Discrete Mathematics, 2016 Spring - Worksheet 3

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In all of the above problems explain your answer in full English sentences.

1. Disprove the following statements:
   (a) If a and b are integers with $a|b$, then $a \leq b$.
   (b) If a, b, and c are positive integers with $a|(bc)$, then $a|b$ or $a|c$.
   (c) If p and q are prime, then $p + q$ is composite.
   (d) Two right triangles have the same area if and only if the lengths of their hypotenuses are the same.
2. What does it mean for an if and only if statement to be false? What properties should a counterexample for an if-and-only-if statement have?
3. Evaluate the following Boolean expressions:
   (a) $True \wedge True \wedge True \wedge True \wedge False$.
   (b) $(\neg True) \vee True$.
   (c) $\neg(True \vee True)$.
   (d) $(True \vee True) \wedge False$.
   (e) $True \vee (True \wedge False)$.
4. Prove the following Boolean identities by truth tables:
   (a) $\neg(x \wedge y) = (\neg x) \vee (\neg y)$ and $\neg(x \vee y) = (\neg x) \wedge (\neg y)$ (DeMorgan's laws).
   (b) $x \to y = (\neg x) \vee y$.
   (c) $x \leftrightarrow y = (\neg x) \leftrightarrow (\neg y)$.

   where $=$ stands for logically equivalent.
5. Find a logically equivalent Boolean expression to $x \leftrightarrow y$ only in terms of the basic Boolean operations $\wedge, \vee, \neg$.
6. How many different binary operation can there be?

7. A person's inititals are the two-element lists consisting of the initial letters of their first and last names. For example, mines are ZP.
   (a) How many possible initials are there?
   (b) How many initials are there where the two letters are different?
8. A club has 10 members.
   (a) A club has 10 members who wish to elect a president and a vice-president. How many ways can these positions be filled (assuming the club is not a cult-of personality dictatorship and one person can only have one title)?
   (b) Now suppose the club also wants to elect a secretary and a treasurer. How many outcomes are there for the election then?
9. In how many ways can a black rook and a white rook be placed on different squares of a chess board such that neither is attacking the other?

Further problems to practice:

10. License plates in a cetrtain state consist of six characters: The first three characters are uppercase letters (A-Z), and the last three characters are digits (0-9).
    (a) How many license plates are possible?
    (b) How many license plates are possible if no character maybe repeated on the same plate?
11. A telephone number (in the US and Canada) is a ten digit number whose first digit cannot be a 0 or a 1. How many telephone numbers are possible?
12. A US Social Security number is a nine-digit number. The first digit may be zero.
    (a) How many of these are even?
    (b) How many have all of their digits even?
    (c) How many read the same backwards? (e.g. 122979221)