به نام خدا

محسن کریمی ٤٠١٢١٩١٣

(١)

$G(s)=\frac{ke^{-s\tau}}{s} \longrightarrow G(j\omega)=k\frac{e^{-j\omega\tau}}{j\omega}$

برای یافتن GM باید فرکانس گذر فاز را بیابیم

$\angle G(j\omega) = -\tau\omega - 90 = -180$

$\to -\tau\omega = 90 \to \omega = 90/\tau = \frac{\pi}{2\tau}$

$|G(j\omega)| = |k/\omega| \xrightarrow[\omega=\pi/2\tau]{if} |G(j\pi/2\tau)| = |k/\pi \times 2\tau| = \frac{2k\tau}{\pi}$

$\to GM = 20\log_{10}\frac{1}{|G(j\omega_p)|} = -20\log_{10}\frac{2k\tau}{\pi}$

برای یافتن PM باید فرکانس گذر بهره را بیابیم:

$|G(j\omega)| = 1 = 0dB \longrightarrow k/\omega = 1 \to \omega = k$

$PM = \angle G(j\omega_g) - 180 \to PM = -k\tau + 90° \longrightarrow PM = -k\tau + \pi/2$

برای پایداری سیستم:

$GM > 0 \to -20\log\frac{2k\tau}{\pi} > 0 \longrightarrow 0 < \frac{2k\tau}{\pi} < 1 \to 0 < k < \pi/2\tau$ (I)

$PM > 0 \to -k\tau + \pi/2 > 0 \longrightarrow k < \pi/2\tau$ (II)

(I) , (II) $\to 0 < k < \pi/2\tau$

(٢)

$G(s) = \frac{k(s+2)}{s^2} \longrightarrow G(j\omega) = \frac{k(j\omega+2)}{-\omega^2}$

$|G(j\omega)| = \frac{k\sqrt{\omega^2+4}}{\omega^2}$, $|G(j\omega)| = 1 \longrightarrow k\sqrt{\omega^2+4} = \omega^2$

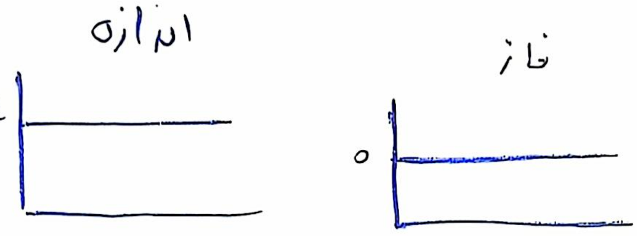
$PM = \angle G(j\omega) - 180 \to \tan^{-1}(\omega_c/2) - 0 - 180 = 45$

$\tan^{-1}(\omega_c/2) = 225 \to \omega_c = 2$

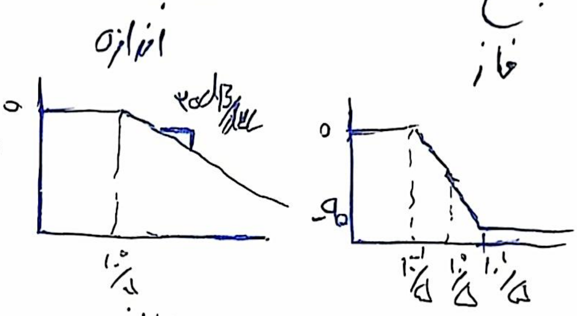
$k\sqrt{4+4} = 4 \longrightarrow k = \sqrt{2}$

۳) جمع تک تک نمودارهای بودی

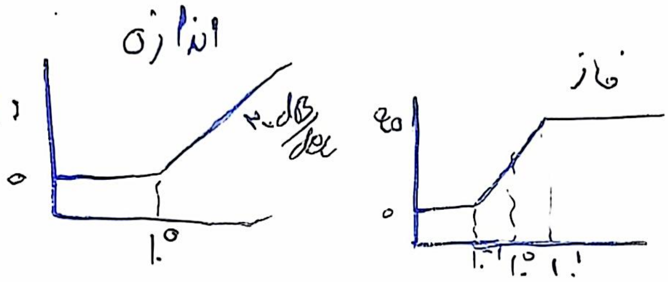
5 ،



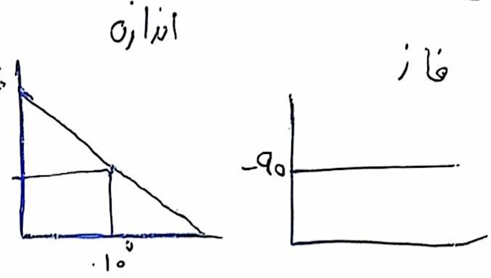
$\frac{1}{5s+1}$:



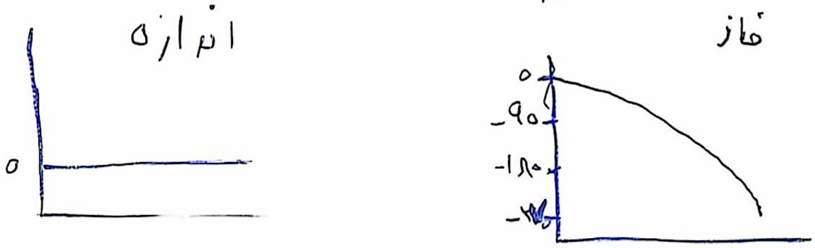
$s+1$ ،



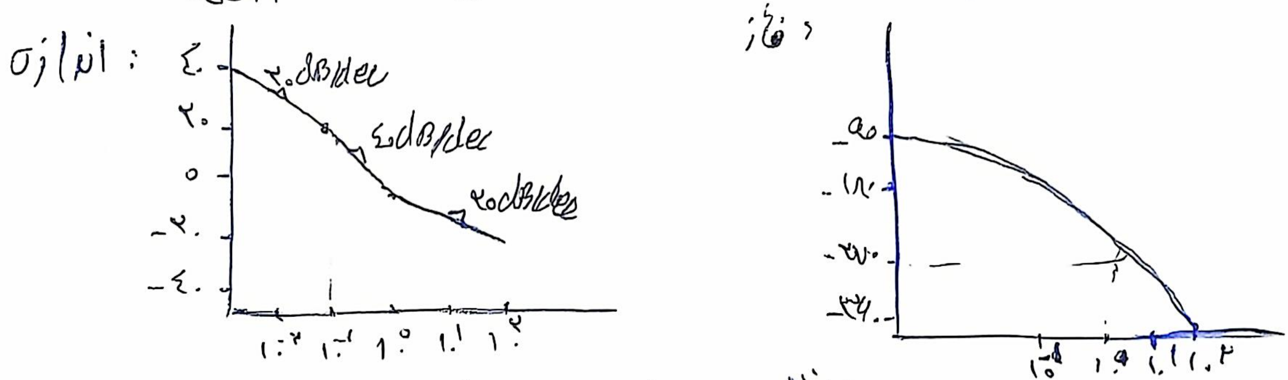
$\frac{1}{s}$ ،



e^{-2s} ،



$$G(s) = \frac{5}{5s+1}\cdot e^{-2}\cdot\frac{s+1}{s}$$



$$G(j\omega) = \frac{5(j\omega+1)e^{-2j\omega}}{j\omega(5j\omega+1)} = \frac{5(j\omega+1)e^{-2j\omega}}{j\omega-5\omega^2}$$

$$\angle G(j\omega) = \tan^{-1}(\omega) - 2\omega - 90 - \tan^{-1}(5\omega) = -180$$

$$\rightarrow \tan^{-1}(\omega) - \tan^{-1}(5\omega) - 2\omega = -90$$

$$|G(j\omega_p)| = \left| \frac{5\times(0.42j+1)e^{-0.84j}}{j\times 0.42(2.1j+1)} \right|$$

$$= \frac{5\sqrt{1+0.42^2}}{0.42\sqrt{1+2.1^2}} = 5.55 \quad , GM = -20\log 5.55 = -14.88\ dB < 0$$

$$|G(j\omega)| = 1 \rightarrow \frac{5\sqrt{\omega^2+1}}{\omega\sqrt{25\omega^2+1}} = 1 \rightarrow \omega_c = 1.34$$

$PM = \angle G(j\omega_c) - 180 \rightarrow \tan^{-1}(\omega) - \tan^{-1}(5\omega) - 2\omega - 90 - 180 = PM$

$\tan^{-1}(1.24) - \tan^{-1}(6.3) - 2 \times 1.24 \times \frac{180}{\pi} - 90 - 180 = PM$

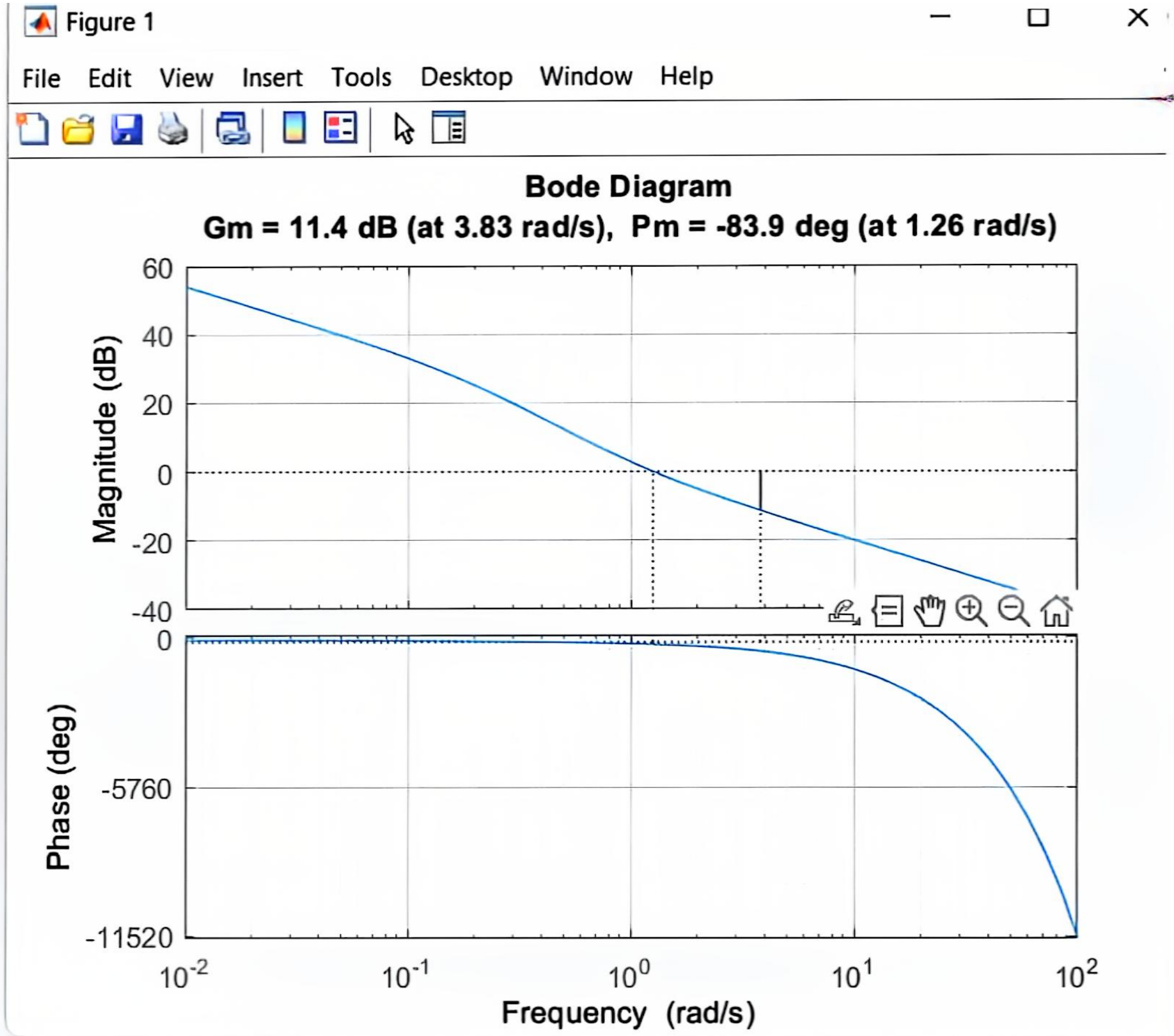
$-443.80 = PM \xrightarrow{+360} -83.8^\circ = PM$

هم PM و هم $GM < 0$

در نتیجه ناپایدار

Scanned with CamScanner

```matlab
clc;
clear;
T = 2;
s = tf('s');
G_no_delay = 5 * (s + 1) / (s * (5 * s + 1));
G = G_no_delay * exp(-T * s);
bode(G);
margin(G), grid
grid on;
```



(۶)

$$G(j\omega) = \frac{4a^2}{(j\omega + a)^2} = \frac{4a^2}{(a^2-\omega^2)+2j\omega a}$$

$$|G(j\omega)| = \frac{4a^2}{\sqrt{(a^2-\omega^2)^2+4\omega^2 a^2}} = \frac{4a^2}{\omega^2+a^2}$$

$$|G(j\omega)| = 1 \longrightarrow 4a^2 = \omega^2 + a^2 \rightarrow \omega^2 = 3a^2 \rightarrow \omega = \sqrt{3}\,a$$

$$P_M = 0 - 2\tan^{-1}(\omega/a) - 180 = -300 \xrightarrow{+360} P_M = 60$$

$$\angle G(j\omega) = 0 - 2\tan^{-1}(\omega/a) = -180 \rightarrow \tan^{-1}(\omega/a) = 90$$

$\rightarrow \omega/a = \infty \longrightarrow$ سیستم حد بهره ندارد $\longrightarrow$ سیستم پایدار است