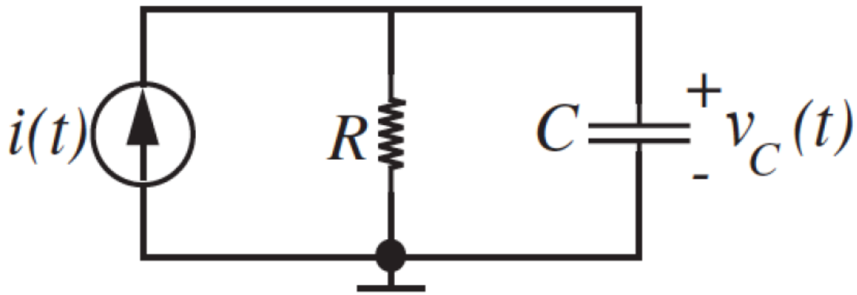


- Find $v_c(t)$ when the circuit is driven by $i(t) = I_0 f(t) = \begin{cases} 0, & t < 0 \\ I_0, & t \geq 0 \end{cases}$
- Initial condition: $v_c(t<0) = 0$ (circuit initially uncharged and at rest)

Solution Steps

1. Use node method to find the differential equation describing v_c
2. Find the homogeneous solution v_{ch} (set the drive to zero)
3. Find the particular solution v_{cp} (educated guess)
4. The total solution is the sum of the homogeneous and particular solutions. Use the initial conditions to solve for the constants.

Review: Simple RC Circuit – Step Response (I_0)



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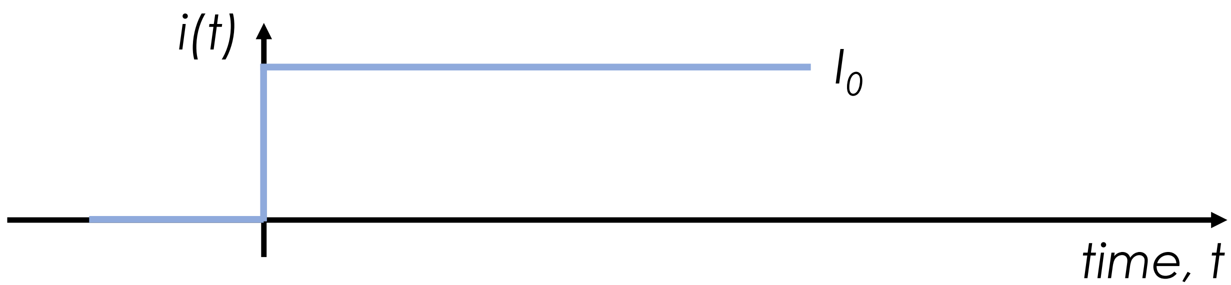

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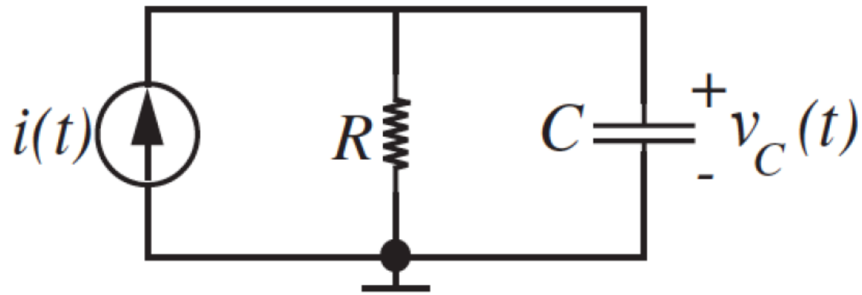

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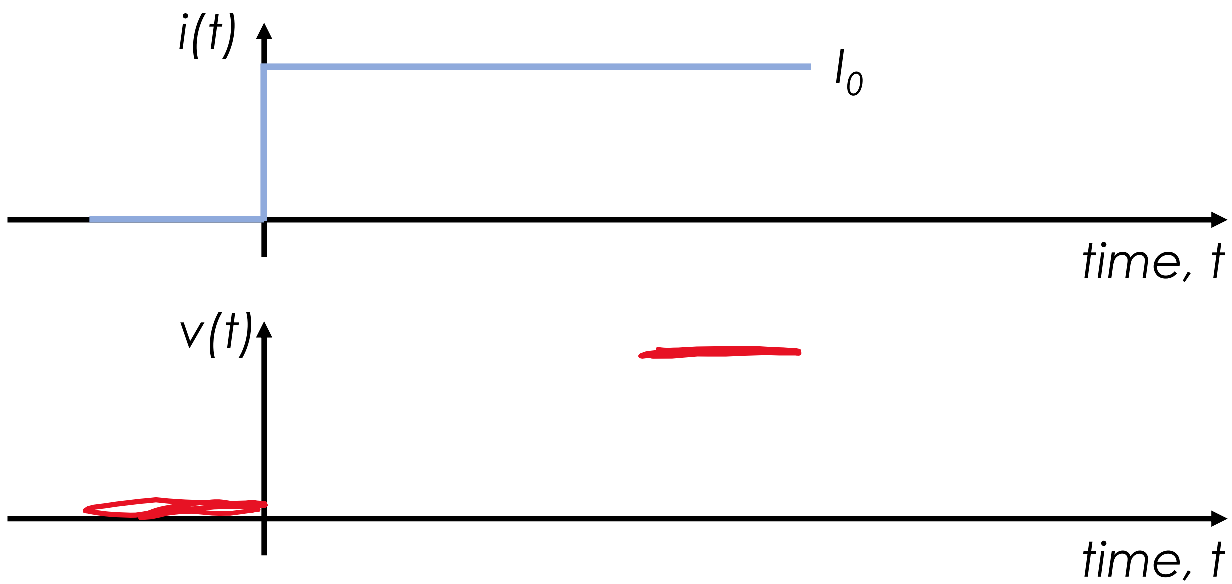

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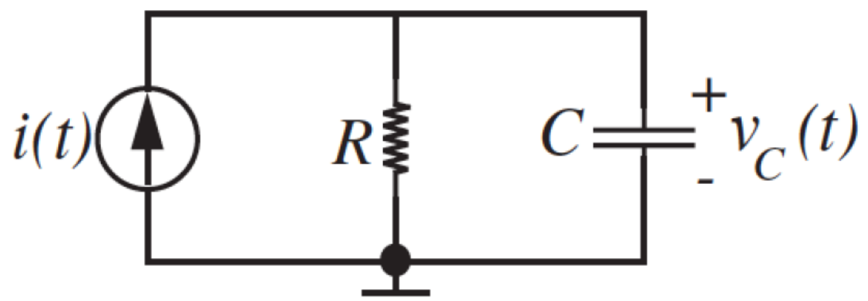

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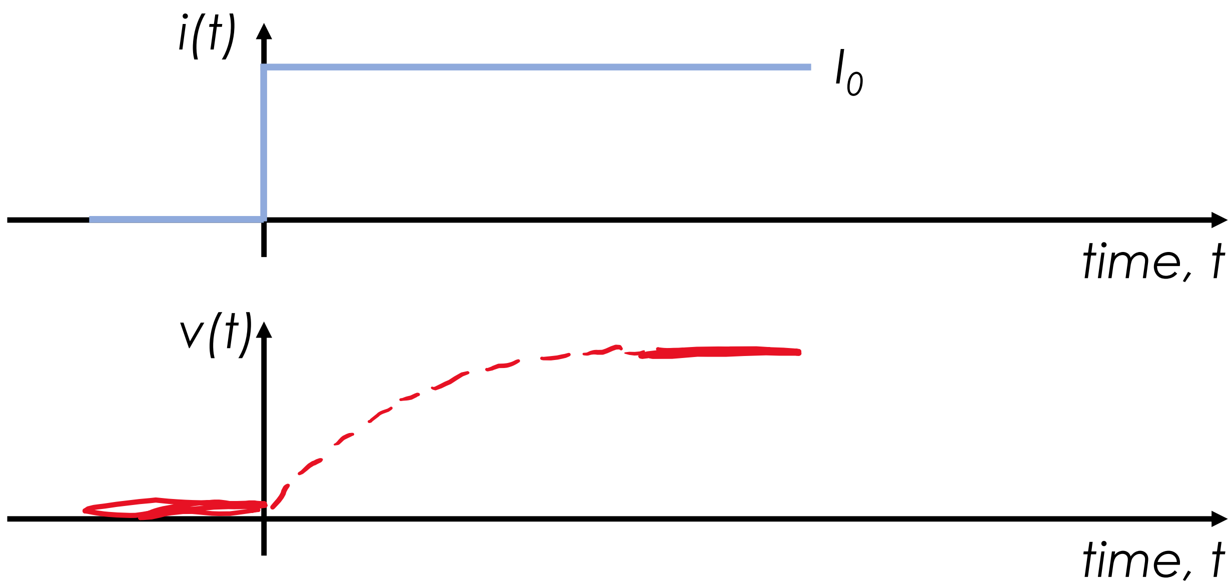
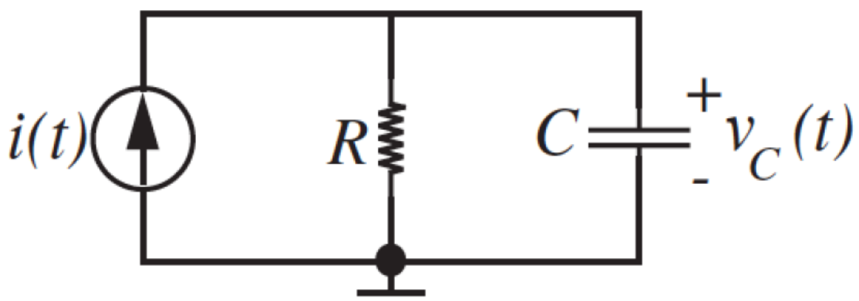
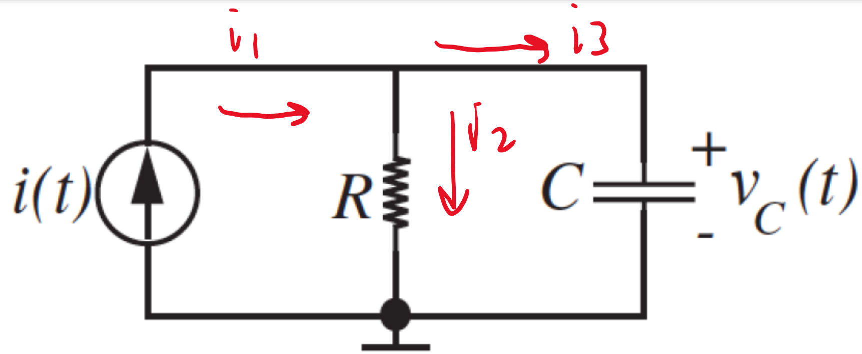


Review: Simple RC Circuit – Step Response (I_0)



Review: Simple RC Circuit – Step Response (I_0)

$i_1 = i_2 + i_3$



$i_c = C\frac{dV_c}{dt}$

$i_2 = \frac{V_c}{R}$

Review: Simple RC Circuit – Step Response (I_0)

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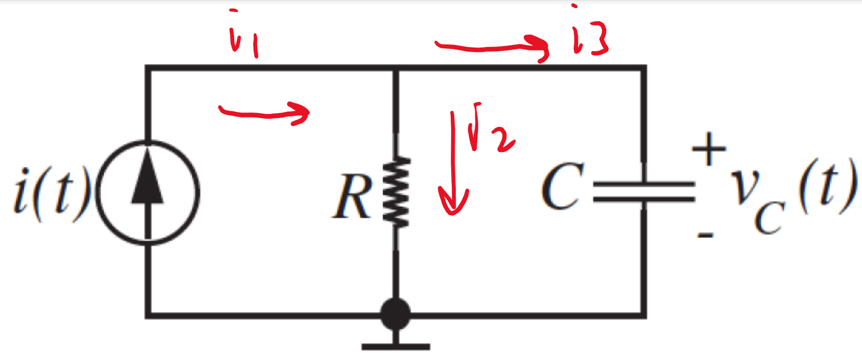

$i_c = C\frac{dV_c}{dt}$

$i_2 = \frac{V_c}{R}$

- From node method/KCL:

$$i(t) = \frac{V_c}{R} + C\frac{dV_c}{dt} \quad (1^{st} \text{ order ODE})$$

Review: Simple RC Circuit – Step Response (I_0)

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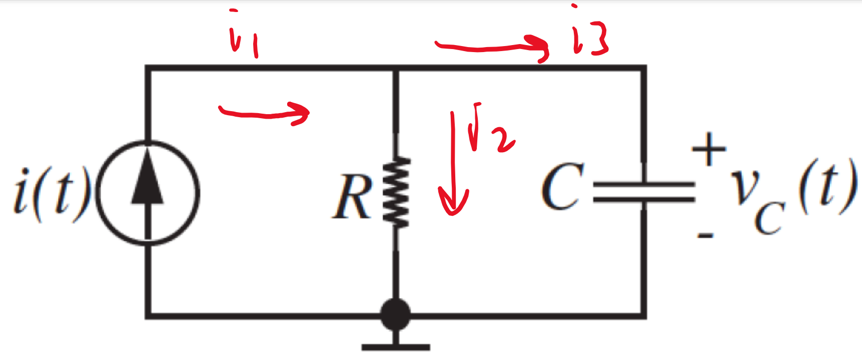

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- From node method/KCL:

$$i(t) = \frac{V_c}{R} + C\frac{dV_c}{dt} \quad (1^{st} \text{ order ODE})$$

- To find the homogeneous solution, set the drive *i(t)* = 0

$$\cancel{0} = \frac{V_c}{R} + C\frac{dV_c}{dt}$$

The homogeneous solution to any constant coefficient ODE is a superposition of functions of the form Ae^{st} (with a caveat*)

* unless the characteristic equation has repeated roots

Review: Simple RC Circuit – Step Response (I_0)

- Plug $v_{ch} = Ae^{st}$ into the differential equation $0 = \frac{v_{ch}}{R} + C\frac{dv_{ch}}{dt}$
- **Characteristic equation:** $0 = \frac{Ae^{st}}{R} + CsAe^{st} \Rightarrow s = -\frac{1}{RC}$

The characteristic equation of an n^{th} degree differential equation is a polynomial of n^{th} degree. We know that an n^{th} degree polynomial will have n roots (including repeated roots or complex roots)

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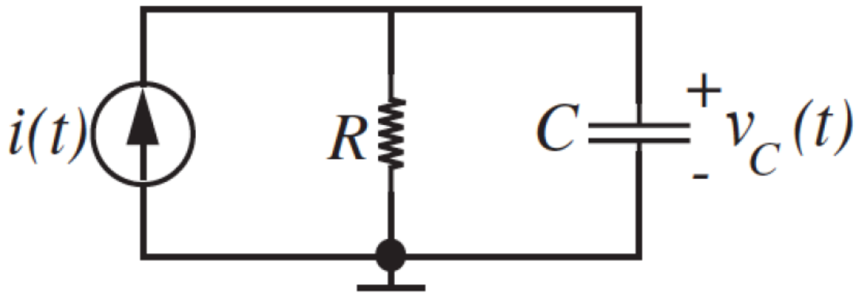
The characteristic equation of an n^{th} degree differential equation is a polynomial of n^{th} degree. We know that an n^{th} degree polynomial will have n roots (including repeated roots or complex roots)

- s ≡ root of characteristic equation, also = "natural frequency" (Chpt 12)
- **Homogeneous Solution:** $V_{ch}(t) = Ae^{-t/RC}$
- Natural response of the circuit (only depends on internal components)
- **RC dimensions = time and is called the *time constant* of the circuit**
- We'll find *A* after solving for the particular solution

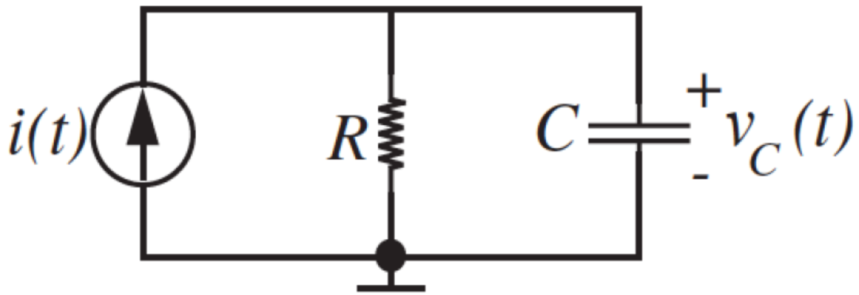
Review: Simple RC Circuit – Step Response (I_0)



- The particular solution satisfies the original equation but doesn't need to satisfy the initial conditions:

$$I_0 = \frac{v_{cp}}{R} + C\frac{dv_{cp}}{dt}$$

Review: Simple RC Circuit – Step Response (I_0)



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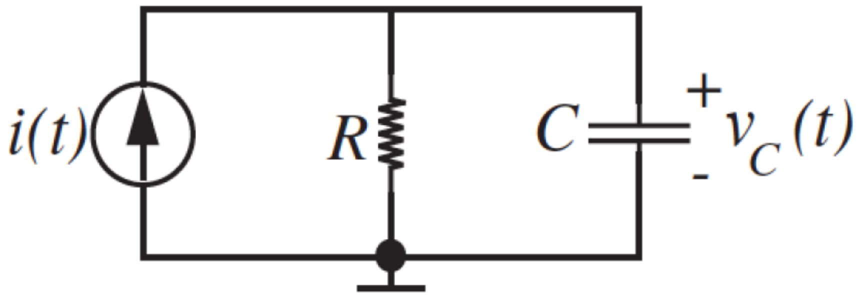
- Since source is constant for $t > 0$, one acceptable solution: $v_{cp} = K$

Review: Simple RC Circuit – Step Response (I_0)



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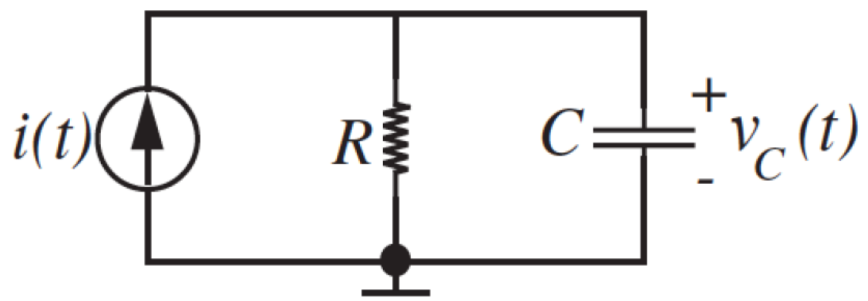
- Total solution? $\mathbf{v_{ch} + v_{cp}}$

$V(t) = I_0 R + A e^{-t/RC}$

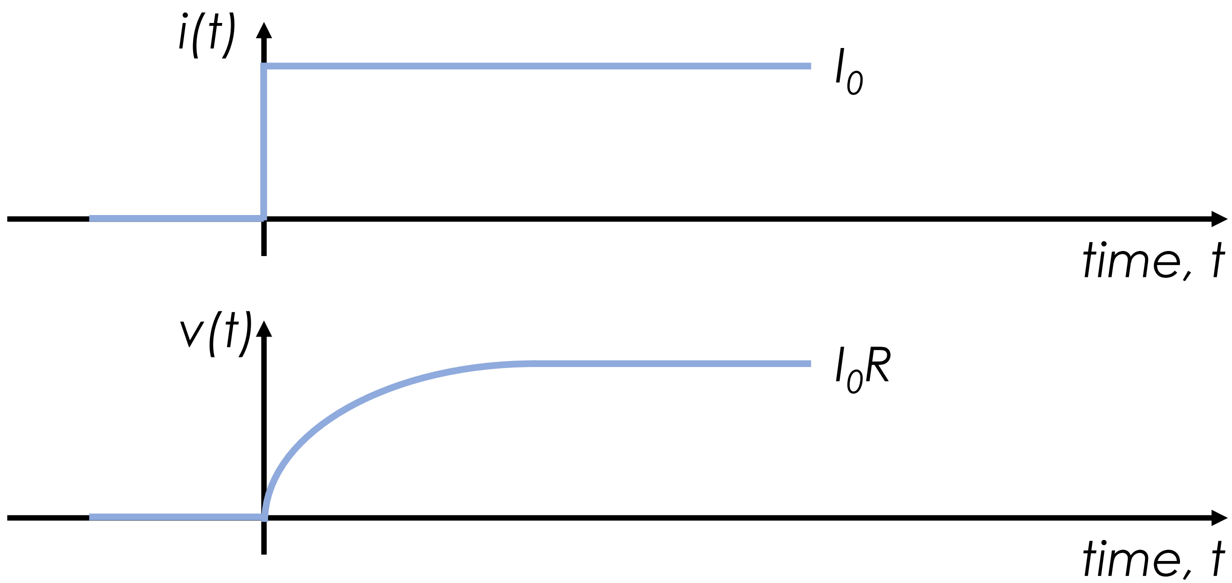
- Initial condition v_C = 0 for t = 0 allows us to find unknown constant A

$A = -I_0 R \rightarrow V(t) = I_0 R - I_0 R e^{-t/RC}$

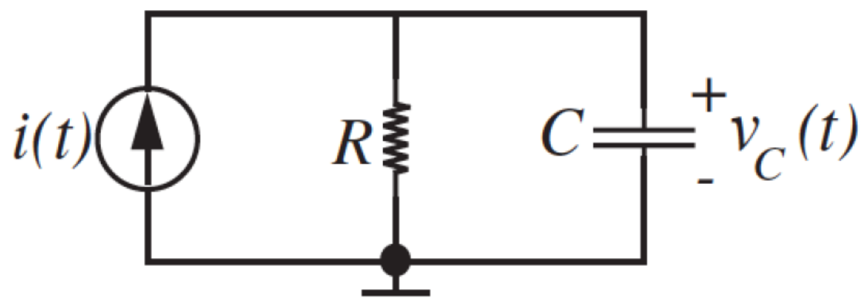
Review: Simple RC Circuit – Step Response (I_0)



$$v_c(t) = -I_0 R e^{-\frac{t}{RC}} + I_0 R$$



Review: Simple RC Circuit – Discharge Transient



- Find $v_c(t)$ when the circuit is driven by $i(t) = I_0 f(t) = \begin{cases} I_0, & t < 0 \\ 0, & t \geq 0 \end{cases}$
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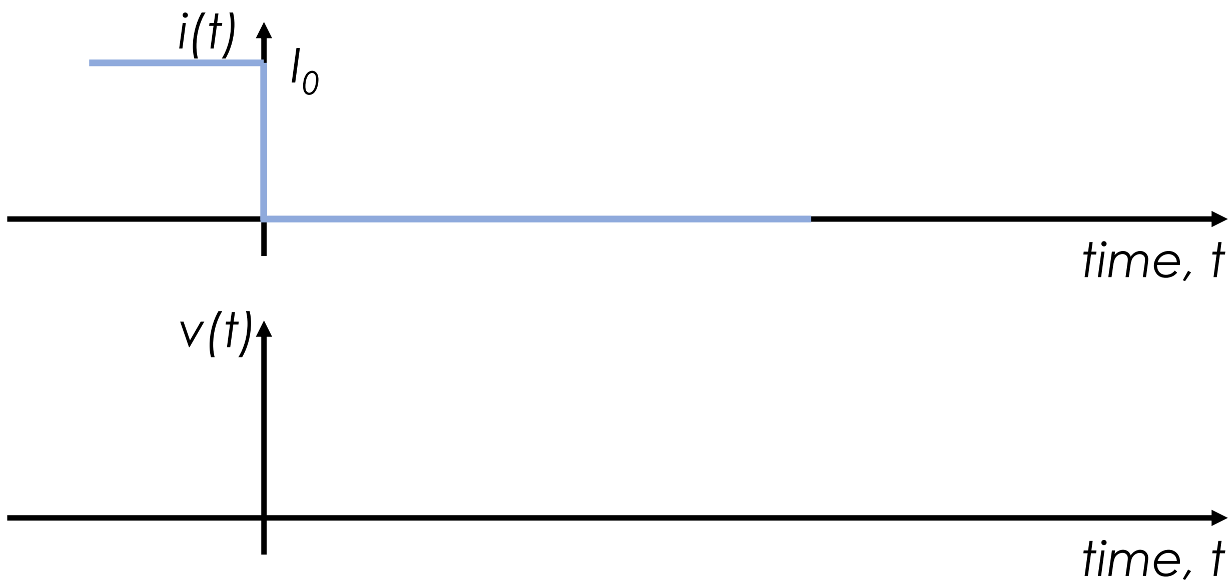

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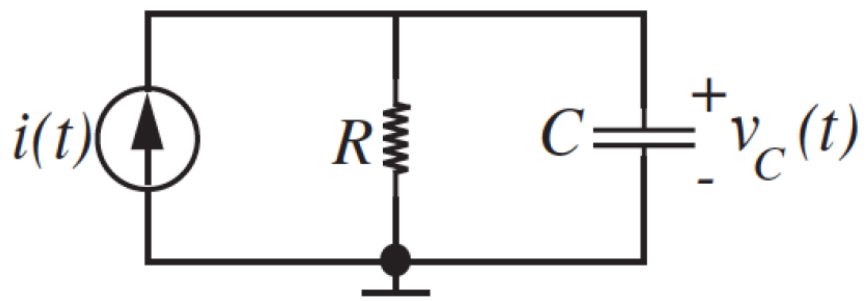

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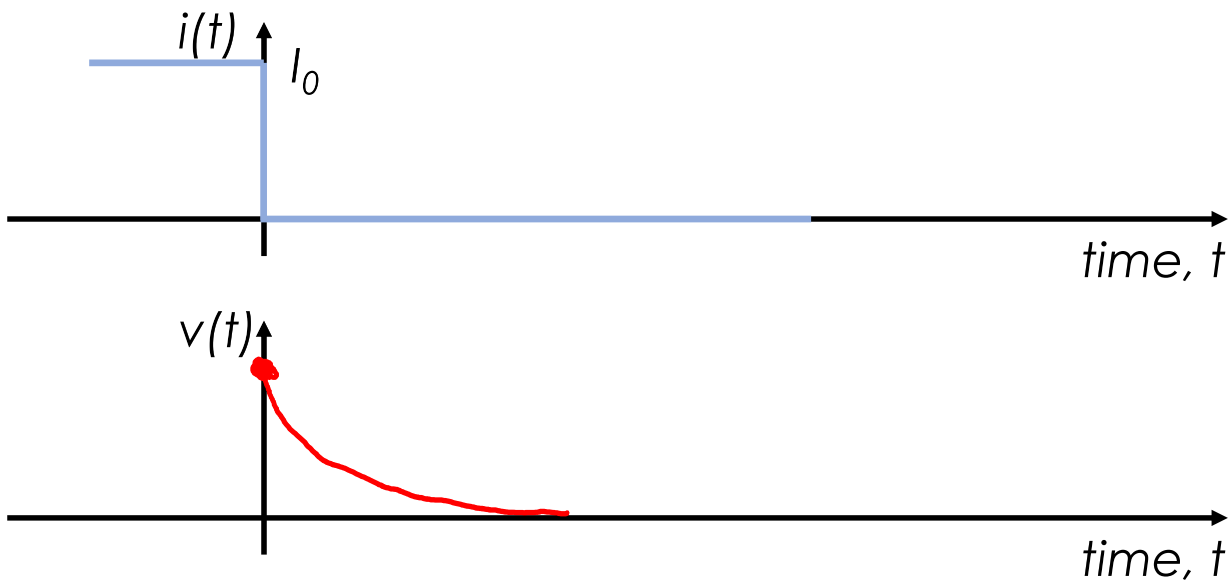

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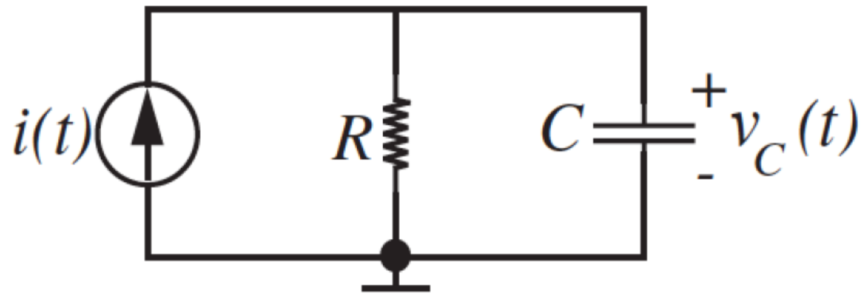

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Homogeneous solution same as before.
Without source, no particular solution.
Thus $v_{ch} = v_c$.

$$v_{ch} = Ae^{-t/RC} = v_c$$

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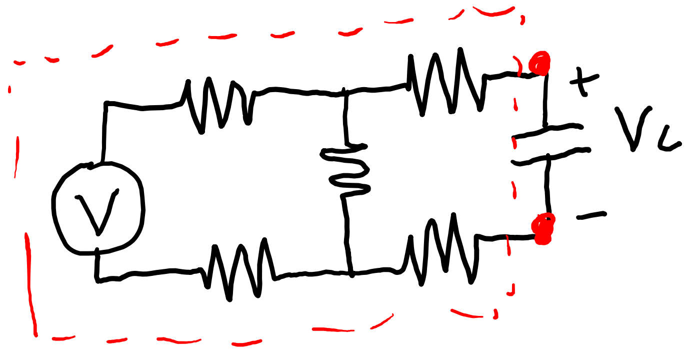
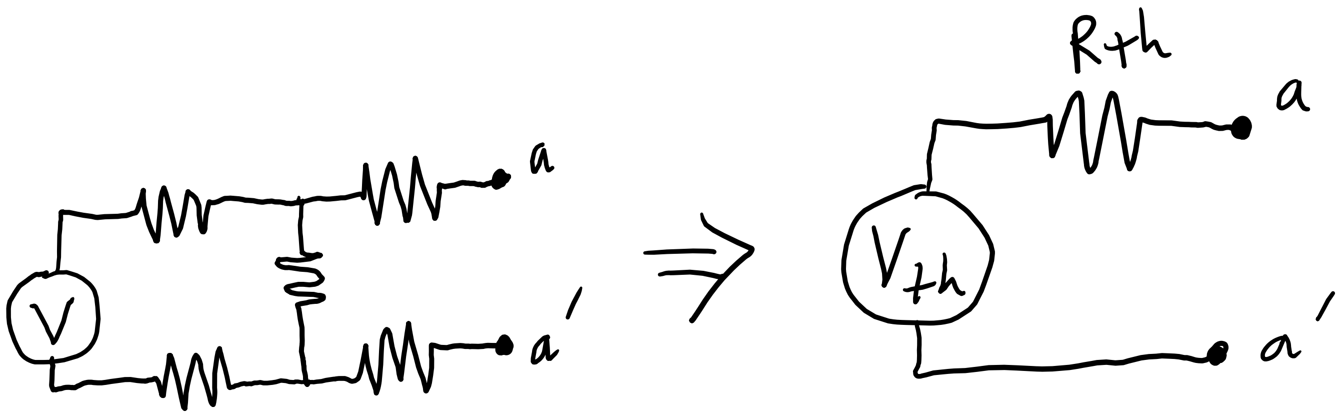
Using initial condition, determine A to get: $v_c = I_0 R e^{-t/RC}$

Review: Let's Add Some Resistors to our RC Circuit



How do we determine the transient behavior?



Determine the Thevenin equivalent circuit

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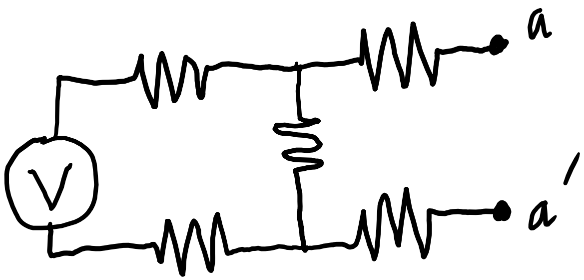

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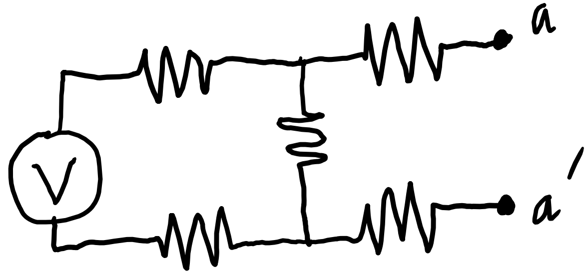

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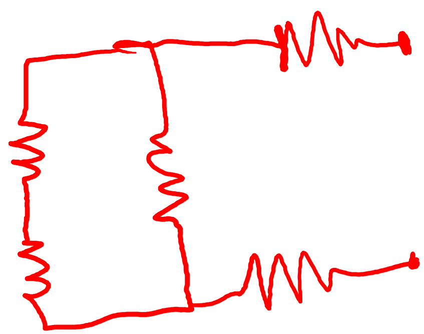
$V_{th} = \frac{V}{3}$

from inspection



$2R // R + R + R$

$= R_{th}$

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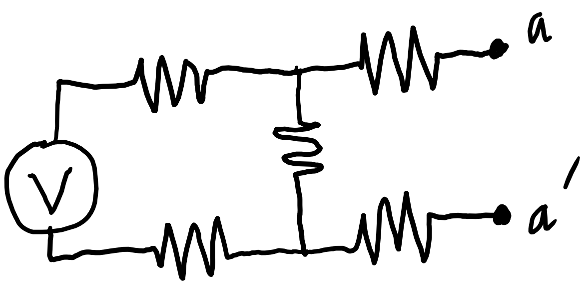

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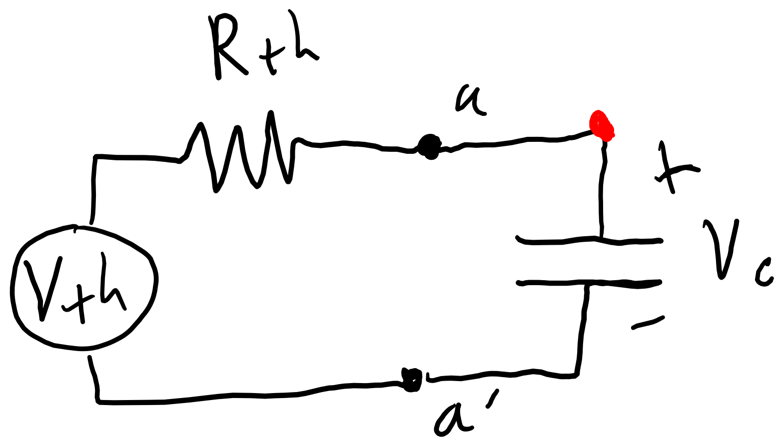


$$V_{th} = \frac{V}{3} \qquad R_{th} = 2R \parallel R + 2R$$

ECE 10C
Fall 2020
Slide Set 1
Instructor: Galan Moody
TA: Kamyar Parto

OHM NEVER FORGOT HIS DYING UNCLE'S ADVICE.

Last Week

- Introduction to course

This Week

- KVL/KCL, node methods
- Thevenin
- 1^{st}-order RC/RL circuits

Important Items:

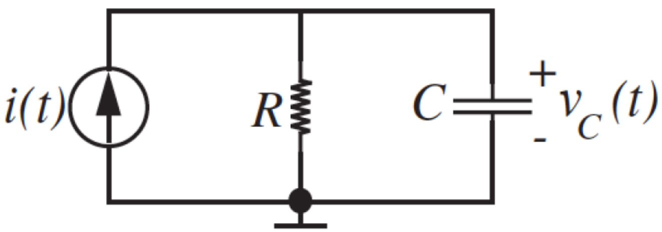
Homework #1 posted
Due Th, Oct 15 by 5 pm

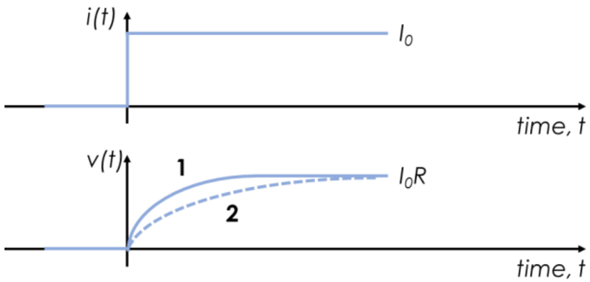
Lab #1 posted
Due Fr, Oct 16

Quiz Time!

Assume capacitors are uncharged at time t = 0.

Q1 [2 point]. For the parallel RC circuit shown on the left, with a constant-current step source shown on the top right, choose the answer that best describes the circuits for the capacitor voltage shown on the bottom right. Curves 1 and 2 correspond to circuits with R_1,C_1 and R_2,C_2, but are otherwise identical.

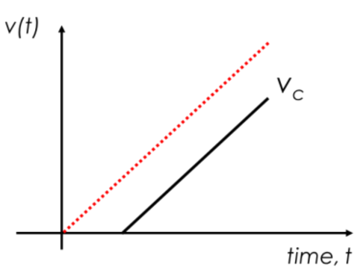




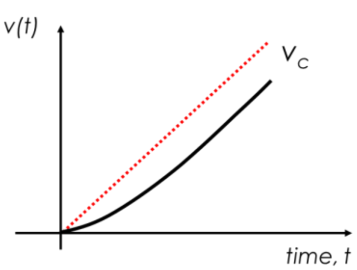
(a) $R_1C_1 = R_2C_2$ **(b)** $R_1 > R_2$ & $C_1 < C_2$ **(c)** $R_1C_1 > R_2C_2$ **(d)** $C_1 < C_2$

Q2 [3 points]. (a) For an RC series circuit with a linear voltage ramp starting at 0V, choose the correct capacitor voltage.

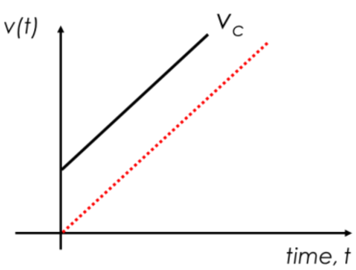
(a)



(b)



(c)



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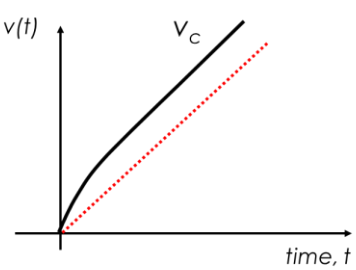

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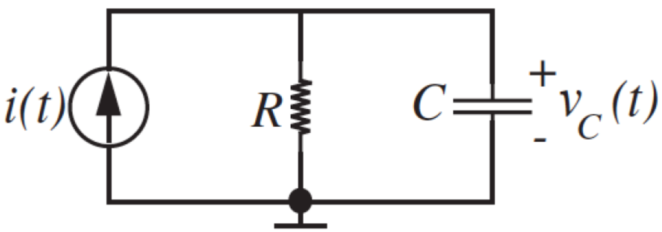

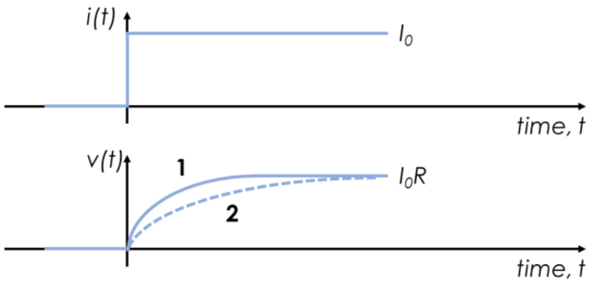

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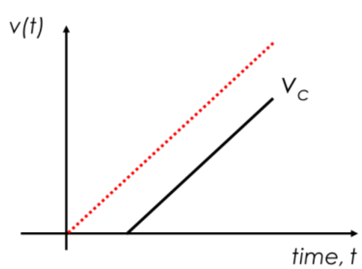

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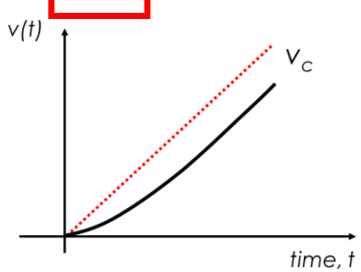

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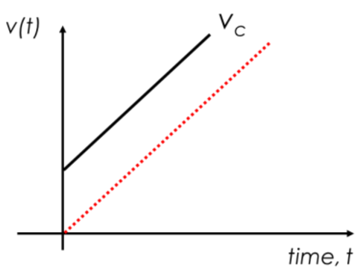

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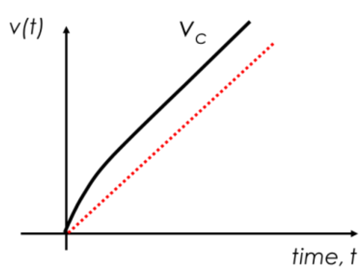
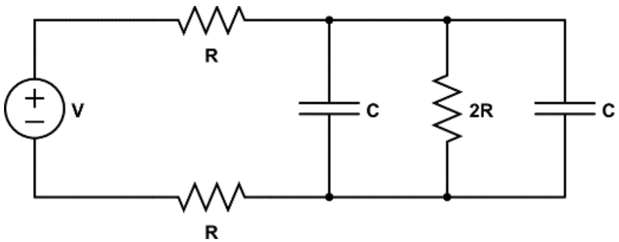


Quiz Time!

Q1 [1 point]. True or False? An inductor stores energy in its magnetic field and reacts to the change in current.

Q2 [2 point]. Pick the correct Thevenin voltage and resistance as would be seen across the two capacitors.

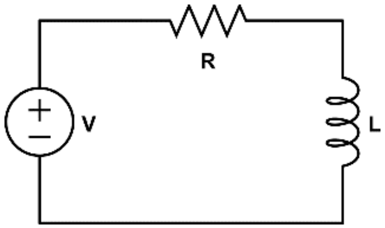


(a) $V_{th} = V/2$, $R_{th} = 2R$

(b) $V_{th} = V$, $R_{th} = R$

(c) $V_{th} = V/2$, $R_{th} = R$

Q3 [2 points]. For a step voltage response from $V = 0$ to $V = V_0$, choose the correct statements for the RL circuit below. Assume the system had been at "rest" for a long time.



At the instance $V = V_0$:

(a) The current through L is zero.

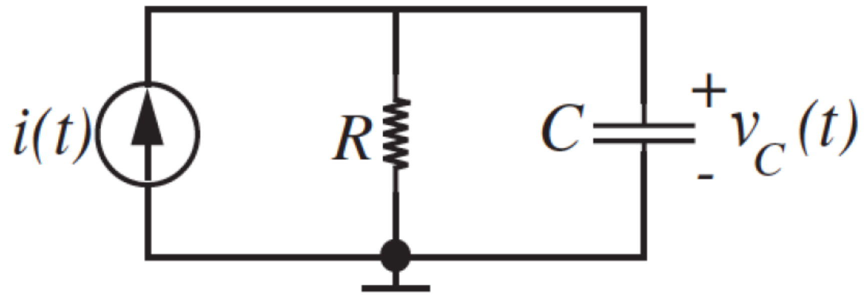
(b) The current through L is not zero.

At long times (steady-state):

(a) The voltage across L is not zero.

(b) The voltage across L is zero.

Review: Simple RC Circuit – Step Response (I_0)

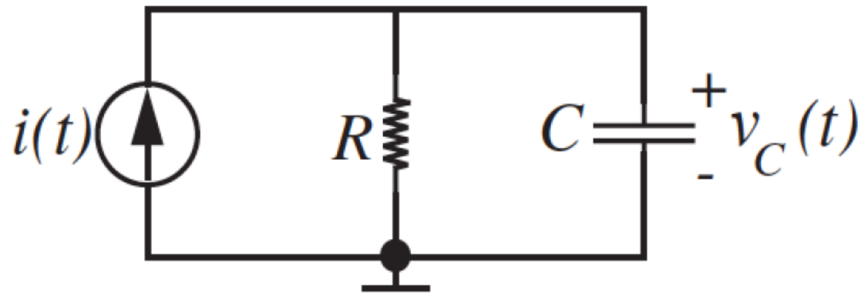


- Find $v_c(t)$ when the circuit is driven by $i(t) = I_0 f(t) = \begin{cases} 0, & t < 0 \\ I_0, & t \geq 0 \end{cases}$
- Initial condition: $v_c(t<0) = 0$ (circuit initially uncharged and at rest)

Solution Steps

1. Use node method to find the differential equation describing v_c
2. Find the homogeneous solution v_{ch} (set the drive to zero)
3. Find the particular solution v_{cp} (educated guess)
4. The total solution is the sum of the homogeneous and particular solutions. Use the initial conditions to solve for the constants.

Review: Simple RC Circuit – Discharge Transient



- Find $v_c(t)$ when the circuit is driven by $i(t) = I_0 f(t) = \begin{cases} I_0, & t < 0 \\ 0, & t \geq 0 \end{cases}$
- Initial condition: $v_c(t<0) = I_0 R$ (capacitor initially charged)

$$i(t) = \frac{v_c}{R} + C\frac{dv_c}{dt}$$

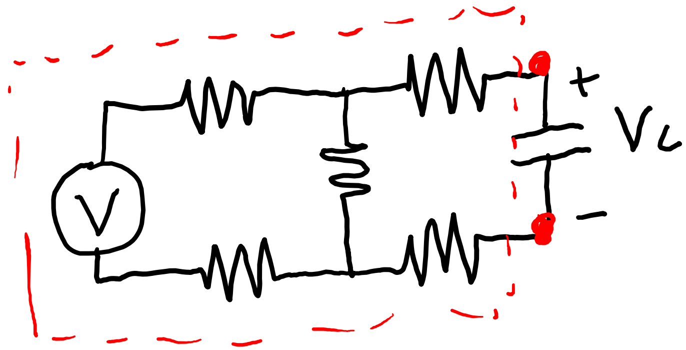
Homogeneous solution same as before.
Without source, no particular solution.
Thus $v_{ch} = v_c$.

$$v_{ch} = Ae^{-t/RC} = v_c$$

$$I_0 R = Ae^{-0/RC}$$

Using initial condition, determine A to get: $v_c = I_0 R e^{-t/RC}$

Review: Let's Add Some Resistors to our RC Circuit



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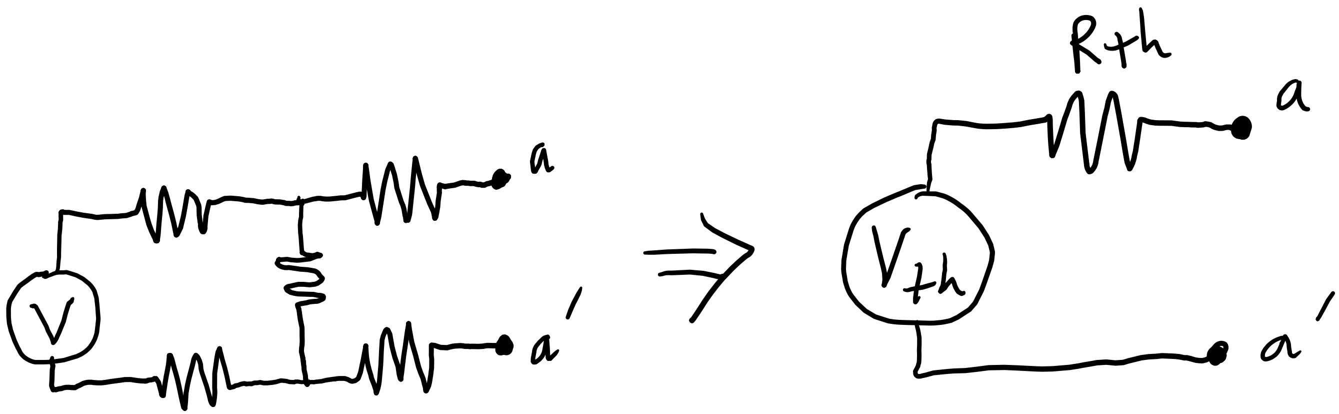
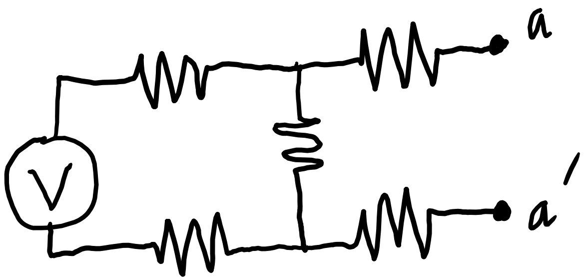


Determine the Thevenin equivalent circuit

Review: Let's Add Some Resistors to our RC Circuit



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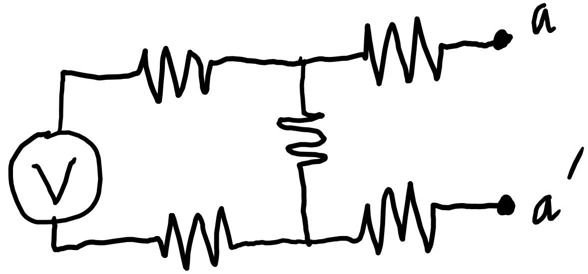

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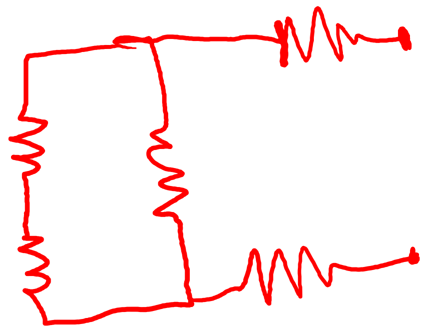
$V_{th} = \frac{V}{3}$

from inspection



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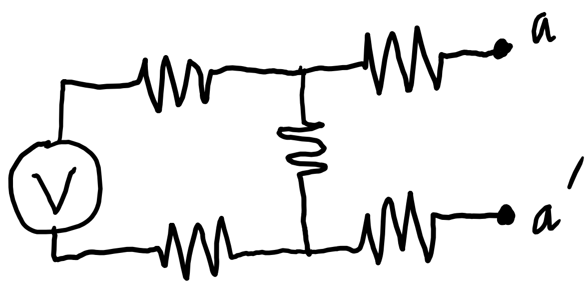

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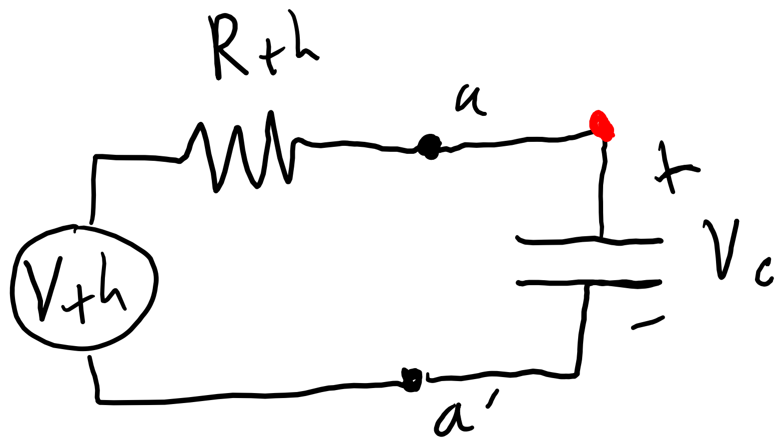
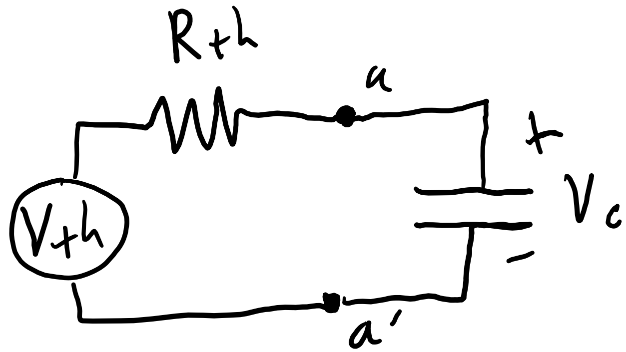


$$V_{th} = \frac{V}{3} \qquad R_{th} = 2R \,//\, R + 2R$$

Review: Let's Add Some Resistors to our RC Circuit



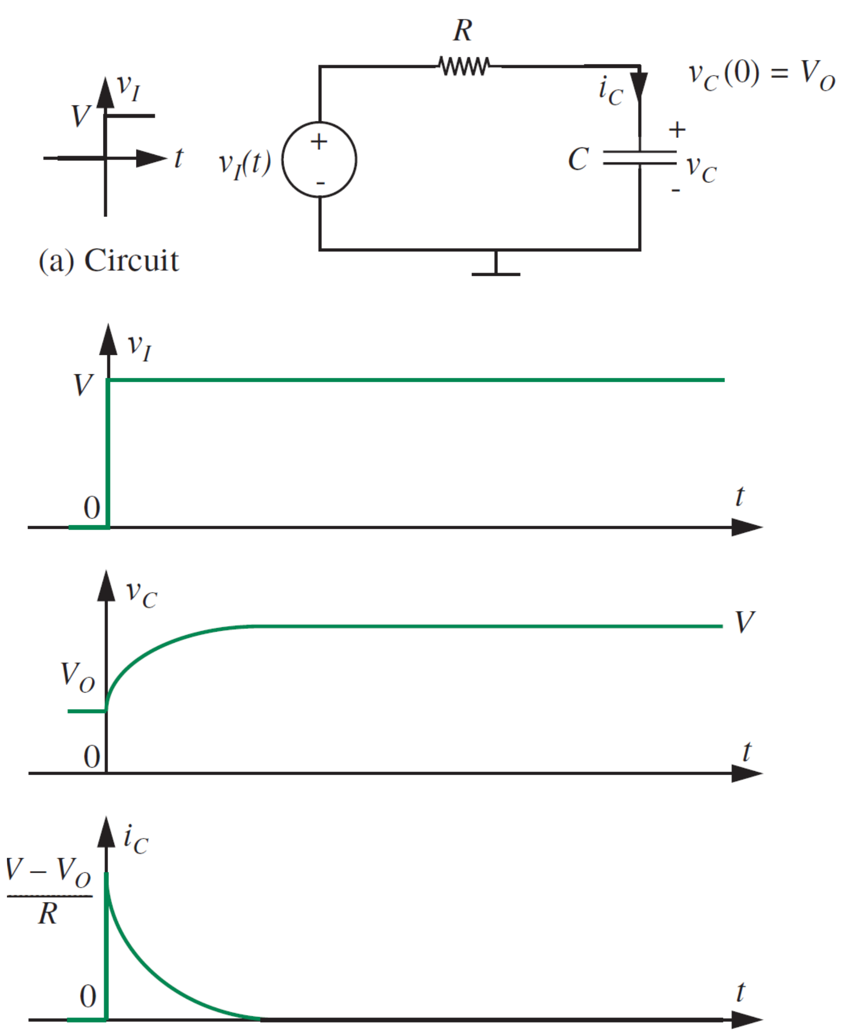
- Find $v_C(t)$ when the circuit is driven by $v(t) = V_0 f(t) = \begin{cases} V_0, & t > 0 \\ 0, & t = 0 \end{cases}$
- Initial condition: $v_C(t<0) = 0$ (capacitor initially charged)
- Use node method: $\dfrac{v_{th} - v_c}{R} - C\dfrac{dv_c}{dt} = 0 \qquad v_{th} = v_c + RC\dfrac{dv_c}{dt}$

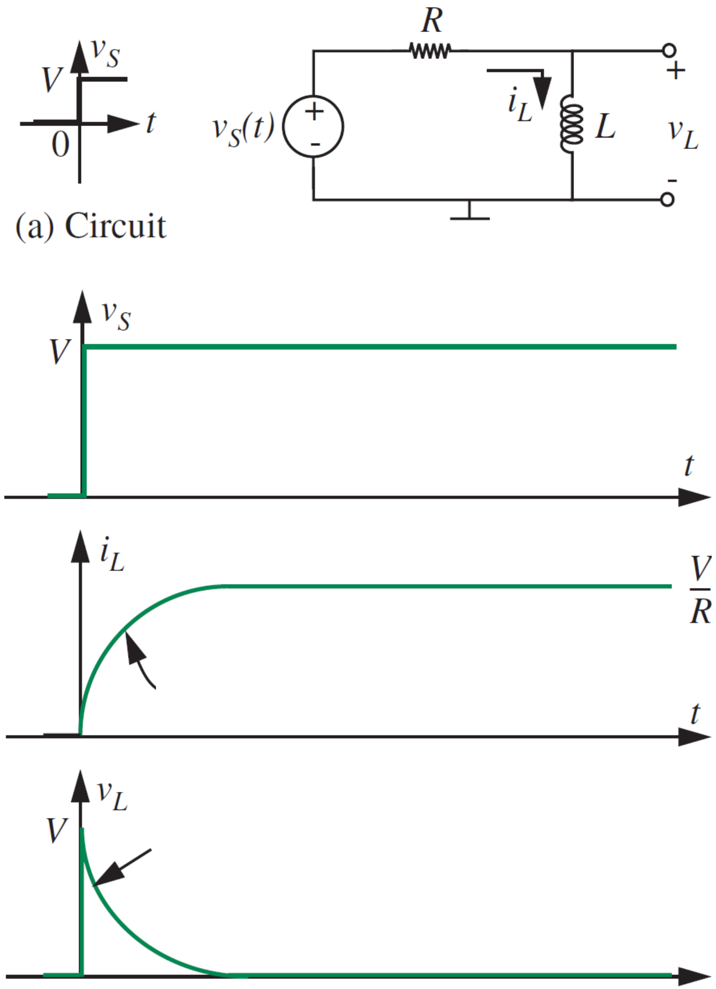
Recall current step response for parallel RC circuit: $I_0 = \dfrac{v_{cp}}{R} + C\dfrac{dv_{cp}}{dt}$

We have a similar expression: $v_c(t) = V_0 - V_0 e^{-\frac{t}{RC}}$

Review: RC versus RL Circuit Transient Response

The transient behavior of an inductor can be understood in a similar way to the capacitor, replacing C → L and v → i





Review: RC versus RL Circuit Transient Response

Often the initial conditions aren't given. Instead, we're told that the circuit has reached a steady state before being disturbed.

Principle #1: Initial response of C and L

A capacitor acts like a short circuit and an inductor like an open circuit when they are "at rest"

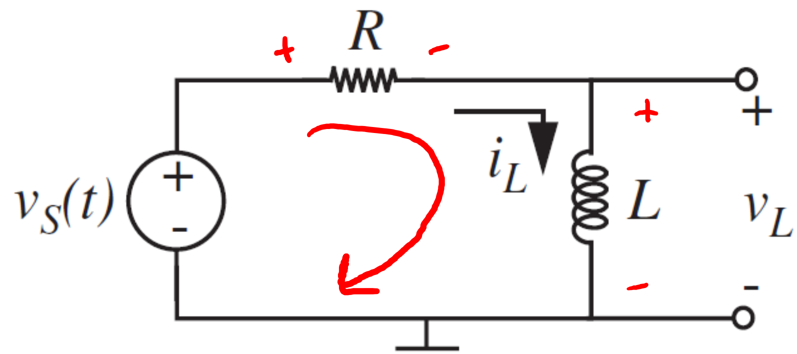
Principle #2: Steady-state of C and L

A capacitor acts like an open circuit and an inductor like a short circuit when they are in their steady state

Principle #3: Continuity

1. Voltage across C is continuous (unless an impulse current)
2. Current through L is continuous (unless impulse voltage)

Review: Simple RL Circuit – Step Response (V_0)

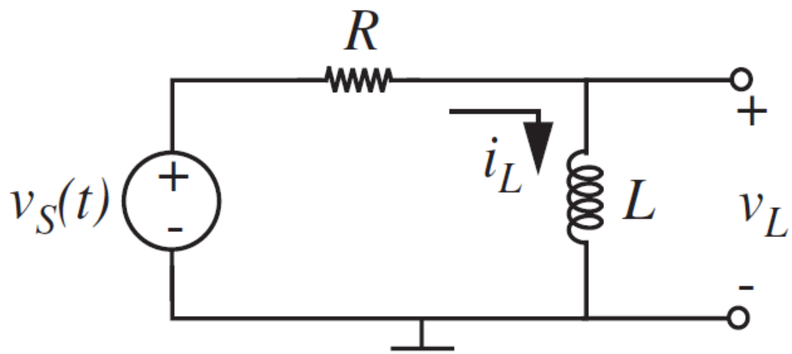


$i = C\frac{dV}{dt}$

$V = L\frac{di}{dt}$

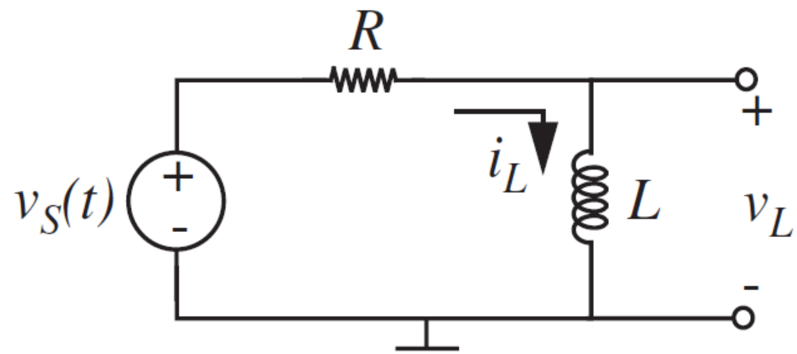
- Find $i_L(t)$ when the circuit is driven by $v(t) = V_0 f(t) = \begin{cases} 0, & t < 0 \\ V_0, & t \geq 0 \end{cases}$
- Initial condition: $i_L(t<0) = 0$ (circuit initially uncharged and at rest)
- From KVL**: $v_s - i_L R - L\frac{di}{dt} = 0 \;\Big|\; v_s = i_L R + L\frac{di_L}{dt}$
- Homogeneous solution: $i_L(t) = Ae^{st} \;\Big|\; 0 = Ae^{st}R + LsAe^{st}$
- Characteristic equation: $s = -\frac{R}{L}$
- Homogeneous solution: $i_L(t) = Ae^{-R/L\, t}$ $\qquad t_c = \frac{L}{R}$

Review: Simple RL Circuit – Step Response (V_0)



- Find $i_L(t)$ when the circuit is driven by $v(t) = V_0 f(t) = \begin{cases} 0, & t < 0 \\ V_0, & t \geq 0 \end{cases}$
- Initial condition: $i_L(t<0) = 0$ (circuit initially uncharged and at rest)
- Particular equation: $$i_L R + L\frac{di_L}{dt} = v_s$$
- Because source is step response, try: $i_{L,p} = K \;\Big|\; KR = V_0 \Rightarrow K = \frac{V_0}{R}$
- Total solution: $i_L(t) = Ae^{-t/t_c} + \frac{V_0}{R}$
- Initial conditions: $i_L(t) = -\frac{V_0}{R} e^{-t/t_c} + \frac{V_0}{R}$

Review: Simple RL Circuit – Step Response (V_0)

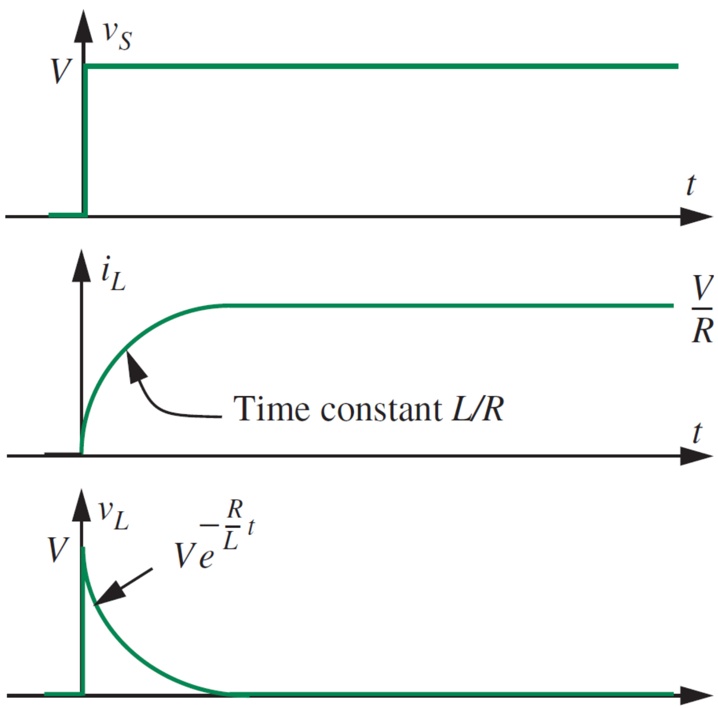


Current response:

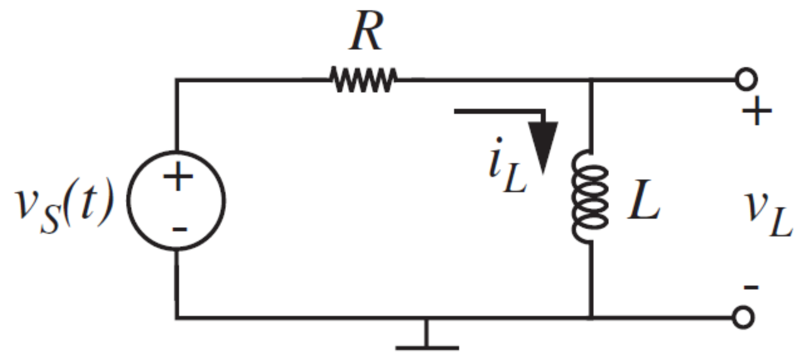
$$i_L = -\frac{V}{R}e^{-\frac{Rt}{L}} + \frac{V}{R}$$

Voltage response:

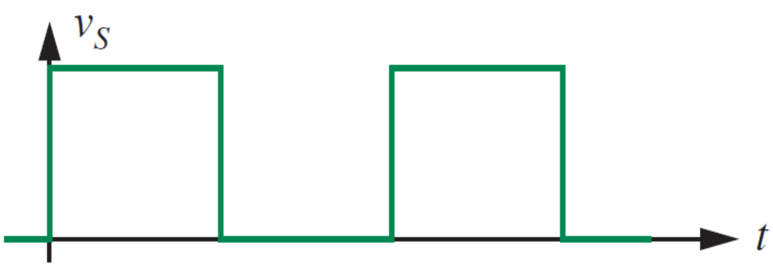
$$v_L = L\frac{di_L}{dt} = Ve^{-\frac{Rt}{L}}$$

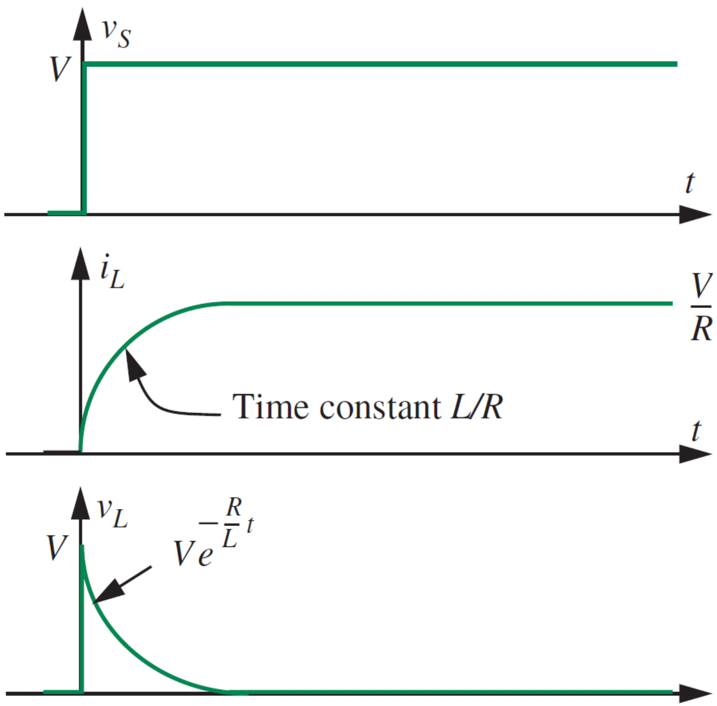


Review: Simple RL Circuit – Pulsed Response (V_0)

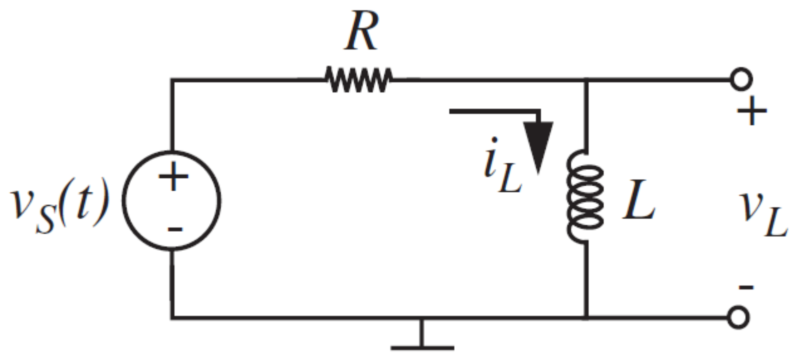


What happens when we use a square wave voltage source?





Review: Simple RL Circuit – Pulsed Response (V_0)



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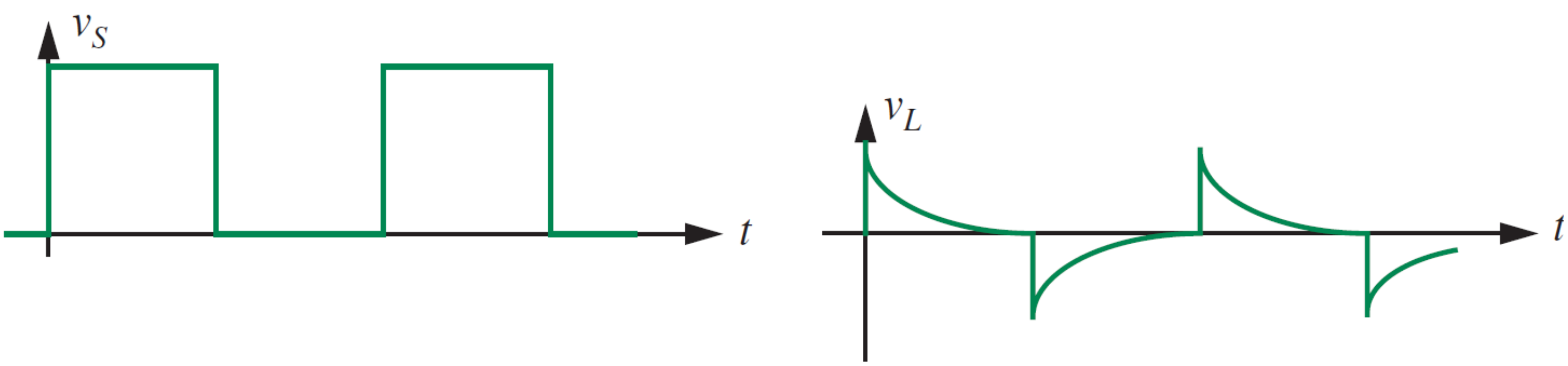
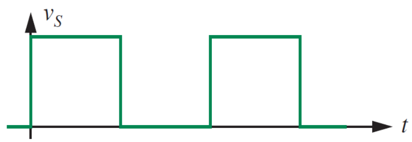
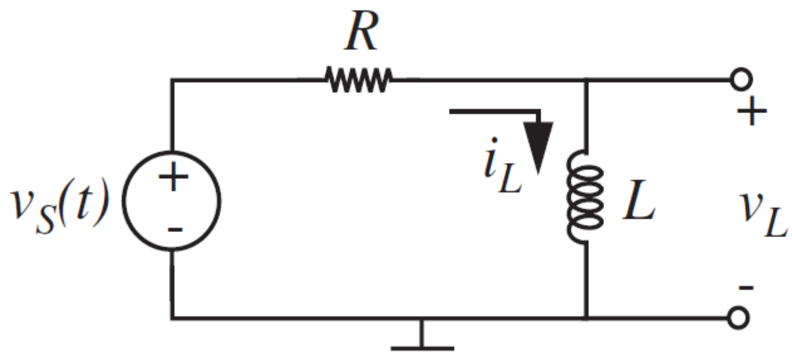
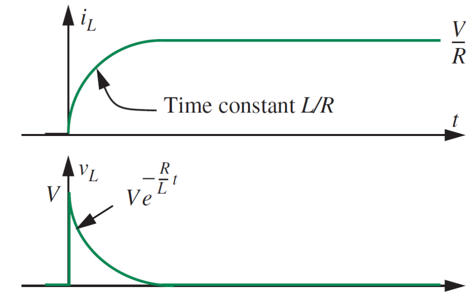


Review: Simple RL Circuit – Pulsed Response (V_0)

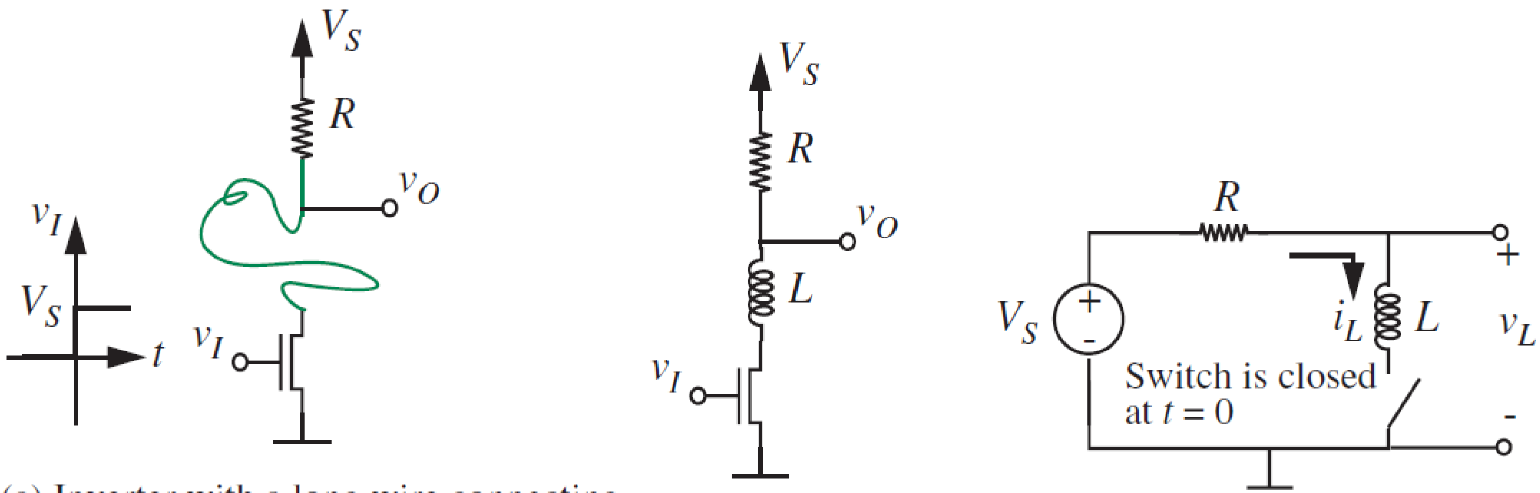






- Initial condition: $i_L(t = 0) = V/R$
- Homogeneous equation: $i_L R + L\dfrac{di_L}{dt} = 0$
- Solution: $i_L(t) = A e^{-t/t_c}$
- Initial conditions: $i_L(t) = \frac{V}{R} e^{-t/t_c}$ | $v = L\frac{di}{dt}$
- Voltage response: $v(t) = L \cdot \frac{V}{R} e^{-t/t_c} \cdot \frac{-R}{L} = -Ve^{-t/t_c}$

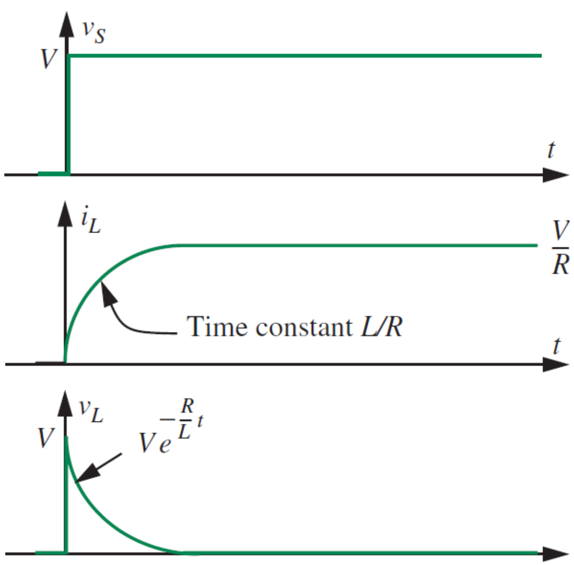
Bigger Picture: Inductance in Digital Circuits



(a) Inverter with a long wire connecting the output and the MOSFET drain

(b) Circuit model

(c) Circuit for step input at v_I



Unwanted inductance introduces a delayed response = unexpected behavior, slower response times of logic circuits

Summary: Capacitor and Inductor 1st Order ODEs

In general, the response of a 1st-order circuit (RC or RL) will be of the form:

$$\mathrm{y(t) = final\ value + (initial\ value\ - final\ value)e^{-t/t_c}}$$

$$v_c(t) = v_c(\infty) + (v_c(0)\ - v_c(\infty))e^{-t/RC}$$

$$i_L(t) = i_L(\infty) + (i_L(0)\ - i_L(\infty))e^{-Rt/L}$$

****applies to branch variables such as C or L current, resistor V, etc.****

Capacitor

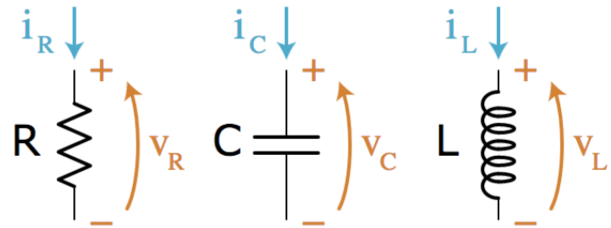
- Behaves as instantaneous **short** circuit when inputs make abrupt changes
- Behaves as **open** circuit at long times when driven by DC voltage source
- If initially charged, behaves like a **voltage source** for abrupt transitions

Inductor

- Behaves as instantaneous **open** circuit when inputs make abrupt changes
- Behaves as **short** circuit at long times when driven by DC current source
- If current is non-zero, behaves like a **current source** for abrupt transitions

Power and Energy

Power = rate of change of energy per unit time



Power and Energy Relation in a Two-Terminal Element

Power: $P(t) = i(t)v(t) = \dfrac{v(t)^2}{R} = i(t)^2 R$ [J/s or W]

where *i(t)* is defined to be positive if it enters at the positive terminal.

Energy: $w(t) = \int_0^T p(t)dt$

circuit elements can consume or release power/energy if $p(t) > 0$ or $p(t) < 0$

Power and Energy

- Battery storage is limited…we want to optimize (reduce) the energy consumption of an IC, while preserving the functionality

- High power consumption = excessive heat = adverse effects

- Where does the consumed power go?
  - Resistors = dissipated as heat
  - Capacitors and inductors = stored

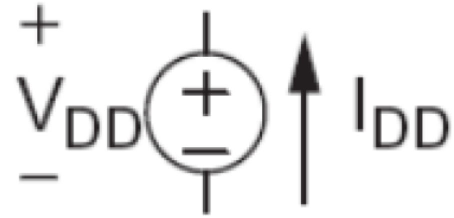
$$w_C(t) = \frac{Cv(t)^2}{2} \qquad w_L(t) = \frac{Li(t)^2}{2}$$

- Where does the consumed power come from? The source

Power and Energy in Circuit Elements

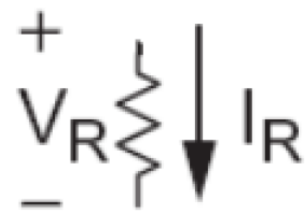
Delivered power: $V_{DD}I_{DD}(t)$

Delivered Energy: $\int V_{DD}I_{DD}dt$



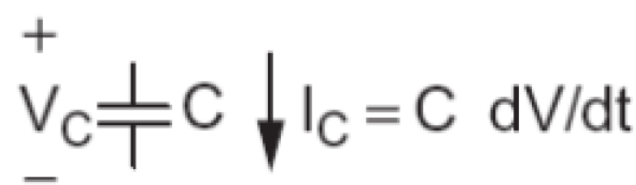
Dissipated power: $V_R(t)IR(t)$

Delivered Energy: $\int V_R(t)IR(t)dt$



Instantaneous power: $CV_C\,dV_C/dt$

Stored energy at *t*: $\frac{1}{2}CV_C^2$



ECE 10C
Fall 2020
Slide Set 1
Instructor: Galan Moody
TA: Kamyar Parto

OHM NEVER FORGOT HIS DYING UNCLE'S ADVICE.

Last Week
- Introduction to course

This Week
- KVL/KCL, node methods
- Thevenin
- 1st-order RC/RL circuits

Important Items:

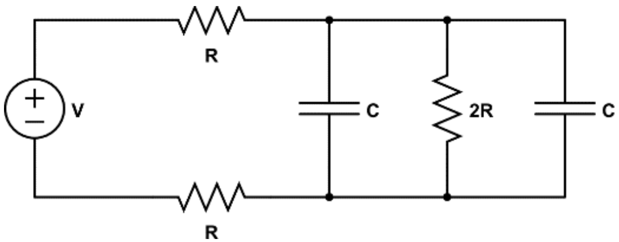
Homework #1 posted
Due Th, Oct 15 by 5 pm

Lab #1 posted
Due Fr, Oct 16

Last Quiz

Q1 [1 point]. True or False? An inductor stores energy in its magnetic field and reacts to the change in current.

Q2 [2 point]. Pick the correct Thevenin voltage and resistance as would be seen across the two capacitors.

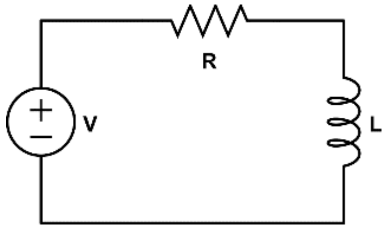


(a) $V_{th} = V/2$, $R_{th} = 2R$

(b) $V_{th} = V$, $R_{th} = R$

(c) $V_{th} = V/2$, $R_{th} = R$

Q3 [2 points]. For a step voltage response from $V = 0$ to $V = V_0$, choose the correct statements for the RL circuit below. Assume the system had been at "rest" for a long time.



At the instance $V = V_0$:

(a) The current through L is zero.

(b) The current through L is not zero.

At long times (steady-state):

(a) The voltage across L is not zero.

(b) The voltage across L is zero.

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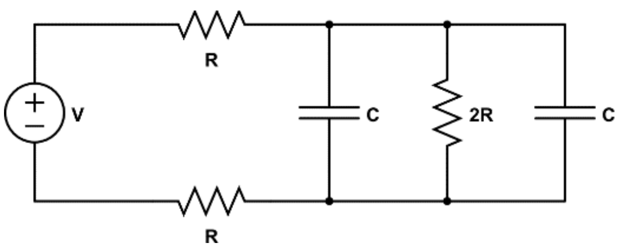

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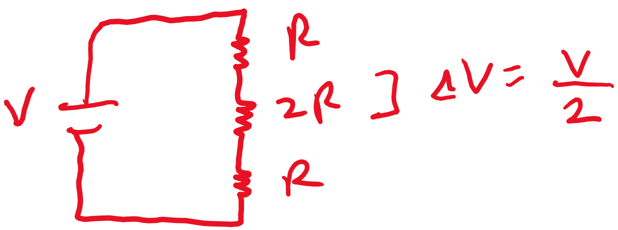

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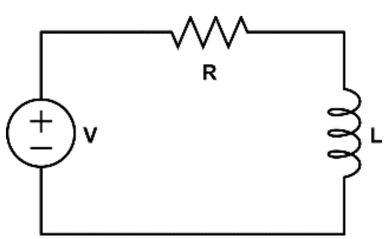

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(a) The current through L is zero.

(b) The current through L is not zero.

At long times (steady-state):

(a) The voltage across L is not zero.

(b) The voltage across L is zero.

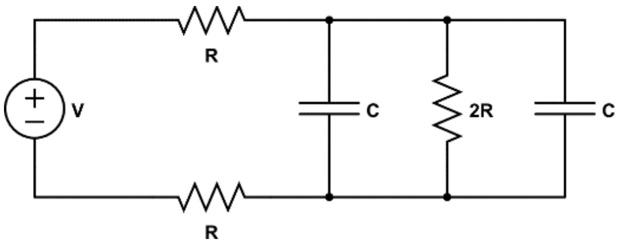
$i_L(0^-) = i_L(0^+)$

at steady state, L = short.
Thus no ΔV across L.

Quiz Time

Q1 [1 point]. True or False? An inductor stores energy in its magnetic field and reacts to the change in current.

Q2 [2 point]. Pick the correct Thevenin voltage and resistance as would be seen across the two capacitors.



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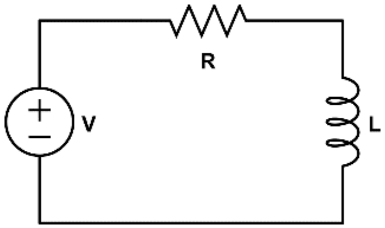

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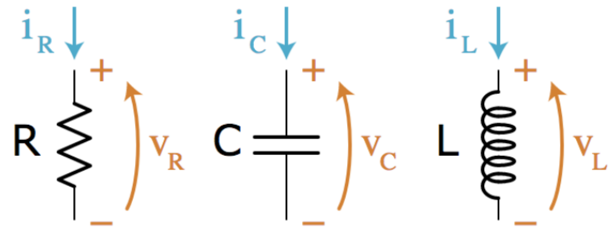

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where *i(t)* is defined to be positive if it enters at the positive terminal.

Energy: $w(t) = \int_0^T p(t)dt$

circuit elements can consume or release power/energy if $p(t) > 0$ or $p(t) < 0$

Note About Time Before/After Switches Open/Close

Time right ***before*** a switch changes position usually denoted as t = 0^-

Time right ***after*** a switch changes position usually denoted as t = 0+

Capacitor

Element law: $i(t) = C\dfrac{dv(t)}{dt}$

Stored energy: $w_E(t) = \dfrac{Cv(t)^2}{2}$

Inductor

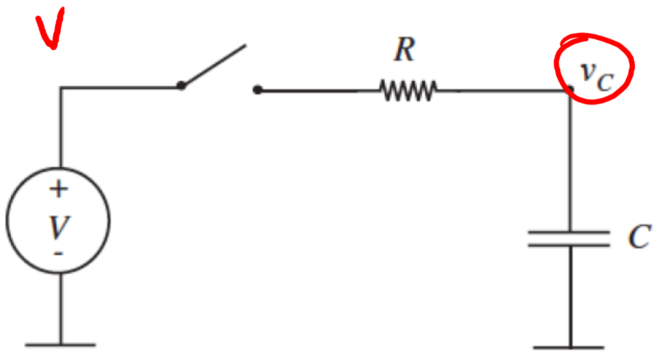
Element law: $v(t) = L\dfrac{di(t)}{dt}$

Stored energy: $w_M(t) = \dfrac{Li(t)^2}{2}$

For a capacitor, voltage $v_c(0^-) = v_c(0^+)$ [*V = Q/C --> charge stored in capacitor is same right before and after a switch opens/closes*]

For an inductor, current $i_L(0^-) = i_L(0^+)$ [*I = Flux/L --> magnetic flux in inductor is same right before and after a switch opens/closes*]

Power and Energy in Simple RC Circuit



$$\frac{V - V_c}{R}$$

- Assume $v_C(0^-) = 0$ (zero state)

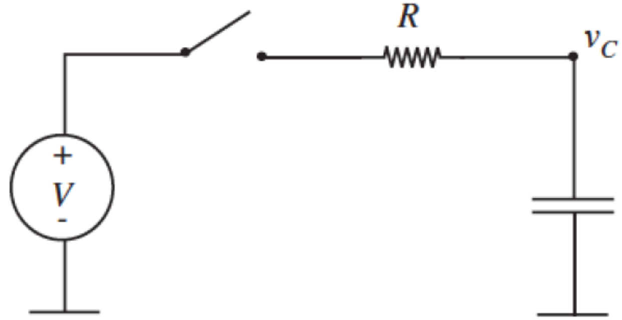
- Let's find the instantaneous power drawn from the voltage source when we close the switch at $t = 0$

- We know: $p(t) = i(t)V$

  where $i(t) = [V - v_c(t)]/R$

- We need to find $v_C(t)$ when $v_C(0^-) = 0$

Power and Energy in Simple RC Circuit

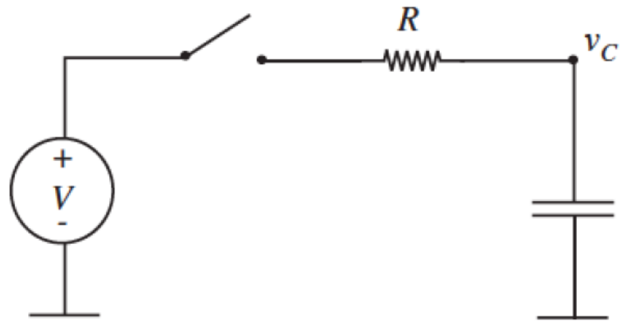


We can find the zero-state response of $v_C(t)$ to the step input $V_0 f(t)$ following the same steps as the parallel RC circuit using the generic expression:

$$v_c(t) = V_0 - V_0 e^{-\frac{t}{RC}}$$

$$\left[v(t) = \underbrace{v(\infty)}_{V_0} + [\underbrace{v(0)}_{0} - \underbrace{v(\infty)}_{V_0}]\, e^{-t/\underbrace{t_c}_{RC}} \right]$$

Power and Energy in Simple RC Circuit



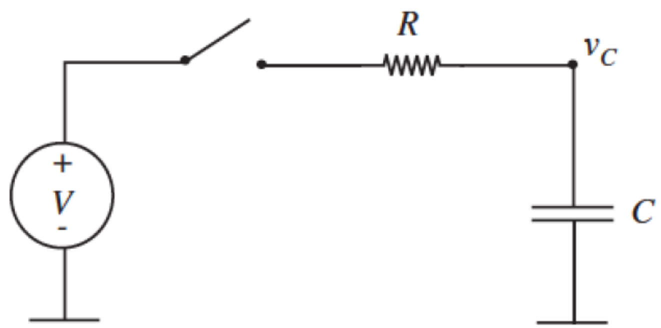
Thus, the instantaneous power ***drawn*** from the source:

$$p(t) = i(t)V = \frac{V_0 - v_c(t)}{R} \times V_0 = \frac{V_0 e^{-\frac{t}{RC}}}{R} \times V_0 = \frac{V_0{}^2 e^{-\frac{t}{RC}}}{R}$$

Energy consumption by an element

Energy: $$w(t) = \int_0^T p(t)dt$$

Power and Energy in Simple RC Circuit



$$p(t) = \frac{V_0{}^2 e^{-\frac{t}{RC}}}{R}$$

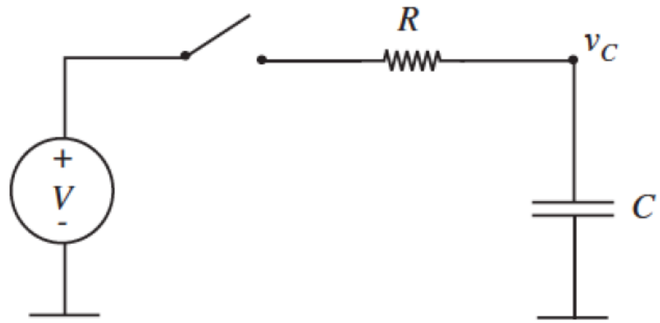
- Total energy **supplied by the voltage source**:

$$w = \int_0^\infty \frac{V_0{}^2 e^{-\frac{t}{RC}}}{R} dt = -\frac{V_0{}^2 RC e^{-\frac{t}{RC}}}{R} = CV_0{}^2$$

- Energy **stored in the capacitor**:

$$w = CV_0{}^2/2$$

Power and Energy in Simple RC Circuit



$$p(t) = \frac{{V_0}^2 e^{-\frac{t}{RC}}}{R}$$

- Total energy **supplied by the voltage source**:

$$w = \int_0^\infty \frac{{V_0}^2 e^{-\frac{t}{RC}}}{R} dt = -\frac{{V_0}^2 RC e^{-\frac{t}{RC}}}{R} = C{V_0}^2$$

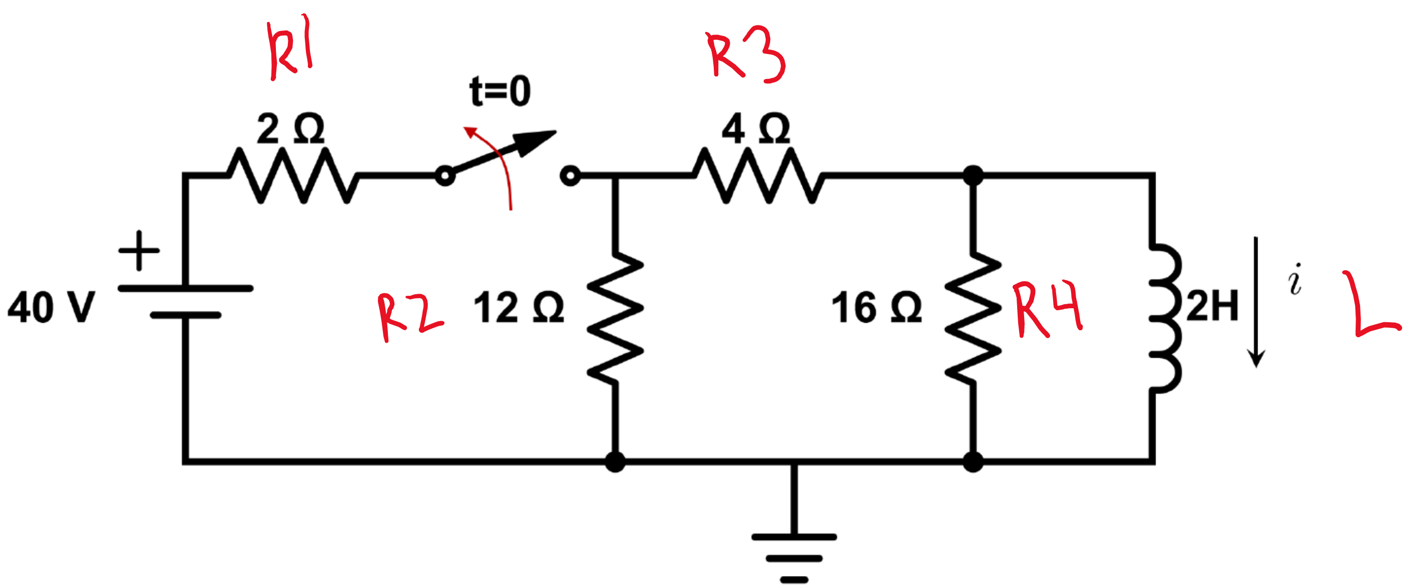
- Energy **stored in the capacitor**:

$w = C{V_0}^2/2$ **Doesn't equal energy supplied by source.**
Why???

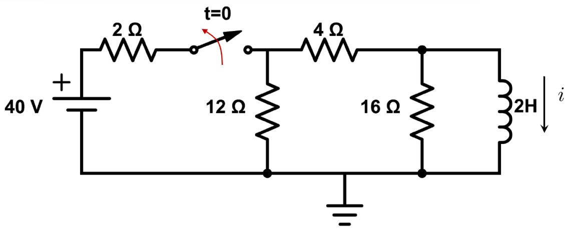
In Class Exercise

The switch in the circuit is opened at $t = 0$ after being closed for a long time.

1. Find $i(0)$
2. Find $i(t)$ for $t > 0$
3. Find the total energy dissipated by the circuit after $t > 0$



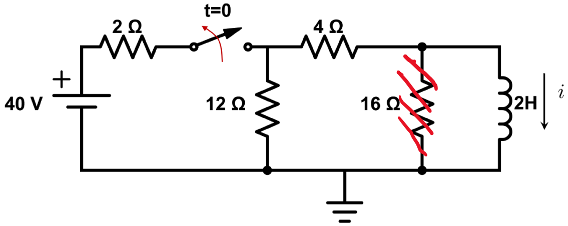
In Class Exercise



Find $i(0)$.

Switch has been closed for a long time.
What's the voltage across L?
What's the current through R4 (16 Ω resistor)?

In Class Exercise



Find *i*(0).

Switch has been closed for a long time.
What's the voltage across L?
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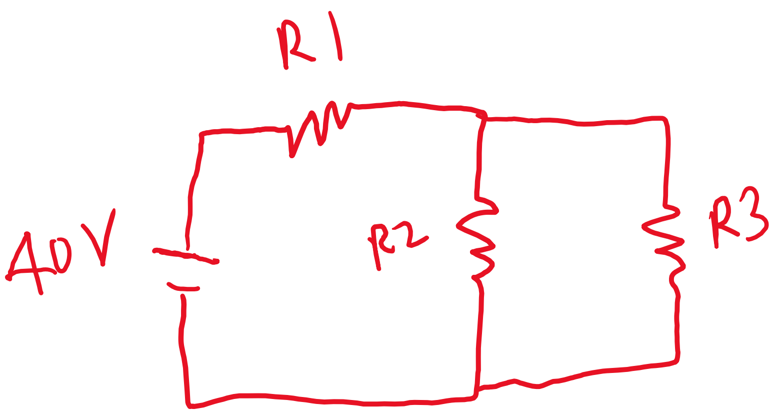
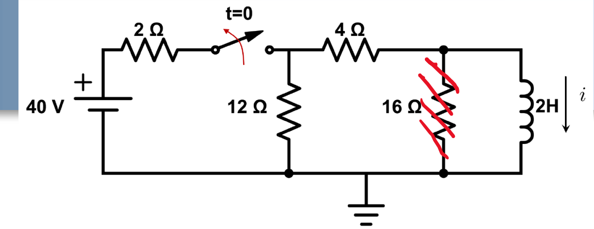


let's find current drawn from V_s,

$R = R_1 + R_2 // R_3 = 2 + \frac{12 \cdot 4}{16} = 5\Omega$

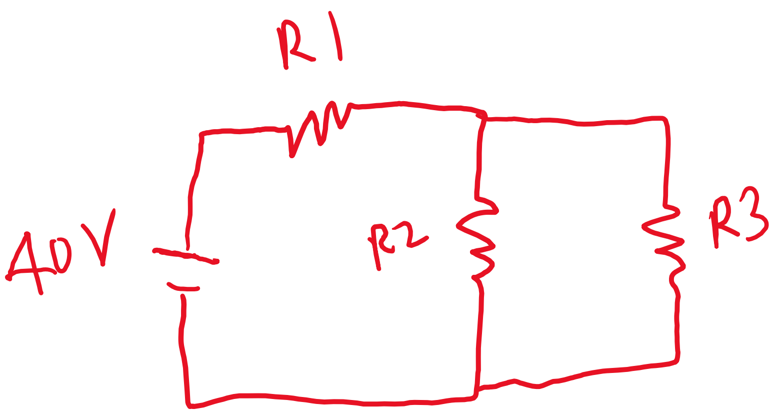
In Class Exercise



Find $i(0)$.

Switch has been closed for a long time.
What's the voltage across L?
What's the current through R4 (16 Ω resistor)?

Let's find current drawn from V_s.



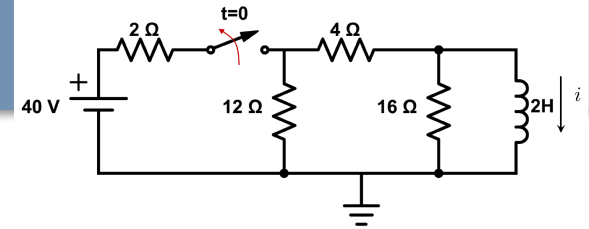
$R = R_1 + R_2 /\!/ R_3 = 2 + \frac{12 \cdot 4}{16} = 5\,\Omega$

$i = \frac{40\,V}{5\,\Omega} = 8\,A$. How much through R3?

Same V across R_2 & R_3. $\frac{R_3}{R_2 + R_3} = \frac{1}{4}$

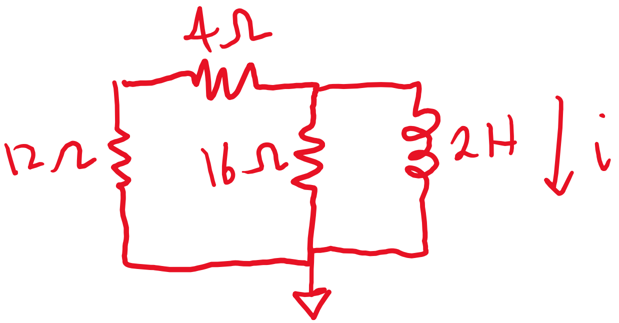
Thus $3/4\ i$ through $R_3 \longrightarrow \boxed{6\,A = i(0)}$

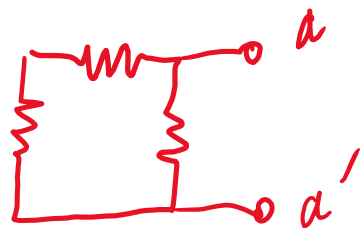
In Class Exercise



Switch opens. Find $i(t)$.

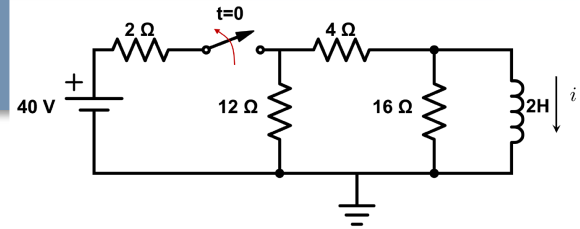
$i_L(0^-) = i_L(0^+) = 6A.$





$\rightarrow R_{th} = 16 \parallel 16 = 8\,\Omega$

In Class Exercise



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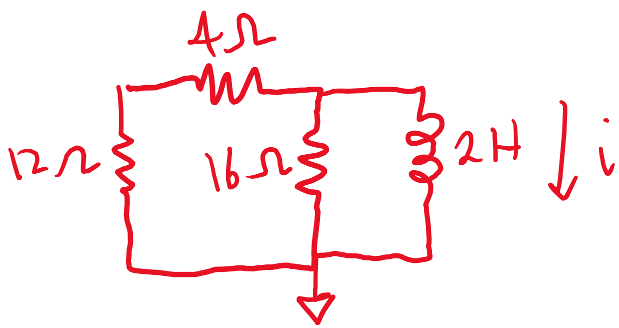
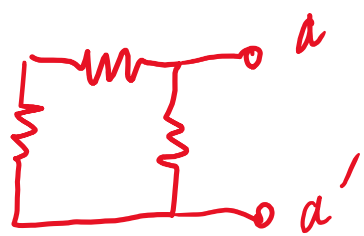
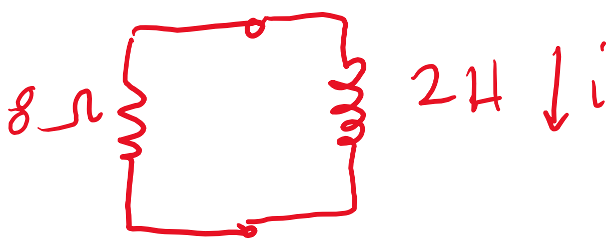




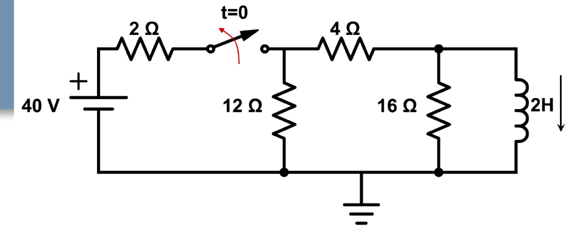
$\rightarrow$ $R_{th} = 16 // 16 = 8\,\Omega$



$$t_c = \frac{L}{R} = 0.25 \text{ sec}.$$

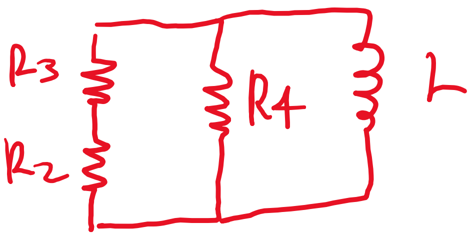
$$i_L(t) = i(\infty) + \left(i(0) - i(\infty)\right) e^{-t/t_c} = 6A\, e^{-t/t_c}$$

In Class Exercise



Determine power and energy dissipated.

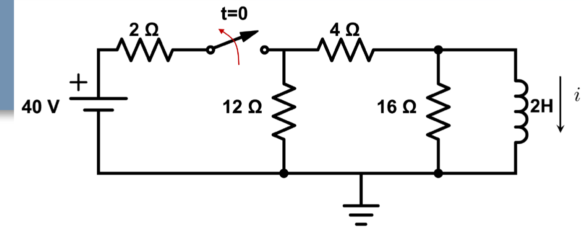
$$p(t) = i(t)\,V(t) = \frac{V(t)^2}{R} \qquad i(t) = 6e^{-4t}\ \text{A}$$



$V_L = V$ across $R_2 + R_3 = V$ across R_4

$$V_L = L\frac{di}{dt} = 2 \cdot 6 \cdot -4e^{-4t} = -48e^{-4t}\ \text{A.}$$

In Class Exercise



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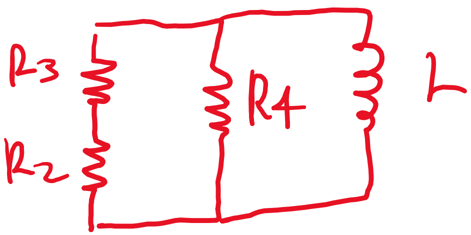

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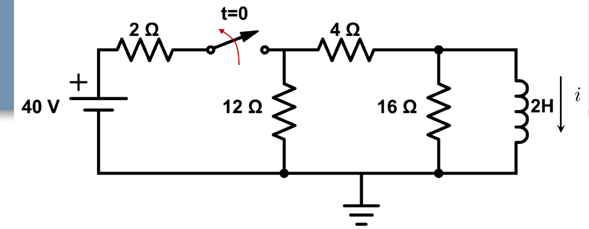
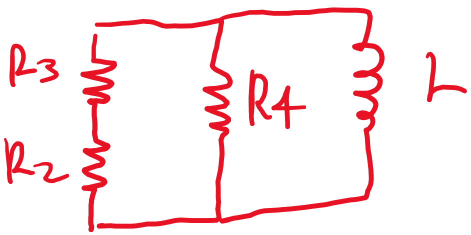
$$p(t) = \frac{(-48e^{-4t})^2}{R} = \frac{(-48e^{-4t})^2}{16\,\Omega}\cdot 2 = 288e^{-8t}$$

In Class Exercise



Determine power and energy dissipated.

$$p(t) = i(t)\,V(t) = \frac{V(t)^2}{R} \qquad i(t) = 6e^{-4t}\ A$$



V_L = V across $R_2 + R_3$ = V across R_4

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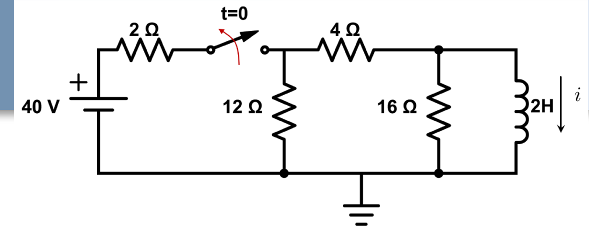
$$p(t) = \frac{(-48e^{-4t})^2}{R} = \frac{(-48e^{-4t})^2}{16\,\Omega} \cdot 2 = 288e^{-8t}$$

$$w = \int_0^\infty p(t)\,dt = \int_0^\infty 288e^{-8t}\,dt = \frac{288}{-8}e^{-8t}\Big|_0^\infty = 36J.$$

In Class Exercise



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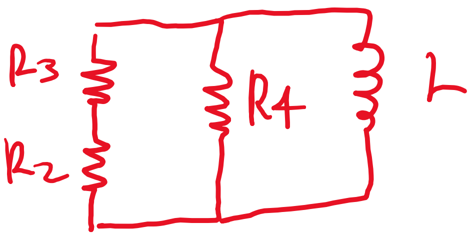


V_L = V across $R_2 + R_3$ = V across R_4

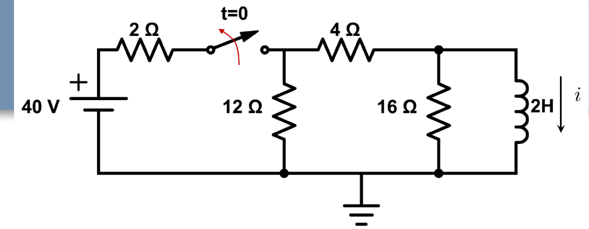
$$V_L = L\frac{di}{dt} = 2 \cdot 6 \cdot -4e^{-4t} = -48e^{-4t}\ A.$$

$$p(t) = \frac{(-48e^{-4t})^2}{R} = \frac{(-48e^{-4t})^2}{16\,\Omega} \cdot 2 = 288e^{-8t}$$

$$w = \int_0^\infty p(t)\,dt = \int_0^\infty 288e^{-8t}\,dt = \frac{288}{-8}e^{-8t}\Big|_0^\infty = 36\ J.$$

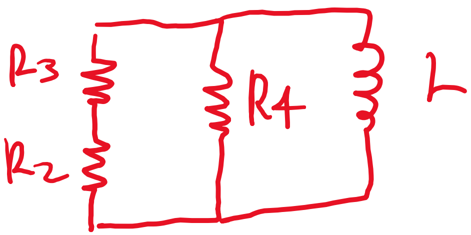
Energy stored in inductor?

In Class Exercise



Determine power and energy dissipated.

$$p(t) = i(t)\,V(t) = \frac{V(t)^2}{R} \qquad i(t) = 6e^{-4t}\ A$$



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$$W = \int_0^\infty p(t)\,dt = \int_0^\infty 288e^{-8t}\,dt = \frac{288}{-8}e^{-8t}\Big|_0^\infty = 36\,J.$$

Energy stored in inductor? $w = \frac{1}{2}L I_0^2 = \frac{1}{2} \cdot 2 \cdot 6^2 = 36J$

Good!

Summary: Capacitor and Inductor 1st Order ODEs

$$y(t) = \text{final value} + (\text{initial value} - \text{final value})e^{-t/t_c}$$

$$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/RC}$$

$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-Rt/L}$$

****applies to branch variables such as C or L current, resistor V, etc.****

Capacitor

- Behaves as instantaneous **short** circuit when inputs make abrupt changes
- Behaves as **open** circuit at long times when driven by DC voltage source

Inductor

- Behaves as instantaneous **open** circuit when inputs make abrupt changes
- Behaves as **short** circuit at long times when driven by DC current source

Power and Energy Relation in a Two-Terminal Element

Power:

$$P(t) = i(t)v(t) = \frac{v(t)^2}{R}$$

Energy:

$$w(t) = \int_0^T p(t)dt$$