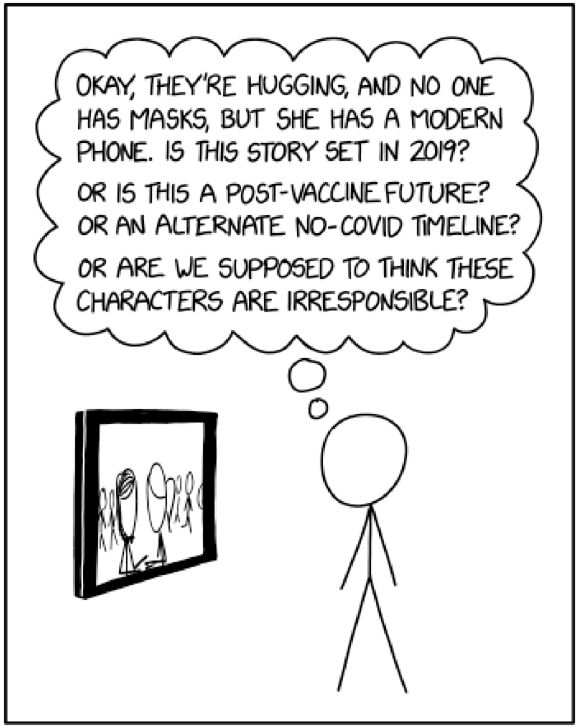
ECE 10C
Fall 2020
Slide Set 8
Instructor: Galan Moody
TA: Kamyar Parto



MOVIES AND SHOWS THAT ARE VAGUELY SET IN "THE PRESENT" WILL BE AWKWARD FOR A WHILE.

Tuesday
- Exam Review
- Op Amps

Today
- Transfer Function
- Logs and dB scale

<u>Important Items:</u>

- HW #4 due Thurs, 11/19
- Lab #4 due 11/20
- **Check your kits to make sure you have all components!**

Quiz Time!

Q1: Let's think about orders of magnitude: The number of electrons/second passing through any point of a wire carrying a 1 A current is approximately equal to (within an order of magnitude or so) [multiple choices]:

a) The estimated age of the universe, in seconds
b) The ratio of 1 second to an attosecond (the shortest laser pulse to date)
c) The ratio of UCSB Turkey Trot 5K distance to the size of an atomic nucleus
d) All of the above
e) None of the above

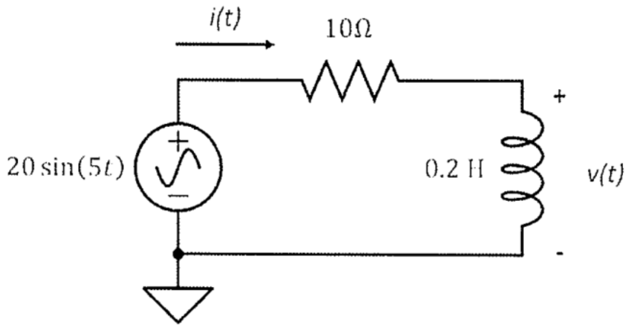
Q2: The decibel (dB) scale is a relative unit of measurement and a convenient way to express the ratio of one value of power to another value. A power ratio of ½ (output/input) approximately corresponds to:

a) ½ dB
b) 2 dB
c) 50 dB
d) 3 dB
e) -3 dB

HW 3: Average 89%, Nice Job!

Problem 2

Using the impedance method, find *v(t)* and *i(t)*. Give a very brief explanation of each step to reach the solution



1. $V(t) = 20\sin(5t) = 20\cos(5t - \pi/2)$. $\rightarrow$ $V_s = 20\angle -\pi/2$ ✓.

2. $Z = R + j\omega L$ $(\omega = 5 \text{ rad/s})$ $\rightarrow$ $Z = 10 + j\ \Omega$.

3. $I = \frac{V_s}{Z} = \frac{20e^{-j\pi/2}}{10+j} = \frac{-20j}{10+j} \cdot \frac{10-j}{10-j} = \frac{-20-200j}{101} = -0.198 - 1.98j$ A.

$\cong 2 \cdot \angle -1.47$ rad.

$i(t) = 2\cos(5t - 1.47)$ A. ✓

4. $V = I \cdot Z_L = (-0.198 - 1.98j) \cdot j = 1.98 - 0.198j$.

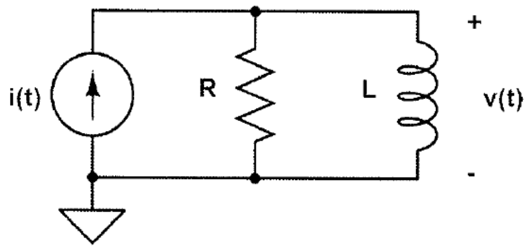
$\cong 2\angle -0.1$ rad.

$V(t) = 2\cos(5t - 0.1)$ V. ✓

HW 3: Average 89%, Nice Job!

Problem 3

Given that $i(t) = I_0\cos(\omega t)$, where $I_0 = 3$ mA and $\omega = 10^6$ rad/sec, determine v(t). Assume R = 1 kΩ and L = 1 mH.



1. $i(t) \rightarrow I_0 \angle \phi.$ $\quad Z_R = 1\,k\Omega \quad Z_L = j\,k\Omega.$

$$Z_T = \frac{Z_R \cdot Z_L}{Z_R + Z_L} = \frac{1E3 \cdot 1E3j}{1E3 + 1E3j}$$

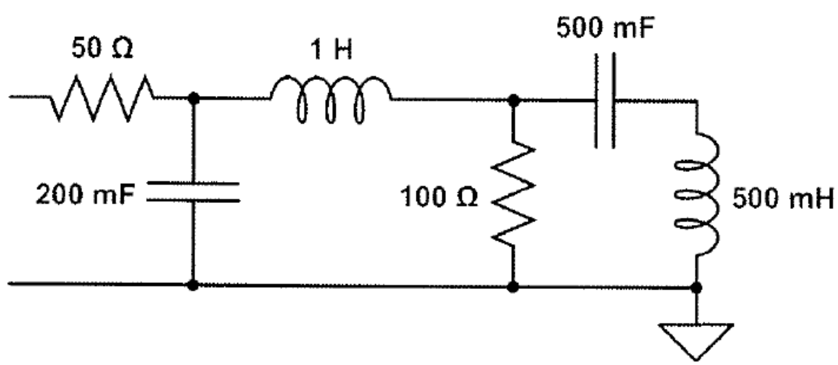
$$V = I \cdot Z_T = \frac{I_0 \cdot Z_R Z_L}{Z_R + Z_L} = \frac{3 \cdot j}{1+j} = \frac{3j}{1+j} \cdot \frac{1-j}{1-j} = \frac{3+3j}{2} = \frac{3}{\sqrt{2}} e^{j\pi/4}$$

Thus: $\quad v(t) = \frac{3}{\sqrt{2}} \cos(10^6 \cdot t + \pi/4)\,V.$ ✓

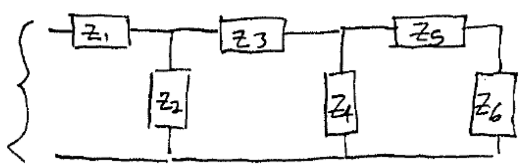
HW 3: Average 89%, Nice Job!

Problem 4

Determine the input impedance of the circuit at frequency $\omega = 2$ rad/s.



$Z_{in} =$



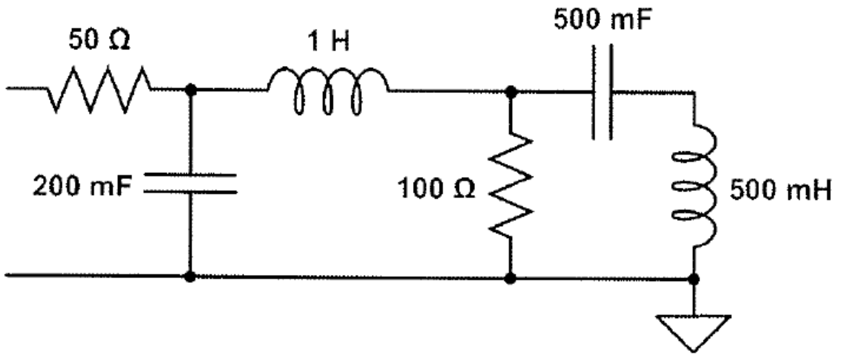
$$Z_{in} = Z_1 + \left[(Z_5+Z_6)//Z_4 + Z_3\right] // Z_2$$

$$= Z_1 + \left\{\left[\frac{(Z_5+Z_6)\cdot Z_4}{Z_5+Z_6+Z_4} + Z_3\right]\cdot Z_2\right\} \Big/ \left\{\left[\frac{(Z_5+Z_6)\cdot Z_4}{Z_5+Z_6+Z_4} + Z_3\right] + Z_2\right\}$$

HW 3: Average 89%, Nice Job!

Problem 4

Determine the input impedance of the circuit at frequency ω = 2 rad/s.



$Z_1 = 50\Omega$

$Z_2 = \cancel{\quad} -2.5j\,\Omega$

$Z_3 = 2j\,\Omega$

$Z_4 = 100\Omega$

$Z_5 = -j\,\Omega$

$Z_6 = j\,\Omega.$

$Z_5 + Z_6 = \phi\,\Omega.$ Thus:

$Z_{in} = Z_1 + [Z_3 \cdot Z_2 / (Z_3 + Z_2)].$

$Z_3 + Z_2 = -0.5j\,\Omega.$

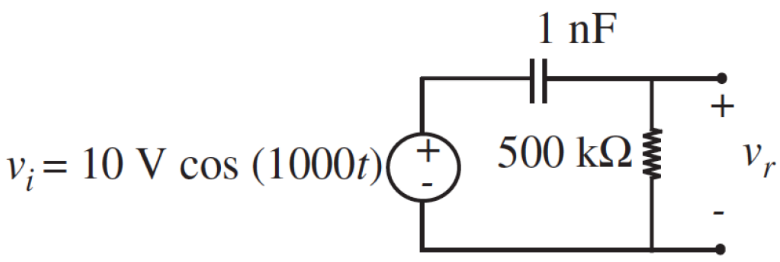
$Z_3 \cdot Z_2 = 5\Omega.$

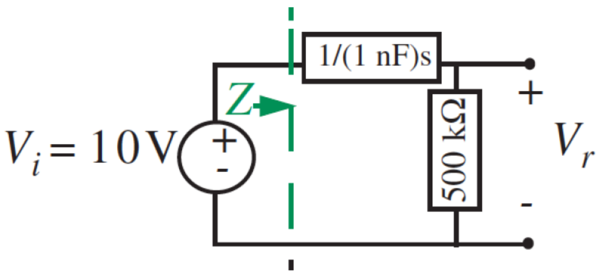
$\Downarrow$

$Z_{in} = 50 + \dfrac{5}{-0.5j} = 50 + 10j.$ ✓

The Impedance Method

Impedance: A generalization of resistance for sinusoidal steady state inputs. Associated with each element is a characteristic impedance that affects the flow of current/voltage drop across the element, analogous to a resistor.





$$v_0(t) = |V_0| \cos(\omega t + \angle V_0) = Re\left\{|V_0| e^{j\angle V_0} e^{j\omega t}\right\}$$

Generalization of Ohm's Law for Sinusoidal Steady State

Resistor: $V = IR \rightarrow Z_R = R$

Capacitor: $V = I\frac{1}{sC} \rightarrow Z_C = \frac{1}{sC} = \frac{1}{j\omega C}$

Inductor: $V = sLI \rightarrow Z_L = sL = j\omega L$

Impedance Method

$$v_0(t) = |V_0| \cos(\omega t + \angle V_0) = Re\{V_0(j\omega)e^{j\omega t}\}$$

1. First, **replace** the (sinusoidal) sources by the complex amplitudes

2. Then, replace the circuit elements by their impedances. The resulting diagram is called **the impedance model of the network.**

3. Now, **determine the complex amplitudes** of the voltages and currents in the circuits (e.g. $V_c(j\omega)$). You can use any standard linear circuit analysis technique you like—Node method, KVL, KCL, Thevenin, intuitive method based on series/parallel simplifications, etc.

4. Finally, we can obtain the time variables from the complex amplitudes and **plug into the general expression for the dynamics**. This isn't usually necessary though. As an example, the time domain response corresponding to the node variable V_c is given by:

Outline for 2nd half of this course

- ***Definition of transfer function [today]***
  - Impedance method simplifies circuit analysis
- ***Frequency domain analysis [today/next week]***
  - Log-log plots, Bode plots
  - Intuitive method for plotting frequency response
- ***Filter designs [next week, week 9]***
  - Arrange R, L, C circuits to achieve high, low, or bandpass filters
- ***Review + Thanksgiving***
- ***Resonant filter designs [week 9]***
- ***Review / final topics TBD [week 10]***

Frequency Domain Analysis

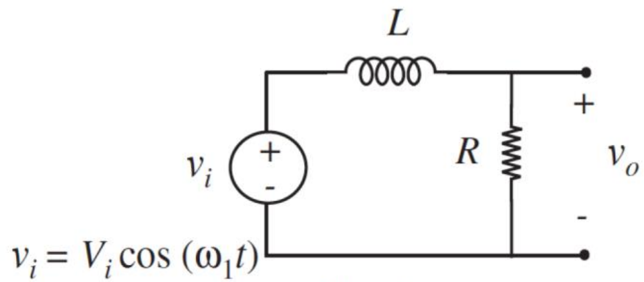
We've seen that a **circuit's behavior** in steady state **depends on the source frequency**. Frequency domain analysis formally studies this dependence

$$v_o(t) = |V_o| \cos(\omega_1 t + \angle V_o)$$

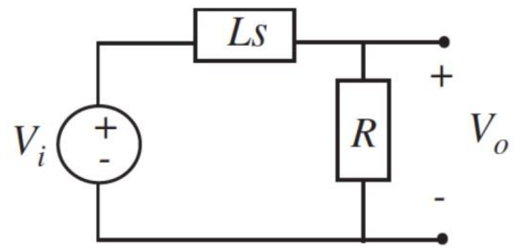
$$V_o = \frac{Z_R}{Z_R + Z_L} V_i$$

$$V_o = \frac{R}{R + j\omega_1 L} V_i$$

$$v_o(t) = \Re\{ V_o e^{j\omega_1 t}\}$$
$$= |V_o| \cos(\omega_1 t + \angle V_o)$$



(a) Circuit



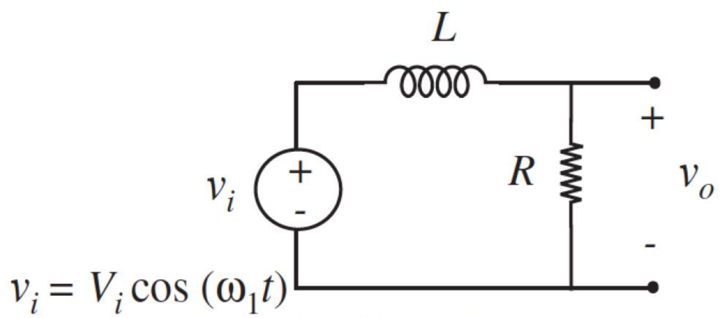
(b) Impedance model

$$v_o(t) = |V_o| \cos(\omega_1 t + \angle V_o) = \frac{R}{\sqrt{R^2 + \omega_1^2 L^2}} V_i \cos(\omega_1 t + \tan^{-1} \frac{-\omega_1 L}{R})$$

RL Circuit – Frequency Dependence of Output

- Let's use some numbers

L = 1 mH
R = 1 kΩ



(a) Circuit

- Using impedances and the voltage divider relation:

$$V_o = \frac{Z_R}{Z_R + Z_L} V_i = \frac{1000}{1000 + j0.001\omega} V_i = H(j\omega)\, V_i$$

$H(j\omega) = \frac{V_0}{V_i}$ is called the **transfer function**

Transfer Function

Definition

The **transfer function** is the ratio of the complex amplitude of the circuit **output** to the complex amplitude of the **input**

$$H(j\omega) = \frac{V_o}{V_i} = \frac{1000}{1000 + j0.001\omega}$$

(only info about the circuit using its complex impedances)

Given the transfer function of a system, we know:

$$H(j\omega) = |H(j\omega)|e^{\angle H(j\omega)}$$

$$V_o = H(j\omega)V_i$$

The output response of the system to a sinusoidal input $\boldsymbol{v_i = V_i \cos(\omega t)}$ is:

$$v_o(t) = |H(j\omega)|V_i \cos(\omega t + \angle H(j\omega))$$

Transfer Function

determined by analyzing the circuit, independent of the source, using the impedance method

$$v_i(t) = V_i \cos(\omega t) \longrightarrow \boxed{H(j\omega)} \longrightarrow v_o(t) = |H(j\omega)| V_i \cos\left(\omega t + \angle H(j\omega)\right)$$

Some naming conventions:

Transfer Type	Output/Input	Units
Transfer Function	V_o/V_i	none
Transconductance (DC) Transadmittance (AC)	I/V	$1/\Omega$
Transresistance/ Transimpedance	V/I	Ω

Frequency Response of a Circuit

$$v_o(t) = |H(j\omega)|V_i \cos(\omega t + \angle H(j\omega))$$

Frequency Response

A plot of the magnitude and phase of the circuit's transfer function as a function of frequency: $|H(j\omega)|$ and $\angle H(j\omega)$

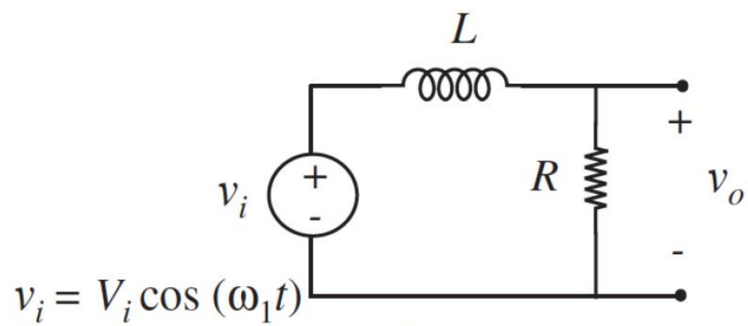
- The magnitude is the ratio of the **amplitudes** of the output and input, and shows the **gain of the circuit** as a function of frequency

$$H(j\omega) = \frac{V_0}{V_i} \qquad |H(j\omega)| = \frac{|V_0|}{|V_i|}$$

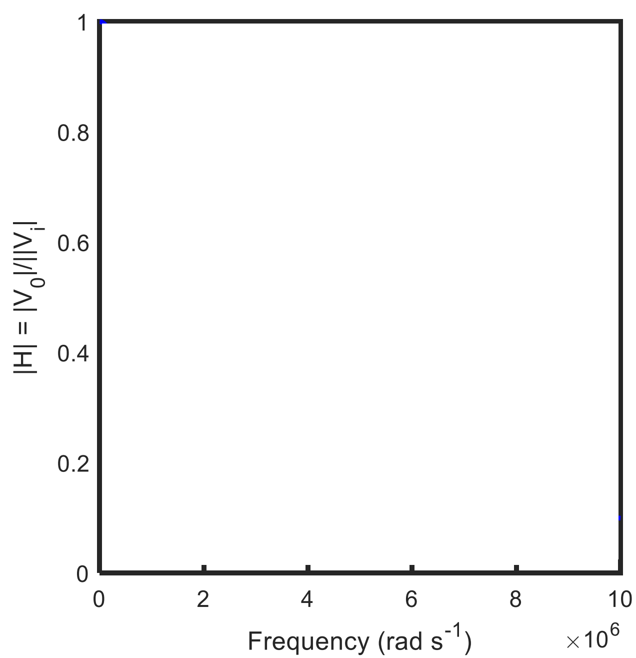
- The phase is the angular difference between the output and the input sinusoids (**phase shift**)

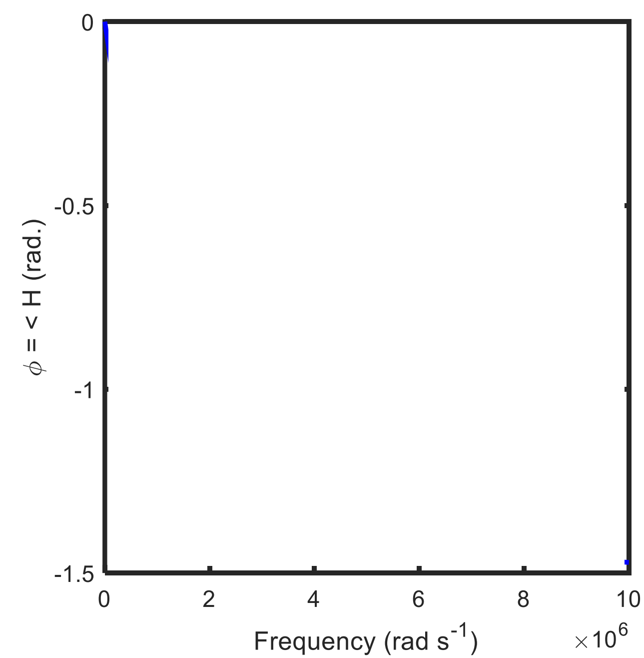
RL Circuit – Frequency Response

$$H(f) = \frac{V_o}{V_i} = \frac{1000}{1000 + j0.001\omega}$$



(a) Circuit



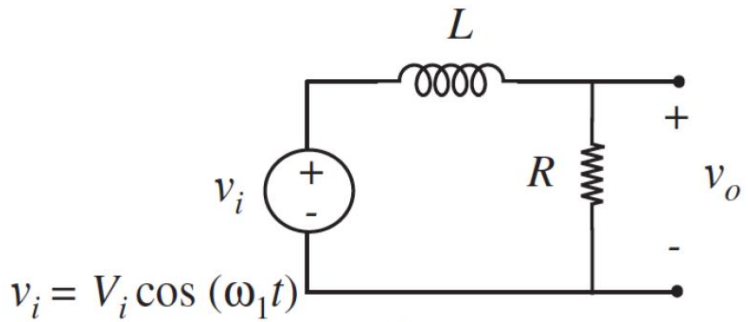


RL Circuit – Frequency Response

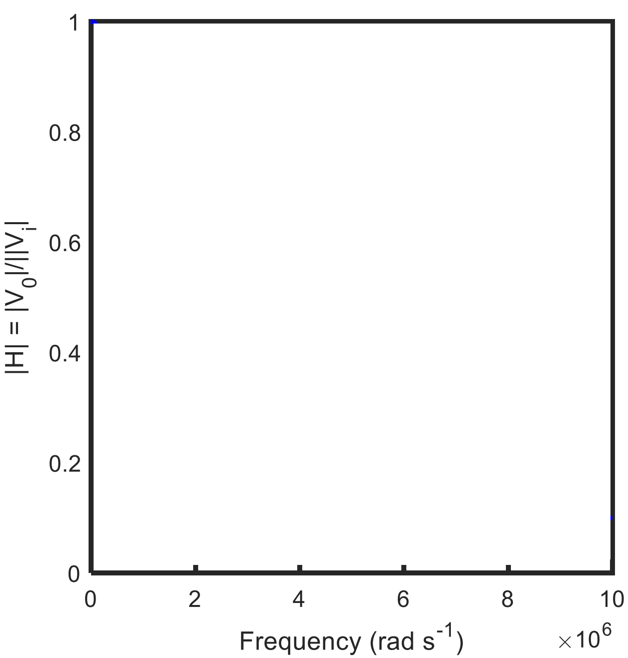
$$H(f) = \frac{V_o}{V_i} = \frac{1000}{1000 + j0.001\omega}$$

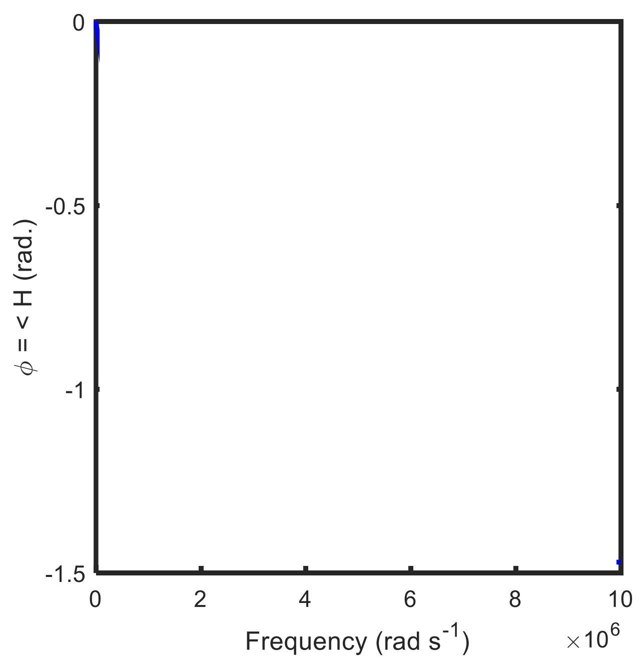
$$\omega \to 0, H \to 1 = r[\cos(\varphi) + jsin(\varphi)]$$

$$\omega \to \infty, H \to -j/\omega = r[\cos(\varphi) + jsin(\varphi)]$$



(a) Circuit





RL Circuit – Frequency Response

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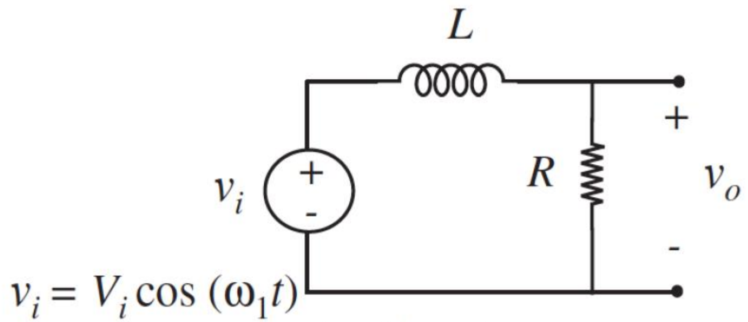
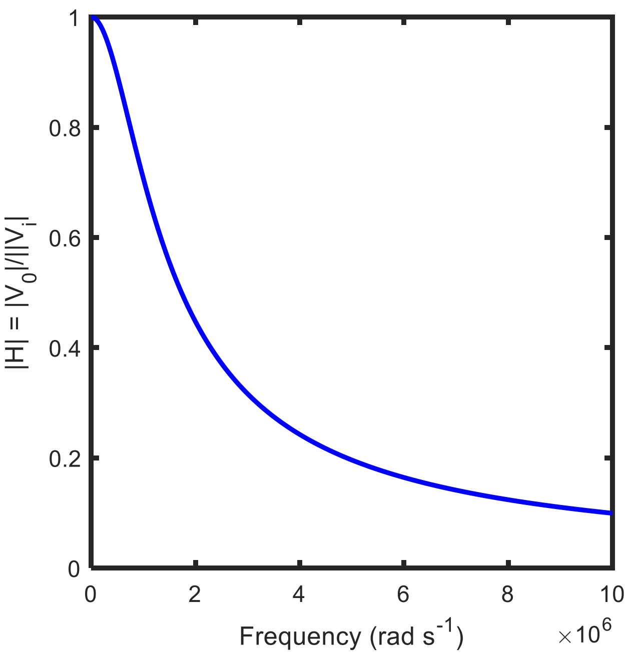
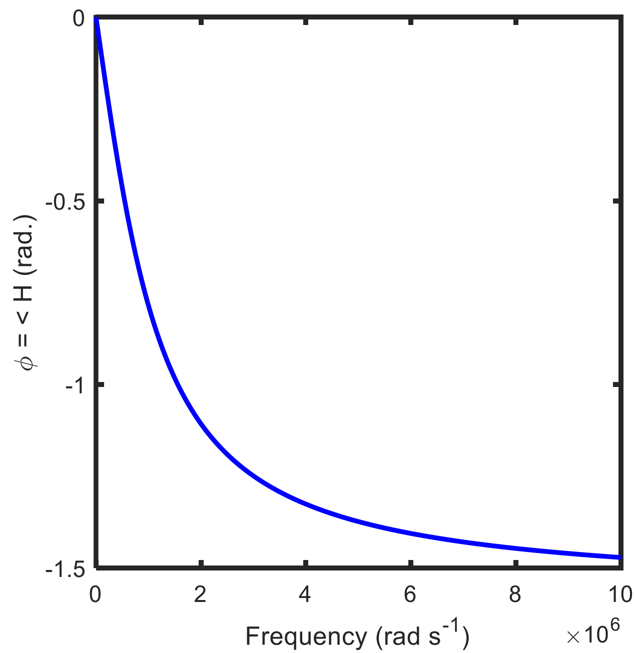


(a) Circuit





RL Circuit – Frequency Response

$$H(f) = \frac{V_o}{V_i} = \frac{1000}{1000 + j0.001\omega}$$

$$\omega \to 0, H \to 1 = r[\cos(\varphi) + j\sin(\varphi)]$$

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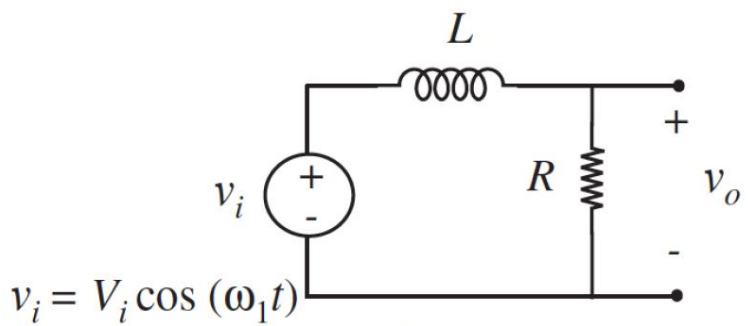
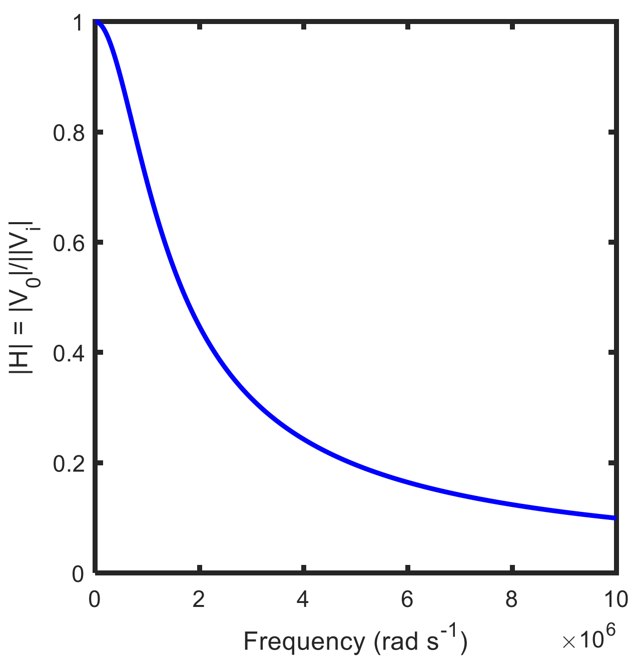
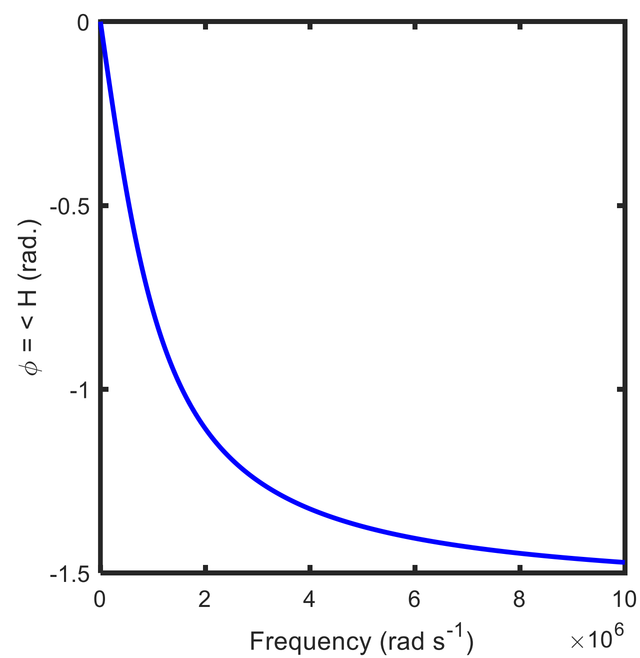


(a) Circuit

Low-pass filter





Finding Magnitude and Phase of Transfer Function

$$H(j\omega) = \frac{V_o}{V_i} = \frac{j\omega}{\left(j\omega + \frac{3}{2}\right)\left(j\omega + \frac{5}{2}\right)} =$$

Finding Magnitude and Phase of Transfer Function

$$H(j\omega) = \frac{V_o}{V_i} = \frac{j\omega}{\left(j\omega + \frac{3}{2}\right)\left(j\omega + \frac{5}{2}\right)} = \frac{A_1}{A_2 A_3} = \frac{|A_1|e^{j\angle A_1}}{|A_2|e^{j\angle A_2}|A_3|e^{j\angle A_3}}$$

Finding Magnitude and Phase of Transfer Function

$$H(j\omega) = \frac{V_o}{V_i} = \frac{j\omega}{\left(j\omega + \frac{3}{2}\right)\left(j\omega + \frac{5}{2}\right)} = \frac{A_1}{A_2 A_3} = \frac{|A_1|e^{j\angle A_1}}{|A_2|e^{j\angle A_2}|A_3|e^{j\angle A_3}}$$

$$|H(j\omega)| = \frac{|A_1|}{|A_2||A_3|} = \frac{\omega}{\sqrt{\omega^2 + \frac{9}{4}}\sqrt{\omega^2 + \frac{25}{4}}}$$

$$\angle H(j\omega) = \angle A_1 - \angle A_2 - \angle A_3 = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{\frac{3}{2}}\right) - \tan^{-1}\left(\frac{\omega}{\frac{5}{2}}\right)$$

Plotting the Frequency Response

typically V_0 or I_0

$$\text{transfer function} = H(j\omega) = \frac{complex\ amplitude\ of\ the\ output}{complex\ amplitude\ of\ the\ input}$$

$$v_i(t) = V_i \cos(\omega t) \longrightarrow \boxed{H(j\omega)} \longrightarrow v_o(t) = |H(j\omega)| V_i \cos\left(\omega t + \angle H(j\omega)\right)$$

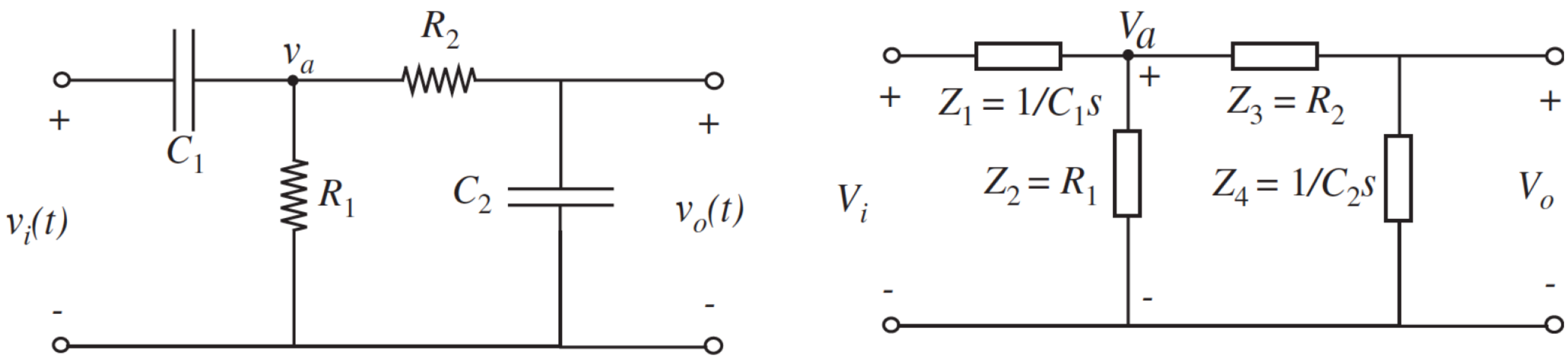
When asked to plot the frequency response of a system, you need to draw the approximate sketches of the magnitude and phase as follows:

Plotting the Frequency Response

- Plot $|H(j\omega)|$: the log magnitude plotted against log frequency
- Plot $\angle H(j\omega)$: the angle in linear scale plotted against log frequency

Example 1

We find the complex amplitude V_o of the output as a function of V_i:



- You would need to apply the voltage divider relation two times:

$$V_a = \frac{\left(R_2 + \frac{1}{C_2 s}\right) \| R_1}{\left(R_2 + \frac{1}{C_2 s}\right) \| R_1 + \frac{1}{C_1 s}} V_i$$

$$V_o = \frac{\frac{1}{C_2 s}}{\frac{1}{C_2 s} + R_2} V_a = \left(\frac{\frac{1}{C_2 s}}{\frac{1}{C_2 s} + R_2}\right) \left(\frac{\left(R_2 + \frac{1}{C_2 s}\right) \| R_1}{\left(R_2 + \frac{1}{C_2 s}\right) \| R_1 + \frac{1}{C_1 s}}\right) V_i$$

Example 1

$$V_o = \frac{\frac{1}{C_2 s}}{\frac{1}{C_2 s} + R_2} V_a = \left(\frac{\frac{1}{C_2 s}}{\frac{1}{C_2 s} + R_2} \right) \left(\frac{\left(R_2 + \frac{1}{C_2 s}\right) \| R_1}{\left(R_2 + \frac{1}{C_2 s}\right) \| R_1 + \frac{1}{C_1 s}} \right) V_i$$

kΩ kΩ

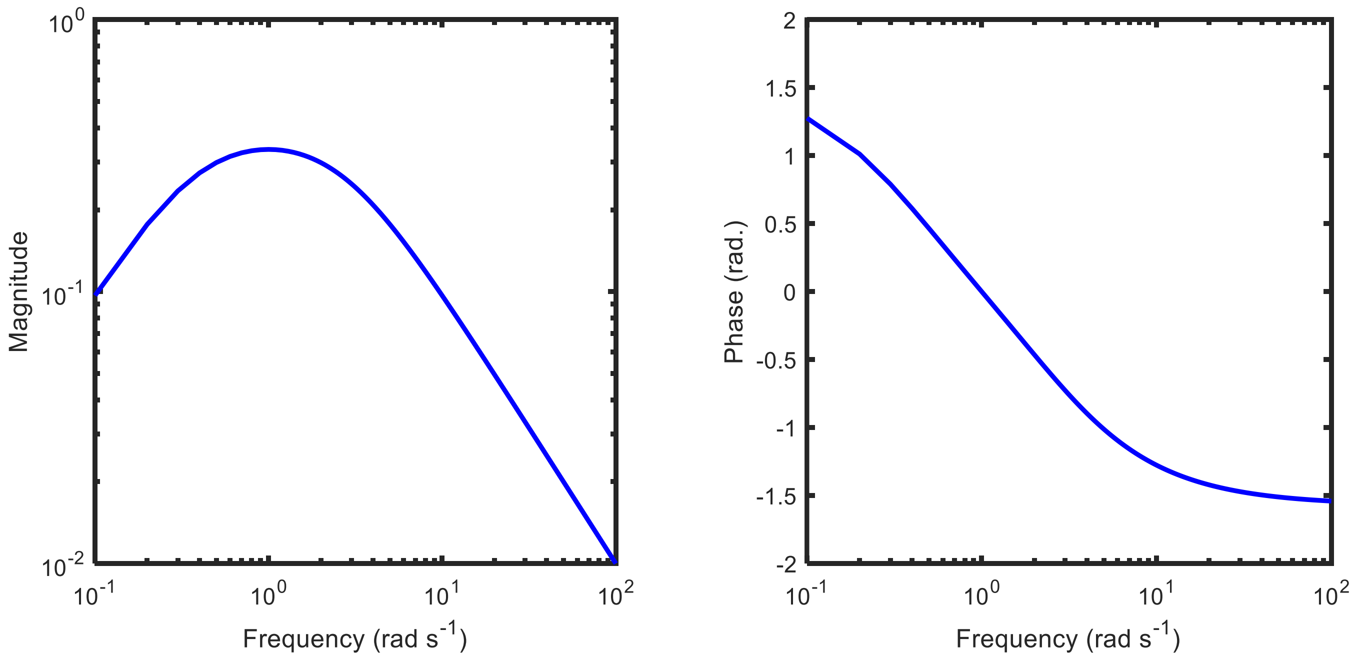
- Assume $R_1 = 1\ \cancel{k}$, $R_2 = 1\ \cancel{k}$, $C_1 = 1\ \mathrm{mF}$, $C_2 = 1\ \mathrm{mF}$

$$\begin{aligned} V_o &= \frac{R_1 C_1 s}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2) s + 1} V_i \\ &= \frac{s}{s^2 + 3s + 1} \\ &\overset{s=j\omega}{=} \frac{j\omega}{\left(j\omega - \frac{-3-\sqrt{5}}{2}\right)\left(j\omega - \frac{-3+\sqrt{5}}{2}\right)} V_i \\ &= \frac{|A_1|}{|A_2||A_3|} e^{j(\phi_1 - \phi_2 - \phi_3)} V_i \end{aligned}$$

Example 1

$$H(j\omega) = \frac{V_o}{V_i} = \frac{j\omega}{\left(j\omega - \frac{-3-\sqrt{5}}{2}\right)\left(j\omega - \frac{-3+\sqrt{5}}{2}\right)}$$

Let us plot $|H(j\omega)|$ and $\angle H(j\omega)$:



Example 1

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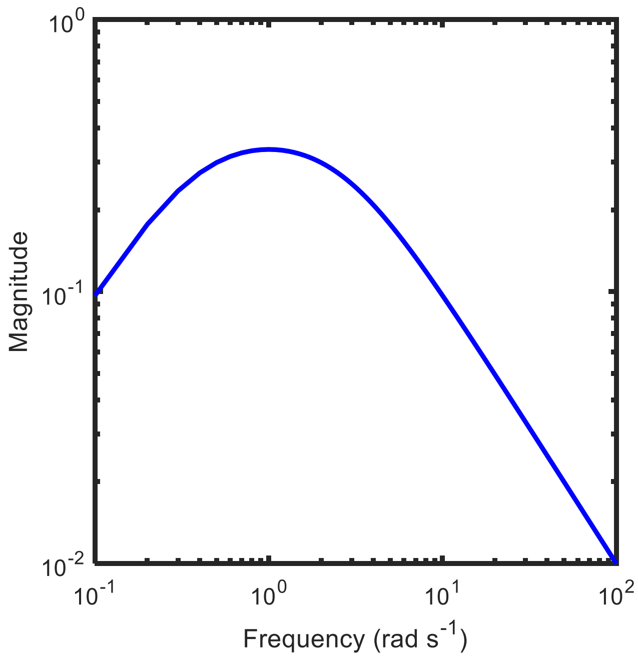
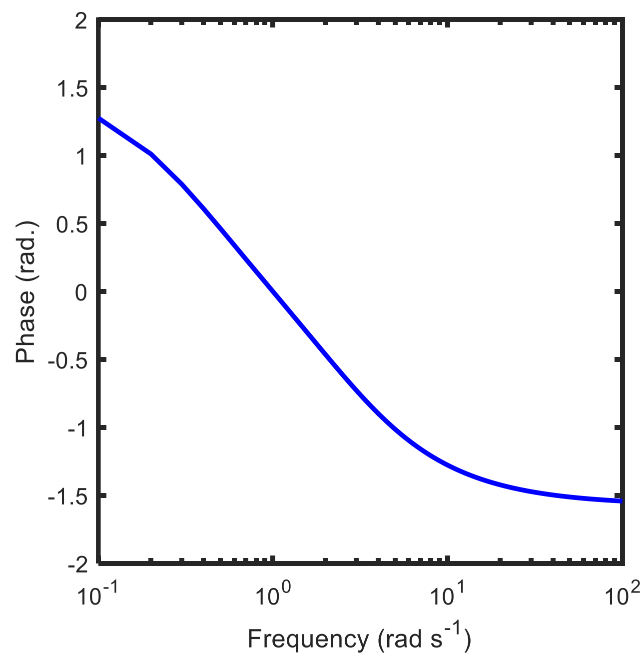




attenuates both low and high frequencies

Plotting the Frequency Response

- Let's review log-log plots
- Plot transfer functions for R, L, and C on their own
- Intuitive sketching of RL and RC plots
- Examples

dB	power ratio	amplitude ratio
100	10 000 000 000	100 000
90	1 000 000 000	31 620
80	100 000 000	10 000
70	10 000 000	3 162
60	1 000 000	1 000
50	100 000	316.2
40	10 000	100
30	1 000	31.62
20	100	10
10	10	3.162
3	1.995	1.413
1	1.259	1.122
0	1	1
-10	0.1	0.316 2
-20	0.01	0.1
-30	0.001	0.031 62
-40	0.000 1	0.01
-50	0.000 01	0.003 162
-60	0.000 001	0.001
-70	0.000 000 1	0.000 316 2
-80	0.000 000 01	0.000 1
-90	0.000 000 001	0.000 031 62
-100	0.000 000 000 1	0.000 01

Decibels:

$$P_2 = 10^{L/10\ dB} P_1$$

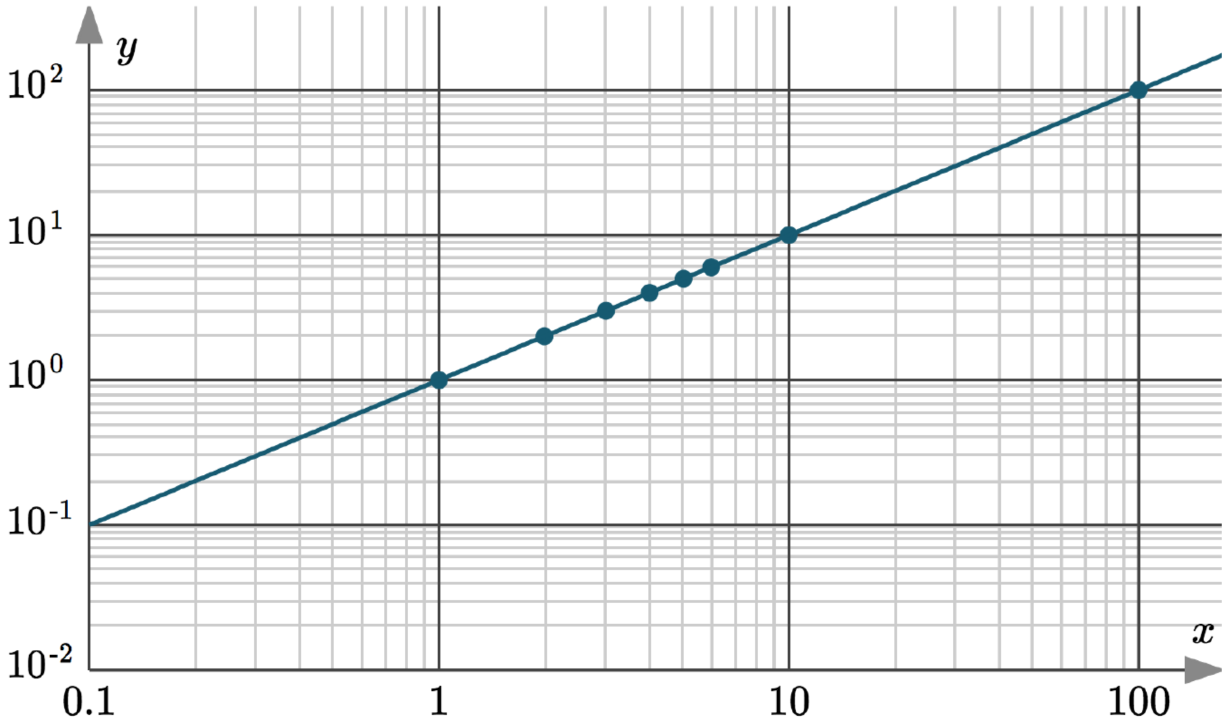
$$\boldsymbol{L = 10\log(P_2/P_1)}\ \textbf{dB}$$

$$P = \frac{V^2}{R} = i^2 * R$$

$$\boldsymbol{L = 20\log(V_2/V_1)}\ \textbf{dB}$$

Review of Log Plots

- In log-log plots, both axes are on logarithmic (base 10) scale



- Easier to see details for small values of y as well as large values of y

Review of Log Plots

Power relations of the form

$$y = kx^p, \qquad p \lesseqgtr 0$$

may be rendered straight by plotting on log-log paper

Take the logarithm of both sides

$$\log y = \log K + p \log x$$

Now if y is plotted against x on log-log paper we get a line of slope:

$$\frac{\log y_2 - \log y_1}{\log x_2 - \log x_1} = p$$

To draw a line, you need the slope + a point the line passes through

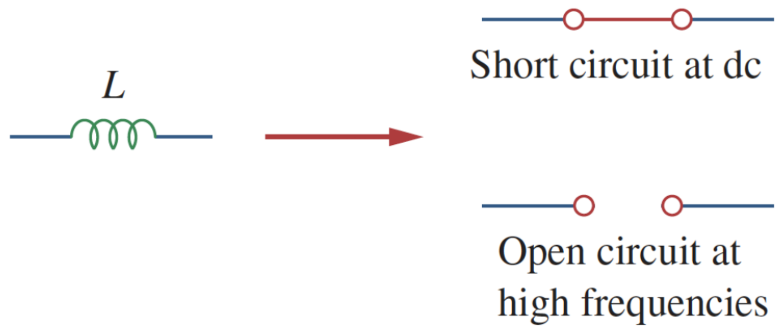
On the log-log paper, neither intercept has any useful meaning ($\log x$ never has a zero value)

Pick another point to draw the line

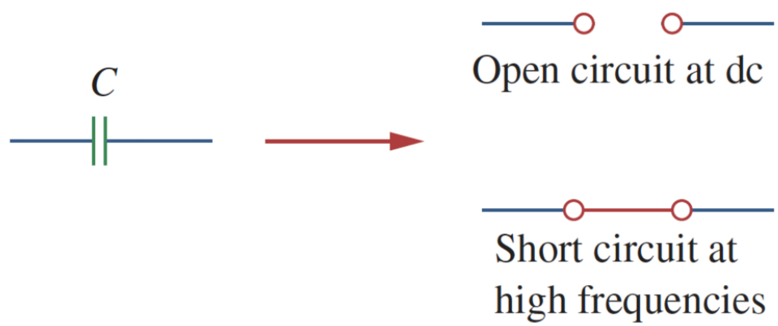
Examples: R, L, and C

- First, we'll see what the frequency response of resistors, capacitors, and inductors looks like to build some intuition
- Then we'll study a general method of approximately sketching the frequency response of complex circuits

$Z = j\omega L$



$Z = 1/j\omega C$



Examples: R, L, and C

We know the element laws for resistors, inductors, and capacitors in terms of complex amplitudes of V and I:

$$\begin{aligned}
\text{Resistor:}\quad & V_o = RI_o \\
\text{Inductor:}\quad & V_o = sLI_o = j\omega LI_o \\
\text{Capacitor:}\quad & V_o = \frac{1}{sC}I_o = \frac{1}{j\omega C}I_o
\end{aligned}$$

So the elements' frequency response is:

$$\begin{aligned}
\text{Resistor:}\quad & H(j\omega) = \frac{V_o}{I_o} = R \\
\text{Inductor:}\quad & H(j\omega) = \frac{V_o}{I_o} = j\omega L \\
\text{Capacitor:}\quad & H(j\omega) = \frac{V_o}{I_o} = \frac{1}{j\omega C}
\end{aligned}$$

Examples: R, L, and C

So here is the frequency response. How do we plot them?

$$\text{Resistor:} \quad H(j\omega) = \frac{V_o}{I_o} = R$$

$$\text{Inductor:} \quad H(j\omega) = \frac{V_o}{I_o} = j\omega L$$

$$\text{Capacitor:} \quad H(j\omega) = \frac{V_o}{I_o} = \frac{1}{j\omega C}$$

Let's calculate the magnitude and phase of the frequency response:

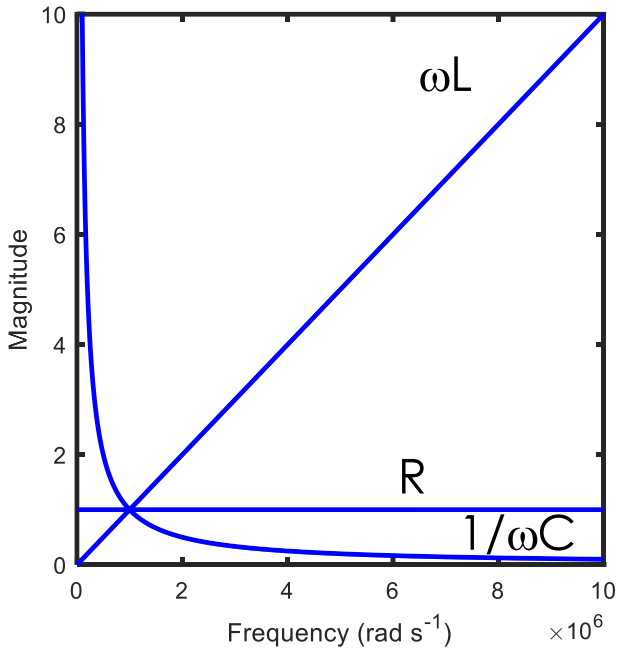
$$\text{Resistor:} \quad \left|\frac{V_o}{I_o}\right| = R, \quad \angle\frac{V_o}{I_o} = 0$$

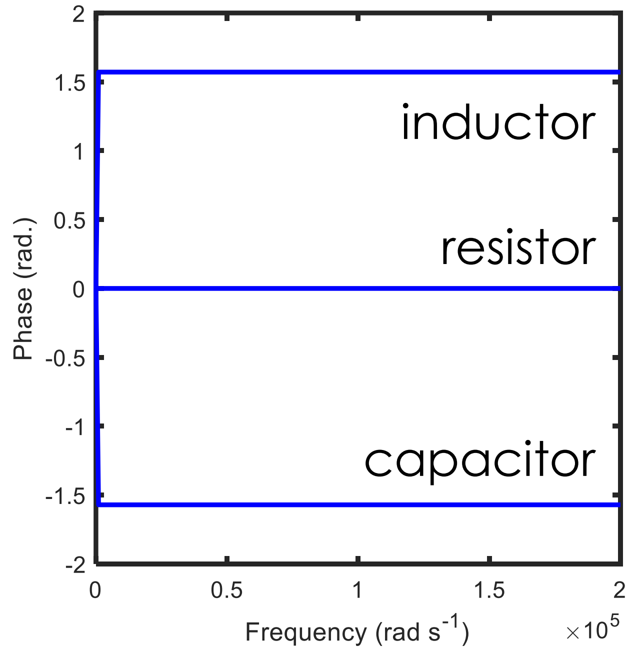
$$\text{Inductor:} \quad \left|\frac{V_o}{I_o}\right| = \omega L, \quad \angle\frac{V_o}{I_o} = \frac{\pi}{2}$$

$$\text{Capacitor:} \quad \left|\frac{V_o}{I_o}\right| = \frac{1}{\omega C}, \quad \angle\frac{V_o}{I_o} = -\frac{\pi}{2}$$

Examples: R, L, and C on Linear Scale

Plot the frequency response for R = 1Ω, L = 1 μH, and C = 1 μF





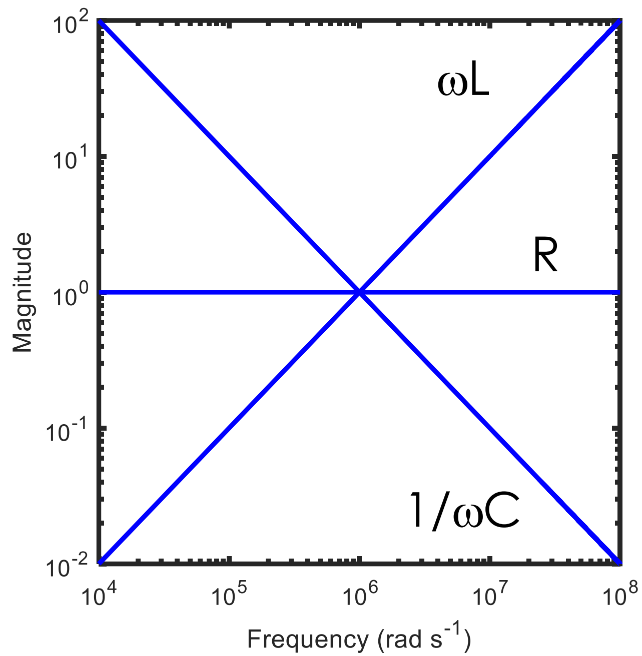
Resistor: $\left|\dfrac{V_o}{I_o}\right| = R, \quad \angle\dfrac{V_o}{I_o} = 0$

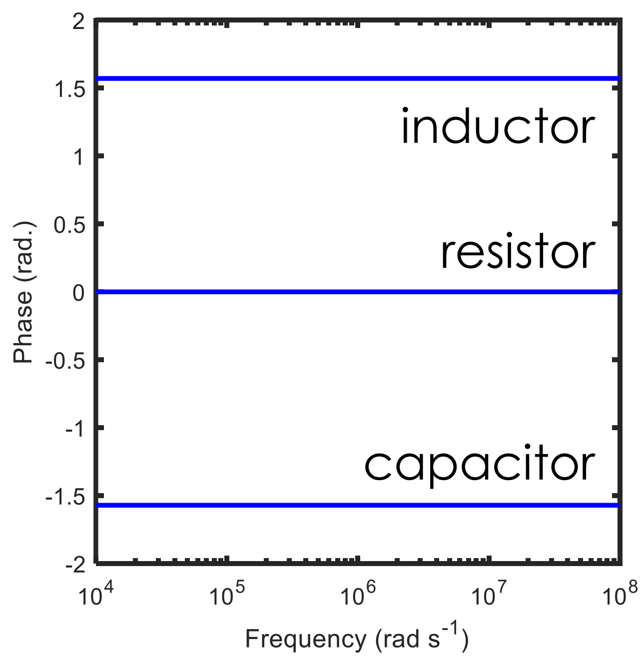
Inductor: $\left|\dfrac{V_o}{I_o}\right| = \omega L, \quad \angle\dfrac{V_o}{I_o} = \dfrac{\pi}{2}$

Capacitor: $\left|\dfrac{V_o}{I_o}\right| = \dfrac{1}{\omega C}, \quad \angle\dfrac{V_o}{I_o} = -\dfrac{\pi}{2}$

Examples: R, L, and C on Log Scale

Plot the frequency response for R = 1Ω, L = 1 μH, and C = 1 μF





If we use log scales:

Resistor: $\log \left| \frac{V_o}{I_o} \right| = \log R$

Inductor: $\log \left| \frac{V_o}{I_o} \right| = \log L\omega = \log L + \log \omega$

Capacitor: $\log \left| \frac{V_o}{I_o} \right| = \log \frac{1}{\omega C} = -\log C - \log \omega$

Summary of Log Plots

- Imagine if you wanted to plot:

$$|H(j\omega)| = |\frac{j\omega + 1}{2j\omega + 1}|$$

- Summing terms is easy to do graphically; products are harder
- On a log scale, products (and divisions) turn into a sum:

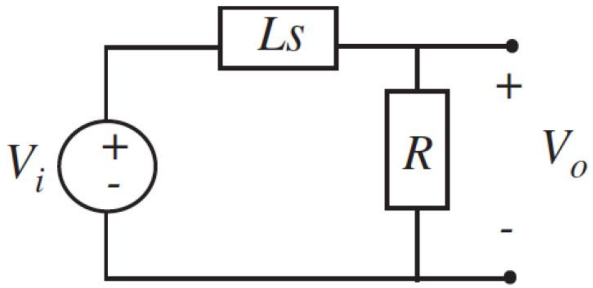
$$\log|\frac{j\omega + 1}{2j\omega + 1}| = \log|j\omega + 1| - \log|2j\omega + 1|$$

- You'll see shortly how we plot terms that appear in RL and RC circuits

Intuitive Sketching for RL and RC Circuits

- Let's first consider the series RL circuit:

$$H(j\omega) = \frac{V_o}{V_i} = \frac{R}{R + j\omega L}$$
$$= \frac{\frac{R}{L}}{\frac{R}{L} + j\omega}$$



(b) Impedance model

- So we get:

$$|H(j\omega)| = \left| \frac{\frac{R}{L}}{\frac{R}{L} + j\omega} \right|$$

$$\angle H(j\omega) = \tan^{-1} \frac{-\omega L}{R}$$

Intuitive Sketching, Magnitude $|H(j,\omega)|$

- First, find the asymptotes of the magnitude plot:

$$|H(j\omega)| = \left| \frac{\frac{R}{L}}{\frac{R}{L} + j\omega} \right|$$

- At low frequencies ($\omega \to 0$):

$$|H(j\omega)| \approx 1$$

  - Hence the magnitude appears as a horizontal line at low frequencies

- At high frequencies $\omega \to \infty$, ω dominates in the denominator

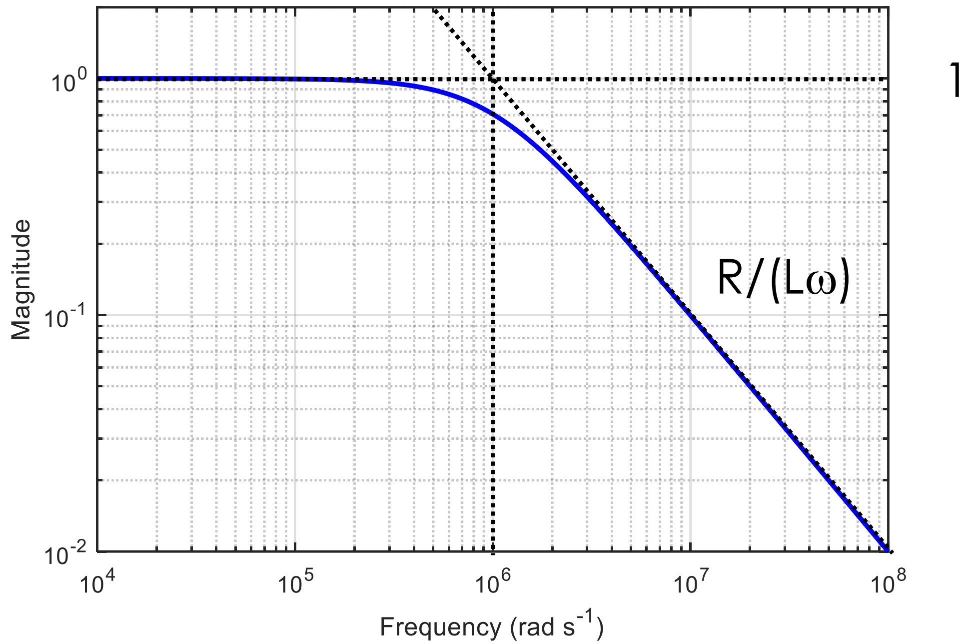
$$|H(j\omega)| \approx \frac{R/L}{\omega} \to \log|H(j\omega)| \approx \log R/L - \log\omega$$

  - **On a log scale, a line with slope = -1 passing through (R/L, 1)**

Intuitive Sketching, Magnitude $|H(j,\omega)|$

- At low frequencies ($\omega \to 0$): $|H(j\omega)| = 1$
- At high frequencies $\omega \to \infty$, ω dominates in the denominator

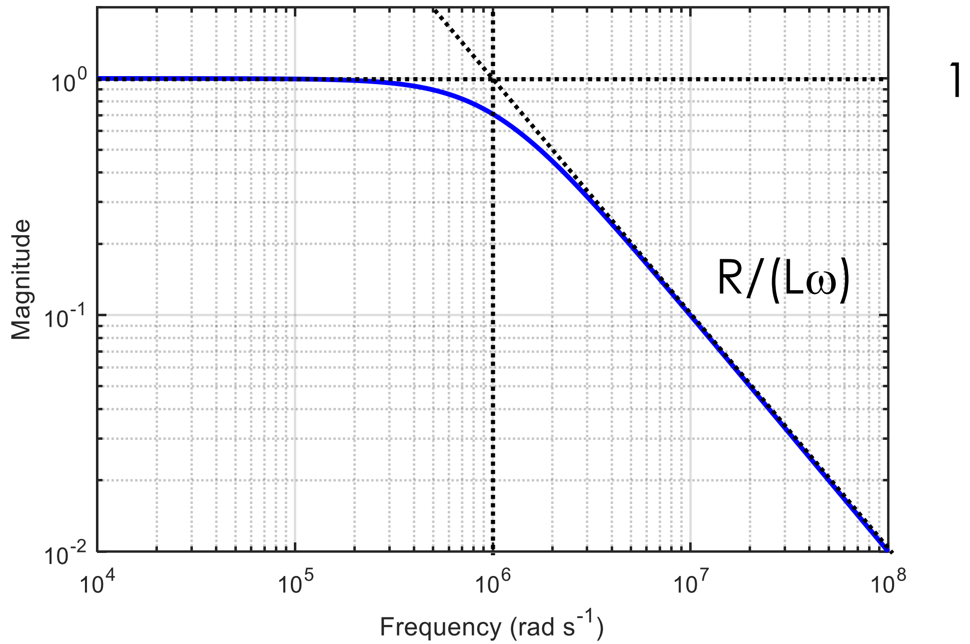
$$|H(j\omega)| \approx \frac{R/L}{\omega} \to \log|H(j\omega)| \approx \log R/L - \log \omega$$



Intuitive Sketching, Magnitude $|H(j,\omega)|$

- The two asymptotes intersect at: $\omega = \dfrac{R}{L}$
- This frequency is called the "corner" or "cutoff" frequency

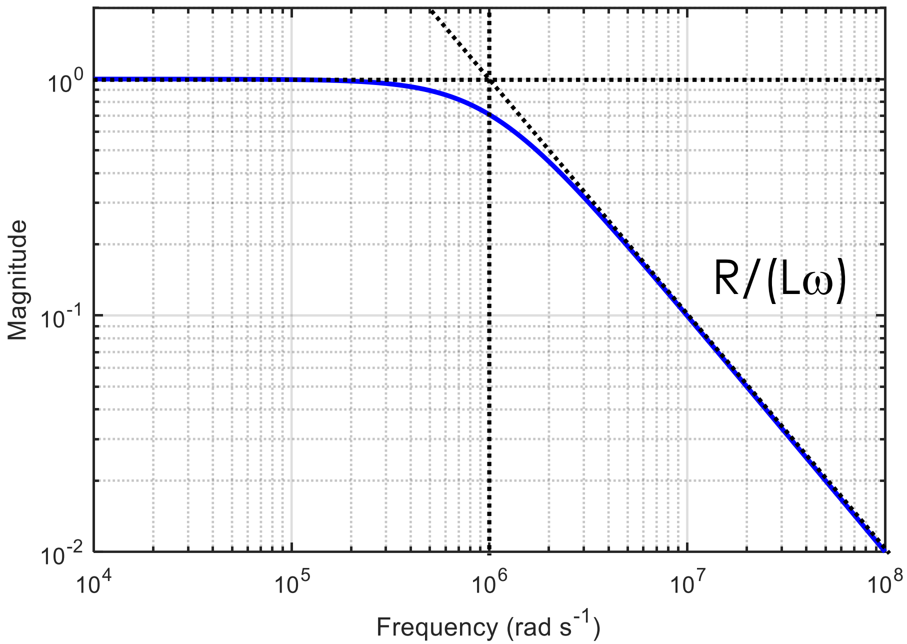
At the cutoff frequency, the real and imaginary components of $H(j,\omega)$ are equal, and $|H(j,\omega)| = \frac{1}{\sqrt{2}} = 0.707$



$$|H(j\omega)| \approx \frac{R/L}{\omega} \rightarrow \log|H(j\omega)| \approx \log R/L - \log\omega$$

Intuitive Sketching, Magnitude $|H(j,\omega)|$

At the cutoff frequency, the real and imaginary components of $H(j,\omega)$ are equal, and $|H(j,\omega)| = \frac{1}{\sqrt{2}} = 0.707$



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Decibels:

$$P_2 = 10^{L/10\,dB} P_1$$

$$L = 10\log(P_2/P_1)\text{ dB}$$

$$10\log\left(\frac{1}{2}\right) \sim 3\text{ dB}$$

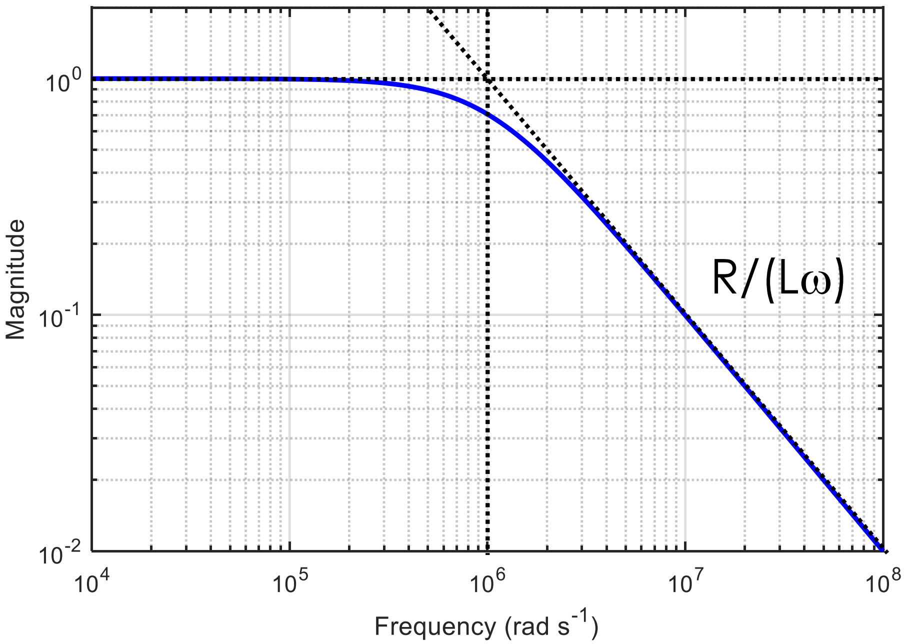
Recall that power dissipated through impedance Z is $\frac{V_0^2}{Z}$, and $V_0^2 \propto |H(j,\omega)|^2$. At the cutoff frequency, $|H(j,\omega)|^2 = 1/2$. **Thus the cutoff frequency also tells you the "3 dB" point**, i.e. the frequency at which the output power is halved.

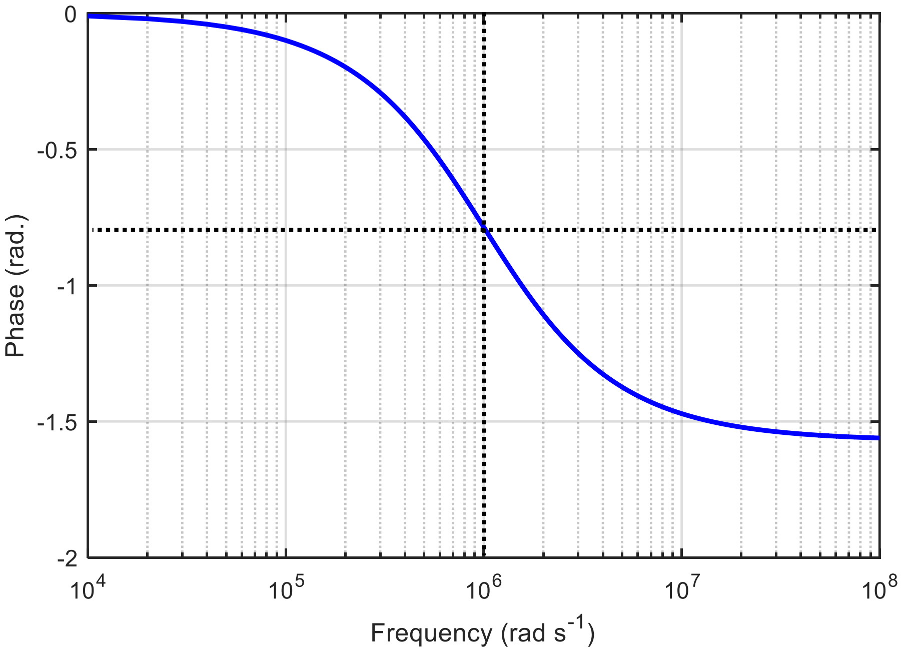
Intuitive Sketching, Phase $\angle H(j,\omega)$

$$H(j\omega) = \frac{\frac{R}{L}}{\frac{R}{L} + j\omega}$$

- At low frequencies ($\omega \to 0$): $\angle H(j\omega) \approx \angle 1 = 0$
- At high frequencies $\omega \gg R/L$, ω dominates in the denominator

$$\angle H(j\omega) \approx \angle \frac{1}{j\omega} = -\frac{\pi}{2}$$





Summary

typically V_0 or I_0

$$\text{transfer function} = H(j\omega) = \frac{\textit{complex amplitude of the output}}{\textit{complex amplitude of the input}}$$

$$v_i(t) = V_i \cos(\omega t) \longrightarrow \boxed{H(j\omega)} \longrightarrow v_o(t) = |H(j\omega)| V_i \cos\left(\omega t + \angle H(j\omega)\right)$$

When asked to plot the frequency response of a system, you need to draw the approximate sketches of the magnitude and phase as follows:

Plotting the Frequency Response

- Plot $|H(j\omega)|$: the log magnitude plotted against log frequency
- Plot $\angle H(j\omega)$: the angle in linear scale plotted against log frequency

Next Week

- Finish up Transfer Function, Log and Bode Plots
- Begin Filter Design
- Don’t forget HW 4 due next Thursday
- Don’t forget Lab 4 due next Friday