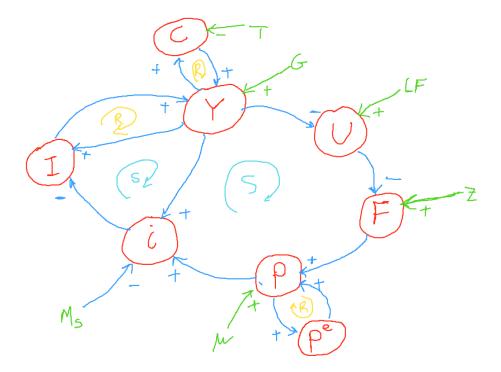
The following CLD represents a simplified model for macroeconomic dynamics in the medium run.



The links connecting the variables C, Y, I and i with one another and with exogenous inputs are faster, with delay equal to one time step. On the other hand, the links that involve U, F, P and  $P^e$  are significantly slower. We will represent this by averaging their inputs over the previous  $\tau$  time steps. To begin with, let  $\tau=4$ .

At each time step, all the variables are updated simultaneously, according to the following formulas for each link:

$$\begin{split} &Y \leftarrow C + I + G \text{, i.e. } Y(t+1) = C(t) + I(t) + G(t) \\ &C \leftarrow \gamma \max(Y-T,0) \text{, where } \gamma \geq 0 \\ &I \leftarrow \frac{\beta Y}{1+i} \text{, where } \beta \geq 0 \\ &i \leftarrow \frac{\beta YP}{M_s} \\ &P^e \leftarrow P \text{, i.e. } P^e(t+4) = \frac{1}{4}(P(t) + P(t+1) + P(t+2) + P(t+3)) \\ &U \leftarrow \max\left(1 - \frac{Y}{LF},0\right) \\ &F \leftarrow \frac{z}{U} \text{, where } 0 \leq z \leq 1 \\ &P \leftarrow (1+\mu)P^e F \end{split}$$

- 1. Do the state variables experience oscillations?
- 2. Do these oscillations die off or do they persist?
- 3. How do these oscillations depend on the values of the parameters  $\tau$ ,  $\beta$  and  $\gamma$ ?
- 4. Do there exist fixed values of the exogenous variables that minimize the oscillations?
- 5. Are there ways to dynamically manage the exogenous variables in order to minimize the resulting oscillations even further?

For concreteness, you can consider starting the economy at Y=1, C=0.75, U=0.05, i=0.04, I=0.2, P=1, P=0.6, with  $\gamma=0.3$ ,  $\beta=0.03$ ,  $\tau=4$ , and  $M_s=0.8$ ,  $\mu=0.4$ , z=0.35, LF=1.2, LF=0.25.