Discrete Mathematics, 2016 Fall - Worksheet 6

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In all of the above problems explain your answer in full English sentences.

1. Similar to what we did on the slide for $<$, define the corresponding relation set to two of the integer relations $\leq, =, >, \geq$.

2. Write the following relations on the set $\{1, 2, 3, 4, 5\}$ as sets of ordered pairs.

   (a) The $\leq$ relation.
   (b) The 'divides' relation.
   (c) The $=$ relation.

3. Each of the following is a relation on the set $\{1, 2, 3, 4, 5\}$. Express these relations in words and then find their inverses.

   (a) $\{(1, 2), (2, 3), (3, 4), (4, 5)\}$
   (b) $\{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (4, 4), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5)\}$.
   (c) $\{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$.

4. What is the inverse of the following relations?

   (a) $\leq$.
   (b) $\{(x, y) : x, y \in \mathbb{Z}, x - y = 1\}$.
   (c) $\{(x, y) : x, y \in \mathbb{Z}, xy > 0\}$.

5. For each of the following relations on $\{1, 2, 3, 4, 5\}$ determine whether the relation is reflexive, irreflexive, symmetric, antisymmetric, and/or transitive:

   - $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$.
   - $R = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$.
   - $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5)\}$.
   - $R = \{(1, 1), (1, 2), (2, 1), (3, 4), (4, 3)\}$.
   - $R = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$.

6. For the following relations on the set of humans beings, please determine whether the relation if reflexive, irreflexive, symmetric, antisymmetric, and/or transitive.

   (a) has the last name as
   (b) is the child of
   (c) is married to
   (d) has a common parent as

7. Consider the relation | (divisible) on

   (a) On the naturals.
   (b) On the integers.

   Decide what properties do they have.

8. Show that the following relation is an equivalence relation:

$$A = \{B \in 2^{\mathbb{Z}} : |B| < \infty\}, \qquad R = \{(B, C) : B, C \in A, |B| = |C|\}$$

9. Which of the following are equivalence relations?

   (a) $R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$ on the set $\{1, 2, 3\}$.
   (b) $|$ on $\mathbb{Z}$.
   (c) $\leq$ on $\mathbb{Z}$.
   (d) Is-an-anagram-of on the set of English words.

10. For each of the following congruences, find all integers N, with $N > 1$, that make the congruence true

   (a) $23 \equiv 13 \pmod{N}$
   (b) $10 \equiv 5 \pmod{N}$
   (c) $6 \equiv 60 \pmod{N}$