Discrete Mathematics, 2016 Fall - Worksheet 14

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In all of the above problems explain your answer in full English sentences.

1. Which of the following relations are functions?

   (a) $\{(1,2),(3,4)\}$

   (b) $\{(x,y) : x,y \in \mathbb{Z}, y = 2x\}$

   (c) $\{(x,y) : x,y \in \mathbb{Z}, x + y = 0\}$

   (d) $\{(x,y) : x,y \in \mathbb{Z}, xy = 0\}$

   (e) $\{(x,y) : x,y \in \mathbb{Z}, y = x^2\}$

   (f) $\emptyset$

   (g) $\{(x,y) : x,y \in \mathbb{Q}, x^2 + y^2 = 1\}$

   (h) $\{(x,y) : x,y \in \mathbb{Z}, x|y\}$

   (i) $\{(x,y) : x,y \in \mathbb{N}, x|y, \text{ and } y|x\}$

   (j) $\{(x,y) : x,y \in \mathbb{N}, \binom{x}{y} = 1\}$

2. For those relations that are functions in Problem 1, find their domain and image.

3. For each of the following functions f, find the image of the function, im.

   (a) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 2x + 1$.

   (b) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{1}{1+x^2}$.

   (c) $f : [-1,1] \to \mathbb{R}$ defined by $f(x) = \sqrt{1 - x^2}$.

4. Which of the functions in Problem 1 are one-to-one? What are the inverses of these functions?

5. For each of the functions, determine whether the function is one-to-one, onto, or both. Prove your assertions.

   (a) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 2x^2$.

   (b) $f : \mathbb{N} \to \mathbb{Z}$ defined by $f(x) = (-1)^x \left(\lfloor x/2 \rfloor + 1\right)$, where $\lfloor . \rfloor$ is the integer part function.

6. Give an example of a set A and a function $f : A \to A$ where f is onto but not one to one. Also give one where f is one-to-one but not onto.