

Work Problems Chapter 3

Suppose that I wanted to know something about the age of customers at my new pizza palace. I take the first 6 people that walk through the door and ask their age in years. I get the following scores:

6 20 34 42 50 56

1. Calculate and report the range of this distribution.

$$56 - 6 = 50$$

2. Calculate and report the variance of this distribution.

First we need to calculate the mean: $208/6 = 34.67$

Then we calculate squared deviations for each score:

$$\begin{aligned}(6 - 34.67)^2 &= 821.97 \\ (20 - 34.67)^2 &= 215.21 \\ (34 - 34.67)^2 &= 0.45 \\ (42 - 34.67)^2 &= 53.73 \\ (50 - 34.67)^2 &= 235.01 \\ (56 - 34.67)^2 &= \underline{454.97} \\ \text{SUM: } &1781.33\end{aligned}$$

Then we divide this sum of squared deviations by $n - 1$: $1781.33/5 = 356.27$. This is the variance.

3. Calculate and report the standard deviation of this distribution.

The standard deviation is just the square root of the variance.

$$s = \sqrt{356.27} = 18.88. \text{ This is the standard deviation of the sample.}$$

4. Write a sentence or two in which you explain, using your own words, what a standard deviation is (in general, not this particular standard deviation).

A standard deviation is roughly the average difference between the individual scores in a distribution and the average for the distribution. A sample standard deviation is an estimate of the population standard deviation.

5. Pretend that this distribution of scores represents a population rather than a sample. Calculate and report the standard deviation and explain what it tells you.

We can use the sum of the squared deviations from Question 2: 1781.33

Divide this by N and we get $1781.33/6 = 296.89$.

Find the square root of this variance: $\sqrt{296.89} = 17.23$. This is the standard deviation of the population. In this population, the average difference between the scores in the distribution and the population mean is 17.23 years.

6. Explain why the standard deviation you reported for Question 3 differs from the standard deviation you reported for Question 5.

In Question 3 we divided the sum of squares by $n - 1$ because it was sample data, producing a larger variance and standard deviation than we found in Question 5, when we used N as the denominator in the variance and standard deviation formulas.

7. Think of an example that illustrates why it is important to know about the standard deviation of a distribution.

There are many possible answers for this one. As a teacher, one example that is often on my mind is student evaluations of my teaching. If I get an average rating of 4 on a scale from 1 to 5, I want to know whether almost all of my students rated me a 4 (producing a small standard deviation) or is most of my students rated me a 5 but a few really hated me and gave me a 1 (producing a larger standard deviation). Did all of my students feel lukewarm about my teaching or did most of them love me and a few of them hate me?