Due by the beginning of class, Oct. 8.

1. Define a *compacted cartesian tree* to be one where every edge between child and a parent with the same value is compacted. That is, suppose node u has value k, and so does node v, where v is u's parent. Suppose that u's children are $c_1, c_2, \ldots, c_\ell$. Then compacting means removing u from the tree and making $c_1, c_2, \ldots, c_\ell$ children of v, without changing the order of nodes.
   (a) Give an algorithm for constructing the compacted cartesian tree of an array.
   (b) Given a tree T, let array E be its Euler Tour and let array D be the depths of the nodes in E. Prove or give a counter example: the compacted cartesian tree of D is (isomorphic to) T.

2. Given an array B of bits, given an algorithm to compute the integer represented by the concatenation of the bits of B. You can assume that $B[0]$ is the highest order bit and $B[\log n]$ is the lowest order bit. What if the values of B are numbers in the range 0 to $2^k - 1$? What if the values of B are in the range 0 to $b - 1$, that is, the array B represents a base b number? In each case, what is the time complexity?
   Let the word size be $\Theta(\log n)$; that is, the basic operation $(+, -, \times, /)$ made on 2 operands of $\mathcal{O}(\log n)$ bits can be done in $\mathcal{O}(1)$ time. Let further $b < n$ and $k < n$.

3. Given a rooted tree T, define $T(v)$ to be the set of leaves below node v in T. For a set of nodes V', define $LCA_T(V')$ to be the shallowest node u in T such that $u = LCA(w, x)$, for $w, x \in V'$.
   (a) Given two rooted trees S and T with the same leaf labels, define, for each node v in S, $M(v) = LCA_T(S(v))$, so it's the least common ancestor in T of all leaves below v in S.
   (b) Give an algorithm to compute M.

4. The Set Cover (SC) problem is defined as follows:
   Input: A collection C of subsets of a finite set S, and a postive integer $K \leq |C|$.
   Output: Yes, if there is a subsets C' of C such that $|C'| \leq K$ and $\cup_{c \in C'} c = S$, No, otherwise.
   The Vertex Cover (VC) problem is defined as follows:
   Input: A graph $G = (V, E)$ and a postive integer $K \leq |V|$.
   Output: Yes, if there is a subsets V' of V such that $|V'| \leq K$ and for every edge $\{a, b\} \in E$, $\{a, b\} \cap V' \neq 0$ No, otherwise.
   Show that if there is a polynomial time algorithm for SC, there is a polynomial time algorithm for VC.