MATH-UA.120.001 - Discrete Mathematics

Final Exam 2016 Fall

Thursday, Dec 17, 2016

This exam is scheduled for 110 minutes, to be done individually, without calculators, notes, textbooks, and other outside materials.

Show all work to receive full credit, except where specified.

Name:

NYU NetID (email):

I pledge that I have observed the NYU honor code, and that I have neither given nor received unauthorized assistance during this examination.

Signature:

Problem	Points
TF	/20
1	/20
2	/15
3	/20
4	/25
Total	/100

Answer Sheet (for Problem 1 True/False)

You must record your final answers on the following answer sheet. Only this page will be graded.

1	T F
2	T F
3	T F
4	T F
5	T F
6	T F
7	T F
8	T F
9	T F
10	T F

TOTAL

Make sure that you have marked your final answers on the answer sheet on page 2. Only answers on that page will be graded.

True or False

(2 points each) Determine if each of the following statement is TRUE or FALSE.
Choose TRUE if the statement is always true, and choose FALSE if it is not always true. Each question is worth 3 points.

Indicate your solution in the answer sheet on page 2. You need not provide any justification.

1. There are integers x and y satisfying the equation $15x + 18y = 9$.

2. There is an element of $\mathbb{Z}_{12}$ that does not have a reciprocal. (Hint: Trying to compute the inverse of all elements is a very inefficient approach.)

3. Prime factorization of integers is easy and therefore it is easy to break public key encryption.

4. If G is a subgraph of H, then $\alpha(G) \leq \alpha(H)$.

5. A group is a pair $(G, *)$ where G is a set and $*$ is an operation on it that is commutative but not necessarily associative.

6. No tree with more than one vertex has an Eulerian trail.

7. Let G be a graph. Then either G or $\bar{G}$ must be connected.

8. If a graph has the same number of vertices as edges then it is a cycle.

9. There is a graph with exactly one vertex with an odd degree.

10. $\chi(G) \leq \Delta(G)$ for all graphs G.

Make sure that you have marked your final answers on the answer sheet on page 2. Only answers on that page will be graded.

Free Response

Show all work and justification.

1. (a) (3 points) Give the precise definition of the greatest common divisor of two positive numbers a and b.

   (b) (7 points) Use Euclid's algorithm to compute $gcd(230, 123)$. (No partial credit will be given for guessing.)
   HELP: $107 = 6 \cdot 16 + 11$

   (c) (7 points) Compute the reciprocal of 123 in $\mathbb{Z}_{230}$.

   (d) (3 points) Solve the equation $123 \otimes x = 2$ in $\mathbb{Z}_{230}$.

2. In this problem we prove the following proposition.

   Proposition. *Prove that consecutive perfect squares are relatively prime.*

   (a) (5 points) First show that if there are two numbers a and b such that no prime divides them both, then $gcd(a, b) = 1$.

   (b) (8 points) Assume FTSC that there are consecutive perfect squares and a prime that divides them both. Reach a contradiction.
   HINT: Can you drop the square in $p|a^2$?

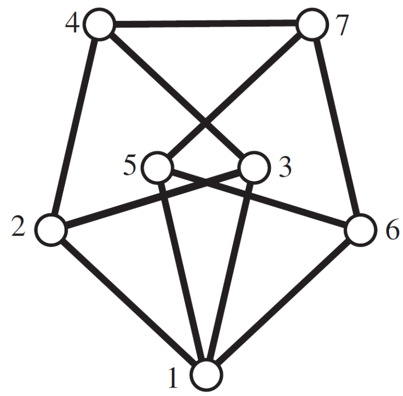
   (c) (2 points) Conclude from this that the Proposition is true.

3. (20 points) Find all simultaneous solution of the following pair of congruences:

$$x \equiv 5 \pmod{9}$$
$$x \equiv 7 \pmod{16}.$$

4. **Graph theory**

Consider the following graph G:



(a) (4 Points) Draw the subgraph $G[1, 4, 5, 6]$ induced on the vertex set $\{1, 2, 4, 5, 6\}$.

(b) (4 Points) Find the minimum and maximum degree $\delta(G)$ and $\Delta(G)$.

(c) (2 Points) Find a cut edge in $G[1, 2, 4, 5, 6]$.

(d) (4 Points) Does G contain a spanning tree (with justification) and if yes find it (Draw. WARNING: Practice on the sketch paper, if there are multiple versions here, it will not be graded.)!

(e) (2 Points) Is this graph Eulerian? (with justification) If yes, exhibit an Euler tour (Draw).

(f) (4 Points) Show that this graph is 4 colorable by exhibiting such a coloring. (Draw the graph again and indicate the colors by labels. WARNING: Practice on the sketch paper, if there are multiple versions here, it will not be graded.)

(g) (5 Points) Show that this graph is not 3 colorable by assuming that it is and then arguing to a contradiction. HINT: Assign a color to vertex one and then discuss the colors of the other vertices in numerical order.

Extra paper