Chapter 5: Standardization and *Z* Scores

I. Standardization

   a. Standardizing scores is the process of converting each raw score in a distribution to a *z* score (or standard deviation units)
      i. Raw Score: the individual observed scores on measured variables

II. *z* Scores (also known as a *standard scores*)

   a. Helps to understand where a score lies in relation to other scores in the distribution
      i. Indicates how far above or below the mean a given score in the distribution is in standard deviation units.
         1. For example, if you know that an individual in a sample has a *z* score of .75, you would know that the individual's score was .75 standard deviations above the mean for that sample.

   b. Calculated using mean and standard deviation
      i. *z* = (raw score - mean) / standard deviation

   c. Using *z* scores to determine probabilities
      i. You can calculate a *z* score using either sample data OR population data
         1. You can only calculate percentiles using Appendix A when you know...
            a. The population standard deviation, or
            b. The sample data are normally distributed
         2. *z* scores let you compare performances on two measures with different scales of measurement
            a. e.g. height and weight, grade point average and standardized test scores

   d. *z* scores can also be calculated for the difference between a sample mean and a population mean, in *standard error* units. (This is discussed further in Chapter 6.)

III. Calculating Probabilities using *z* Scores and the Normal Distribution

   a. With normal distribution of scores, you can calculate probabilities
      i. Example: Given a distribution with a mean = 100 and a standard deviation = 15
         1. What is the probability of getting an IQ score of 130 or higher?
         2. What is the probability of getting an IQ score between 90 and 100?
         3. Process: Find *z* score
            a. (raw score - mean) / standard deviation
            b. Look up probability in Appendix A
         4. Answer: Probability of getting a score of 130 or higher = 0.0228
            a. Probability of getting a score between 90 and 100 = 0.2486

   b. The *standard normal distribution* refers to the normal distribution divided into standard deviation units, with the proportion of the normal distribution that is contained within each standard deviation unit.

IV. Finding the Raw Score that Marks a Given Percentile of a Normal Distribution

a. Percentile Score: A score that divides a distribution in two at a particular percentile

b. Process
    i. Find z value from Appendix A
    ii. Apply Formula: $X = (\mu) + (z) * (s)$

c. Example 1: What score marks the 20th percentile of a distribution with a mean of 100 and a standard deviation of 15?
    i. Answer: As per Appendix A, z value is between <u>*.84*</u> and <u>*.85*</u>
        1. X = 100 – (.845 * 15) = <u>*87.325*</u>
        2. Therefore, the raw score that marks the 20th percentile of this distribution is <u>*87.325*</u>

V. Converting a Raw Score into a Percentile Score

a. Step 1: Convert raw score into z score using formula

b. Step 2: Find the percentage of the normal distribution that falls beyond the z score

c. Step 3: State Findings
    i. Example: Suppose the average SAT score for males is 517, with standard deviation of 100, forming normal distribution
    ii. Suppose a student has a raw score = 425
    iii. How can you convert the raw score into a percentile score?
        1. Note: The percentile score will be below 50% because the raw score is below the mean score.
        2. $z = (425\text{-}517)/100$
        3. $z = -88/100$
        4. $z = -0.88$ *In Appendix A, find the area beyond z. We can see in Appendix A that .8106 falls below a z score of .88, so 1 - .8106 = .1894. This is the proportion that falls beyond a z score of .88. Because the normal distribution is symmetrical, this is also the proportion that falls beyond a z score of -.88.*
        5. Percentile = 0.1894 or 18.94% of the population would be expected to get an SAT score of 425 *or lower*.

VI. Finding the Proportion of Scores Between Two Raw Scores

a. Process: Convert raw scores into z scores
    i. Find z score from Appendix A for each raw score
    ii. Find corresponding proportions for each
        1. If they are on opposite sides of the mean, ADD
        2. If they are on the same side of the mean, SUBTRACT

b. Example: Proportion of the scores expected to fall between 110 and 140 in a population with a mean of 100, standard deviation of 15?

c. Answer: z scores are .67 and 2.67
    i. Proportion between mean and 0.67 = 0.2486
    ii. Proportion between mean and 2.67 = 0.4962
    iii. Proportion between 110 and 140: .4962 - .2486 = 0.2494

VII. Finding the Extreme Scores in a Distribution

a. Process: Divide the extreme percentile by 2 (e.g., “Extreme 10% divided by 2 is 5% at the top, 5% at the bottom”)
    i. Apply formula: X = (mean) ± (z score) * (standard deviation)

b. Example: What scores mark the extreme 10% of distribution with a mean of 100 and a standard deviation of 15?
    i. As per Appendix A, z score is between *1.64* and *1.65*, for a score of *1.645*
    ii. Scores: $= 100 + [(1.645) * (15)] = 124.675$
    $= 100 - [(1.645) * (15)] = 75.325$

VIII. Summary

a. Standardization and standard scores (i.e., z scores) are very important in statistics. They allow researchers to put variables that are originally measured using different scales of measurement onto the same scale. They also allow researchers to calculate probabilities and percentile scores that are based on the normal distribution.