

[2번 해답]

$$y(t) = h(t) * u(t) \xleftrightarrow[\text{Transform}]{\text{Laplace}} Y(s) = H(s)U(s) \text{ 이를 이용}$$

(1) $u(t) = 1(t) \rightsquigarrow U(s) = \frac{1}{s}$

$$r(s) = H(s) \cdot U(s) = \frac{1}{(s+2)s} = \left(\frac{1}{s} - \frac{1}{s+2} \right) \frac{1}{2}$$

$$y(t) = \mathcal{L}^{-1}\{r(s)\} = \frac{1}{2} (1 - e^{-2t}) \cdot 1(t) \quad \therefore y(t) = \frac{1}{2} (1 - e^{-2t}) \cdot 1(t)$$

(2) $u(t) = t \cdot 1(t)$

$$\Rightarrow t \cdot 1(t) \xleftrightarrow{\mathcal{L}} -\frac{d}{ds}\left(\frac{1}{s}\right) = \frac{1}{s^2} = U(s)$$

$$\Rightarrow Y(s) = H(s) \cdot U(s) = \frac{1}{s+2} \cdot \frac{1}{s^2} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+2} \quad (As+B)(s+2)$$

$$= \frac{1}{s^2(s+2)} \left\{ \underbrace{As^2 + (2A+B)s + 2B + Cs^2}_=1 \right\}$$

$$\Rightarrow A+C=0, 2A+B=0, 2B=1 \quad B=\frac{1}{2}, A=-\frac{1}{4}, C=\frac{1}{4}$$

$$\Rightarrow r(s) = -\frac{1}{4} \frac{1}{s^2} + \frac{1}{2} \frac{1}{s} + \frac{1}{4} \frac{1}{s+2}$$

$$\therefore y(t) = \left(-\frac{1}{4}t + \frac{1}{2} + \frac{1}{4}e^{-2t} \right) 1(t)$$

(3) $u(t) = \begin{cases} t & 0 \leq t < 1 \\ 1 & t \geq 1 \end{cases}$

$$\Rightarrow u(t) = t \cdot 1(t) - (t-1)1(t-1)$$

linear {
(1) $\alpha u_1 \mapsto \alpha y_1$
(2) $\alpha u_1 + \beta u_2 \mapsto \alpha y_1 + \beta y_2$
time-invariant (3) $u(t-\tau) \mapsto y(t-\tau)$

$$\Rightarrow \text{LTI system 이므로, (2)의 } y(t) \text{를 } y_2(t) \text{라 했을 때, } y(t) = y_2(t) - y_2(t-1)$$

$$\therefore y(t) = \left(\frac{1}{2} - \frac{1}{4}t + \frac{1}{4}e^{-2t} \right) 1(t) - \left(\frac{1}{2} - \frac{1}{4}(t-1) + \frac{1}{4}e^{-2(t-1)} \right) 1(t-1)$$