Discrete Mathematics, 2016 Fall - Worksheet 18

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In all of the above problems explain your answer in full English sentences.

1. Solve the single congruence
$$7k + 1 \equiv 3 \pmod{11}$$
2. Solve the following system of equation
$$x \equiv 4 \pmod{5}, \qquad x \equiv 7 \pmod{11}$$
3. Factor the following positive integers into primes.
   (a) 25
   (b) 4200
   (c) 10^{10}
   (d) 19
   (e) 1
4. Let a and b be positive integers. Prove that a and b are relatively prime if and only if there is no prime p such that $p|a$ and $p|b$.
5. Let a and b be positive integers. Prove that 2^a and $2^b - 1$ are relatively prime by considering their prime factorizations.
6. Prove that if $a, p \in \mathbb{Z}$ with p prime and $p|a^2$, then $p|a$.

Optional programming exercise:
Write a function that factorizes integers into primes, i.e. one that for a given integer input n returns a list containing the prime factors of n (a prime factor can appear in the list multiple times).
Look up the sieve of Erastothenes on Wikipedia and implement that algorithm. Compare the speed of this algorithm with your approach for large n.