Chapter 8: *t* Tests

I. Overview

a. Technically, a *t* test is any test of statistical significance involving the family of *t* distributions.
    i. Test of correlation coefficients, regression coefficients are also *t* tests.

b. In this chapter, we focus on *t* tests that compare two means to each other.
    i. One-sample *t* tests compare the sample mean to a population mean. (See Chapter 7.)
    ii. Independent samples *t* tests compare the sample means of two mutually exclusive groups.
    iii. Dependent, or paired, samples *t* tests compare the sample means of two paired sets of scores.

II. *t* Distributions

a. The family of *t* distributions are similar in shape and properties to the normal distribution, but their exact characteristics depend on sample size.
    i. These distributions are bell shaped, symmetrical and have the mean, median, and mode in the middle, just as do normal distributions.

b. The shape of each *t* distribution depends on the degrees of freedom in the analysis.
    i. $n - 1$ for a 1-sample *t* test
    ii. $n + n - 2$ for an independent samples *t* test.
    iii. Number of pairs of scores – 1 for a dependent samples *t* test.

c. As the degrees of freedom decrease, the peak of the distributions gets lower and flatter, and the tails get higher above the X axis, indicating a higher frequency of extreme *t* values and fewer *t* values near the mean.

d. As the degrees of freedom increase, the shape of the *t* distribution looks more and more like the normal distribution.
    i. When the degrees of freedom are greater than 120, the *t* distribution is identical to the normal distribution.

III. 1-Sample *t* Test

a. Purpose: To determine whether the difference between a sample mean and a population mean is statistically significant.
    i. E.g., Does my random sample of 100 high school seniors differ from the population of high school seniors in their average score on a standardized math test?

b. Types of variables used: One dependent variable measured on an interval scale compared between the sample and the population.

c. How it works: The standard error of the mean is calculated using the sample size and the sample standard deviation. This is the denominator of the *t* value formula and is used to

divide the difference between the sample mean and the population mean (the numerator of the *t* value formula). The result is the observed *t* value, and this is compared to the critical *t* value from Appendix B to determine statistical significance.

i. Degrees of freedom: $n - 1$.

IV. Independent Samples *t* Test

a. Purpose: To determine whether the sample means of two mutually exclusive samples (e.g., men and women) are statistically significantly different from each other.
   i. E.g., Do men and women differ in how many hours a day they sleep, on average?

b. Types of variables used: One dependent variable measured on an interval scale and one independent, categorical variable.

c. How it works: Each sample has a mean and a standard error of the mean. These two standard errors are combined, or pooled, to create one average standard error.
   i. If the sample sizes differ for the two groups, the contribution of each sample's standard error is weighted according to sample size in the pooled standard error.

d. The pooled standard error is used as the denominator for the *t* test formula. The numerator of this formula is the difference between the two sample means.

e. The calculated, or observed, *t* value is then compared to a critical *t* value found in Appendix B.
   i. The critical *t* value is found by using the degrees of freedom, the alpha level, and deciding whether it is a 1-tailed or 2-tailed test.
      1. Degrees of freedom = $n + n - 2$.
      2. 1-tailed tests are for directional alternative hypotheses, such as "I wonder whether girls sleep more than boys, on average."
      3. 2-tailed tests are for non-directional alternative hypotheses, such as "I wonder whether boys and girls differ in the amount they sleep, on average."
   ii. If the observed *t* value exceeds the critical *t* value, the result is considered statistically significant.
      1. The researcher rejects the null hypothesis and concludes that the difference between the sample means represents a genuine difference between the population means.

f. Effect size

   i. Calculate a Cohen's *d* statistic to determine the effect size and whether the results have *practical* significance.

   ii. Calculate a confidence interval for the *difference* between the sample means to find the range within which the difference between the population means is likely contained.

V. Dependent (i.e., Paired) Samples *t* Test

a. To determine whether the paired means of two variables, or one variable measured twice, differ significantly.
   i. E.g., 1: Is a sample of adult daughters taller, on average, than their mothers?

1. Two paired samples compared on one variable: Height.

ii. E.g., 2: I wonder whether a sample of adults in the U.S. is willing to donate more money to charity before or after watching a video about how donations help people.

1. One sample measured twice on the same variable.

b. How it works: As with the independent samples *t* test, the formula for the dependent, or paired, *t* test has the difference between the two means as the numerator and the standard error of the difference between the means in the denominator.

i. The calculation of the standard error involves calculating the difference between each pair of scores, squaring them, summing these squared differences, then dividing by the degrees of freedom and taking the square root.

c. Once the observed *t* value has been calculated, the same procedures as described above for the independent *t* test are used to determine whether the result is statistically significant.

i. All is the same, except the degrees of freedom is the number of pairs of scores minus one.

d. Effect size

i. Calculate a Cohen's *d* statistic to determine the effect size and whether the results have *practical* significance.

ii. Calculate a confidence interval for the *difference* between the sample means to find the range within which the difference between the population means is likely contained.

VI. Summary

a. *t* tests are used quite often to test a variety of hypotheses about several statistics, including to compare means of a sample and a population, two independent samples, and of paired samples.

b. *t* tests are based on a family of *t* distributions that are similar to the normal distribution but vary depending on the size of the sample(s) involved in the analysis.

c. One-sample, independent samples, and dependent samples (i.e., paired) *t* tests all rely on the same basic formula for calculating the observed *t* value: Difference between the means in the numerator, standard error in the denominator.

d. Statistically significant results reveal that the difference between the sample means represents a meaningful difference between the population means.

e. Effect sizes and confidence intervals for the differences between the means provide information about the practical significance and magnitude of the effects.