# Discrete Mathematics, Section 001, Fall 2016

## Lecture 1: Introduction, Definitions

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Courant Institute of Mathematical Sciences

September 7, 2016

# Outline

1. Introduction
   - Welcome to discrete mathematics
   - Assessment
   - Course policies
2. Definitions

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| **Office** | WWH 1105A |
| **Office hours** | Mon 7:50-8:50am, 4:00-5:00pm |
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**Textbook:**
Scheinerman, *Mathematics: A Discrete Introduction. (3rd Ed)*

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- Write clear mathematical statements using standard notation and terminology.
- Understand and execute a variety of proof techniques (contradiction, induction, etc.).
- Show fluency in the language of basic set theory and Boolean logic.
- Understand the basic theorems and their implications in a variety of (discrete) fields including:
  - Function theory
  - Number theory
  - Graph theory

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- Final (25%): Date: December 19, 8:00-10:50am

# Grades

$$FS = 0.25 \cdot HW\% + 0.1 \cdot Q\% + 0.2 \cdot M1\% + 0.2 \cdot M2\% + 0.25 \cdot F\%$$

| **Cutoff** | **Letter Grade** |
|---|---|
| 93 | A |
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| 87 | B+ |
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- No letter grades are assigned to any individual midterms.

# Qualifying reasons for accommodations

- Religious holidays.
- University sponsored event.
- Illness.

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- NO ACCOMMODATION FOR MORE CONVENIENT TRAVEL!

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  - Use the internet, but cautiously.

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- **Post-rigorous:**
  - Comfortable with rigorous foundations of one's field.
  - Intuition is back but this time well-founded in rigorous theory.
  - Easy transition between intution and rigorous arguments.

# Why do rigorous math?

**This class is the first step in**

$$\text{Pre-rigorous} \quad \rightarrow \quad \text{Rigorous}$$

This is usually rather traumatic but here is why you should do it:

- Pre-rigorous approach becomes inadequate as complexity of the subject increases.
- We need to be able to tell if a statement is true, even if it is not obvious by immediate intuition.
- This can be done through a chain of obviously true implications from an obviously true statement (Proof).

$$\text{True statement} \xrightarrow{\text{logical implication}} \text{True statement}$$

# Outline

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For example:

**Definition**

An integer is called **even** provided it is divisible by two.

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- This is done in an unambiguous way, in terms of previously well defined mathematical objects using English.

For example:

**Definition**

An integer is called **even** provided it is divisible by two.

What are integer, divisible, two?

# Fundamental objects (for this class)

Some objects and their properties have to be considered given.

- Numbers:
  - **Natural numbers:** $\mathbb{N} = \{0, 1, 2, 3, \dots\}$
  - **Integer numbers:** $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
  - **Rational numbers:** $\mathbb{Q}$
  - **Real numbers:** $\mathbb{R}$
- Operations: $+, \cdot$.
- Order relations: $<, \leq, >, \geq, =$.

If you want to dig deeper: Mathematical logic

**Definition**

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An integer is called **even** provided it is divisible by two.

- No need to define integer or two.
- However, what does divisible mean?

e.g. Is 3 divisible by 2?

$$\text{'}3 \quad \text{divided by} \quad 2 \quad \text{is} \quad \frac{3}{2}.\text{'}$$

But this is not what we want!

**Definition**

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For example:

- Is 12 divisible by 4?

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- 12 is divisible by 4 because there is an integer 3 such that $4 \cdot 3 = 12$.

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- Is 12 divisible by 5?

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For example:

- 12 is divisible by 4 because there is an integer 3 such that $4 \cdot 3 = 12$.
- 12 is not divisible by 5 because there is no integer $x$ for which $5x = 12$.

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## Terminology and notation

If $a$ is divisible by $b$, we write $b|a$ and also say

- $b$ divides $a$,
- $b$ is a factor of $a$,
- $b$ is a divisor of $a$.

**Definition**

An integer is called **even** provided it is divisible by two.

What about odd numbers?

**Alternative 1**

An integer $a$ is called **odd** provided it is not even.

**Alternative 2 (We go with this one)**

An integer $a$ is called **odd** provided there is an integer $x$ such that $a = 2x + 1$.

That these two are equivalent is a statement that requires verification!

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**Definition**

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For example:

- Is 12 even?

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**Definition**

An integer $a$ is called **odd** provided there is an integer $x$ such that $a = 2x + 1$.

For example:

- 12 is even because $2 \cdot 6 = 12$.

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- 12 is even because $2 \cdot 6 = 12$.
- Is 13 odd?

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**Definition**

An integer $a$ is called **odd** provided there is an integer $x$ such that $a = 2x + 1$.

For example:

- 12 is even because $2 \cdot 6 = 12$.
- 13 is odd because $2 \cdot 6 + 1 = 13$.

# Prime numbers, composite numbers

**Definition**

An integer $p$ is called **prime** provided $p > 1$ and the only positive divisors of $p$ are 1 and $p$.

For example:

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- Is 1 a prime number?

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- 1 is not a prime because $1 \not> 1$.

# Prime numbers, composite numbers

**Definition**

A positive integer $a$ is called **composite** provided there is an integer $b$ such that $1 < b < a$ and $b|a$.

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For example:

- Is 12 a composite number?

# Prime numbers, composite numbers

**Definition**

A positive integer $a$ is called **composite** provided there is an integer $b$ such that $1 < b < a$ and $b|a$.

For example:

- 12 is a composite number because e.g. $1 < 3 < 12$ and $3|12$.

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- Is 360 a composite number?

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For example:

- 12 is a composite number because e.g. $1 < 3 < 12$ and $3|12$.
- 360 is a composite number as e.g. $1 < 180 < 360$ and $180|360$.
- Is 1 a composite number?

# Prime numbers, composite numbers

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A positive integer $a$ is called **composite** provided there is an integer $b$ such that $1 < b < a$ and $b|a$.

For example:

- 12 is a composite number because e.g. $1 < 3 < 12$ and $3|12$.
- 360 is a composite number as e.g. $1 < 180 < 360$ and $180|360$.
- 1 is not a composite number as there is no number $b$ such that $1 < b < 1$.

# Prime numbers, composite numbers

**Definition**

An integer $p$ is called **prime** provided $p > 1$ and the only positive divisors of $p$ are 1 and $p$.

**Definition**

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Comparison:

- A prime $p$ is not composite as there is no integer $b$ with $1 < b < p$ with $b|p$.

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Comparison:

- A prime $p$ is not composite as there is no integer $b$ with $1 < b < p$ with $b|p$.
- A composite $a$ is not a prime as there is an integer $b \neq 1, a$ such that $b|a$.

# Prime numbers, composite numbers

**Definition**

An integer $p$ is called **prime** provided $p > 1$ and the only positive divisors of $p$ are 1 and $p$.

**Definition**

A positive integer $a$ is called **composite** provided there is an integer $b$ such that $1 < b < a$ and $b|a$.

Comparison:

- A prime $p$ is not composite as there is no integer $b$ with $1 < b < p$ with $b|p$.
- A composite $a$ is not a prime as there is an integer $b \neq 1, a$ such that $b|a$.
- However, 1 is neither a prime, nor a composite.