Work Problems Chapter 7

Suppose that in the population of adults in the United States, the average number of hours spent per year riding a bicycle is 20. I select a random sample of 36 adults from Ohio to see if their average amount of time riding a bike differs from the population in the U.S. The mean for the Ohio sample is 18 with a standard deviation of 3. Please answer the following questions using this information.

1. Write the null and alternative hypotheses for this problem, once using symbols and again in sentence form.

   Null: The population mean of Ohio equals the population mean of the U.S. ($\mu_{\text{-OH}} = \mu_{\text{-U.S.}}$)

   Alternative: The population mean of Ohio does not equal the population mean of the U.S. ($\mu_{\text{-OH}} \neq \mu_{\text{-U.S.}}$)

2. Is the difference between the sample mean and the population mean statistically

   significant?

   We do not know the population standard deviation, so we will calculate a *t* value here.

   First, we need to calculate the standard error of the mean: S.E. = $3/\sqrt{36} \rightarrow 3/6 = .50$

   Next, we calculate the *t* value: $t = (18 - 20)/.50 = -2/.5 = -4.00$.

   With *df* = 35, alpha level of .05, and a 2-tailed test, the critical value of *t* is ± 2.042. (Note that we used 30 degrees of freedom in Appendix B because that was the closest, but smaller, value to our *df* of 35. Because 35 is exactly between 30 and 40 *df*, we could have used the average of 2.042 and 2.021, 2.0315. But out observed *t* value was well beyond our critical *t* value, so no need to be quite so precise with the critical *t* value.

   Because our observed *t* value exceeds our critical *t* value, the result is statistically significant. On average, people in Ohio spend less time riding bikes than do the general population of Americans.

3. Calculate a Cohen's *d* statistic and interpret it. Would this be considered a small, moderate, or medium effect size?

   First, we need to convert the standard error into a standard deviation:

s = .5√36 = .5(6) = 3

Next, we plug that standard deviation into the formula for *d:*

***d* = (18 – 20) / 3 = -2/3 = -.67. This would be considered a fairly large effect size.**

4. Calculate two confidence intervals and explain what they indicate.

    a. First, calculate a 95% confidence interval for the sample mean.

CI_{95} = 18 ± (.5)(2.042)

CI_{95} = 18 ± 1.021

CI_{95} = 16.979, 19.021. We are 95% confident that the Ohio population mean is contained within the interval between 16.979 and 19.021 hours per year spent riding a bike.

(NOTE: We used the critical *t* value with *df* = 30, 2-tailed, alpha = .05)

    b. Then calculate a 99% confidence interval for the difference between the sample mean and the population mean, assuming the population mean is actually 20 hours of bike riding per year.

CI_{99} = (18 – 20) ± (.5)(2.750)

CI_{99} = –2 ± 1.375

CI_{99} = -3.375, -.625 We are 99% confident that the difference between the Ohio population mean and the U.S. population mean is contained within the interval between Ohioans riding their bikes an average of 3.375 to .625 more hours per year than does the American population, on average.

(NOTE: We used the critical *t* value with *df* = 30, 2-tailed, alpha = .01)

5. What, exactly, does a *p* value tell you?

It tells us the probability of obtaining our result *by chance*, or random sampling error.

6. What does it mean to say that a result is statistically significant?

7. **It means that the probability of obtaining our result *by chance* was smaller than our alpha level. ($p < \alpha$). It also means that we are deciding to reject the null hypothesis and conclude that our results *did not* occur by chance.**

8. If we know that a result is statistically significant, why should we calculate an effect size and a confidence interval? What additional information is gained from these statistics?

   We calculate an effect size to decide whether the difference between the means is large enough to have any practical (i.e., real-world) significance. Tests of statistical significance can be heavily influenced by sample size, with larger samples producing statistically significant results even for small differences between the means. Practical significance takes sample size out of the equation, letting us know about the difference between the means in standard deviation units. This is a useful metric for deciding whether the results are large enough to care about. In addition, confidence intervals provide useful information about the potential range of the population parameter (e.g., the mean), which provides information about how well our sample statistic represents the population parameter.

   a. Please discuss the difference between practical significance and statistical significance in this answer.

   Statistical significance is simply an indication of whether the result obtained from the sample data was due to chance, or random sampling error. Practical significance, which is not strongly influenced by sample size, provides information about whether the results provide useful, or important, information in the real world. How large is the observed effect in the sample data?

9. How does sample size affect whether a result is statistically significant?

   Larger samples produce smaller standard errors. Smaller standard errors (which are the denominator in many statistics) produce larger *t* values and *z* scores. Larger *t* and *z* values are less likely to occur by chance, and therefore more likely to be considered statistically significant.