Discrete Mathematics, 2016 Spring - HW 4

September 21, 2016

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To get full credit in all of the problems, use rigorous justification and unless otherwise indicated, make sure that your solution reads as a perfect English sentence. You should only assume integers, operations and order relations as given. If you use a statement or a definition from the textbook, make sure to indicate it.

Section 14

11) Consider the relation $\subseteq$ on $2^{\mathbb{Z}}$. Which of the properties does $\subseteq$ have? Prove your answers.

13) The property **irreflexive** is not the same as being not reflexive. To illustrate this, please do the following:

    (a) Give an example of a relation on a set that is neither reflexive nor irreflexive.

    (b) Give an example of a relation on a set that is both reflexive and irreflexive.

    Part (a) is not too hard, but for (b), you need to create a rather strange example.

Section 15

6) Show that $\equiv$ is a transitive relation.

7) For each equivalence relation below, find the requested equivalence class.

    (a) R is has-the-same-parents-as on the set of human beings. Find [you].

    (b) R is has-the same-size-as on $2^{|\{1,2,3,4,5\}}$. Find $[\{1,3\}]$.

8) For each of the following equivalence relations, determine the number of equivalence classes that relation has.

    (a) Congruence modulo 10 (for integers)

    (b) Has-the-same-birthday-as (for human beings). Here same birthday means same month/day, not necessarily same year.

12) Prove the following proposition

    Proposition 1. *Ler R be an equivalence relation on a set A and let $a, x, y \in A$. If $x, y \in [a]$, then xRy.*

Section 16

2) How many different anagrams (including nonsensical words) can be made from each of the following

   (a) MATHEMATICS

   (b) MISSISSIPPI

16) How many partitions of the set $\{1, 2, 3, \ldots, 100\}$ are there that satisfy the following two properties.

   (a) There are exactly three parts.

   (b) Elements $1, 2, 3$ are in different parts.

18) Two different coins are placed on squares of a standard 8×8 chess board; they may both be placed on the same square. Let two arrangements of these coins be equivalent if we can move the coins diagonally to get from one arrangement to another. How many different (inequivalent) ways can the coins be placed on the board?