Q1

July 17, 2020

[1]:
```
from scipy.io import loadmat
import matplotlib.pyplot as plt
import numpy as np
from IPython.core.display import display, HTML
display(HTML("<style>.container { width:100% !important; }</style>"))
```

```
<IPython.core.display.HTML object>
```

[2]:
```
data = loadmat('allFaces.mat')
```

[3]:
```
faces = data['faces'][:,:64]
w = int(data['n'][0])
h = int(data['m'][0])
```

[4]:
```
faces.shape
```

[4]:
```
(32256, 64)
```

[5]:
```
plt.figure()
plt.imshow(faces[:,0].reshape((h,w)).T, cmap='gray', vmin=0, vmax=255)
plt.axis('off')
plt.show()
```

[6]:
```python
def rPCA(D, lam=1e-2, tol=1e-3):
    m,n = D.shape

    Y = D.copy()
    norm2 = np.linalg.norm(Y, 2)
    normInf = Y.max()/lam

    Y = Y / normInf

    A_hat = np.zeros((m,n))
    E_hat = np.zeros_like(A_hat)

    mu = 1.25 / norm2
    mu_bar = mu*1e7
    rho = 1.5
    d_norm = np.linalg.norm(D, 'fro')
    iter_ = 0
    total_svd = 0
    converged = False
    while not converged:
        iter_ += 1
        X = D - A_hat + Y/mu
        # temp_T = D-A_hat+ Y/mu
        # Soft threshold on scalars (this implementation matches slides better)
        E_hat = np.sign(X)*np.maximum(np.abs(X)-lam/mu, 0)
        # Equivalent, as per MATLAB code
        # E_hat = np.maximum(X-lam/mu,0)
```

```
        # E_hat = E_hat+np.minimum(X+lam/mu,0)
        # Soft threshold SVD
        u, s, vh = np.linalg.svd(D-E_hat+Y/mu,full_matrices=False)
        svp = (s > 1/mu).sum()
        A_hat = (u[:, :svp]*(s[:svp]-1/mu))@vh[:svp]  # reconstruct
        total_svd += 1
        Z = D-A_hat-E_hat  # Constraint violation
        Y = Y + mu*Z  # update lagragian multipliers
        mu = min(mu*rho, mu_bar)  # mu is augmented penalty coefficient
        # % stop Criterion
        stopCriterion = np.linalg.norm(Z, 'fro') / d_norm
        if stopCriterion < tol:
            converged = True

    return A_hat, E_hat
```

[7]:
```
a,e = rPCA(faces)
```

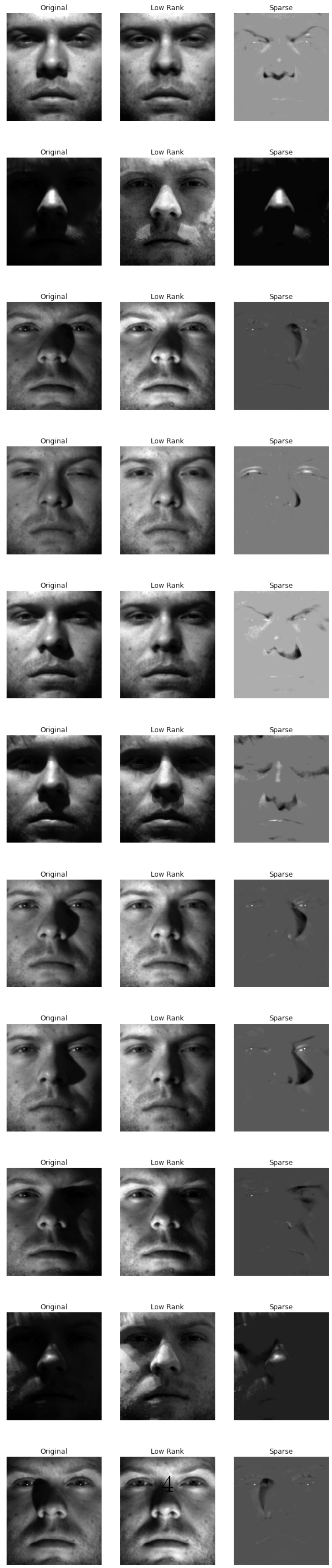
[10]:
```
def plotit(main, low, sparse, cols, figsize):
    plt.subplots(nrows=len(cols), ncols=3, figsize=figsize)
    for pos,col in enumerate(cols):

        base = pos * 3
        plt.subplot(len(cols), 3, base+1)
        plt.imshow(main[:,col-1].reshape((h,w)).T, cmap='gray', vmin=0,␣
↪vmax=255)
        plt.axis('off')
        plt.title('Original')

        plt.subplot(len(cols), 3, base+2)
        plt.imshow(low[:,col-1].reshape((h,w)).T, cmap='gray')
        plt.axis('off')
        plt.title('Low Rank')

        plt.subplot(len(cols), 3, base+3)
        plt.imshow(sparse[:,col-1].reshape((h,w)).T, cmap='gray')
        plt.axis('off')
        plt.title('Sparse')
    plt.show()
```

[11]:
```
plotit(faces,a,e,cols=(3,4,14,15,17,18,19,20,21,32,43), figsize=(10,50))
```



Original
Low Rank
Sparse
Original
Low Rank
Sparse
Original
Low Rank
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Original
Low Rank
Sparse
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Original
Low Rank
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Original
Low Rank
Sparse

0.1 Results

rPCA really shows it promises for the images that are of very low pixel intensity. Image 4 and 32, which is mostly black, has the entire face revealed upon utilizing rPCA. I also found it interesting that the algorithm is intelligent enough to remove the glare (sparkle) from the camera flash in the eyes of the original image when viewing the Low Rank response.It even manages to highlight facial imperfections (freckles, pimples, etc) when being hidden by shadow casting on the original image

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[ ]:
```