

# EXPERIMENTAL EVALUATION OF FEEDFORWARD AND COMPUTED TORQUE CONTROL

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**Abstract.** Trajectory tracking errors resulting from the application of various controllers have been experimentally determined on the MIT Serial Link Direct Drive Arm. The controllers range from simple analog PD control applied independently at each joint to feedforward and computed torque methods incorporating full dynamics. It was found that trajectory tracking errors decreased as more dynamic compensation terms were incorporated. There was no significant difference in trajectory tracking performance between the feedforward controller using independent digital servos and the full computed torque controller.

## 1 Introduction

Despite voluminous publications on the control of robot arms, there have been few experimental evaluations of the performance of proposed controllers. A major reason for this is the lack of a suitable manipulator for testing that fits these publications' modeling assumptions. Commercial robots are characterized by high gear ratios, substantial joint friction, and slow movement. As a result, their dynamics are approximated well by single-joint dynamics (Goor, 1985; Good, Sweet, and Strobel, 1985). Moreover, hardly any commercial robots allow the control of joint torque, which is required in many of the proposed controllers.

Direct drive arms are increasingly overcoming some of the performance limitations of highly geared robots (Asada and Youcef-Toumi, 1984; Curran and Mayer, 1985; Kuwahara et al., 1985). The manipulator dynamics are made more ideal by the reduction of joint friction and backlash effects, and the control of joint torques becomes more feasible. Hence direct drive arms have the potential for serving as good experimental devices for testing advanced arm control strategies. When gearing is eliminated, however, the full nonlinear dynamic interactions between moving links are manifested.

This paper reports on two sets of experiments with the MIT Serial Link Direct Drive Arm (SLDDA) involving a subset of proposed control strategies. The first set of experiments is based on a hybrid control system. There is an independent analog servo for each joint with the position and velocity references, and feedforward commands generated by a microprocessor. Since most commercial arms are controlled by simple independent PID controller for each joint, an independent PD controller was tested on this arm to provide a baseline for comparison. The PD controller was augmented by feeding forward first gravity compensation and then the complete rigid body dynamics to ascertain any trajectory following improvements attained by taking the nonlinear dynamics more fully into account. The second set of experiments shows the preliminary results of digital servo implementation, using one Motorola 68000 based microprocessor to control all the joints of the SLDDA. The on-line computed

torque approach is compared to the PD and to the feedforward approaches using the digital servo.

The accuracy of the manipulator dynamic model impinges on the performance of feedforward and computed torque control. Since friction is negligible for direct drive arms, and presuming that one has good control of joint torques, the issue of accuracy reduces to how well the inertial parameters of the rigid links are known. In our previous work, we developed an algorithm for estimating these inertial parameters for any multi-link robot as a result of movement, and applied it to the SLDDA (An, Atkeson, and Hollerbach, 1985). The present paper presents results of utilizing the estimated model to control the robot by both off-line (feedforward) and on-line (computed torque) computation of the joint torques.

## 1.1 Control Algorithms

The full dynamics of an  $n$  degree-of-freedom manipulator are described by

$$\mathbf{n} = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) \quad (1)$$

where  $\mathbf{n}$  is the vector of joint torques (for rotational joints),  $\mathbf{q}$  is the vector of joint angles,  $\mathbf{J}$  is the inertia matrix,  $\mathbf{V}$  is the vector of coriolis and centripetal terms,  $\mathbf{G}$  is the gravity vector, and  $\mathbf{F}$  is a vector of friction terms. The simplest and most common form of robot control is independent joint PD control, described by

$$\mathbf{n} = \mathbf{K}_v(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q}) \quad (2)$$

where  $\dot{\mathbf{q}}_d$  and  $\mathbf{q}_d$  are the desired joint velocities and positions, and  $\mathbf{K}_p$  and  $\mathbf{K}_v$  are  $n \times n$  diagonal matrices of position and velocity gains.

Feedforward control augments the basic PD controller by providing a set of nominal torques  $\mathbf{n}_{ff}$ :

$$\mathbf{n}_{ff}(\mathbf{q}_d, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d) = \hat{\mathbf{J}}(\mathbf{q}_d)\ddot{\mathbf{q}}_d + \hat{\mathbf{V}}(\mathbf{q}_d, \dot{\mathbf{q}}_d) + \hat{\mathbf{G}}(\mathbf{q}_d) + \hat{\mathbf{F}}(\mathbf{q}_d, \dot{\mathbf{q}}_d) \quad (3)$$

where the hat (^) refers to the modelled values. When this equation is combined with (2), the feedforward controller results:

$$\mathbf{n} = \mathbf{n}_{ff}(\mathbf{q}_d, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d) + \mathbf{K}_v(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q}) \quad (4)$$

The feedforward term  $\mathbf{n}_{ff}$  can be thought of as a set of nominal torques which linearize the dynamics (1) about the operating points  $\mathbf{q}_d$ ,  $\dot{\mathbf{q}}_d$ , and  $\ddot{\mathbf{q}}_d$ . Therefore, it is reasonable to add the linear feedback terms  $\mathbf{K}_v(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q})$  as the control for the linearized system. These feedforward terms can be computed off-line, since they are functions of the parameters of the desired trajectory only.

On the other hand, the computed torque controller computes the dynamics on-line, using the sampled joint position and velocity data. The control equation is:

$$\mathbf{n}_{ct}(\mathbf{q}_d, \mathbf{q}, \dot{\mathbf{q}}_d, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_d) = \hat{\mathbf{J}}(\mathbf{q})\ddot{\mathbf{q}}^* + \hat{\mathbf{V}}(\mathbf{q}, \dot{\mathbf{q}}) + \hat{\mathbf{G}}(\mathbf{q}) + \hat{\mathbf{F}}(\mathbf{q}, \dot{\mathbf{q}}) \quad (5)$$

where  $\ddot{\mathbf{q}}^*$  is given by,

$$\ddot{\mathbf{q}}^* = \ddot{\mathbf{q}}_d + \mathbf{K}_v(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q}). \quad (6)$$

If the robot model is exact, then each link of the robot is decoupled, and the trajectory error goes to zero. Gilbert and Ha (1984) have shown that the computed torque control method is robust to small modelling errors.

Previously, Liegeois, Fournier, and Aldon (1980) suggested feedforward control as an alternative to the on-line computation requirements of computed torque control, although they did not present any experimental results. Golla, Garg, and Hughes (1981) discussed different linear state-feedback controller using a linearized model of a manipulator. Asada, Kanade, and Takeyama (1983) presented some results of applying a feedforward control to the early version of a direct drive arm at the Robotics Institute of CMU, though for quite slow movements and for inertial parameters derived by CAD modeling. The computed torque method has been considered by several other investigators (Paul, 1972; Markiewicz, 1973; Bejczy, 1974; Luh, Walker, and Paul, 1980). Although simulation results have been presented, there has been no published report on the actual implementation of this controller, mainly due to the lack of an appropriate manipulator or on-line computational facility.

In this paper, we first use the feedforward controller to evaluate the accuracy of our estimates of the link inertial parameters, and to compare its performance against several other simpler control methods for high speed movements. Then, we present some preliminary results on the implementation of a computed torque controller, again using the estimated inertial parameters of the links.

## 1.2 Estimates of inertial parameters

The inertial parameters in the feedforward computation for the SLDDA were estimated by an algorithm developed previously (An, Atkeson, and Hollerbach, 1985). It was shown that the unknown inertial parameters of each link (mass, center of mass, and moments of inertia) appear linearly in the rigid body dynamics of a manipulator. A least squares algorithm was used to compute the estimates of these parameters.

The accuracy of the inertial parameters was verified initially by comparing the measured joint torques to the torques computed from the estimated parameters. This comparison, together with the torques computed from parameters derived by CAD modelling, is shown in Figure 1 for a 1.3s trajectory of all the joints moving 250 degrees. The results were actually superior for the dynamically estimated parameters than for the CAD-modelled parameters. A more practical verification of the estimated parameters is in generating feedforward torques as part of a control algorithm. The results of such experiments are presented in the next section.

## 2 Robot Controller Experiments

In this section, performances of several different controllers for

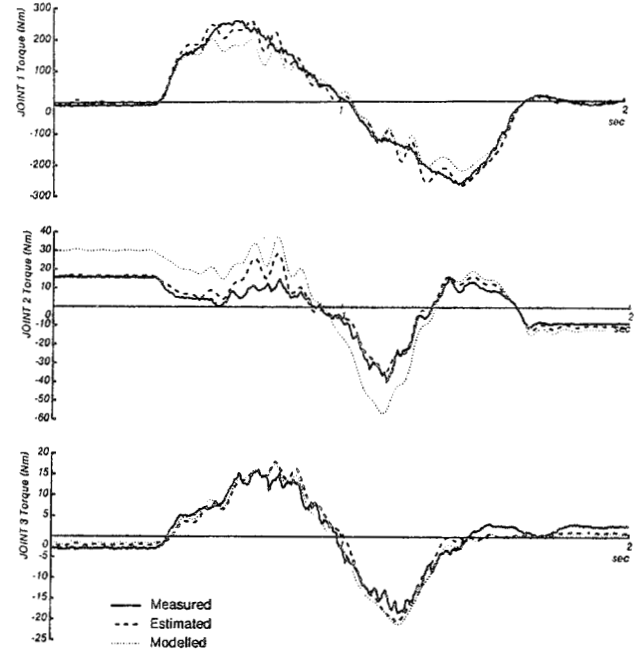


Figure 1: The measured, CAD-modelled, and estimated joint torques.

full motion of the SLDDA are evaluated using the hybrid controller. The reference positions and velocities, and the feedforward torques are generated by a microprocessor and input to three independent joint analog servos. We present evaluations of the following five control methods used for high speed movements of all three joints of the manipulator:

1. PD controller with position reference only:

$$\mathbf{n} = -\mathbf{K}_v\dot{\mathbf{q}} + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q})$$

2. PD controller with position reference and feedforward of gravity torques:

$$\mathbf{n} = \hat{\mathbf{G}}(\mathbf{q}_d) - \mathbf{K}_v\dot{\mathbf{q}} + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q})$$

3. PD controller with position and velocity references:

$$\mathbf{n} = \mathbf{K}_v(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q})$$

4. PD controller with position and velocity references plus feedforward of gravity torques:

$$\mathbf{n} = \hat{\mathbf{G}}(\mathbf{q}_d) + \mathbf{K}_v(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q})$$

5. PD controller with position and velocity references plus feedforward of full dynamics:

$$\mathbf{n} = \hat{\mathbf{J}}(\mathbf{q}_d)\ddot{\mathbf{q}}_d + \hat{\mathbf{V}}(\mathbf{q}_d, \dot{\mathbf{q}}_d) + \hat{\mathbf{G}}(\mathbf{q}_d) + \mathbf{K}_v(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q})$$

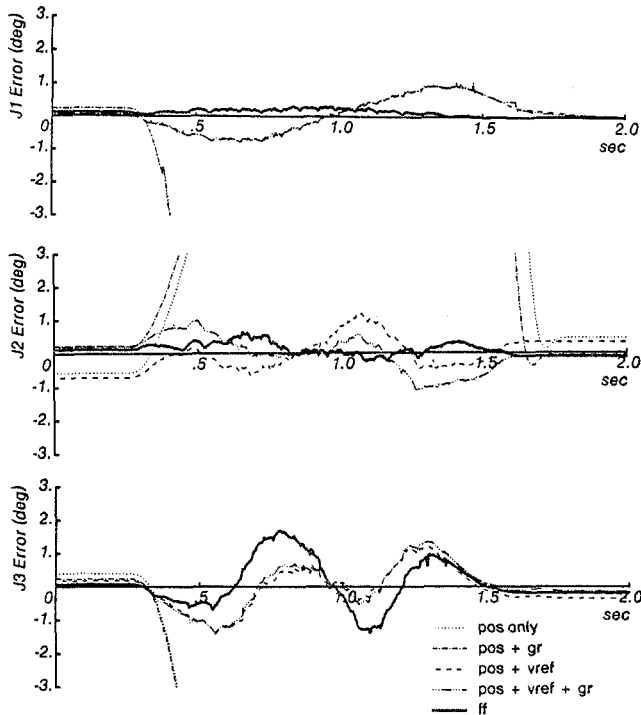


Figure 2: Trajectory errors of the 5 controllers for full 1.3s motion.

In these experiments, friction was neglected. The nominal position and velocity gains were adjusted experimentally to achieve high stiffness and overdamped characteristics without the feedforward terms.

A fifth order polynomial in joint space was used to generate the reference trajectory. The joints moved from  $(80^\circ, 269.1^\circ, -30^\circ)$  to  $(330^\circ, 19.1^\circ, 220^\circ)$  in 1.3s, with peak velocities of  $360 \text{ deg/sec}$  and the peak accelerations of  $854 \text{ deg/sec}^2$  for each joint. For control methods (2), (4), and (5), the estimates of the link inertial parameters given by An, Atkeson, and Hollerbach (1985) were used to compute the feedforward torques.

The trajectory errors for the above 5 controllers are shown in Figure 2. The errors for the first controller are very large and are out of the graph range. Adding a gravity feedforward term does not help very much, and the trajectory errors for Controller 2 are also very large. This was expected since gravity feedforward is a static correction to Controller 1, and the dynamic effects dominate the response for high speed movements. Modifying the first controller by adding a velocity reference signal improved the response greatly. As with Controller 2, adding a gravity feedforward term did not reduce the trajectory errors very much, and influenced mainly the steady state errors for joints 2 and 3.

The full feedforward controller reduced the trajectory errors significantly for joints 1 and 2, with peak errors of only  $0.33^\circ$  and  $0.64^\circ$ , respectively. For joint 3, the feedforward torques did not help because of the light inertia and the dominance of unmodelled dynamics in the motor and in bearing friction. The high feedback gains make this joint somewhat robust to these unmodelled dynamics; yet, the trajectory errors could not be reduced below  $1.4^\circ$  with the feedforward torques based on the ideal rigid body model of the link.

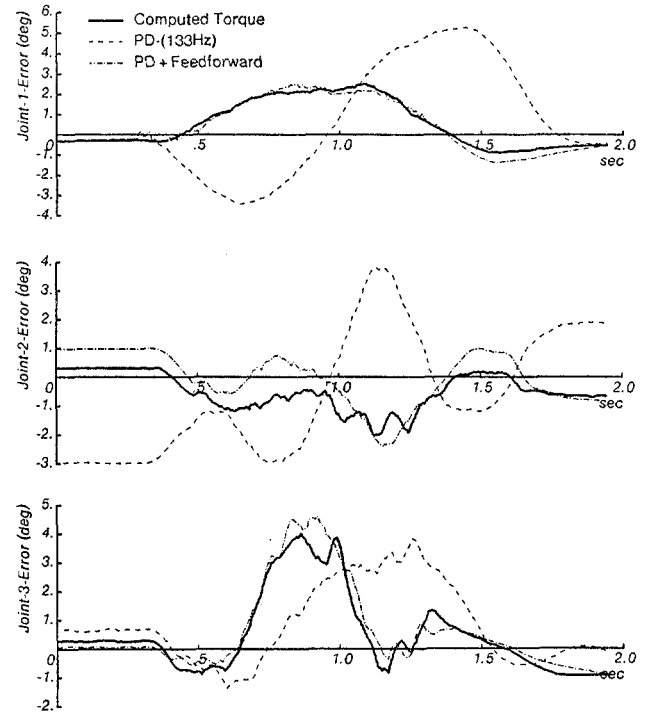


Figure 3: Trajectory errors of the three digital controllers for full 1.3s motion.

## 2.1 Computed Torque Controller Experiment

In this section, some preliminary results are presented for the computed torque method implemented on the SLDDA. In this implementation, the analog servos are disabled, and the feedback computation is done digitally by one Motorola 68000 based microprocessor using scaled fixed-point arithmetic. Written in the C language, the controller, including the full computation of the robot dynamics, runs at a 133 Hz sampling frequency. Although further improvements in computation time are possible, this speed was adequate in demonstrating the efficacy of dynamic compensation. The details of this implementation are discussed in (Griffiths, 1986).

A similar fifth order polynomial trajectory as in the previous section was used for this experiment. Figure 3 shows the trajectory errors for three controllers: the digital PD controller, the feedforward controller using a digital servo, and the computed torque controller. The computed torque and the feedforward controllers both show a significant reduction in tracking errors for joints 1 and 2 compared with the PD control, with no clear distinction between feedforward and computed torque. The tracking errors for joint 1 range from  $4.4^\circ$  to  $2.2^\circ$  and for joint 2 go from  $3.5^\circ$  to  $2.0^\circ$  with the addition of dynamic component. As before, the trajectory errors for joint 3 were not reduced by the computed torque or the feedforward controller. Again, this seems to indicate that our model for the third link may not be very good.

The trajectory tracking performance of the computed torque controller is not as good as that of the analog feedforward controller of the previous section. The main reason for this is the slow sampling frequency (133 Hz) of the digital controller, as compared to the 1 KHz sampling frequency at which the reference inputs were given to the analog servos. Improvements in the computation time should also improve the tracking performance of the computed torque controller.

### 3 Conclusions

We have presented experimental results of using an estimated dynamic model of the manipulator for dynamic compensation via feedforward and computed torque control methods. The results indicate that dynamic compensation can improve trajectory accuracy significantly and that the estimated rigid body model of the manipulator is quite accurate and adequate for control purposes for joints 1 and 2. The unmodelled dynamics of the light third link, including the motor dynamics and friction, are dominant and yield larger trajectory errors than at the other two joints. Therefore, for joint 3, it may be necessary to use a more complete model to improve trajectory following.

The results of the digital implementation of the feedforward and computed torque controllers were not as good as the hybrid feedforward controller. This indicates that if a robot was being used solely for free space movements without significant variation of its loads, then a hybrid controller using an independent analog servo for each joint may be quite adequate. A hybrid controller, however, is not flexible, and cannot handle varying loads or interactions with the environment. Future experiments with the MIT Serial Link Direct Drive Arm will concentrate on improving the computation time for the digital control system and on issues of force control.

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