Work Problems Chapter 5

Suppose that you know that in the population of full-time employees in the United States, the average number of vacation days taken off of work per year is 10 with a standard deviation of 4. Please answer the following questions using this information, assuming that the number of vacation days taken forms a normal distribution.

1. What percentage of the population takes at least 7 vacation days off per year?

   $z = (7-10)/4 \rightarrow -3/4 = -.75$. This is a negative number. From Appendix A, we can see that .7734 of the distribution falls *below* a z score of .75. Because the normal distribution is symmetrical, the positive side of the distribution is a mirror image of the negative side. So if 77.34% of the normal distribution falls *below* a z score of .75, that same percentage will fall *above* a z score of -.75. So our answer is 77.34% of the normal distribution would be expected to take *at least* 7 days of vacation per year.

2. What is the number of vacation days that marks the 30^{th} percentile of this distribution?

   $X = 10 - (4)(.525) = 10 - 2.10 = 7.90$

   I already know the mean and the standard deviation. To complete the formula, I must find the z value that is associated with the 30^{th} percentile. The 30^{th} percentile means that 30 percent of the distribution will fall *below* this score. But this value will be *below* the mean, which would produce a negative z value. So we use the fact that the normal distribution is symmetrical to find the z value that is associated with 30% of the distribution falling *above* it, and 70% falling below. In Appendix A, we look for the proportion of the distribution closest to .70. We can see it is between a z score of .52 (proportion equals .6985) and a z score of .53 (proportion equals .7019). The average of .52 and .53 is .525. So I plug this z value into the formula.

   Note: The 30^{th} percentile is BELOW the mean, so you know your answer should be something below the mean of 10.

3. What proportion of the distribution takes between 6 and 10 vacation days off per year?

   $z = (6-10)/4 \rightarrow -4/4 = -1.00$. I know that 50% of the distribution falls below the mean of 10, and I know that the normal distribution is symmetrical. If I can find the proportion of the distributions that falls below a z score of 1.00 and subtract .50 from that, I'll have the proportion that falls between the mean and a z score of 1.00. In Appendix A, I find that the proportion of the normal distribution that falls below a z score of 1.00 is .8413. Subtract .50 from that and I get .8413 - .50 = .3413. So the proportion of the normal distribution that takes between 6 and 10 days of vacation is .3413.

4. What percentage of the distribution takes between 6 and 15 vacation days off per year?

   From the previous problem, we know that .3413 of the distribution falls between the mean (10) and 6.

   Now we just need to figure out the proportion that falls between the mean and 15 and add that proportion to .3413.

   ***z* = (15 – 10) / 4 → 5/4 = 1.25. The proportion of the normal distribution that falls below a *z* score of 1.25 is .8944. If we subtract .50 (the area below the mean) from this, we get .8944 - .50 = .3944.**

   This is the proportion that falls between 10 and 15. Now we add the two proportions together: .3944 + .3413 = .7357. This is the proportion of the normal distribution that falls between the scores of 6 and 15.

5. Suppose that you randomly select an individual from the population who takes 20 vacation days off per year. Calculate that person's *z* score and interpret it. What does it tell you?

   ***z* = (20 – 10)/4 → 2.50. This individual's score (i.e., number of vacation days taken) is 2.50 standard deviations above the mean.**

6. What is the probability of randomly selecting an individual from the population who takes 8 vacation days or more off?

   ***z* = (8 – 10)/4 = -.50. The proportion the normal distribution that falls below a *z* score of .50 is .6915 (see Appendix A). Because the normal distribution is symmetrical, if .6915 falls below *z* score of .50, .6915 also falls *above* (i.e., to the right) of a *z* score of -.50. So the answer is .6915.**