The University of Windsor

ELEC3240: Control Systems I

Summer 2020

Lab 1

Simulating RLC Circuit using MATLAB/Simulink

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Introduction

Throughout this lab, we will perform a handful of calculations and plot the current versus time relationships for two RLC circuit's zero-state response of varying capacitor parameters. In doing so, we highlight the relationship between how current changes with an increase in capacitance. With this knowledge, we can predict how an RLC circuit's current might respond if there are no capacitors involved. This lab's main purpose is to facilitate a better understanding for RLC circuits and how we can solve and graph them using MATLAB/Simulink. We will show how to calculate the transfer function for an RLC circuit and how adjusting the capacitance parameter changes the current response. To conduct such an experiment, we are given an arbitrary input voltage value in the time domain which will allow us to isolate our transfer function for current. To switch from the frequency domain, back to the time domain, we will use MATLAB/Simulink functions such as "ilaplace()".

Procedure

Hand-Calculation Procedure:

1) Start by applying Kirchhoff's voltage law to the loop of the RLC circuit
2) After finding the voltage equation, take the derivative with respect to time, d/dt
   - This will allow us to easily move into the frequency domain
3) Transform the differential equation from the time to frequency domain
   - Apply the derivative identities for Laplace: d/dt = s, $d^2/dt^2 = s^2$
4) Factor out the I(s) terms and rearrange to find the transfer function I(s)/V(s)
5) Apply the Laplace transform for V(t) = 1 V and sub in the value for V(s)
   - This allows for the isolation of I(s)

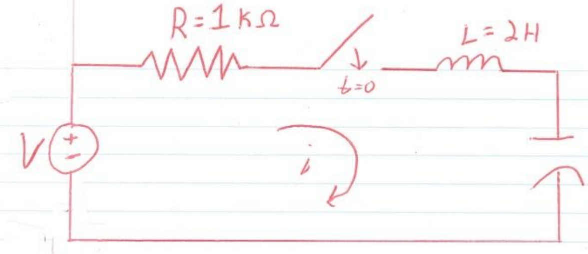
MATLAB/Simulink Procedure:

1) Take the inverse-Laplace of the current function we just derived using "ilaplace()"
2) Set the time domain to a wide range of values so that we can scale it back later
   - t = 0:100:12000 seemed to work best to represent the characteristics
3) Using the "plot()" function, the current versus time graph is achieved

Note: For part 3, Redo the hand-calculations and MATLAB/Simulink procedures for an RLC circuit with the same design as before but double the capacitors (in series).

Summary: Start by first calculating the transfer function for the RLC circuit in terms of I(s)/V(s). After the transfer function is calculated, we take the Laplace for V(t) = 1 and input it into the transfer function we had just calculated to isolate for current I(s). Take the inverse-Laplace of I(s) to get i(t) using MATLAB/Simulink. Lastly, with i(t) calculated, we can graph the current-time response of the RLC circuit for when there are one or two capacitors.

Calculations



1) At $t=0$

KVL: $V = V_R + V_L + V_C$

$$V_L(t) = L\frac{di(t)}{dt} \qquad V_R(t) = R\, i(t)$$

$$V_C(t) = \frac{1}{C}\int_0^t i\,dt + \cancel{V_0}^{\,0}$$

$$V(t) = L\frac{di(t)}{dt} + R\,i(t) + \frac{1}{C}\int_0^t i(t)\,dt$$

Differentiating $\Rightarrow$

$$\frac{d}{dt}v(t) = L\frac{d^2 i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

2) Laplace Q1 $\rightarrow$

$$S\cdot V(S) = L\,S^2 I(S) + R\,S\,I(S) + \frac{I(S)}{C}$$

$$S\,V(S) = I(S)\left(2S^2 + 1000\,S + 1/2\right)$$

Transfer Function

$$G(S) = \frac{I(S)}{V(S)} = \frac{S}{(2S^2 + 1000S + 1/2)}$$

$$\boxed{G(S) = \frac{2S}{4S^2 + 2000S + 1} \quad \text{or} \quad \frac{S}{2S^2 + 1000S + \frac{1}{2}}}$$

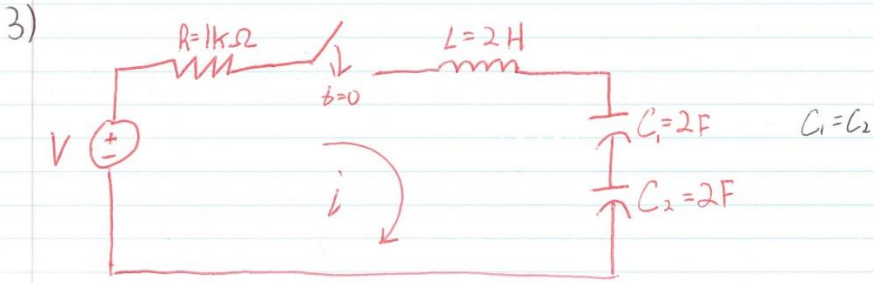
Part2 1) if $V(t) = 1$

$$\mathcal{L}[V(t)] = \mathcal{L}[1] \Rightarrow V(S) = \frac{1}{S} \rightarrow \frac{I(S)}{1/S} = \frac{S}{2S^2 + 1000S + \frac{1}{2}} \Rightarrow I(S) = \frac{1}{2S^2 + 1000S + \frac{1}{2}}$$

Figure 1 RLC calculations using one capacitor

Above, we find 1) The differential equation for an RLC's zero-state response, 2) the transfer function I(s)/V(s), and 2.1) the frequency response of I(s) when V(t) = 1

3)



From KVL: $V = V_R + V_L + V_{C_1} + V_{C_2} = V_R + V_L + 2V_C$

$V_R = i(t)\,R \qquad V_L = L\,\frac{d\,i(t)}{dt} \qquad V_C = \frac{1}{C}\int_0^t i(t)\,dt$

$$v(t) = i(t)R + L\frac{di(t)}{dt} + \frac{2}{C}\int_0^t i(t)\,dt$$

differentiating $\Rightarrow \frac{d\,v(t)}{dt} = 1000\,\frac{di(t)}{dt} + 2\,\frac{d^2 i(t)}{dt^2} + \frac{2}{2}\,i(t).$

Laplace $\rightarrow \; sV(s) = s1000\,I(s) + s^2 2\,I(s) + I(s)$

$I(s)/V(s) = \frac{s}{2s^2 + 1000s + 1} \qquad v(t) = 1 \Rightarrow V(s) = \frac{1}{s}$

$V(s) = \frac{1}{s} \Rightarrow I(s) = \frac{1}{2s^2 + 1000s + 1}$

Figure 2 RLC calculations using two capacitor capacitors

For part 3, we do the same calculations as before except now we double the capacitors voltage value.

Solving for i(t) using MATLAB:

```
%%Author: Emmanuel Mati
%Control Systems 1
%Summer 2020
%Lab 1 RLC Circuit

%Clear everything
close all;
clear all;
clc;

%our symbols
syms t s

%Our time domain
t= 0:100:11000;

%Current functions
I1 = 1/(2*s^2 + 1000*s + 1/2); %One Capactior
I2 = 1/(2*s^2 + 1000*s + 1); % Two Capacitors

%Taking inverse laplace
i1 = ilaplace(I1);
i2 = ilaplace(I2);

%Plotting i(t) with one capacitor
ii1 = subs(i1);
figure
plot(t, ii1)
xlabel('t (seconds)')
ylabel('i(t) Amperes')
title('i(t) with one capacitor')

%Plotting i(t) with two capacitors
ii2 = subs(i2);
figure
plot(t, ii2)
xlabel('t (seconds)')
ylabel('i(t) Amperes')
title('i(t) with two capacitors')

%Displaying current functions
disp('The current equation I(s) in the s-domain with only one capacitor:')
pretty(I1);
disp('The current equation i(t) with only one capacitor:') %Displaying i(t)
pretty(i1);

%Displaying current functions
disp('The current equation I(s) in the s-domain with two capacitors:')
pretty(I2);
disp('The current equation i(t) with two capacitors:') %Displaying i(t)
pretty(i2);
```

```
Command Window
The current equation I(s) in the s-domain with only one capacitor:
        1
-----------------
   2            1
2 s  + 1000 s + -
                2

The current equation i(t) with only one capacitor:
                             / sqrt(249999) t \
sqrt(249999) exp(-250 t) sinh| -------------- |
                             \        2       /
-----------------------------------------------
                    249999

The current equation I(s) in the s-domain with two capacitors:
        1
-----------------
   2
2 s  + 1000 s + 1

The current equation i(t) with two capacitors:
                       / 7 sqrt(2) sqrt(2551) t \
sqrt(2) sqrt(2551) sinh| ---------------------- | exp(-250 t)
                       \            2           /
-------------------------------------------------------------
                           35714

fx >>
```

Figure 3 MATLAB command window showing the I(s) and i(t) functions

In the above code, we calculate the current for two RLC circuits when V(t) = 1 V. The first RLC complex function i1 represents current with one capacitor and i2 represents the current with two capacitors. The function “ilaplace()” was used to take the inverse of the functions and lastly “plot()” was used to graph the functions.

Note: *The MATLAB code is attached to this report for easier access*

Diagrams

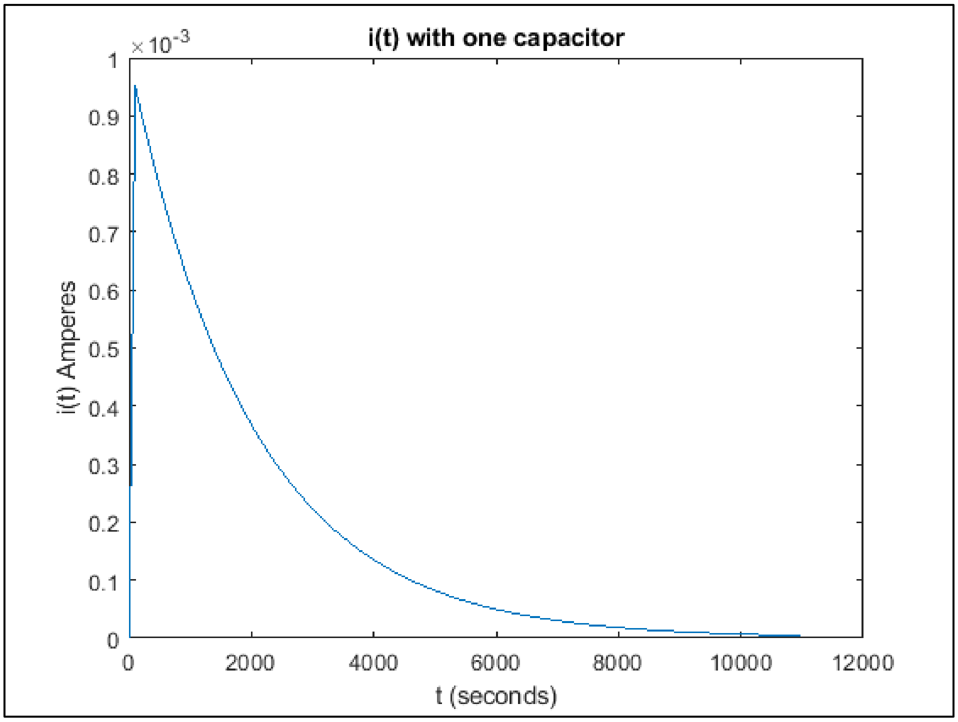


Figure 4 Current Versus time of an RLC with only one capacitor

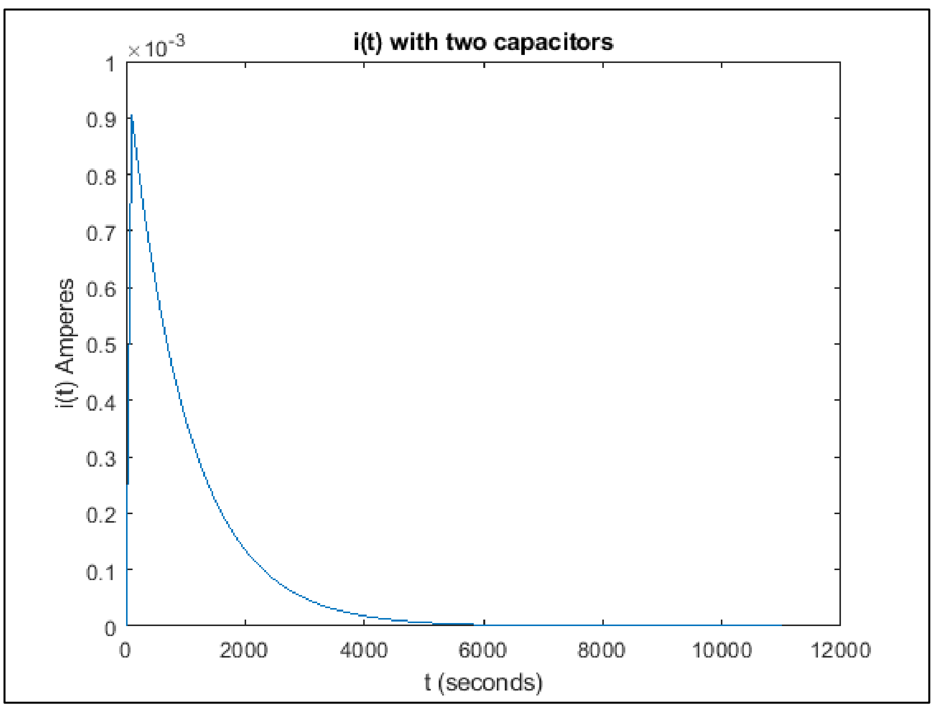


Figure 5 Current Versus time of an RLC with two capacitors

Conclusion

After finding the graphs for i(t) vs t, we see that as the capacitance increased in our RLC circuit, the current reached zero faster. The current reached zero in half the time which means that capacitance is inversely related to response time. The reason the current reached zero is that the capacitor is charging up and eventually stops allowing current to pass through it when it is fully charged. This means that the capacitor eventually reaches the same potential as the input voltage source (1 volt) and stops current from flowing. With the capacitors being removed, we can predict that the current levels will stop dropping and they will eventually reach a stable constant level; easily found using Ohm's law (I = V/R = 1 / 1000 = 1 mA). Initially, we will see a sharp rise in current as can be seen in figures 4 and 5 until eventually the current reaches 1 mA and levels off. Inductors under DC eventually act shorted and thus, its effects on the RL circuit will only be noticed when the initial switch is closed.