# Chapter 10

Jeffrey Wubbenhorst

April 9, 2016

- To describe rotation of a rigid body about a fixed axis (rotation axis), we assume a reference line is fixed in the body, perpendicular to the axis and rotating with the body. The angular position $\theta$ of the line is measured relative to fixed direction, and is given as $\theta = \frac{s}{R}$, where $s$ is the arc length, and $R$ is the radius.
- Angular displacement is given as $\Delta\theta = \theta_2 - \theta_1$, where $\theta$ is positive for counterclockwise rotation and negative for counterclockwise. *Note: NOT to be treated as a vector*
- Angular velocity is given as $\omega_{avg} = \frac{\Delta\theta}{\Delta t}, \omega = \frac{d\theta}{dt}$, which are both vectors, wit direction given by the right-hand rule.
- Angular acceleration is given as $\frac{\Delta\omega}{dt}$
- Constant angular acceleration ($\alpha =$ constant has properties that adhere to the following kinematic equations:

$$\omega = \omega_0 + \alpha t$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$
$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha(\theta - \theta_0)$$
$$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)$$
$$\theta - \theta_0 = \omega t - \frac{1}{2}at^2$$

- A point in a rigid rotating body at a perpendicular distance $r$ from the rotation axis, moves in a circle with radius $r$. If the body rotates through an angle $\theta$, the point moves along an arc with length $s$ given by $s = \theta r$.
- Linear velocity is tangent to circle; point's linear speed is given by $v = \omega R$, where $\omega$ is the angular speed of the body.
- The linear acceleration $\vec{a}$ of the point has both tangential and radial components. The tangential component is $a_r = \frac{v^2}{r} = \omega^2 r$. *Note that this quantity is given in radians.*
- If a point moves in uniform circular motion, the period $T$ is given as $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$
- Kinetic energy of a rigid body rotating about a fixed axis is given by $k = \frac{1}{2}I\omega^2$, where $I$ is the rotational inertia of the body, defined as $I = \sigma m_i r^2$ for a system of particles
- $I$ is the rotational inertia (also known as the moment of inertia with respect to the axis of rotation. The SI unit for $I$ is the kg m$^2$
- $I$ is the rotational inertia of a body defined as $I = \sum m_i r^2$ for a system of discrete particles and defined as $I = \int r^2 dm$ for a body with $r_i$ represent the perpendicular distance from the axis of rotation to each mass element in the body.
- The parallel-axis theorem states that the rotational inertia $I$ of a body about any axis to that of the same body about a parallel axis through the center of mass is given as $I = I_{com} + Mh^2$, where $h$ is the perpendicular distance between the two axes, and $I_{com}$ is the rotational inertia of the body about the axis through the com. $h$ can also be described as the distance the actual rotation axis has been shifted from the rotational axis through the com.
- Torque ($\tau$) is defined as $\tau = (r)(F\sin\theta)$ where $\theta$ is the angle between $\vec{F}$ and $\vec{r}$ (the position vector relative to the rotation axis). The units of torque are Newtons-meter ($N\dot{m}$).
- The rotational analog of Newton's second law is $\tau_{net} = I\alpha$
- Work is defined rotationally as $W = \int_{\theta i}^{\theta} \tau d\theta$
- Power is given as $P = \frac{dW}{dt} = \tau\omega$
- When $\tau$ is constant, the integral reduces to $W = \tau(\theta(\theta_r - \theta_i)$

| **Pure translation** | | **Fixed Direction** | |
|---|---|---|---|
| Position | $x$ | Angular position | $\theta$ |
| Velocity | $v = dx/dt$ | Angular velocity $\omega = d\theta/dt$ | |
| Acceleration | $a = dv/dt$ | Angular acceleration | $\alpha = d\omega/dt$ |
| Mass | m | Rotational Inertia | $I$ |
| Newton's second law | $F_net = ma$ | Newton's second law | $\tau_{net} = I\alpha$ |
| Work | $W = \int F dx$ | work $W = \int \tau d\theta$ | |
| Kinetic energy | $K = \frac{12}{2} mv^2$ | Kinetic energy | $K = \frac{1}{2} I\omega^2$ |
| Power (constant force) | $P = Fv$ | Power (constant torque) | $P = \tau\omega$ |
| Work-kinetic energy theorem | $W = \Delta K$ | work-kinetic energy theorem | $W = \Delta K$ |

Table 1: Corresponding relations

- The work-energy theorem for rotating bodies is given as $\delta k = k_f - k_i = \frac{1}{2} I\omega_i^2 = W$ where $\omega_i$ and $\omega_f$ are the angular speeds of the body before and after the work is done.
- Below are corresponding relations for translational and rotational motion: