Discrete Mathematics, Sect 002, 2016 Fall - Quiz 4

October 2, 2016

Name:

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This quiz is scheduled for 10 minutes. No outside notes or calculators are permitted. To get full credit in all of the problems, use rigorous justification and unless otherwise indicated, make sure that your solution reads as a perfect English sentence. You should only assume the notion of integers, operations, order relations and geometrical objects as given. If you use a statement or a definition from the textbook, make sure to indicate it.

1. (10 points) Identify whether the following relation on the set $\{1,\ 2,\ 3,\ 4\}$ is reflexive, irreflexive, symmetric, antisymmetric, and/or transitive. Is it an equivalence relation? When the answer is no, support why.

$$R = \{(1,2),(3,2),(4,3),(2,3),(3,4),(2,1)\}$$

- Reflexive:No, there are elements not in relation with themselves (actually none of them are)
- Irreflexive: Yes
- Antisymmetric: No, e.g $(1,2)$ and $(2,1)$ are in R but clearly $1 \neq 2$.
- Transitive: No, e.g. $(1,2)$ and $(2,1)$ are in R but $(1,1)$ is not.
- Equivalence relation: No, it's not reflexive nor transitive so it cannot be an equivalence relation.
- Symmetric: Yes

2. (10 points) 20 people are to be divided into two teams with ten players on each team. In how many ways can this be done? Define an equivalence relation on the set of all arrangements of the 20 people and then count how many elements are in the equivalence classes.

Let A be the set of all arrangements of the 20 people. Then $|A| = 20!$. Define an equivalence relation R on A under which xRy provided the first and the last 10 people are the same for both x and y but perhaps differently arranged or when the first and last 10 people are swapped between x and y (and perhaps also differently arranged within themselves). If we choose the teams by selecting the first 10 people and the last 10 people into the two teams respectively, then two arrangements are equivalent under R exactly if they give the same two teams.

Every equivalence class consists of $|[.]| = 10! \cdot 10! \cdot 2$ arrangements and therefore the number of possible different teams is

$$\frac{|A|}{[.]} = \frac{20!}{2(10!)^2}$$