Discrete Mathematics, 2016 Fall - Worksheet 23

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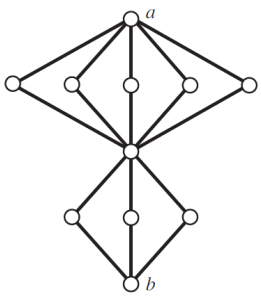
In all of the above problems explain your answer in full English sentences.

1. Let G be the graph in the figure.



   (a) How many different paths are there from a to b?

   (b) How many different walks are there from a to b?

2. Prove that K_n is connected.

3. Suppose G is a connected graph in which each vertex has even degree. Then, G has no cut edges.

4. List all the trees

   (a) with vertex set $\{1, 2, 3\}$

   (b) with vertex set $\{1, 2, 3, 4\}$

5. a) Let T be a tree with $n \geq 1$ vertices. Prove that T has $n - 1$ edges.

   b) Prove the converse, i.e. that if G is a connected graph that has exactly $n - 1$ edges then G must be a tree.

6. Let T be a tree. Prove that the average degree of a vertex in T is less than 2.