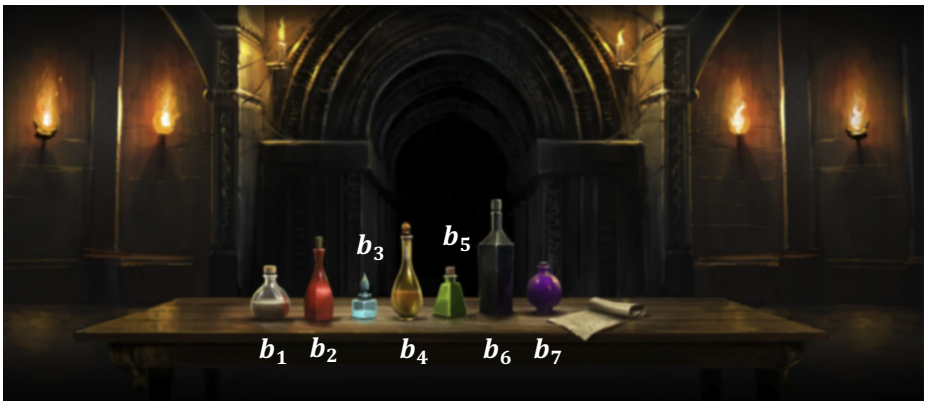
Pb.4: Dungeon & Potions Riddle

Sol^{n}: *Label bottle ordering on image:*



Definitions & Interferences:

From Label Ordering: $\boldsymbol{B} = \{\boldsymbol{b_1}, \boldsymbol{b_2}, \boldsymbol{b_3}, \boldsymbol{b_4}, \boldsymbol{b_5}, \boldsymbol{b_6}, \boldsymbol{b_7}\}$ where $\boldsymbol{B}$ is the universal set of bottles
From Line **3**: so, the number of bottles as $n(\boldsymbol{B}) = 7$
From Line **2**, **3**: $\boldsymbol{G} = \{\boldsymbol{b_g} \in \boldsymbol{B} \mid \boldsymbol{g} \in [\mathbf{1}, \mathbf{7}]\}$, where $\boldsymbol{G}$ is the set of 1 bottle bringing you back *(target bottle 1)*
From Line **4**: $\boldsymbol{F} = \{\, \boldsymbol{b_f} \in \boldsymbol{B} \mid \boldsymbol{f} \in [\mathbf{1}, \mathbf{7}]\}$ where $\boldsymbol{F}$ be the set of 1 bottle moving you forward *(target bottle 2)*
From Line **5**: $\boldsymbol{N} = \{\boldsymbol{b_n}, \boldsymbol{b_m} \in \boldsymbol{B} \mid \boldsymbol{n},\ \boldsymbol{m} \in [\mathbf{1}, \mathbf{7}]\}$ where $\boldsymbol{N}$ is the set of **nettle** wine bottles
From Line **6**: $\boldsymbol{P} = \{\boldsymbol{b_p}, \boldsymbol{b_q}, \boldsymbol{b_r} \in \boldsymbol{B} \mid \boldsymbol{p},\ \boldsymbol{q},\ \boldsymbol{r} \in [\mathbf{1}, \mathbf{7}]\}$ where $\boldsymbol{P}$ is the set of **poisonous** bottles
$\boldsymbol{G}, \boldsymbol{F}, \boldsymbol{N}, \boldsymbol{P}$ *are all* ***disjoint*** *sets.*

Solve for the ordering of: f, g

(1) From **Clue 1**: $\exists p \in \boldsymbol{P},\ \exists n \in \boldsymbol{N},\ \boldsymbol{p} = \boldsymbol{b_i},\ \boldsymbol{n} = \boldsymbol{b_{i+1}}$

(2) From **Clue 2**: either $\boldsymbol{b_1}$ *or* $\boldsymbol{b_7} \in \boldsymbol{G}$

(3) From **Clue 3**: $\boldsymbol{b_{largest}}, \boldsymbol{b_{smallest}} \in \boldsymbol{P'}$
As seen in **the image**: $\boldsymbol{b_3}, \boldsymbol{b_6} \in \boldsymbol{P'}$
$\boldsymbol{b_3}, \boldsymbol{b_6} \in \boldsymbol{G} \cup \boldsymbol{F} \cup \boldsymbol{N}$

(4) From **Clue 4**: $\boldsymbol{b_2}, \boldsymbol{b_6}$ are from the same set where $\boldsymbol{n} = \mathbf{2}$. *The only set that has* **2** *elements* ***is N.***
By **Simplification** ***(Logic)***: Hence, $\boldsymbol{N} = \{\boldsymbol{b_2}, \boldsymbol{b_6}\}$

(4.1) $\boldsymbol{b_4} \in \boldsymbol{G} \cup \boldsymbol{P} \cup \boldsymbol{F}$
Remaining uncertain elements = 7-2 = ***5***

(4.2) (3) and *(4)*: $\boldsymbol{b_3} \in \boldsymbol{G} \cup \boldsymbol{F}$

(4.3) (1) and *(4)*: $\boldsymbol{b_1}, \boldsymbol{b_5} \in \boldsymbol{P}$
Remaining uncertain elements = 5-2 = ***3 (1P, 1F, 1G) and (3, 4, 7)***

(5) *(2)* and (4.3): $\boldsymbol{b_7 \in G}$
Remaining uncertain elements = 3-1 = ***2 (1P, 1F) and (3, 4)***

(6) *(5)* and (4.2) and (4.1): $\boldsymbol{b_3 \in F}$, $\boldsymbol{b_4} \in \boldsymbol{P}$

Answer: $\boldsymbol{b_3}$ **คือขวดที่กินแล้ววาร์ปไปข้างหน้าและ** $\boldsymbol{b_7}$ **คือขวดที่กินแล้ววาร์ปกลับบ้าน**