CTRL 연구 참여 2주차

Stability of the System

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Contents

Pole & Zero

- Rational transfer function

$$H(s) = \frac{b_1 s^m + b_2 s^{m-1} + \cdots + b_{m+1}}{s^n + a_1 s^{n-1} + \cdots + a_n} = \frac{N(s)}{D(s)}$$

$$H(s) = K \frac{\prod_{i=1}^{m} (s - z_i)}{\prod_{i=1}^{n} (s - z_i)}$$

K: transfer function gain

- Pole: $H(s)|_{s=p_i} = \infty$

-related to the system's stability

- Zero: $H(s)|_{s=z_i} = 0$

Pole & Zero

- Effect of pole locations

- The **negative real part of the pole** determines **the decay rate** of the exponential envelope

complex poles: $s = -\sigma \pm j\omega_d$

transfer function: $H(s) = \dfrac{\omega_n^2}{(s+\sigma)^2 + \omega_d^2}$

$\zeta = \dfrac{\sigma}{\omega_n}$: damping ratio

ω_n: undamped natural frequency

ω_d: damped natural frequency

impulse reponse : $h(t) = \dfrac{\omega_n}{\sqrt{1-\zeta^2}} e^{\boxed{-\sigma t}} (sin\omega_d t) 1(t), (\sigma = \zeta\omega_n)$

- Stability of complex poles

$\sigma < 0$: unstable (real part is positive)

$\sigma = 0$: neutrally stable

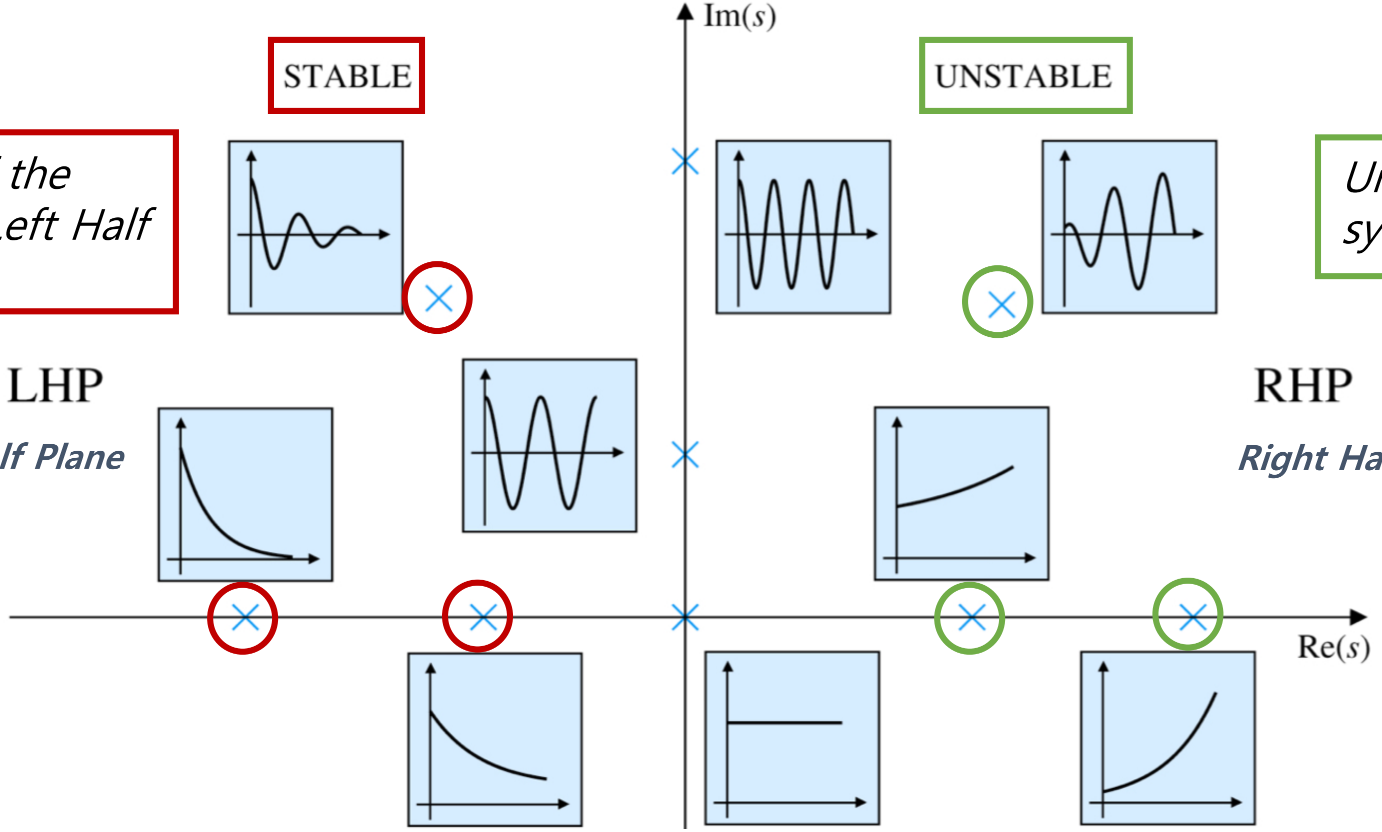
$\sigma > 0$: stable (real part is negative)

Pole & Zero

- Summary of pole location

- Pole locations determine the way of it behaves for impulse response of the system

Stable if all the poles of the system in OLHP (Open Left Half Plane)



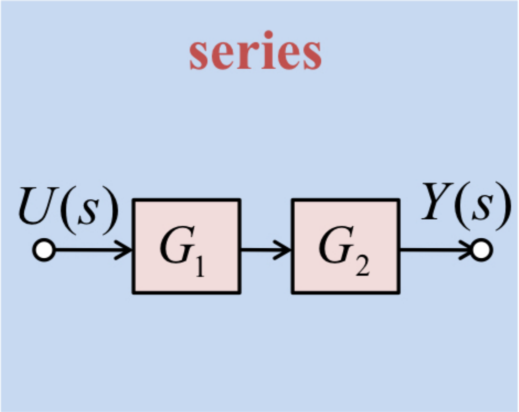
Unstable if any pole of the system in RHP, or CRHP

Special case of unstable system: oscillatory system

Transfer function corresponding to the block diagram

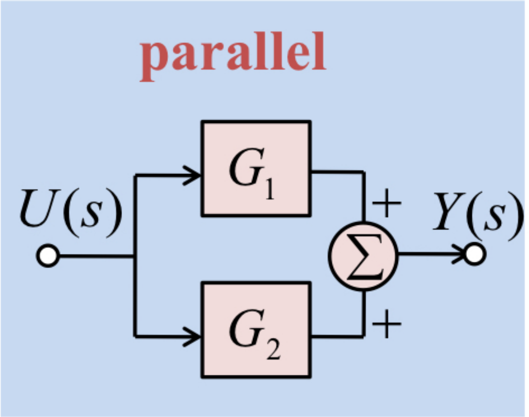
- Block diagram
- It can be used to illustrate the relationship between the components of given system

- Series



$$\frac{Y(s)}{U(s)} = G_2(s)G_1(s)$$

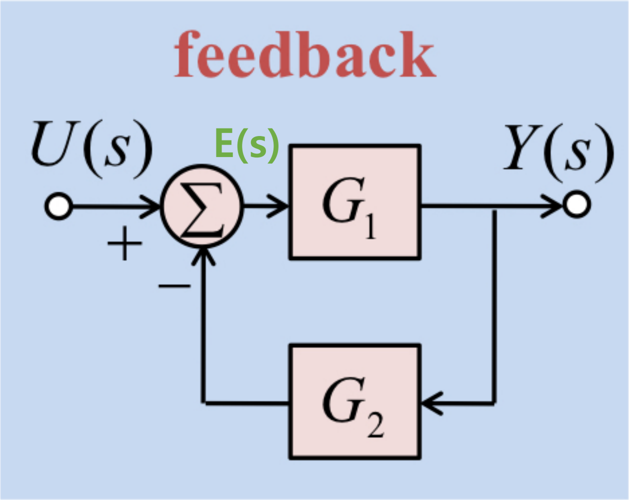
- Parallel



$$\frac{Y(s)}{U(s)} = G_1(s) + G_2(s)$$

Transfer function corresponding to the block diagram

- Feedback



$$\frac{Y(s)}{U(s)} = \frac{G_1(s)}{1 + G_1(s)G_2(s)}$$

$$Y(s) = G_1(s)E(s)$$
$$E(s) = U(s) - G_2(s)Y(s)$$

$$\rightarrow Y(s) = G_1(s)U(s) - G_2(s)G_1(s)Y(s)$$
$$\rightarrow (1 + G_1(s)G_2(s))Y(s) = G_1(s)U(s)$$

Transfer function corresponding to the block diagram

- Relation between state-space equation and transfer function

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

$$\Updownarrow$$

$$G(s) = \frac{b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} + d$$

- Poles of G(s) = Eigenvalues of A

$$G(s) = C(sI - A)^{-1} + D$$

eigenvalues of A: $|\lambda I - A| = 0$

Root locus

- Motivation

- Poles of the feedback systems are closely related to the responses of the feedback systems

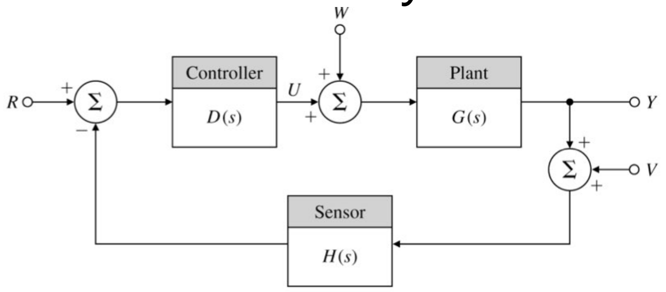


Figure 5.1 Basic closed loop block diagram

- Concepts

- closed-loop transfer function: $\frac{Y(s)}{R(s)} = T(s) = \frac{D(s)G(s)}{\boxed{1 + D(s)G(s)H(s)}}$

- Characteristic equation: $\boxed{1 + D(s)G(s)H(s) = 0} \rightarrow a(s) + Kb(s) = 0$ (K := parameter of interest)

$\rightarrow 1 + KL(s) = 0(L(s) = \frac{b(s)}{a(s)})$

- Root locus: plot the locus of all possible roots of $1 + KL(s)$ as K varies from 0 to ∞

$\rightarrow$ find the pole's location!

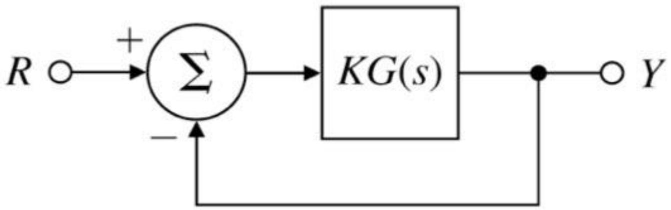
Root locus

- A graphical representation of closed loop poles as a system parameter varied
- Select a particular value of K that will meet the specifications for static and dynamic response

Nyquist plot

- Nyquist stability

- Relationship between the stability of closed loop system and the frequency response of the plant



- For all points s on the root locus

$$1 + KG(s) = 0 \rightarrow |KG(s)| = 1 \text{ and } \angle G(s) = 180°$$

▪ neutral stability condition (at the point of neutral stability)

$$|KG(j\omega_0)| = 1 \text{ and } \angle G(j\omega_0) = 180°$$

→ relates the open-loop frequency response to the number of RHP poles of the closed-loop system

Nyquist plot

- There are some systems where $|KG(j\omega)|$ crosses magnitude=1 more than once

→ use Nyquist stability criterion

- a contour map of KG(s) will encircle -1 **N=Z-P** times (by the argument principle)

where Z: the number of RHP zeros of 1+KG(s)

P: the number of RHP poles of KG(s)

$$1 + KG(s) = 1 + K\frac{b(s)}{a(s)} = \frac{a(s) + Kb(s)}{a(s)}$$

$$\begin{cases} \text{Poles of } G(s) = \text{poles of } 1+KG(s) = \text{poles of } KG(s) \\ \text{Closed loop poles} = \text{zeros of } 1+KG(s) \end{cases}$$

$$\begin{cases} \text{Poles of } G(s) \text{ in RHP} = \text{poles of } (1+KG(s)) \text{ in RHP} = \text{poles of } (1+KG(s)) \text{ in RHP} \\ \text{Closed loop poles in RHP} = \text{zeros of } (1+KG(s)) \text{ in RHP} \end{cases}$$

Nyquist plot

- Nyquist stability criterion $\dfrac{Y(s)}{R(s)} = T(s) = \dfrac{KG(s)}{1+KG(s)}$, we have $Z = N + P$

want to know!

Z: the number of RHP poles of closed-loop system

N: the number of clockwise encirclement of KG(s) about -1

P: the number of RHP poles of open-loop system

→ stability of the closed-loop system can be determined in terms of the number of RHP poles of the open-loop system (KG(s)) and the Nyquist plot (the number of clockwise encirclement of KG(s) about -1)

Nyquist plot

- example $$KG(s) : L(s) = \frac{10(s+1)(s+2)}{(s-3)(s-4)}(K=1)$$

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>> H = tf([10 30 20],[1 -7 12]);
nyquist(H)
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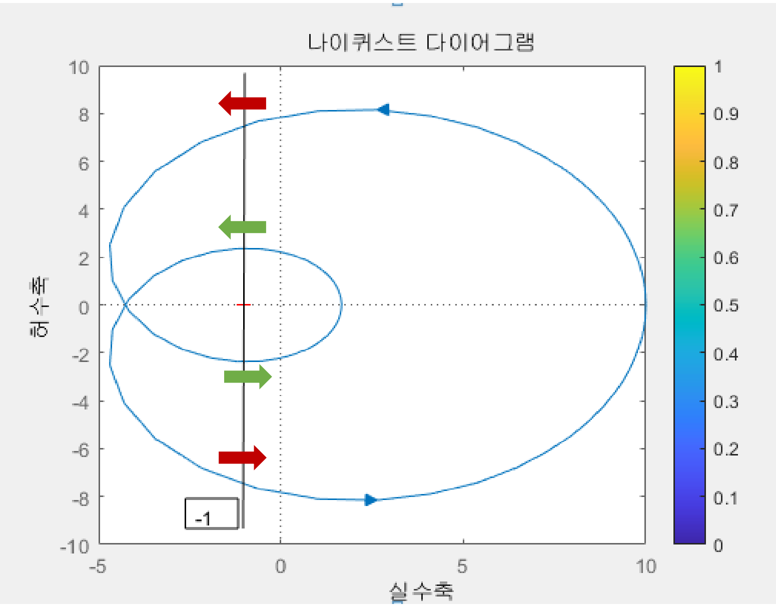

$$Z = N - P \quad N = -2 \quad \text{and} \quad P = 2 \quad \text{so} \quad Z = 0$$
$$\rightarrow \text{stable system}$$

- We can also determine system's gain K using gain margin

Q&A