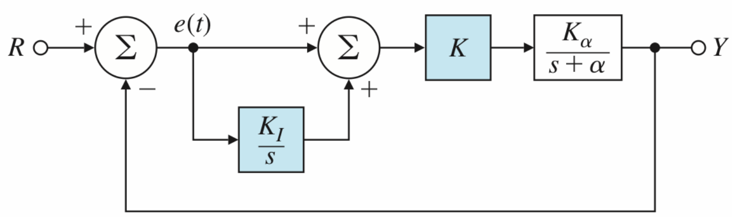
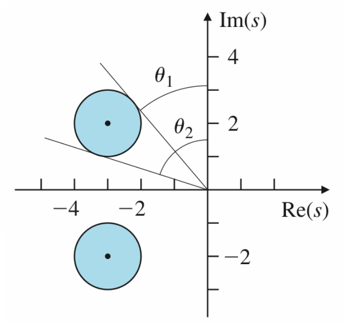
3.31 Suppose you are to design a unity feedback controller for a first-order plant depicted in Fig. 3.56. (As you will learn in Chapter 4, the configuration shown is referred to as a proportional–integral controller.) You are to design the controller so that the closed-loop poles lie within the shaded regions shown in Fig. 3.57.

(a) What values of ω_n and ζ correspond to the shaded regions in Fig. 3.57? (A simple estimate from the figure is sufficient.)

(b) Let $K_\alpha = \alpha = 2$. Find values for K and K_I so that the poles of the closed-loop system lie within the shaded regions.





(a)

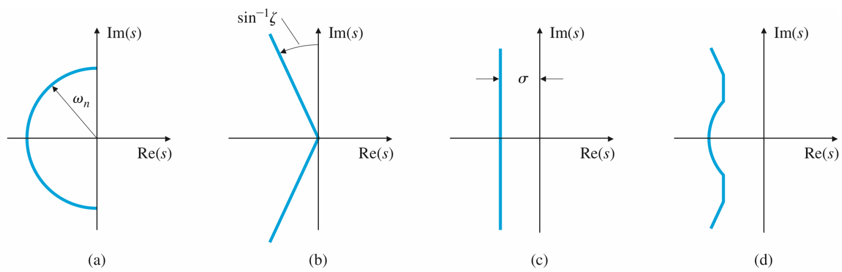


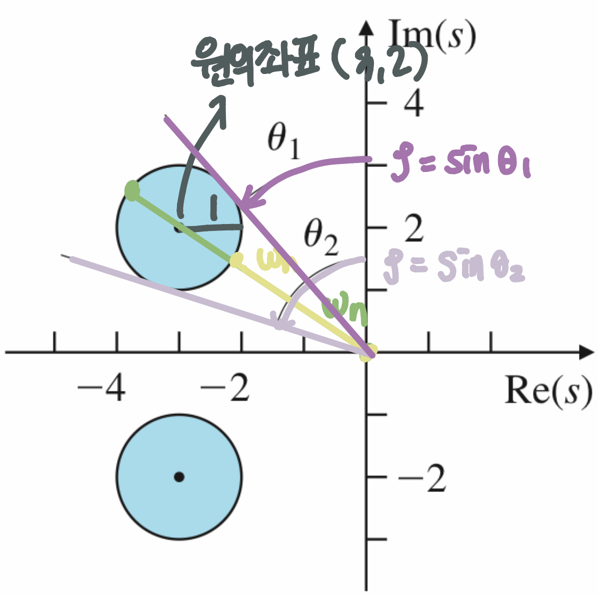
Figure 3.25
Graphs of regions in the *s*-plane delineated by certain transient requirements: (a) rise time; (b) overshoot; (c) settling time; (d) composite of all three requirements

$t_r \simeq \frac{1.8}{\omega_n}$

$\zeta = \frac{\sigma}{\omega_n}$

$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$

$t_s = \frac{4.6}{\sigma}$



① ω_n이 가장클때

$\omega_n = \sqrt{3^2+2^2} + 1 = 4.606$

ω_n이 가장작을때

$\omega_n = \sqrt{3^2+2^2} - 1 = 2.606$

$\therefore\ 2.606 < \omega_n < 4.606$

② ζ가 가장클 때

$\theta_2 \approx 70° \rightarrow \zeta = \sin 70° = 0.940$

③ ζ가 가장 작을 때

$\theta_1 \approx 34° \rightarrow \zeta = \sin 34° = 0.559$

$0.559 < \zeta < 0.940$

(b) system의 w_n과 ζ를 알려면 R에 대한 Y의 T.F를 구해야함.

T.F $T(s) = \dfrac{GH}{1+GH}$ $\quad GH = (1+\frac{k_I}{s})(k)(\frac{2}{s+2})$

$\therefore$ C.E: $1+GH=0$

$$1 + (1+\frac{k_I}{s})(k)(\frac{2}{s+2}) = 0$$

$$s(s+2) + (s+k_I)(k)(2) = 0$$

$$s^2 + 2s + 2ks + 2kk_I = s^2 + (2+2k)s + 2kk_I = 0$$

2차 system의 기본 C.E 형태와 계수비교!

$$s^2 + 2\zeta w_n s + w_n^2 = s^2 + 2(1+k)s + 2kk_I$$

$$w_n^2 = 2kk_I \rightarrow w_n = \sqrt{2kk_I}$$

$$\zeta w_n = (1+k) \rightarrow \zeta = (1+k)/w_n = (1+k)/\sqrt{2kk_I}$$

$\therefore$ $\quad 2.606 < \sqrt{2kk_I} < 4.606$

$\quad 0.559 < (1+k)/\sqrt{2kk_I} < 0.940$