

Discrete Mathematics, 2016 Spring - HW 10

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To get full credit in all of the problems, use rigorous justification and unless otherwise indicated, make sure that your solution reads as a perfect English sentence. You should only assume integers, operations and order relations as given. If you use a statement or a definition from the textbook, make sure to indicate it.

Section 35

- 8) Prove Proposition 35.8 in the textbook.
- 9) Prove that the sum of any three consecutive integers is divisible by 3.

Section 36

1-2) Please calculate:

- (a) $\gcd(54321, 50)$.
- (b) $\gcd(1739, -29341)$.

In each case, find the integers x and y such that $ax + by = \gcd(a, b)$.

- 11) Prove that consecutive integers must be relatively prime.
- 15) Suppose that a and b are relatively prime integers and that $a|c$ and $b|c$. Prove that $(ab)|c$.
- 21) You have two measuring cups. One holds 8 ounces and the other holds 13 ounces. These cups have no marks to show individual ounces. All you can measure is either a full 13 or a full 8 ounces. If you want to measure, say, 5 ounces, you can fill the 13-ounce measuring cup, use it to fill the 8-ounce cup, and you will have 5 ounces left in the larger cup.
 - (a) Show how to measure exactly 1 ounce. You may assume you have a large bowl for holding liquid, but this large bowl has no marks for measuring. At the end, the bowl should contain exactly one ounce.
 - (b) Generalize this problem. Suppose you have a and b ounces where a and b are positive integers. Give a necessary and sufficient conditions on a and b such that it is possible to measure out exactly 1 ounce using these cups. (Hint: Think how this problem relates to the lecture in the first place.)
 - (c) Watch https://www.youtube.com/watch?v=BVtQNK_ZUJg and be proud that you're better than John McClane.