Q1

July 23, 2020

<IPython.core.display.HTML object>

1 Question 1. Regularization (33 points)

In this problem, you build a set of different models to classify different forest types based on their spectral characteristics at visible-to-near infrared wavelengths observed by ASTER satellite imagery over a forest area in Ibaraki Prefecture, Japan (36° 57 N, 140° 38 E). The training data can be found in "training.csv" and test data can be found in "testing.csv". Attribute Information:

4 classes: * 's' ('Sugi' forest) * 'h' ('Hinoki' forest) * 'd' ('Mixed deciduous' forest) * 'o' ('Other' non-forest land)

27 features: * b1 - b9: ASTER image bands containing spectral information in the green, red, and near infrared wavelengths for three days (Sept. 26, 2010; March 19, 2011; May 08, 2011).

- pred_minus_obs_S_b1 pred_minus_obs_S_b9: Predicted spectral values (based on spatial interpolation) minus actual spectral values for the 's' class (b1-b9).
- pred_minus_obs_H_b1 pred_minus_obs_H_b9: Predicted spectral values (based on spatial interpolation) minus actual spectral values for the 'h' class (b1-b9).

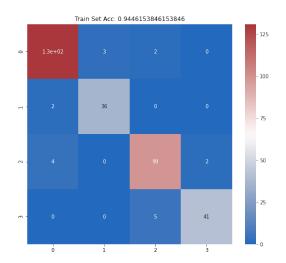
1.1 1.) LOGISTIC REGRESSION

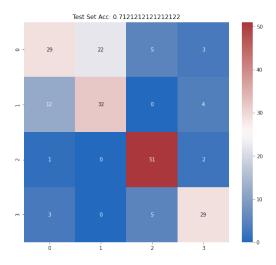
Run a multinomial logistic regression to classify different forest. Present the coefficients obtained. Present the confusion matrix on the test set.

```
[2]: #read in the data
     train = pd.read_csv('Q1_training.csv')
     test = pd.read_csv('Q1_testing.csv')
[3]: train.head()
[3]:
       class
               b1
                    b2
                        b3
                              b4
                                  b5
                                        b6
                                              b7
                                                       b9
                                                               pred_minus_obs_H_b9
                                                  b8
           d
                        68
                                                  31
                                                                               -9.17
     0
               67
                    51
                             115
                                  69
                                       111
                                             136
                                                       67
                                                                               -2.25
     1
               67
                    28
                        51
                              99
                                  50
                                        97
                                              82
                                                  26
                                                       59
     2
           s
               63
                    26
                        50
                              95
                                  49
                                        91
                                              81
                                                  26
                                                       57
                                                                               -0.44
     3
           d
               63
                    42
                        63
                              97
                                  66
                                       108
                                             111
                                                  28
                                                       59
                                                                               -2.34
     4
                    27
                        50
                                        90
                                                  26
                                                                                1.25
               46
                              83
                                  51
                                              76
                                                       56
           S
        pred_minus_obs_S_b1
                                pred_minus_obs_S_b2
                                                        pred minus obs S b3
                                                -1.80
                                                                        -6.32
     0
                       -18.27
                                                                        -6.35
                       -20.13
                                                -2.11
     1
     2
                       -17.64
                                                -1.81
                                                                        -4.70
     3
                                                                        -5.47
                       -20.20
                                                -1.89
     4
                       -18.62
                                                -2.17
                                                                        -7.11
        pred_minus_obs_S_b4
                                pred_minus_obs_S_b5
                                                        pred_minus_obs_S_b6
                                                                        -6.13
     0
                       -20.88
                                                -1.63
                                                -1.22
                                                                        -6.13
     1
                       -21.94
     2
                       -19.39
                                                -0.65
                                                                        -5.01
     3
                       -21.65
                                                                        -5.71
                                                -0.99
     4
                       -21.12
                                                -1.56
                                                                        -6.35
        pred_minus_obs_S_b7
                                pred_minus_obs_S_b8
                                                        pred_minus_obs_S_b9
     0
                       -22.56
                                                -5.53
                                                                        -8.11
     1
                       -22.20
                                                -3.41
                                                                        -6.57
     2
                       -20.89
                                                -3.96
                                                                        -6.85
     3
                       -22.19
                                                -3.41
                                                                        -6.52
     4
                       -22.19
                                                -4.45
                                                                        -7.32
     [5 rows x 28 columns]
[4]:
    test.head()
[4]:
                                                               pred_minus_obs_H_b9
       class
               b1
                    b2
                        b3
                              b4
                                  b5
                                        b6
                                              b7
                                                  b8
                                                       b9
                                  59
                                                  27
                                                                              -2.36
     0
           d
               39
                    36
                        57
                              91
                                       101
                                              93
                                                       60
     1
               84
                    30
                        57
                             112
                                  51
                                        98
                                              92
                                                  26
                                                       62
                                                                              -2.26
           h
                                                  26
     2
                    25
                        49
                              99
                                  51
                                        93
                                              84
                                                       58
                                                                              -1.46
               53
           s
     3
               59
                    26
                        49
                             103
                                  47
                                        92
                                              82
                                                  25
                                                       56
                                                                               2.68
```

```
-2.94
     4
         d 57 49 66 103 64 106 114 28 59 ...
       pred_minus_obs_S_b1 pred_minus_obs_S_b2 pred_minus_obs_S_b3 \
     0
                     -18.41
                                           -1.88
                                                                -6.43
                    -16.27
                                           -1.95
                                                                -6.25
     1
                                                                -4.64
     2
                    -15.92
                                           -1.79
                     -13.77
                                           -2.53
                                                                -6.34
     3
     4
                                                                -4.62
                     -21.74
                                           -1.64
       pred_minus_obs_S_b4 pred_minus_obs_S_b5 pred_minus_obs_S_b6 \
     0
                     -21.03
                                                                -6.18
                                           -1.60
     1
                    -18.79
                                           -1.99
                                                                -6.18
     2
                    -17.73
                                           -0.48
                                                                -4.69
                                                                -6.60
     3
                     -22.03
                                           -2.34
     4
                    -23.74
                                           -0.85
                                                                -5.50
       pred_minus_obs_S_b7 pred_minus_obs_S_b8 pred_minus_obs_S_b9
     0
                                           -5.20
                                                                -7.86
                     -22.50
                    -23.41
                                           -8.87
     1
                                                               -10.83
     2
                     -19.97
                                           -4.10
                                                                -7.07
     3
                     -27.10
                                           -7.99
                                                               -10.81
     4
                     -22.83
                                           -2.74
                                                                -5.84
     [5 rows x 28 columns]
[5]: #create a class mapper to convert target variable to integers
     classes = ('s', 'h', 'd', 'o')
     #encoder - string class to integer
     mapper = {x:pos for pos,x in enumerate(classes)}
     #decoder - integer class to string
     decoder = {pos:x for pos,x in enumerate(classes)}
[6]: # separate independent and dependent variables
     X_train = train.iloc[:,1:].values
     y_train = train.loc[:,'class'].apply(lambda x: mapper[x.strip()]).values
     X_test = test.iloc[:, 1:].values
     y_test = test.loc[:, 'class'].apply(lambda x: mapper[x.strip()]).values
[7]: #scale the train and test data
     scaler = StandardScaler()
     scaler.fit(X_train)
     X_train_scaled = scaler.transform(X_train)
     X test scaled = scaler.transform(X test)
```

```
[8]: #instantiate the classifier - turn off penalty to ensure no regularization
      #set it to multinomial for multi class model
      clf = LogisticRegression(penalty='none', random_state=42, max_iter=50_000,__
       →multi_class='multinomial', fit_intercept=True)
 [9]: #fit the classifier with the training data
      clf.fit(X_train_scaled, y_train)
 [9]: LogisticRegression(C=1.0, class_weight=None, dual=False, fit_intercept=True,
                         intercept_scaling=1, l1_ratio=None, max_iter=50000,
                         multi_class='multinomial', n_jobs=None, penalty='none',
                         random_state=42, solver='lbfgs', tol=0.0001, verbose=0,
                         warm_start=False)
[10]: #predict on train and test set
      pred_train = clf.predict(X_train_scaled)
      pred_test = clf.predict(X_test_scaled)
[11]: #make confusion matrices
      confusion_train = confusion_matrix(y_train, pred_train)
      confusion_test = confusion_matrix(y_test, pred_test)
      #get accuracy scores
      acc_train = accuracy_score(y_train, pred_train)
      acc_test = accuracy_score(y_test, pred_test)
[12]: | plt.subplots(nrows=1, ncols=2, figsize=(20,8))
      plt.subplot(121)
      sns.heatmap(confusion_train, annot=True, cmap='vlag')
      plt.title('Train Set Acc: {}'.format(acc_train))
      plt.subplot(122)
      sns.heatmap(confusion_test, annot=True, cmap='vlag')
      plt.title('Test Set Acc: {}'.format(acc_test))
      plt.show()
```





[13]: #coefficients (rows = features, columns = Class) pd.DataFrame(clf.coef_.T).rename(columns=decoder)

```
[13]:
                                             d
                    S
                                                          0
      0
           22.224101
                        -1.302074
                                     -8.258062
                                                -12.663966
      1
                        77.870134
                                    -50.755042
                                                -79.478259
           52.363167
      2
          -37.773756
                       -48.202318
                                     30.328013
                                                  55.648060
      3
           -4.028496
                        -3.922334
                                      3.051031
                                                   4.899798
      4
         -382.051600 -117.486297
                                    151.175764
                                                348.362133
      5
          168.678234
                         6.637064
                                    -37.272095 -138.043204
      6
          -21.475885
                         8.330583
                                      0.967618
                                                  12.177684
                                                -73.929716
      7
          117.919863
                       -26.546668
                                    -17.443479
         -145.021416
                       -41.425883
                                     53.259859
                                                133.187440
      8
      9
           19.831919
                        -6.826307
                                     -3.046437
                                                  -9.959175
      10
           47.588132
                        92.370554
                                    -53.117671
                                                -86.841015
          -31.014359
                       -47.937564
                                     23.527098
                                                 55.424826
      11
      12
           -0.461811
                        -5.121338
                                      1.755859
                                                   3.827290
      13 -379.175832 -115.552594
                                    148.815355
                                                345.913072
      14
          168.471272
                         5.226777
                                    -36.325404 -137.372645
                                     -1.185658
      15
          -19.879987
                         7.503203
                                                  13.562442
          117.782400
                       -15.882980
                                    -21.302216
                                                -80.597203
      17 -147.145592
                       -44.399188
                                     57.055426
                                                134.489353
      18
            2.256094
                        -3.754036
                                     -0.048547
                                                   1.546488
      19
                                     -0.054149
                                                   0.960502
            2.404994
                        -3.311347
      20
           -1.999296
                         2.184821
                                     -1.080859
                                                   0.895333
      21
                        -0.889348
                                      1.888789
                                                  -1.285923
            0.286482
      22
                        -3.351117
           -4.101728
                                      3.284430
                                                  4.168415
      23
            7.601662
                         1.959644
                                     -2.295833
                                                  -7.265473
      24
            0.400051
                         5.815209
                                     -4.239893
                                                  -1.975367
      25
            5.895548
                         8.229928
                                     -7.382851
                                                  -6.742625
```

1.2 2.) RIDGE LOGISTIC REGRESSION

Use ridge multinomial logistic regression to classify different forest. Explain why we use ridge regression. Present the optimal tuning parameter obtained using cross-validation, the coefficients for this parameter and the confusion matrix for the test set.

```
[14]: #search for hyperaramter from 0.0001 to 10
      alphas = np.linspace(0.0001, 10, 100)
      #instantiate Ridge classifier
      clf = RidgeClassifierCV(alphas=alphas, cv=5)
[15]: #fit the classifier with the training data
      clf.fit(X_train_scaled, y_train)
[15]: RidgeClassifierCV(alphas=array([1.00000000e-04, 1.01109091e-01, 2.02118182e-01,
      3.03127273e-01,
             4.04136364e-01, 5.05145455e-01, 6.06154545e-01, 7.07163636e-01,
             8.08172727e-01, 9.09181818e-01, 1.01019091e+00, 1.11120000e+00,
             1.21220909e+00, 1.31321818e+00, 1.41422727e+00, 1.51523636e+00,
             1.61624545e+00, 1.71725455e+00, 1.81826364e+00, 1.91927273e+00,
             2.02028182e+00, 2.1...
             8.08082727e+00, 8.18183636e+00, 8.28284545e+00, 8.38385455e+00,
             8.48486364e+00, 8.58587273e+00, 8.68688182e+00, 8.78789091e+00,
             8.88890000e+00, 8.98990909e+00, 9.09091818e+00, 9.19192727e+00,
             9.29293636e+00, 9.39394545e+00, 9.49495455e+00, 9.59596364e+00,
             9.69697273e+00, 9.79798182e+00, 9.89899091e+00, 1.00000000e+01]),
                        class weight=None, cv=5, fit intercept=True, normalize=False,
                        scoring=None, store cv values=False)
[16]: #get the best paramter and retrain model using all training observations with
      \rightarrowbest parameter
      best_alpha = clf.alpha_
      clf = RidgeClassifier(alpha=best_alpha, max_iter=50_000, random_state=42)
      clf.fit(X_train_scaled,y_train)
[16]: RidgeClassifier(alpha=2.6263363636364, class_weight=None, copy_X=True,
                      fit_intercept=True, max_iter=50000, normalize=False,
                      random_state=42, solver='auto', tol=0.001)
[17]: #predict on train and test set
      pred_train = clf.predict(X_train_scaled)
      pred_test = clf.predict(X_test_scaled)
```

```
[18]: #make confusion matrices
      confusion_train = confusion_matrix(y_train, pred_train)
      confusion_test = confusion_matrix(y_test, pred_test)
      #qet accuracy scores
      acc_train = accuracy_score(y_train, pred_train)
      acc_test = accuracy_score(y_test, pred_test)
[19]: plt.subplots(nrows=1, ncols=2, figsize=(20,8))
      plt.subplot(121)
      sns.heatmap(confusion_train, annot=True, cmap='vlag')
      plt.title('Train Set Acc: {}'.format(acc_train))
      plt.subplot(122)
      sns.heatmap(confusion_test, annot=True, cmap='vlag')
      plt.title('Test Set Acc: {}'.format(acc_test))
      plt.show()
                  Train Set Acc: 0.8492307692307692
                                                              Test Set Acc: 0.8686868686868687
[20]: #coefficients (rows = features, columns = Class)
```

```
pd.DataFrame(clf.coef_.T).rename(columns = decoder)
```

```
[20]:
        0.318479 -0.347377 -0.095102 0.124000
     1
     2
        0.250846 -0.065822 -0.108316 -0.076707
     3 -0.315205 0.216851 -0.016188 0.114541
     4 -0.044972 -0.045038 0.047629 0.042380
       0.208850 -0.301712 0.217241 -0.124379
     6 -0.218114 0.046421 0.271023 -0.099330
     7 -0.212579 -0.011921 0.209987 0.014514
     8 0.066341 -0.021506 -0.239206 0.194370
```

```
9
   10 0.041477 0.174915 -0.107648 -0.108744
11 0.817632 -0.315010 -0.528766 0.026144
13 0.189392 -0.158677 0.103722 -0.134437
14
   0.419416 -0.256289 -0.007712 -0.155415
15 0.051944 0.087483 -0.266326 0.126900
16 -0.090577  0.008514  0.052028  0.030035
17 -0.285317 -0.064763 0.521317 -0.171237
18 0.200506 -0.157174 -0.162007 0.118674
19
   0.039114 0.007050 -0.107414 0.061250
20 0.010368 -0.015943 0.077813 -0.072237
21 -0.065267  0.066382  0.060218 -0.061333
23 0.164011 -0.080596 -0.130412 0.046996
24 -0.132086  0.057422  0.089429 -0.014764
25 0.035121 -0.066322 -0.092364 0.123565
26 0.000551 0.050378 0.088519 -0.139448
```

```
[21]: print('Optimal tuning parameter: {}\nSklearn interprets this value as 1/alpha<sub>□</sub> 

→in other models - 1/C: {}'.format(clf.alpha, (1/clf.alpha)))
```

```
Optimal tuning parameter: 2.6263363636364
Sklearn interprets this value as 1/alpha in other models - 1/C: 0.3807585402409855
```

Why Ridge? We use Ridge regression to prevent training a model that is of high variance. A high variance model is one in which the learned parameters reflect the trends in the training data plus the randomness that is also inherit in the training data. Therefore it fails to generalize to unseen data. We see this in part 1.) where the training accuracy is 94% but the test data accuracy is only 71%. After applying Ridge regression in this step we can see that the training accuracy has dropped to 85% but the test accuracy has jumped to 86% - meaning we have improved the model's capability to generalize

1.3 3.) LASSO LOGISTIC REGRESSION

Use lasso multinomial logistic regression to classify different forest. Explain why we use lasso. Present the optimal tuning parameter obtained using cross-validation, the coefficients for this parameter and the confusion matrix on the test set.

```
[22]: #search for hyperaramter from 0.0001 to 10

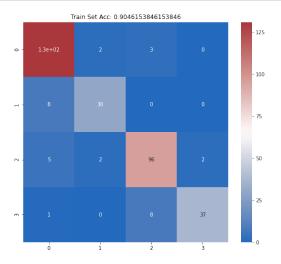
alphas = np.linspace(0.0001, 10, 100)

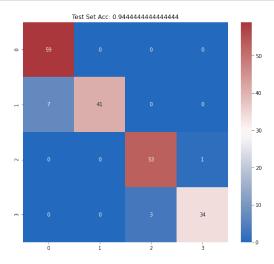
#instantiate Lasso classifier by setting penalty to L1 and solver to saga and

class to mulitnomial
```

```
clf = LogisticRegressionCV(penalty='l1', solver='saga', Cs=alphas, cv=5,_
       →max_iter=50_000, random_state=42, multi_class='multinomial')
[23]: #fit the classifier with the training data
      clf.fit(X_train_scaled, y_train)
[23]: LogisticRegressionCV(Cs=array([1.00000000e-04, 1.01109091e-01, 2.02118182e-01,
      3.03127273e-01,
             4.04136364e-01, 5.05145455e-01, 6.06154545e-01, 7.07163636e-01,
             8.08172727e-01, 9.09181818e-01, 1.01019091e+00, 1.11120000e+00,
             1.21220909e+00, 1.31321818e+00, 1.41422727e+00, 1.51523636e+00,
             1.61624545e+00, 1.71725455e+00, 1.81826364e+00, 1.91927273e+00,
             2.02028182e+00, 2.12...
             9.29293636e+00, 9.39394545e+00, 9.49495455e+00, 9.59596364e+00,
             9.69697273e+00, 9.79798182e+00, 9.89899091e+00, 1.00000000e+01]),
                           class_weight=None, cv=5, dual=False, fit_intercept=True,
                           intercept_scaling=1.0, l1_ratios=None, max_iter=50000,
                           multi_class='multinomial', n_jobs=None, penalty='l1',
                           random_state=42, refit=True, scoring=None, solver='saga',
                           tol=0.0001, verbose=0)
[24]: #get the best paramter and retrain model using all training observations with
       \rightarrowbest parameter
      best C = clf.C[0]
      clf = LogisticRegression(C=best_C, penalty='l1', solver='saga',__
      →multi_class='multinomial', max_iter=50_000, random_state=42)
      clf.fit(X_train_scaled,y_train)
[24]: LogisticRegression(C=0.5051454545454546, class_weight=None, dual=False,
                         fit_intercept=True, intercept_scaling=1, l1_ratio=None,
                         max_iter=50000, multi_class='multinomial', n_jobs=None,
                         penalty='11', random_state=42, solver='saga', tol=0.0001,
                         verbose=0, warm_start=False)
[25]: #predict on train and test set
      pred_train = clf.predict(X_train_scaled)
      pred_test = clf.predict(X_test_scaled)
[26]: #make confusion matrices
      confusion_train = confusion_matrix(y_train, pred_train)
      confusion_test = confusion_matrix(y_test, pred_test)
      #qet accuracy scores
      acc_train = accuracy_score(y_train, pred_train)
      acc_test = accuracy_score(y_test, pred_test)
```

```
[27]: plt.subplots(nrows=1, ncols=2, figsize=(20,8))
   plt.subplot(121)
   sns.heatmap(confusion_train, annot=True, cmap='vlag')
   plt.title('Train Set Acc: {}'.format(acc_train))
   plt.subplot(122)
   sns.heatmap(confusion_test, annot=True, cmap='vlag')
   plt.title('Test Set Acc: {}'.format(acc_test))
   plt.show()
```





```
[28]: #coefficients (rows = features, columns = Class)
pd.DataFrame(clf.coef_.T).rename(columns=decoder)
```

```
[28]:
                           h
                                     d
      0
          1.145691
                   0.000000 -1.027796
                                        0.000000
          0.000000 -2.918075
                              0.000000
                                        0.021869
      1
          0.000000 0.000000
                              0.000000
      2
                                        0.000000
      3
         -0.619734 0.864181
                              0.000000
                                        0.000000
      4
          0.000000
                   0.000000
                              0.000000
                                        0.528908
      5
          0.000000
                   0.000000
                              0.000000
                                        0.156077
      6
          0.000000
                   0.000000
                              1.241439
                                        0.000000
      7
                   0.000000
                              0.000000
          0.000000
                                        0.673786
          0.000000 0.000000
                              0.000000
                                        0.982671
          0.000000 -2.122027
                              0.000000
      9
                                        0.000000
      10
         0.352270 0.000000 -0.742273
                                        0.000000
                                        0.000000
      11
          3.034156 0.000000
                              0.000000
      12
         0.000000
                   0.000000
                              0.000000
                                        0.113993
      13
                   0.000000
         0.425488
                              0.000000
                                        0.000000
      14
          0.574503
                    0.000000
                              0.000000
                                        0.000000
      15
          0.000000
                    0.000000
                              0.000000
                                        0.000000
      16
         0.000000
                   0.968612
                              0.000000
                                        0.000000
```

```
17
          0.000000 0.000000 0.910250
                                        0.000000
      18
         0.325084 -0.524258
                             0.000000
                                        0.000000
      19
         0.000000 0.000000 -0.384115
                                        0.000000
      20
         0.111678
                   0.000000
                             0.000000
                                        0.000000
      21 -0.097382 0.000000
                             0.151162
                                        0.000000
      22
         0.000000
                   0.000000
                              0.159336 -0.006871
      23
         0.000000
                   0.000000
                              0.000000
                                       0.000000
      24
         0.000000
                   0.000000
                              0.000000
                                        0.000000
      25
         0.139475
                   0.000000
                             0.000000
                                        0.000000
         0.000000
                   0.000000
                             0.000000 -0.212842
      26
[29]:
     print('Optimal tuning parameter: {}'.format(clf.C))
```

Optimal tuning parameter: 0.50514545454546

Why Lasso? We use lasso for feature importance and dimensionality reduction. The original input to the model had 27 features plus bias term (28 total features). That is a high dimensional complex model. By utilizing Lasso regularization, we are able to identify those parameters that bear no consequence to the dependent variable and set their coefficient to 0. This reduces dimensionality and therefore reduces complexity. For example, in the coefficients presented above, for class 3, we went from a 27 dimensional model to a 8 dimensional model by applying Lasso and removing the non important features. In doing so, we have increased model to performance to 90% and 94% accuracy for the train and test sets respectively

1.4 4.) ADAPTIVE LOGISTIC REGRESSION

Use adaptive lasso multinomial logistic regression to classify different forest. Explain why we use adaptive lasso regression. Present the optimal tuning parameter obtained using cross-validation, the coefficients for this parameter and the confusion matrix on the test set.

```
[30]: #according to docs this library only works with py3 and linux for those who⊔
want to run this code
#also need lib fortran
# sudo apt-get install libgfortran3

from glmnet_python import cvglmnet, glmnetCoef, glmnetPlot,⊔
→glmnetPredict
import scipy as sp
```

```
[31]: gamma = 2

[32]: #get coeffs from Logistic Regression
lr_clf = LogisticRegression()
lr_clf.fit(X_train_scaled, y_train)

b_lr = lr_clf.coef_
```

```
[33]: #qet best coeff from Ridge CV by using best tunning param from RidgeCV
      #search for hyperaramter from 0.0001 to 10
      alphas = np.linspace(0.0001, 10, 100)
      #instantiate Ridge CV classifier
      ridge_clfcv = RidgeClassifierCV(alphas=alphas, cv=5)
      ridge_clfcv.fit(X_train_scaled, y_train)
      #get best tuning param
      l_ridge = ridge_clfcv.alpha_
      #get best coeff from Ridge
      ridge_clf = RidgeClassifier(alpha=l_ridge, max_iter=50_000, random_state=42)
      ridge_clf.fit(X_train_scaled, y_train)
      b_ridge = ridge_clf.coef_
[34]: #w1 and w2
      w1 = sp.float64([1/np.abs(x)**gamma for x in b_lr])
      w2 = sp.float64([1/np.abs(x)**gamma for x in b_ridge])
[35]: #find best lambdas for each model
      alasso1 = cvglmnet(x=X_train_scaled.copy().astype(sp.float64), y=y_train.copy().
       →astype(sp.float64), family='multinomial', alpha=1, standardize=False, 
      →penalty_factor=w1.copy(), mtype='grouped')
      alasso2 = cvglmnet(x=X_train_scaled.copy().astype(sp.float64), y=y_train.copy().
       →astype(sp.float64), family='multinomial', alpha=1, standardize=False,
       →penalty_factor=w2.copy(), mtype='grouped')
      #lambda 1 and lambda2
      lambda1 = alasso1['lambda_min']
      lambda2 = alasso2['lambda_min']
[36]: #fit lasso models
      alasso1 = glmnet(x=X_train_scaled.copy().astype(sp.float64), y=y_train.copy().
       →astype(sp.float64), family='multinomial', alpha=1, penalty_factor=sp.
       →float64(w1.copy()), mtype='grouped')
      alasso2 = glmnet(x=X_train_scaled.copy().astype(sp.float64), y=y_train.copy().
       →astype(sp.float64), family='multinomial', alpha=1, penalty_factor=sp.
       →float64(w2.copy()), mtype='grouped')
[37]: #predict using the fitted models and the optimal lambdas found in cross__
       \rightarrow validation
      #logistic regression weight penalties
      pred_train1 = glmnetPredict(alasso1, newx=X_train_scaled.copy().astype(sp.
       →float64), ptype='class',s=lambda1)
```

```
[38]: #make confusion matrices
    confusion_train1 = confusion_matrix(y_train, pred_train1)
    confusion_train2 = confusion_matrix(y_train, pred_train2)

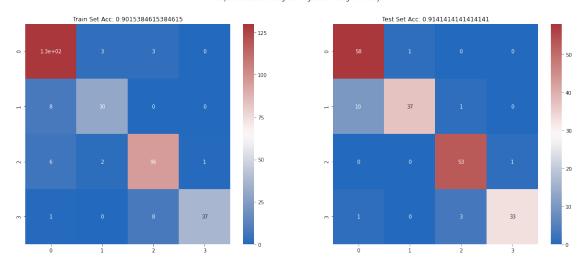
confusion_test1 = confusion_matrix(y_test, pred_test1)
    confusion_test2 = confusion_matrix(y_test, pred_test2)

#get accuracy scores
    acc_train1 = accuracy_score(y_train, pred_train1)
    acc_train2 = accuracy_score(y_train, pred_train2)

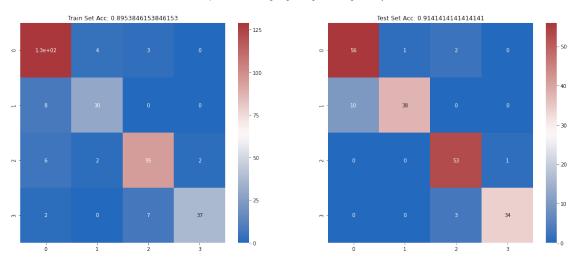
acc_test1 = accuracy_score(y_test, pred_test1)
    acc_test2 = accuracy_score(y_test, pred_test2)
```

```
[39]: plt.subplots(nrows=2, ncols=2, figsize=(20,8))
      plt.subplot(121)
      sns.heatmap(confusion_train1, annot=True, cmap='vlag')
      plt.title('Train Set Acc: {}'.format(acc_train1))
      plt.subplot(122)
      sns.heatmap(confusion_test1, annot=True, cmap='vlag')
      plt.title('Test Set Acc: {}'.format(acc_test1))
      plt.suptitle('Adpative Lasso 1 - Logistic Regression Weight Penalty')
      plt.show()
      plt.subplots(nrows=1, ncols=2, figsize=(20,8))
      plt.subplot(121)
      sns.heatmap(confusion_train2, annot=True, cmap='vlag')
      plt.title('Train Set Acc: {}'.format(acc_train2))
      plt.subplot(122)
      sns.heatmap(confusion_test2, annot=True, cmap='vlag')
      plt.title('Test Set Acc: {}'.format(acc_test2))
      plt.suptitle('Adpative Lasso 2 - Ridge Logistic Regression Weight Penalty')
      plt.show()
```

Adpative Lasso 1 - Logistic Regression Weight Penalty



Adpative Lasso 2 - Ridge Logistic Regression Weight Penalty



```
[41]: print('Best lambda LR Adaptive: {}\nBest lambda Ridgde Adaptive: {}'.

→format(lambda1[0], lambda2[0]))
```

```
Best lambda LR Adaptive: 0.14344714715106677
Best lambda Ridgde Adaptive: 0.08312375690553696
```

Why Adaptive? Adaptive LASSO is said to have the oracle property where an oracle estimator has both consistent parameter estimation and variable selection. When using simple LASSO, the shrinkage parameter must be larger for selection than for prediction. Also large nonzero parameters will be too small so that bias ends up being too large. Thus LASSO is only consistent for variable selection nder some conditions on the shrinkage parameters. Therefore Aapative LASSO acts as a "secondary stage" of estimation which helps control the bais of LASSO estimates.

1.5 5.)

Which model you select? Why?

Answer

Adaptive LASSO effectively handled the bis/variance trade off while reducing overall model complexity. It also achieved the highest accuracy on the test set without the need for intermediate steps (as required in Adaptive LASSO). Therefore Adaptive LASSO is the winner