Discrete Mathematics, 2016 Spring - HW 7

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To get full credit in all of the problems, use rigorous justification and unless otherwise indicated, make sure that your solution reads as a perfect English sentence. You should only assume integers, operations and order relations as given. If you use a statement or a definition from the textbook, make sure to indicate it.

Section 23

2) Solve each of the following recurrence relations by giving an explicit formula for a_n.

   (a) $a_n = 10a_{n-1}, a_0 = 3$.
   (b) $a_n = a_{n-1} + 3, a_0 = 2$.
   (c) $a_n = 7a_{n-1} - 2, a_0 = -1$.
   (d) $F_n = F_{n-1} + F_{n-2}, F_0 = 1, F_1 = 1$. (Fibonacci sequence)
   (e) $a_n = 2a_{n-1} - a_{n-2}, a_0 = 2, a_1 = -1$.

15) Extrapolate the discussion during lecture and solve the following third-order recurrence relation.

$$a_n = 4a_{n-1} - a_{n-2} - 6a_{n-3}, a_0 = 8, a_1 = 3, a_2 = 27.$$

17) There are many types of non-linear recurrence relations that are of different forms from those presented in the lecture. Most of them can be really hard to solve, however sometimes a little guesswork reveals the solution. Try your hand at conjecturing a solution to the following ones and then prove them by induction.

   (a) $a_n = na_{n-1}, a_0 = 1$.
   (b) $a_n = a_{n-1}^2, a_0 = 2$.
   (c) $a_n = a_0 + a_1 + a_2 + \cdots + a_{n-1}, a_0 = 1$.

3) Each of the following sequences is generated by a polynomial expression. For each, find the polynomial expression that gives a_n.

   (a) $4, 4, 10, 28, 64, 124, 214, 340, 508, 724$
   (b) $5, 16, 41, 116, 301, 680, 1361, 2476, 4181, 6656$

5) Let k be a positive integer and let $a_n = \binom{n}{k}$. Prove that $a_0 = \Delta a_0 = \Delta^2 a_0 = \cdots = \Delta^{k-1} a_0 = 0$ and that $\Delta^k a_0 = 1$.

7) Find a polynomial formula for $1^4 + 2^4 + 3^4 + \cdots + n^4$.

N/A) Read and understand the proof of Theorem 23.17 on page 161. You do not need to hand in anything for this problem, the point of this problem is to practice reading proofs even if otherwise you don't look at the textbook.