Discrete Mathematics, 2016 Spring - Worksheet 2

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In all of the above problems explain your answer in full English sentences.

1. Recast the following statements in the $if - then$ form.

   (a) The product of an odd integer and an even integer is even.

   (b) The square of a prime number is not a prime.

   (c) The product of two negative integers is negative.

   (d) The sum of three consecutive integers is divisible by three.

2. Consider the claim: 'If a guinea pig has a tail, its eyes are blue'. True or False? (Hint: Guinea pigs don't have tails.)

3. Below you will find pairs of statements A and B. For each pair, please indicate which of the following three sentences are true and which are false:

   - If A, then B.
   - If B, then A.
   - A if and only if B.

   You may just write True or False.

   (a) A: $x > 0$, B: $x^2 > 0$.

   (b) A: $x < 0$, B: $x^3 < 0$.

   (c) A: $xy = 0$, B:$x = 0$ or $y = 0$.

   (d) A: $xy = 0$ B: $x = 0$ and $y = 0$

4. Consider the two statements:

   (a) If A, then B.

   (b) If (not B), then not A.

   Under what circumstances are these statements true? When are they false? Explain why these statements are, in essence, identical.

5. Write a proof of the following result:

   Proposition 1. *Let a, b, and c be integers. If $a|b$ and $b|c$, then $a|c$.*

6. Write a proof of the following result:

   Proposition 2. *Let x be an integer. Then x is even if and only if $x + 1$ is odd.*

7. Using Proposition 1, write a proof of the following result:

   Proposition 3. *Let a, b, c, and d be integers. If $a|b$, $b|c$, and $c|d$, then $a|d$.*

8. Write a proof for the following statements:

   (a) The sum of two odd integers is even.

   (b) If n is an odd integer, then $-n$ is also odd.

   (c) The product of an even integer and an odd integer is even.

9. Suppose you are asked to prove a statement of the form 'A iff B'. The standard method is to prove both $A \Rightarrow B$ and $B \Rightarrow A$. Consider the following alternative proof strategy: Prove both $A \Rightarrow B$ and (not$A$) $\Rightarrow$ (not$B$). Explain why this would give a valid proof.