

Work Problems Chapter 8

- 1) I want to know whether Californians, who tend to be pretty health conscious, eat fewer hot dogs than the general population of Americans. Suppose that in the population of adults, the average number of hot dogs eaten per year is 15. I select a random sample of 25 adults from California and find that they eat an average of 17 hot dogs with a standard deviation of 3.

- a. State the null and alternative hypotheses

H_0 : CA mean = U.S. mean

H_A : CA mean < U.S. mean

- b. Chose an alpha level.

.05

- c. Decide whether you are doing a 1-tailed or 2-tailed test (explain why)

1-tailed because question suggests Californian's will eat fewer hot dogs than U.S. population due to health consciousness.

- d. State your degrees of freedom

$64 - 1 = 63$ (Note: Using 60 *df* in Appendix B because it is closest to 63, and smaller)

- e. Find and report your critical value for t .

$t_c = -1.671$. NOTE: I am reporting a *negative* critical value for t because my alternative hypothesis is that my sample mean will be *less than* my population mean. If this is true, then my *observed* t value will be negative. In other words, I am only looking at the negative side of the distribution.

- f. Compute your observed t -value.

First I need to calculate a standard error of the mean: $s.e. = 3/\sqrt{25} \rightarrow 3/5 \rightarrow .60$

$t = 17 - 15 / .60 \rightarrow 2/.60 \rightarrow 3.33$.

- g. Decide whether to reject or fail to reject H_0

I am going to fail to reject (i.e., RETAIN) the null hypothesis.

- h. Describe your results. Is this a statistically significant difference?

Even though my observed t value is larger than the absolute value of my critical t value (and therefore my p value is smaller than my alpha level), I was only looking for *negative* t values. My observed t value is positive, so I disregard it. This is strange, I know, but it is the way it works with a 1-tailed alternative hypothesis. This is why many statisticians recommend conducting 2-tailed tests as a rule.

- 2) Suppose you want to know whether men and women who take a college statistics class differ in their enjoyment of statistics. Enjoyment of statistics is measured by asking people to rate, on a 10 point scale (1 = “I hate it” and 10 = “I love it”) “How much do you enjoy statistics?” I select a random sample of 20 women who took statistics and 20 men who took statistics. I find the women had a mean of 4 and a standard deviation of 1.5 whereas the men had a mean of 4.5 and a standard deviation of 2.

- a. State the null and alternative hypotheses

H₀: $\mu_{\text{men}} = \mu_{\text{women}}$

H_A: $\mu_{\text{men}} \neq \mu_{\text{women}}$

- b. Chose an alpha level.

Either .05 or .01. I'll solve for both.

- c. Decide whether you are doing a 1-tailed or 2-tailed test (explain why)

Two-tailed because I am looking for whether men and women “differ” without speculating about which group might be higher.

- d. Decide whether to reject or fail to reject H₀.

Degrees of freedom = 20 + 20 - 2 = 38.

If $\alpha = .05$, $t_c = 2.021$ (2-tailed), if $\alpha = .01$, $t_c = 2.704$ (2-tailed) NOTE: This was using 40 df in Appendix B.

**I first need to calculate two standard errors: For women, $s.e. = 1.5/\sqrt{20} \rightarrow 1.5/4.47 \rightarrow .36$
For men, $s.e. = 2/\sqrt{20} \rightarrow 2/4.47 \rightarrow .48$**

Now, to find the standard error of the difference between the means I need to square each standard error, then add them together, then square root this sum:

$$(.36)^2 = .13$$

$$(.48)^2 = .23$$

$$.13 + .23 = .36. \sqrt{.36} = .60$$

Now I can compute my observed t value: $4 - 4.5/.60 \rightarrow -0.5/0.60 \rightarrow -0.83$.

I fail to reject H₀ if I have an alpha level of .01 because my observed t value (absolute value, because this is a 2-tailed test) is smaller than my critical t value (i.e.,

0.83 < 2.704). If my alpha level is .05, I also fail to reject the null hypothesis (0.83 < 2.021) and conclude that the results are not statistically significant.

e. Describe your results. What do they tell you?

Using an alpha level of .01 or .05, my results are not statistically significant. In the populations that these samples represent, men did not differ from women in their average liking of statistics class.

1. Is this a statistically significant difference?

The difference is not statistically significant using an alpha level of .05 or an alpha level of .01.

f. Calculate a 99% confidence interval for the difference between the means and explain what this confidence interval tells you.

$$CI_{99} = -0.5 \pm (.60)(2.704) \rightarrow -0.5 \pm 1.62 = -2.12, 1.12$$

We are 99% confident that the difference between the population means is between men liking stats 2.12 points more than women and women liking stats 1.12 points more than men.

g. Calculate an effect size (d) and describe what it means.

First, we need to calculate a standard deviation of the difference between the means:

$$s = \sqrt{20} * (.60) \rightarrow 4.47(.60) = 2.68.$$

Then I plug that into the denominator: $d = (4 - 4.5)/2.68 \rightarrow -0.5/2.68 = -.19$.

Effect sizes are treated as absolute values. This effect size of .19 would be considered a small effect size, so there is a not much practical significance to this difference between the means of men and women in their liking of statistics.

3) I think people are getting taller. I select a random sample of 15 fathers and their adult sons to be in a study. I find that the fathers have an average height of 71 inches and their sons have an average height of 73 inches. The standard *deviation* of the difference between the means is 1.3.

a. Write the null and alternative hypotheses.

$H_0: \mu_x = \mu_y$ Note that we need to specify here that the X population is fathers and the Y

$H_1: \mu_x < \mu_y$ population is sons. Need to be clear on which population is which.

b. Select an alpha level.

Alpha is .05, meaning that there must be less than a 5% probability that my results are due to chance, or random sampling error, before I will consider them statistically significant.

- c. Is this a statistically significant difference? (do the calculations and explain what your results mean).

First I need to find the standard error of the differences between the sample means. I'm already given the standard deviation of the difference. So I take that number and divide it by the square root of N , which is the number of *pairs* in the sample. So:

$$s_{\bar{D}} = 1.3/\sqrt{15} \rightarrow 1.3/3.87 \rightarrow .36$$

Now I calculate the observed t value:

$$t = \frac{\bar{X} - \bar{Y}}{s_{\bar{D}}} \rightarrow (71 - 73)/.36 \rightarrow -2/.36 \rightarrow t = -5.56.$$

Using Appendix B, one-tailed test, .05 alpha, $df = 15 - 1 = 14$, I find a critical value of -1.761 . NOTE: Again, my critical value is negative because if my alternative hypothesis is true, I should get a negative observed t value. This is a 1-tailed test because I speculated that sons would be taller than their fathers.

Because my observed t value *exceeds* (i.e., it is beyond it, in the negative direction) my critical t value, and is in the region of rejection, my result is statistically significant. In the populations of fathers and their sons, the sons are taller, on average. So it looks like people are getting taller—if we define people as men.