Discrete Mathematics, 2016 Fall - Worksheet 16

November 7, 2016

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In all of the above problems explain your answer in full English sentences.

1. For each of the pair of functions below, determine which of $g \circ f$ and $f \circ g$ is defined. If one or both are defined, find the resulting functions. If both are defined, determine wheteher $g \circ f = f \circ g$.

   (a) $f = \{(1,2),(2,3),(3,4)\}$ and $g = \{(1,3),(2,4),(3,1)\}$

   (b) $f = \{(1,2),(2,3),(3,4)\}$ and $g = \{(2,1),(3,1),(4,1)\}$

   (c) $f = \{(1,4),(2,4),(3,3),(4,1)\}$ and $g = \{(1,1),(2,1),(3,4),(4,4)\}$.

   (d) $f(x) = 1 - x$ and $g(x) = 2 - x$ for $x \in \mathbb{Q}$.

2. Suppose A, B, and C are sets and $f : A \to B$ and $g : B \to C$. Prove the following:

   (a) If f and g are one-to-one, so is $g \circ f$.

   (b) If f and g are onto, so is $g \circ f$.

   (c) If f and g are bijections, so is $g \circ f$.

3. Define the operation $*$ on the integers defined by $x * y = |x - y|$.

   (a) Is $*$ closed on the integers?

   (b) Is $*$ commutative?

   (c) Is $*$ associative?

   (d) Does $*$ have an identity element? If so, does every integer have an inverse?