Discrete Mathematics, 2016 Spring - Worksheet 5

September 21, 2016

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In all of the above problems explain your answer in full English sentences.

1. Rewrite the following sentences using the quantifier notation.
   (a) Every integer is a prime.
   (b) There is an integer whose square is 2.
   (c) All integers are divisible by 5.
   (d) Some integer is divisible by 7.
   (e) For every integer x, there is an integer y such that $xy = 1$.
   (f) There are an integer x and an integer y such that $x/y = 10$.
   (g) There is an integer that, when multiplied by any integer, always gives the result 0.
   (h) No matter what integer you choose, there is always another integer that is larger.
2. Write the negation of each of the sentences in the previous problem, first with quantifiers and then in plain English. In the first way, move the $\neg$ symbol as far to the right as possible.
3. Label each of the following sentences about integers as either true or false. (No need to prove them)
   (a) $\forall x, \forall y, xy = 0$
   (b) $\forall x, \exists y, xy = 0$
   (c) $\exists x, \forall y, xy = 0$
   (d) $\exists x, \exists y, xy = 0$.
4. Let $A = \{1, 2, 3, 4, 5\}$ and let $B = \{4, 5, 6, 7\}$. Compute
   (a) $A \cup B$
   (b) $A \cap B$
   (c) $A - B$
   (d) $B - A$
   (e) $A \Delta B$

(f) $A \times B$

(g) $B \times A$

5. Prove the following theorem and illustrate it with a Venn-diagram (you can look at p57 for what this means). (First DeMorgan's Law)

Theorem 1. *Let A, B, and C sets. Then*

$$A - (B \cup C) = (A - B) \cap (A - C)$$

6. Let A and B be sets with $|A| = 10$ and $|B| = 7$.

(a) Calculate $|A \cap B| + |A \cup B|$.

(b) Find an upper bound y and a lower bound x for $|A \cup B|$, that are sharp. That is

$$x \leq |A \cup B| \leq y.$$

To show that your answer is sharp, find sets such that $|A \cup B| = x$ and $|A \cup B| = y$ exactly.

7. Prove the following proposition:

Proposition 1. *Let n be an integer. Then*

$$2^0 + 2^1 + ... + 2^{n-1} = 2^n - 1$$

Hints:

- How many subsets does the set $N = \{1, 2, \ldots, n\}$ have?
- How many subsets are there whose largest element is j? (Write this out for $j = 1, 2, 3, 4$ to see the pattern.)
- How do the two answers to the previous questions relate?