Discrete Mathematics, 2016 Fall - Worksheet 10

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In all of the above problems explain your answer in full English sentences.

1. Please state the contrapositive of each of the following statements:
   (a) If x is odd, then x^2 is odd.
   (b) If x is non-zero, then x^2 is positive.
2. Prove by the contrapositive method that if a does not divide b, then the equation $ax^2 + bx + b - a = 0$ has no positive integer solution for x.
3. For each of the following statements, write the first sentences of a proof by contradiction (do not attempt to complete the proofs). Please use the phrase "for the sake of contradiction".
   (a) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
   (b) The sum of two negative integers is a negative integer.
   (c) If the square of a rational number is an integer, then the rational number must also be an integer.
4. Prove the following statements by contradiction.
   (a) Consecutive integers cannot be both even.
   (b) Consecutive integers cannot be both odd.
   (c) If the sum of two primes is prime, then one of the primes must be 2 (you may assume that every integer is either even or odd, but never both.)
   (d) Suppose n is an integer that is divisible by 4. Then $n + 2$ is not divisible by 4.
   (e) Let A and B be sets. Then $(A - B) \cap (B - A) = \emptyset$.
5. Prove by the method of smallest counterexample that $1 + 2 + 3 + \cdots + n = n(n+1)/2$ for all positive integer n.
6. Prove by the method of smallest counterexmaple that $n < 2^n$ for all $n \in \mathbb{N}$.
7. Prove by the method of smallest counterexmaple that when $a \neq 0, 1$, then
$$a^0 + a^1 + a^2 + \cdots + a^n = \frac{a^{n+1} - 1}{a - 1}, \qquad \forall n \in \mathbb{N}.$$
8. For all integers $n \geq 5$, we have $2^n > n^2$.