Discrete Mathematics, Sect 001, 2016 Fall - Quiz 7

November 27, 2016

Name:

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This quiz is scheduled for 15 minutes. No outside notes or calculators are permitted. To get full credit in all of the problems, use rigorous justification and unless otherwise indicated, make sure that your solution reads as a perfect English sentence. You should only assume the notion of integers, operations, order relations and geometrical objects as given. If you use a statement or a definition from the textbook, make sure to indicate it.

1. (10 points) Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. Let $f : A \to B$ and $g : B \to A$ defined by $f = \{(1, 2), (2, 3), (3, 4)\}$ and $g = \{(2, 1), (3, 1), (4, 2)\}$. Please answer the following:
   (a) What is $f(2)$? What is $g(3)$?
   $$f(2) = 3, \qquad g(3) = 1$$
   (b) What is $\mathrm{dom}(f)$? What is $\mathrm{dom}(g)$?
   $$\mathrm{Dom}(f) = \{1, 2, 3\}, \qquad \mathrm{Dom}(g) = \{2, 3, 4\}$$
   (c) What is $\mathrm{im}(f)$? What is $\mathrm{im}(g)$?
   $$\mathrm{Im}(f) = \{2, 3, 4\}, \qquad \mathrm{Im}(g) = \{1, 2\}$$
   (d) Which one of f, g are one-to one and which ones are onto?
   f is one to one and onto, while g is not one to one and it is not onto either.
   (e) Find f^{-1} and g^{-1}. Are they functions? Justify your answer.
   $$f^{-1} = \{(2, 1), (3, 2), (4, 3)\}, \qquad g^{-1} = \{(1, 2), (1, 3), (4, 2)\}$$
   and f^{-1} is a function while g^{-1} is not.

2. (8 points) Let $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x^3$. Is f one-to-one? If yes, find f^{-1}. Is f onto?
   f is both one to one and onto with $f^{-1}(x) = x^{1/3}$.