Discrete Mathematics, 2016 Fall - Worksheet 17

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In all of the above problems explain your answer in full English sentences.

1. Please express the following permutations in disjoint cycle form.

   (a) $\pi = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 1 \end{bmatrix}$.

   (b) $\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 1 & 3 & 5 \end{bmatrix}$

2. Prove that in the cycle decomposition produced by the algorithm discussed on the slides, the resulting cycles are pairwise disjoint.

3. How many permutations in S_n have exactly one cycle?

4. Let π, σ be given by

   $$\pi = (1)(2,3,4,5)(6,7,8,9), \qquad \sigma = (1,3,5,7,9,2,4,6,8)$$

   Calculate the following

   (a) $\pi \circ \sigma$

   (b) $\sigma \circ \pi$

   (c) π^{-1}

   (d) $\pi^{-1} \circ \pi$.

5. Write the following permutations as the composition of transpositions and determine whether the permutation is even or odd.

   (a) $(1,3)(2,4,5)$

   (b) $(1,2,4,3)(5)$

   (c) $[(1,3)(2,4,5)]^{-1}$.

6. Prove the following group facts:

   (a) If $(G,*)$ is a group and $g \in G$, then $(g^{-1})^{-1} = g$.

   (b) If $(G,*)$ is a group with identity element e, then $e^{-1} = e$.

   (c) If $(G,*)$ is a group and $g, h \in G$, then $(g * h)^{-1} = h^{-1} * g^{-1}$.

7. Show that the alternating group $(A_n, \circ)$ is indeed a group.