

Lec 16 Math 300 4/2/11

Non-linear linear models fit with OLS

- polynomial terms eg  $x^2$  term: <sup>quadratic</sup> slope increasing in  $x$  linearly.
- log terms  $y = b_0 + b_1 \ln(x)$  this means that  $b_1$  represents a percent increase in the change of one unit in  $x$ . How?

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \approx x$$

$$\Rightarrow \ln(x) = \ln(x+1) - 1 \approx x - 1$$

$$\text{e.g. } \ln(1.07) = .019 \approx .02$$

$$x_f = 1.07 \\ x_0 = 1.00$$

% increase

$$\frac{x_f}{x_0} - 1 = 0.07 = 7\%$$

$$b_1 \Delta \ln(x) = b_1 (\ln(x_f) - \ln(x_0)) = b_1 \ln\left(\frac{x_f}{x_0}\right) = b_1 \left(\frac{x_f}{x_0} - 1\right)$$

$b_1$ : change in  $y$  for 100% change in  $x$  (doubling)

Some variable work this way!

• Interactions

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_1 x_2$$

$$= b_0 + (b_1 + b_3 x_2) x_1 + b_2 x_2$$

the slope of  $x_1$  is linear in  $x_2$  with coefficient  $b_3$

much more expressible here...

$$X = \begin{bmatrix} 1 & \ln(x_{11}) \\ \vdots & \vdots \\ 1 & \ln(x_{n1}) \end{bmatrix}$$

$$p=3, df=4$$

$$\begin{bmatrix} 1 & x_{11} & x_{22} & x_{11}x_{22} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & x_{n1}x_{n2} \end{bmatrix}$$