MATH-UA.120.001 - Discrete Mathematics

Midterm 2

Monday, November 14, 2016

This exam is scheduled for 110 minutes, to be done individually, without calculators, notes, textbooks, and other outside materials.

Show all work to receive full credit, except where specified.

Name:

NYU NetID (email):

I pledge that I have observed the NYU honor code, and that I have neither given nor received unauthorized assistance during this examination.

Signature:

Problem	Points
TF	/20
1	/20
2	/10
3	/20
4	/30
Total	/100

Answer Sheet (for Problem 1 True/False)

You must record your final answers on the following answer sheet. Only this page will be graded.

1	(T) (F)
2	(T) (F)
3	(T) (F)
4	(T) (F)
5	(T) (F)
6	(T) (F)
7	(T) (F)
8	(T) (F)
9	(T) (F)
10	(T) (F)

TOTAL

Make sure that you have marked your final answers on the answer sheet on page 2. Only answers on that page will be graded.

True or False

(2 points each) Determine if each of the following statement is TRUE or FALSE.
Choose TRUE if the statement is always true, and choose FALSE if it is not always true. Each question is worth 3 points.

Indicate your solution in the answer sheet on page 2. You need not provide any justification.

1. The set $[1, 2] \cup \mathbb{Z}$ satisfies the well ordering principle.

2. Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 3, 4, 5, 6\}$. Then the relation $\{(1, 5), (2, 2), (3, 1), (4, 2)\}$ is a one to one function $f : A \to B$.

3. Let A and B be finite sets. Assume $f : A \to B$ is onto and that there is a function $g : B \to A$ that is onto. Then f is a bijection.

4. $(h \circ g \circ f)^{-1} = f^{-1} \circ g^{-1} \circ h^{-1}$

5. For any five points in the unit square $[0, 1] \times [0, 1]$, the distance between the two closes points must be less than or equal to $\frac{1}{\sqrt{2}}$.

6. There is no onto function from $\mathbb{N}$ to the set of all (finite or infinite) sequences of the natural numbers where no number appears more than once.

7. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ such that g is one-to-one and f is onto. Then $f \circ g$ is necessarily onto.

8. Suppose S is a set of 8 integers. There exist distinct $a, b \in S$ such that $a - b$ is a multiple of 7.

9. A triangle that is isosceles but not equilateral (two sides are the same length, but the third one is different has 2 symmetries.

10. Suppose $f, g : \mathbb{R} \to \mathbb{R}$ are defined by

$$f(x) = x^2 + 4x - 2, \qquad g(x) = x + 2$$

Then $(f \circ g)(x) - (g \circ f)(x) = 2x^2 - 4x + 4$.

Make sure that you have marked your final answers on the answer sheet on page 2. Only answers on that page will be graded.

Free Response

Show all work and justification.

1. (20 points) Solve the following recurrence relations.

   (a) $a_n = 4a_{n-1} - 3$; $a_0 = -1$.

   (b) $a_n = 6a_{n-1} - 9a_{n-2}$; $a_0 = -1$, $a_1 = 2$.

2. (10 points) Suppose a_n is a polynomial sequence with athe following entries.

$$a_0 = 10, \qquad a_1 = 14, \qquad a_2 = 14, \qquad a_3 = 10,$$

$$a_4 = 2, \qquad a_5 = -10, \qquad a_6 = -26.$$

Find a_{100}.

3. (20 points)

   (a) (10 points) Show that for all $n \geq 2$,

$$\prod_{i=2}^{n}\left(1-\frac{1}{i^2}\right)=\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\dots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}.$$

   (b) (10 points) Prove that if $f:\{1,2,\dots\}\to\{1,2,\dots\}$ is a function and

$$f(n+1)>f(n), \qquad \forall n\geq 1$$

   then $f(n)\geq n$ for every $n\geq 1$.

4. (a) (10 points) Write the cycle decomposition of $(2,4,5,3)(1,6)\circ(1,5,2,6)(3,4)$. Also write this permutation as a product of transpositions. Is this permutation even or odd?

   (b) (10 points) Recall that S_{11} is the set of all permutations on the set $\{1,2,3,\ldots,11\}$. Define the function $f : S_{11} \to S_{11}$ by the formula

$$f(\pi) = (1,2,3)\circ\pi.$$

   Prove that f is an injective function.

   HINT: What is the inverse of $(1,2,3)$?

   (c) (10 points) Let now $g : S_{11} \to S_{11}$ by the formula

$$g(\pi) = \pi\circ\pi$$

   Show that g is not one to one.

   HINT: Transpositions.

Extra paper