Problem Set 04: Linear Regression

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Contents

Background

For this problem set you will first run through an example of a simple linear regression, answering a few questions on the way. Then you will work through a regression analysis independently. Knit this file…and you can read through all the instructions. You can do your work in this `.Rmd` file; simply insert your answers below each Question, knit your final copy, and turn in the PDF file.

We will look at some demographic data from the `fivethirtyeight` package recorded for 48 voting areas in the US states just after the 2016 presidential election. We will investigate what variables within those regions might be tied to the percentage of US voters that supported Donald Trump, and in turn, which variables might be useful to predict Trump support in other regions (i.e. to a wider US population).

Setup

Load packages

We will read the data in with the `readr` package, explore the data using the `dplyr` package and visualize it using the `ggplot2` package. The `moderndive` package includes some nice functions to show regression model outputs.

```
library(dplyr)
library(ggplot2)
library(readr)
library(moderndive)
```

The data

Copy, paste, and run the following in a code chunk to load the data from where it is published on the web.

```
trump <- read_csv("https://docs.google.com/spreadsheets/d/e/2PACX-1vT8qHdvTPaRc62hU94ShBcSh04HP3c11b6XZ
```

Take a moment to look at the data in the viewer or by using `glimpse()`. The explanatory variables include:

- `hs_ed` - the percentage of the adults in the region with a high school education.
- `poverty`- the percentage of the "white" households in the region in poverty.
- `non_white`- the percentage of humans in a region that identify as a person of color.

The outcome variable `trump_support` is the percentage of votes for Trump in 2016 in each region.

Observe that all percentages are expressed as values between 0 and 100, and not 0 and 1.

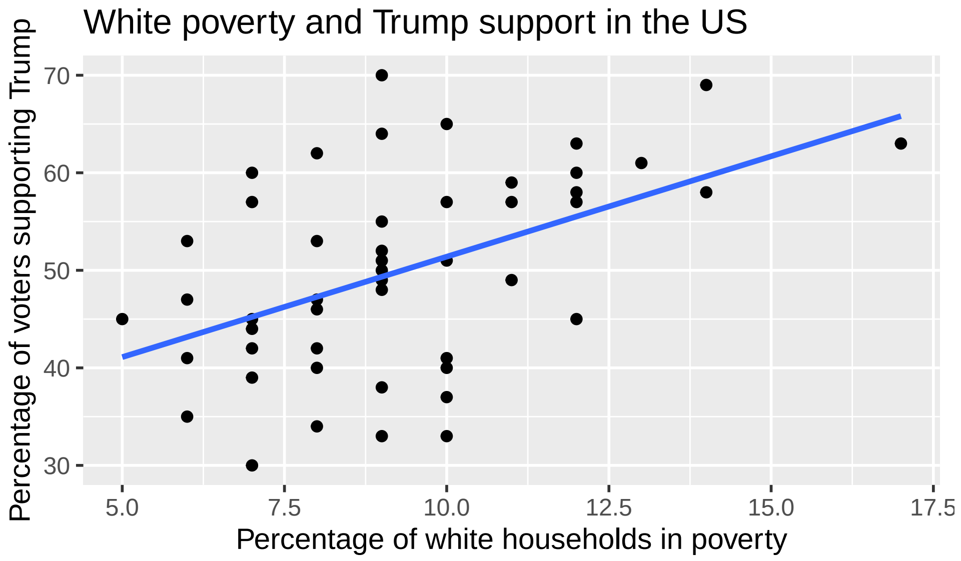
An Example/Demo

Visualization

We will start by investigating the relationship between white poverty levels and support for Trump.

We'll do this by creating a scatterplot with `trump_support` as the outcome variable on the y-axis and `poverty` as the explanatory variable on the x-axis. Note the use of the `geom_smooth()` function, that tells R to add a regression line. While the points do scatter/vary around the blue regression line, of all possible lines we can draw in this point of clouds, the blue line is the "best-fitting" line in the sense that it minimizes the sum of the squared residuals (see moderndive 5.3.2 for a fuller explanation).

```
ggplot(data = trump, aes(y = trump_support, x = poverty)) +
  geom_point() +
  geom_smooth(method = "lm", se = FALSE) +
  labs(x = "Percentage of white households in poverty",
       y = "Percentage of voters supporting Trump",
       title = "White poverty and Trump support in the US")
```



QUESTION 1

Does the relationship appear to be positive or negative? Does it look to be reasonably linear?

Answer:

The correlation coefficient (r)

We can numerically quantify the strength of the linear relationship between the two variables with the correlation coefficient. The following tells R to `summarize()` the correlation coefficient between the numerical variables `poverty` and `trump_support`. Note that the correlation coefficient only exists for pairs of numerical variables.

```
trump %>%
  summarise(r = cor(trump_support, poverty))
```

```
## # A tibble: 1 x 1
##       r
##   <dbl>
## 1 0.486
```

Running a linear regression model

In R we can fit a linear regression model (a regression line), like so:

```
poverty_mod <- lm(trump_support ~ poverty, data = trump)
```

Note that:

- the function `lm()` is short for "linear model"
- the first argument is a *formula* in the form `y ~ x` or in other words `outcome variable ~ explantory variable`.

- the second argument is the data frame in which the outcome and explanatory variables can be found.
- we **SAVED THE MODEL RESULTS** as an object called `poverty_mod`

This object `poverty_mod` contains all of the information we need about the linear model that was just fit and we'll be accessing this information again later.

Get the regression table

The `get_regression_table()` function from the `moderndive` package will output a regression table. Let's focus on the value in the second column: an estimate for 1) an intercept, and 2) a slope for the `poverty` variable. We'll revisit what the other columns mean in a future problem set.

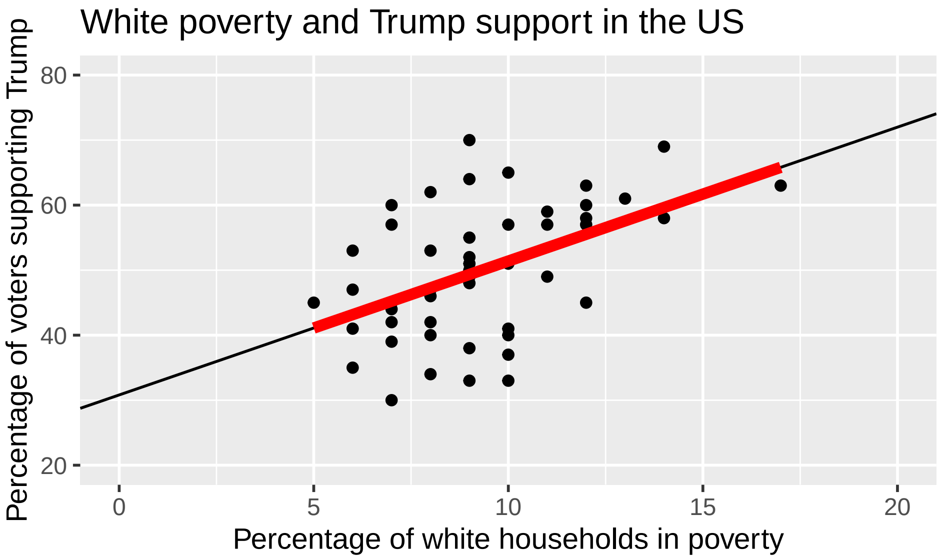
```
get_regression_table(poverty_mod)
```

```
## # A tibble: 2 x 7
##   term      estimate std_error statistic p_value lower_ci upper_ci
##   <chr>        <dbl>     <dbl>     <dbl>   <dbl>    <dbl>    <dbl>
## 1 intercept    30.8       5.22      5.90       0   20.3       41.3
## 2 poverty       2.06      0.545     3.78       0    0.961      3.16
```

We can interpret the `intercept` and `poverty` slope like so:

- When the poverty level is 0, the predicted average Trump support is 30.81%
- For every increase in poverty level of 1 percentage point, there is an **associated increase** in Trump support of 2.059 percentage points.

Revisiting the plot from earlier, we can see that the best-fit line hits the y axis at 30.8064 (if we extend it). This is the intercept…the y value at which poverty = 0 (note, a value that is sadly not close to the range of values for "percentage of white households in poverty").



QUESTION 2

We found a positive correlation coefficient. Is it reasonable for us to conclude that social policies that increase white poverty will **cause** an increase in Trump support? Explain why or why not?

Answer:

Making predictions

Based on the R output of our model, the following is our least squares regression line for the linear model:

$$\widehat{trump\_support} = 30.806 + 2.059 \times poverty$$

We can use the line from our graph of the `poverty`, `trump_support` relationship to **visually** make predictions…for instance at 15% white poverty, the line shows a value of just over 60% Trump support.

To get a **more accurate** prediction, we could actually plug 15% into the regression equation like so:

```
y_hat = 30.8064 + 2.0591 * 15
y_hat
```

```
## [1] 61.6929
```

QUESTION 3

What percent of Trump support would you expect at a value of 6% white poverty?

Answer:

QUESTION 4

Do you think it is a good idea to predict Trump support at 85% white poverty, based on this regression equation? Explain your reasoning.

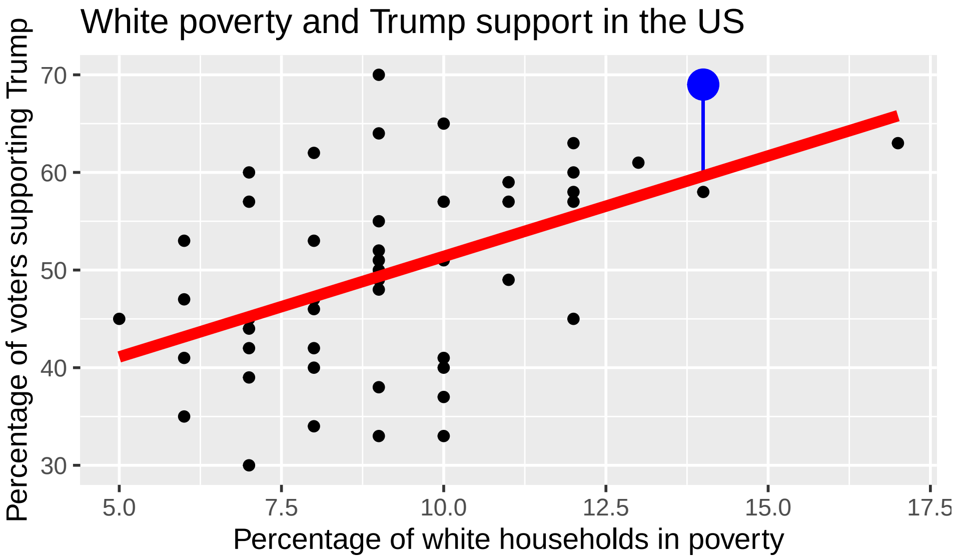
Answer:

Residuals

Recall that model residuals are the difference between the **observed values in your data set** and the **values predicted by the line**:

residual $= y - \hat{y}$

For instance, below, one data point is highlighted in blue…the residual is the difference between the y value of the **data point** (here 69), and the y value **predicted** by the line (roughly 59). Here the residual is roughly 10 ($69 - 59 = 10$). The regression equation has under-estimated Trump support in this voting area.



The following function `get_regression_points()` gives you the **fitted** (also known as **predicted**) value for every data point, and the **residual** for every data point. The first row in the output is the first data point. You can see that in this voting area Trump support was 30%, white poverty was 7%, the regression equation predicted 45.22% Trump support, and the residual was -15.22 ($30 - 45.22$).

```
get_regression_points(poverty_mod)
```

Put your skills to practice independently!

Use the same `trump` data set for the following questions:

QUESTION 5

Generate a scatterplot with a best-fitting line with `non_white` as the explanatory variable, and `trump_support` as the response. Be sure to include an informative title and axis labels to your plot. This will help contextualize it.

QUESTION 6

Do you expect the correlation coefficient (for `non_white` and `trump_support`) to be positive or negative? Test your prediction using R (it is OK if your expectation was wrong!).

Answer:

QUESTION 7

Run a linear regression using `non_white` as the **explanatory** variable, and `trump_support` as the **outcome** variable. Interpret the Intercept and slope estimates.

Answer:

QUESTION 8

Make a numerical prediction for the level of trump support for a region that has 70% of humans that identify as a person of color. In other words, use **math** not a visual prediction.

Answer:

QUESTION 9

Based on the evidence you have so far (scatterplots and correlation coefficients), which of the explanatory variables we have considered (`non_white` or `poverty`) seems to be a better explanatory variable of Trump support? Explain.

Answer:

QUESTION 10

If Representative Ocasio-Cortez saw the regression line and not the actual data:

A. What would her prediction of Trump support be for a region in which 61% of the people identify as non-white?

B. Would her prediction be an overestimate or an underestimate (compared to the observed data), and by how much?

C. In other words, what is the residual for this prediction?

Answers: