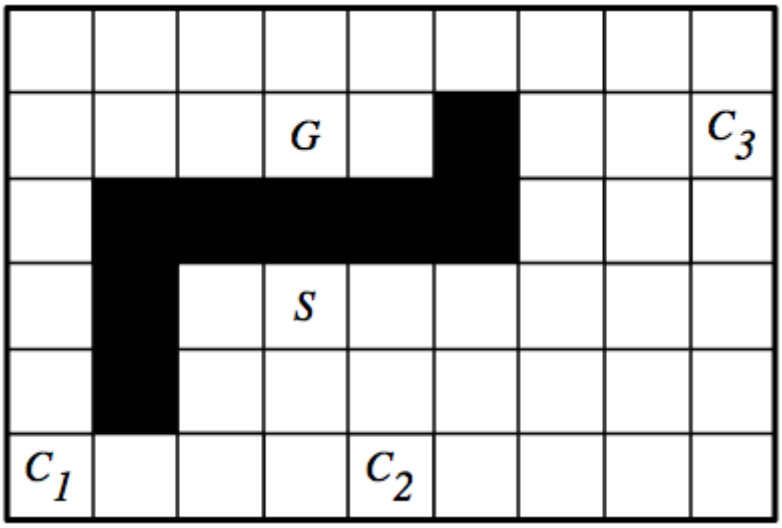
Assignment One: Search

Due: 12:30pm, Monday 25 September 2006

Some of the following problems will be easier to do if you use the CIspace applets at http://www.cs.ubc.ca/labs/lci/CIspace/

Question One

Consider the grid world in the following figure:

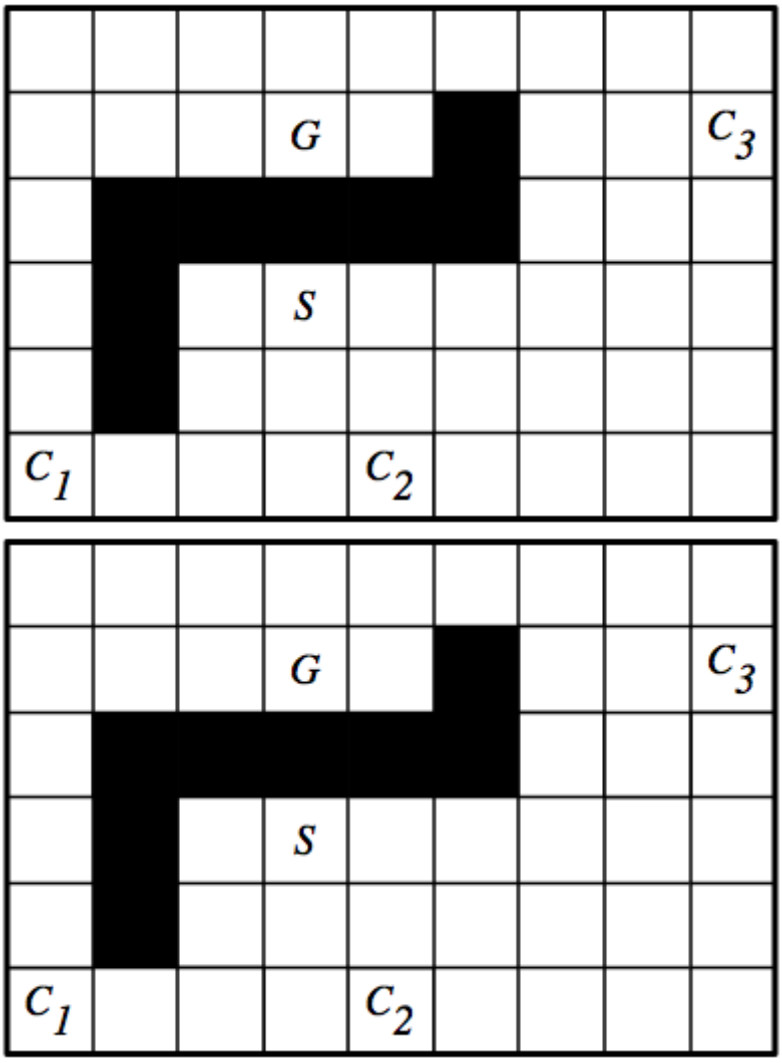


Assume that the robot can be in any of the white squares, and can do one step up, down, left, or right at each time. It cannot step into one of the black squares or outside of the boundary. The cost of a path is the number of steps in the path. At the squares marked with C_i are coffee shops where, if the robot goes to the square, it will be given coffee. Suppose the robot starts at the square marked as *S*, without coffee, and must end up at the square marked *G* carrying coffee.

a. What is the state space? How many states are there?
b. Draw the graph that represents the search space. You can use whatever notation you like as long as you explain it.
c. Give an admissible heuristic. Your heuristic can take into account where the coffee shops are, but not where the blocked squares are. Hint: a useful concept is the Manhattan distance between two points (x_1,y_1) and (x_2,y_2) which is $|x_1-x_2|+|y_1-y_2|$.
d. On your graph number the nodes in the order they are expanded in an A^* search with multiple path pruning.
e. How will your heuristic and A^* change if we assume that there is a hill going down towards the right so that steps to the right cost *0.5* and steps to the left cost *1.5*, and steps up and down cost 1 each.

Solution:

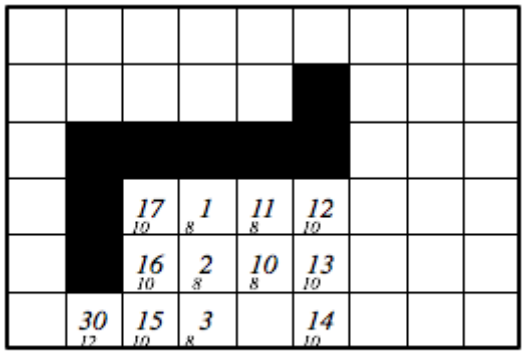
a. The state space can be seen as a triple $\langle x,y,c \rangle$ where x is the horizontal position, y is the vertical position and c is a Boolean variable that specifies whether the robot has coffee. There are 89 states (there are 46 squares the robot could be at with coffee and 43 squares the robot could be without coffee).
b. A graph that represents the search space:



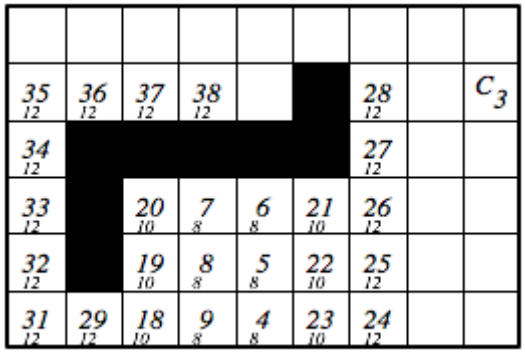
On the left is the states without coffee, and on the right are the states with coffee. There is a state for each white square that is not labeled with a C_i on the left graph, and a state for each white square on the right graph. The neighbors of a node are the white squares adjacent to it, except for the nodes that would lead to one of the C_i squares in the left graph--these lead to the corresponding C_i node on the right graph.

c. An admissible heuristic. For a node <*x,y,true*> (i.e., a state where the robot has coffee), *h(<x,y,true>)* is the Manhattan distance from <*x,y*> to the location *G*. For a node <*x,y,false*> (i.e., a state where the robot doesn't have coffee) *h(<x,y,false>)* is the minimum (over $i \in \{1,2,3\}$) value of the Manhattan distance from <*x,y*> to C_i plus the Manhattan distance from C_i to *G*. This would be an exact distance, if not for the walls.

d. Here are the states with the nodes in the order they are expanded in an A^* search with multiple path pruning. The number in the center is the order the node is expanded. The number at the bottom is the *f*-value of the node. This assumes that for the nodes with the same *f*-value, the node added last is expanded first.



without coffee



with 8 coffee

Note that the path found goes straight down from *S*, then turns left to C_1 then up and right to *G*.

e. How will your heuristic and A^* change if we assume that there is a hill going down towards the right so that steps to the right cost *0.5* and steps to the left cost *1.5*, and steps up and down cost 1 each.

You can modify the Manhattan distance by seeing whether you go right or left and use 1.5 for the cost when you have to go left and 0,5 when it is right. A^* search does not change as every node has the same *f*-value (the costs and the *h*-values are different, but the differences cancel out when summing).

Question Two

Progress is made in science by observing a phenomenon of interest, making hypotheses and testing the hypothesis either empirically or by proving theorems.

For this question you are to think about the effect of heuristic accuracy on A^* search. That is, you are to experiment with, and think about how close *h(n)* is to the actual distance from node *n* to a goal affects the efficiency and accuracy of A^*. To get full marks you must at least invent and prove one theorem and show some empirical evidence for you answer. Your answers need to be precise (e.g, don't say "it works better", but say something like "it always works better", "it sometimes works better" or "it works better in a majority of cases").

a. What is the effect of reducing *h(n)* when *h(n)* is already an underestimate?
b. How does A^* perform when *h(n)* is the exact distance from *n* to a goal?
c. What happens if *h(n)* is not an underestimate?

One way to vary *h(n)* is to use the applet with the automatic node heuristics (under the search options menu) turned on, then turn it off and change heuristic values. There are ways to change all heuristic values.

Solution:

a. What is the effect of reducing *h(n)* when *h(n)* is already an underestimate?

At the top level of abstraction the answer is "more nodes are expanded", however, it could be the case that the same number of nodes are expanded. "no fewer nodes are expanded". However, if you try this on lots of examples with multiple paths to the node you will find this this isn't true either! The reason is that a different optimal path may be found. One true statement is that any sub-optimal path explored with h_1 will be will be explored with h_2 if $h_2(n) <= h_1(n) <= cost(n,g)$, where $cost(n,g)$ is the actual cost from n to g.

b. How does A^* perform when $h(n)$ is the exact distance from n to a goal?

This depends on what happens when there are multiple optimal paths to the goal. If the frontier acts as a stack for nodes with equal f-values, then it will proceed to the goal without expanding any node off a single optimal path.

c. What happens if $h(n)$ is not an underestimate?

It need not find an optimal path. However there are some general statements that are true:

If $h(n) <= cost(n,g)+eps$ (for $eps >= 0$), the first path found will be at most eps from optimal.

If $h(n) <= cost(n,g) \times gamma$ (for $gamma >= 1$), the first path found will be at most $gamma$ times optimal.

Question Three

For each of the following, give a graph that is a tree (there is at most one arc into any node), contains at most 15 nodes, and has at most two arcs out of any node.

a. Give a graph where depth-first search is much more efficient (expands fewer nodes) than breadth-first search.
b. Give a graph where breadth-first search is much better than depth-first search.
c. Give a graph where A^* search is more efficient than either depth-first search or breadth-first search.
d. Give a graph where depth-first search and breadth-first search are both more efficient than A^* search.

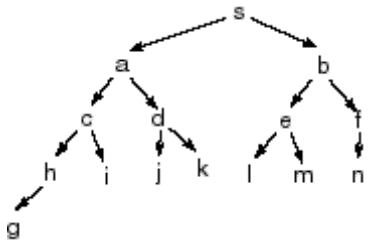
You must draw the graph and show the order of the neighbors (this is needed for the depth-first search). Either give the arc costs and heuristic function or state explicitly that you are drawing the graph to scale and are using Euclidean distance for the arc costs and the heuristic function.

Solution:

In all of these graphs, we assume that we are on a plane, with Euclidean distance (straight-line distance) as the arc cost and as the heuristic function. We also assume that the neighbors are ordered from left to right. The start node is s and the goal node is g.

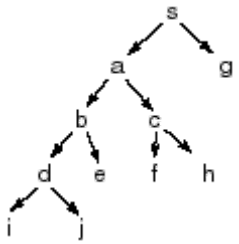
a. Give a graph where depth-first search is much more efficient (expands fewer nodes) than breadth-first search.

Here depth-first search expands every node, whereas breadth-first search expands three nodes:



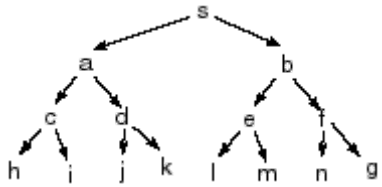
b. Give a graph where breadth-first search is much better than depth-first search.

Here depth-first search expands every node, whereas breadth-first search expands three nodes:



c. Give a graph where A* search is more efficient than either depth-first search or breadth-first search.

Here depth-first search and breadth-first search expand every node, whereas A* search expands 4 nodes.



d. Give a graph where depth-first search and breadth-first search are both more efficient than A* search.

Here depth-first search expands three nodes, breadth-first search expands 4, yet A* search expands every nodes.