The University of Windsor

ELEC3240: Control Systems I

Summer 2020

Lab 2

Concentrates on a Virtual PID controller Design using the Ziegler-Nichols Rule

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Emmanuel Mati

Student ID: mati1

Student Number:104418019

Part 1: Unity Feedback Control System

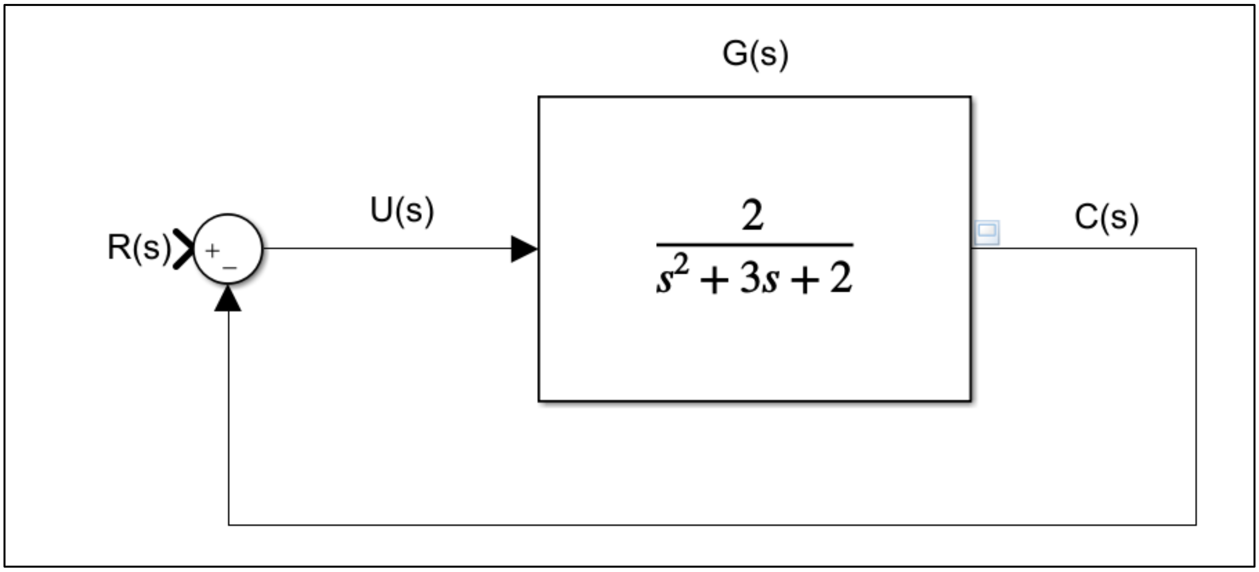

Figure 1 Unity Feedback Control System diagram made with Simulink

Part 2: Calculating L and T

Control Systems Lab 2

2. $G(S) = \frac{C(S)}{U(S)} = \frac{2}{S^2+3S+2}$

Input: Unit Step

$u(t) = 1$

$U(S) = \frac{1}{S}$

$$U(S) \longrightarrow \boxed{\overset{G(S)}{\frac{2}{S^2+3S+2}}} \longrightarrow C(S)$$

$$C(S) = \frac{2}{S^3+3S^2+2S} = \frac{2}{S(S+1)(S+2)}$$

Integration by Part: $\frac{2}{S(S+1)(S+2)} = \frac{A}{S} + \frac{B}{(S+1)} + \frac{C}{(S+2)} \Rightarrow 2 = A(S+1)(S+2) + BS(S+2) + CS(S+1)$

$S=-1 \Rightarrow \boxed{B=-2}$ $\quad S=0 \Rightarrow \boxed{A=1}$ $\quad S=-2 \Rightarrow \boxed{C=1}$

$$C(S) = \frac{1}{S} - \frac{2}{(S+1)} + \frac{1}{(S+2)} \Rightarrow C(t) = L^{-1}(C(S)) = 1 - 2e^{-t} + e^{-2t}$$

$\boxed{C(t) = (e^{-2t} - 2e^{-t} + 1)u(t)}$ $\quad C'(t) = -2e^{-2t} + 2e^{-t}$

$u(t) = 1$ when $t \geq 0$

$u(t) = 0$ when $t < 0$

$C''(t) = 4e^{-2t} - 2e^{-t}$ $\quad$ when $C''(t) = 0 \Rightarrow 2e^{-2t} = e^{-t}$

$-t = \ln(\frac{1}{2}) \Rightarrow t = 0.6931$

Slope of tangent: $C'(0.6931) = 0.5$

Coordinates: $(0.6931, 0.25)$

y coordinate of tangent: $C(0.6931) = 0.25$

$y - y_1 = m(x - x_1)$

$y - 0.25 = 0.5(x - 0.6931)$

$\boxed{y = 0.5x - 0.097}$

Change to be in terms of C & t

$\boxed{C_t(t) = 0.5t - 0.097}$ tangent Line

$C_t(t) = 0.5L - 0.097 = 0 \Rightarrow \boxed{L = 0.1940}$

$C_t(t) = 0.5(T+L) - 0.097 = 1 \Rightarrow \boxed{T = 2.00}$

$C(t) = (e^{-2t} - 2e^{-t} + 1)u(t)$

tangent: $C_t(t) = 0.5t - 0.097$

Figure 2 Calculations for L and T

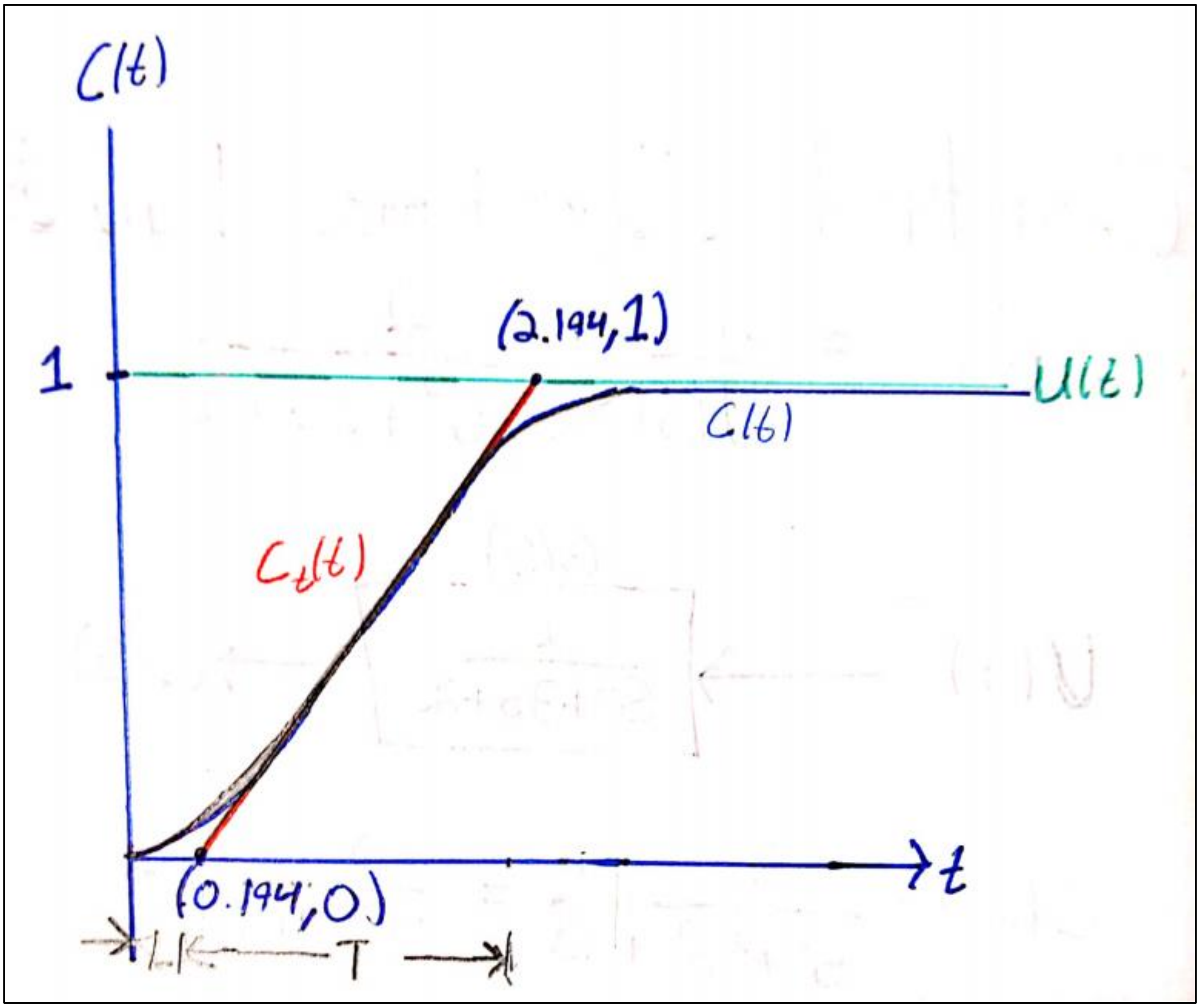

Figure 3 Graph showing the tangent of the inflection

As seen above, L is calculated to be 0.194 and T is equal to 2.00

Part 3: Reference response

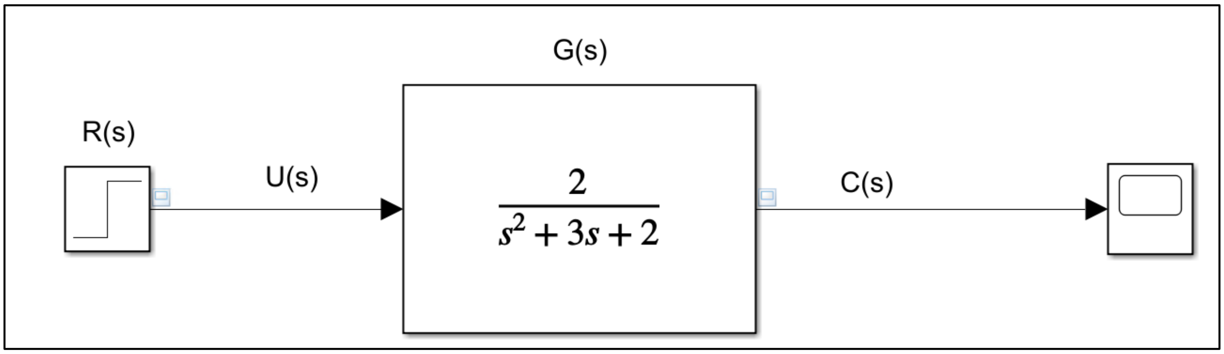

Figure 4 Reference response without control and feedback

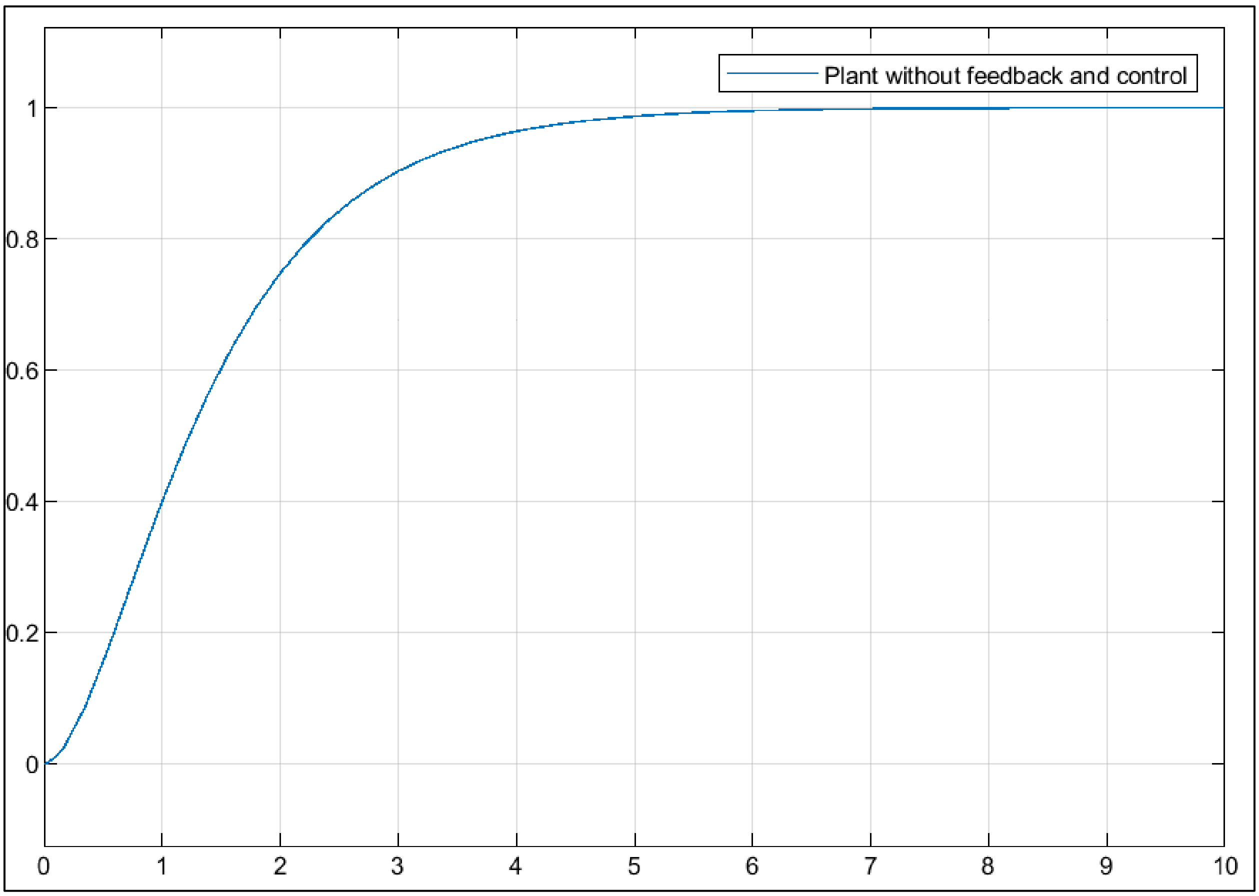

Figure 5 Plant without feedback and control

Part 3-1: P control with closed-loop unit-step Response

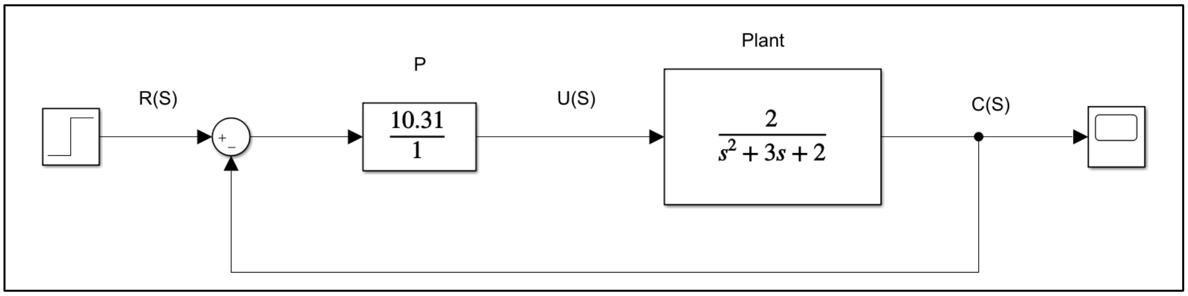

Figure 6 P control with plant and feedback

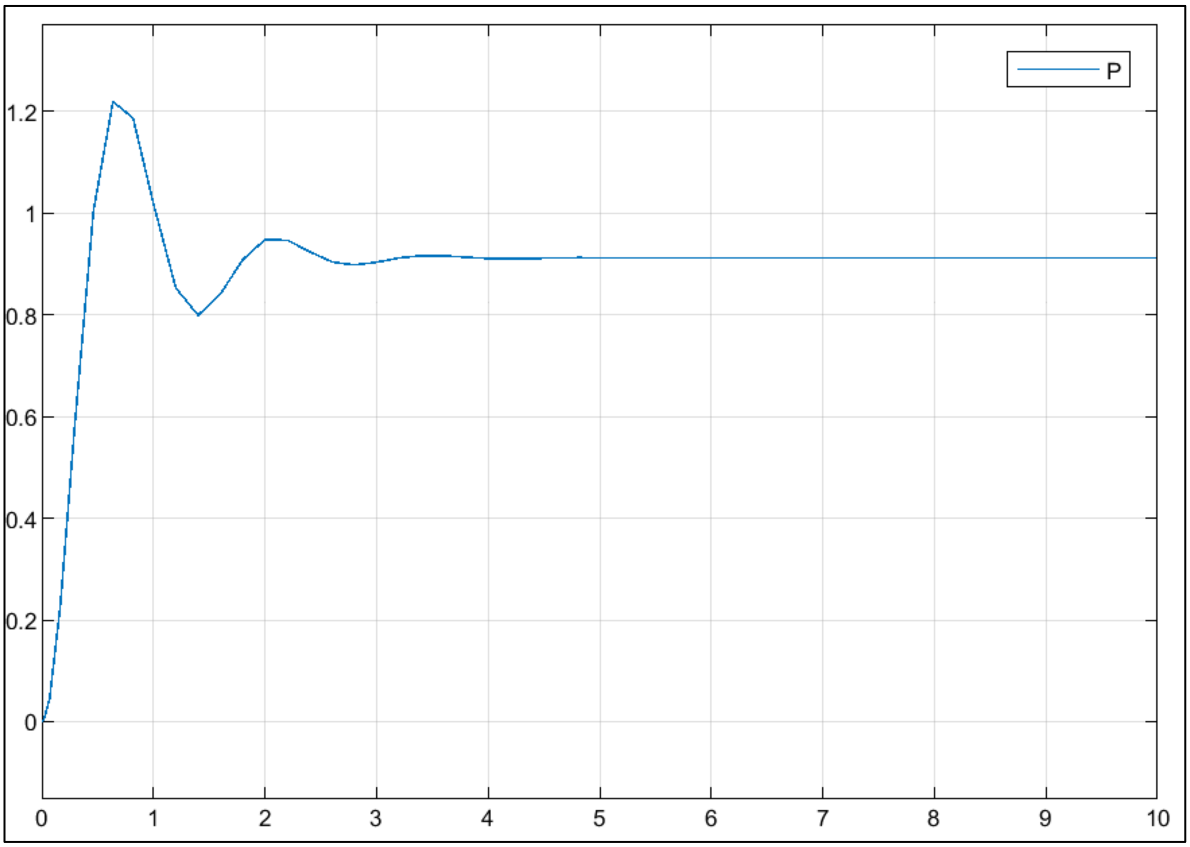

Figure 7 Response of the P control

Evaluating the P control results and comparing it with the reference response shows a couple of key changes. Tracking accuracy does not match our reference signal until after we reach 4 seconds; the signal is amplified to 1.2 which is .2 above what we are trying to achieve. Response time has improved quite a lot as it reaches a value of 1 before the first second. Unfortunately, stability has worsened as it takes 4 seconds before the signal becomes stable.

Part 3-2: PI control with closed-loop unit-step Response

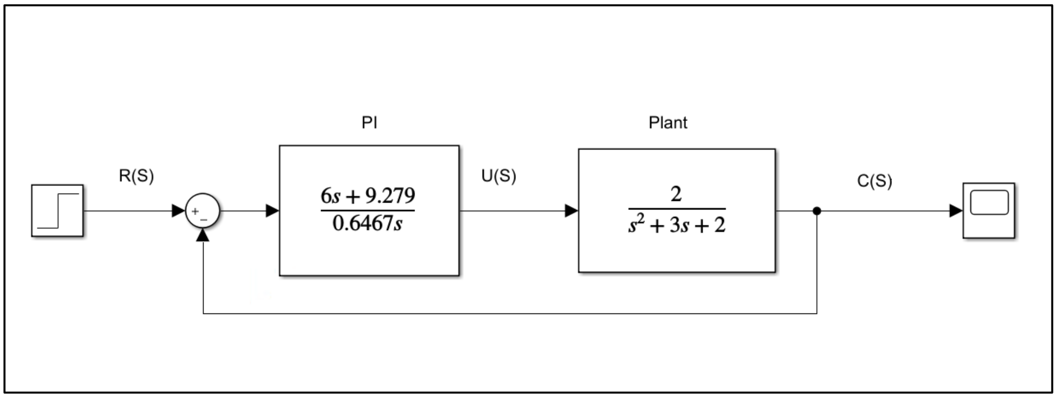

Figure 8 PI control with plant and feedback

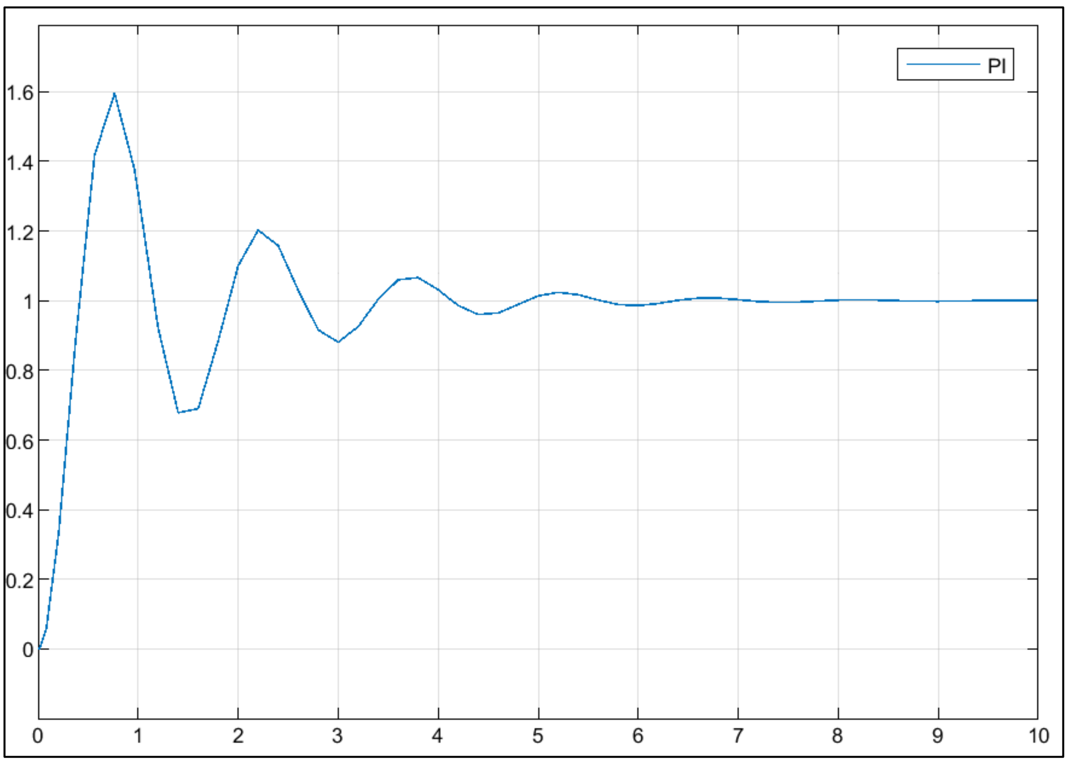

Figure 9 Response of the PI control

When comparing the PI control response with the reference response and P control shows a couple of key changes. Tracking accuracy has worsened quite a lot; the signal oscillates much more. Response time has essentially remained the same as the P control as it reaches 1 at approximately 0.5 seconds. The stability has worsened a lot as it takes almost 7 seconds to stabilize the output signal.

Part 3-3: PID control with closed-loop unit-step Response

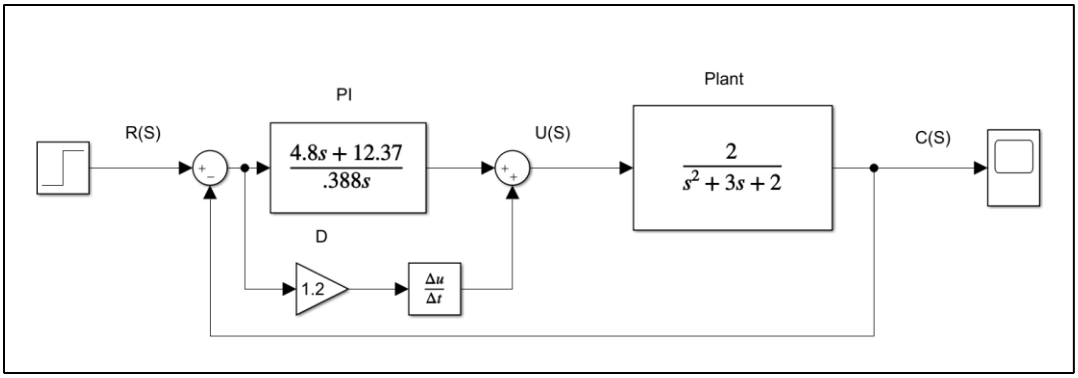

Figure 10 PID control with plant and feedback

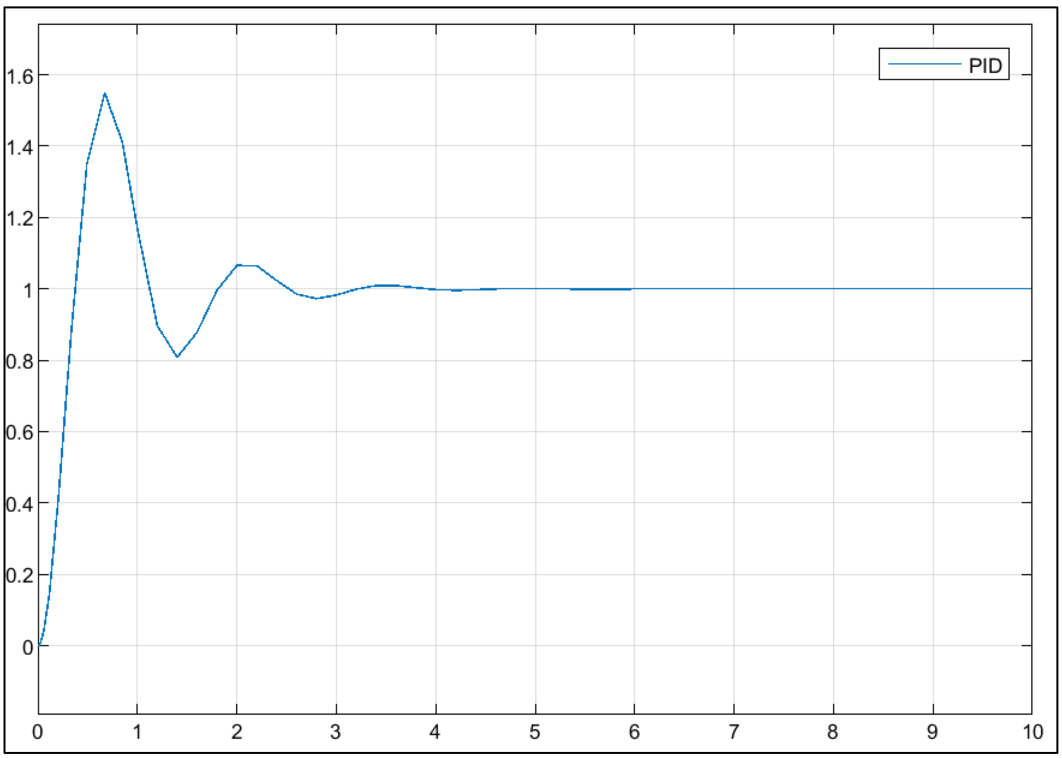

Figure 11 Response of the PID control

Finally, comparing the PID control response with the reference response, P, and PI control shows a couple of key improvements. Tracking accuracy has slightly improved; the signal has started to oscillate less and is getting closer to 1. Response time has improved to where the signal reaches 1 at around 0.3 seconds. The stability has improved a lot; it reaches stability at approximately 4 seconds.

Part 4: PID control with closed-loop unit-step Response

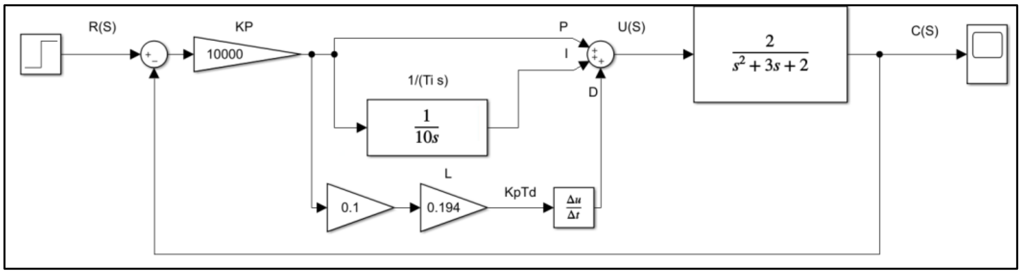

Figure 12 PID control with plant and feedback and tuned parameters

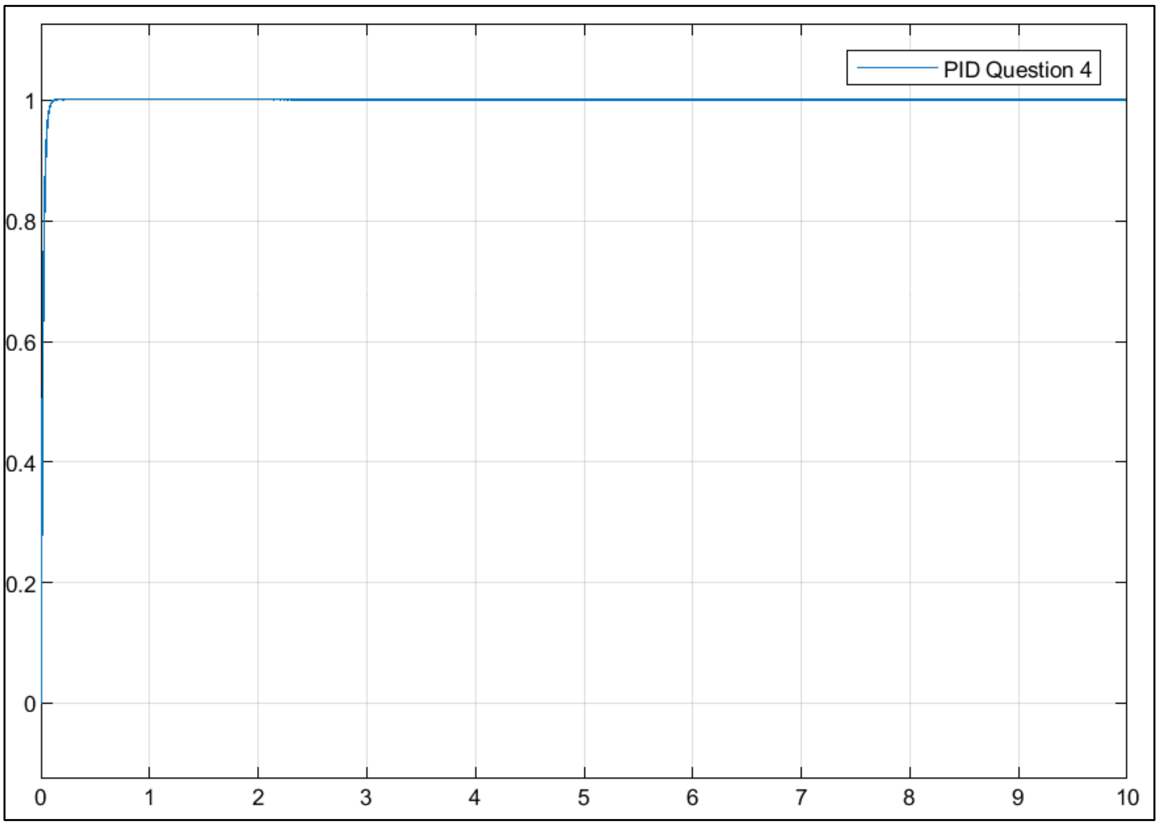

Figure 13 Response of the PID control with adjusted parameters

Table 1 Adjusted parameters used to calculate the response

L =	0.194			
T =	2			
		Kp	**Ti**	**Td**
		970*T/L	5*L	0.1*L

After adjusting parameters to reach a Kp value of 1000, Ti of 0.97 and Td of 0.0194, the output response begins to look very identical to the input unit step value.

Part 5: Findings and Conclusion

Observing the response with a P control and comparing it to that of the reference response proves that stability can be reached quicker. However, tracking performance is hurt in the long-run and thus, additional controls are needed. The I control in combination with the P control does not improve the results from the P control. The PI control makes the signal arguably worse but to counteract this, a D control must be added. It is found that adding a D control to the PI controller significantly improves the signal's response in multiple ways. For starters, both response time and stability are significantly improved. The only setback is that poor tracking performance remains at the start of the response. However, with fine-tuning the PID's parameters, an ideal response can be reached as seen in part 4.

After finding the response with a P control, PI control, and PID control, it can be concluded that after adjusting key parameters, an ideal output response can be reached. For starters, it is observed that increasing the Kp value significantly improves the tracking of the output signal. Increasing Ti improved the response time by reaching the unit step value quicker. Decreasing Td also improved the stability of the response. Through trial and error, it was found that the best results for our controller were when Kp had a value of 1000, Ti was set to 0.97, and Td was set to 0.1. In conclusion, using a tuned PID controller can help in achieving ideal output response parameters to nearly match that of the input parameters.