Discrete Mathematics, 2016 Fall - HW 2

Due: September 21, 2016

Instructor: Zsolt Pajor-Gyulai

Courant Institute of Mathematical Sciences, NYU

To get full credit in all of the problems, use rigorous justification and unless otherwise indicated, make sure that your solution reads as a perfect English sentence. You should only assume integers, operations and order relations as given. If you use a statement or a definition from the textbook, make sure to indicate it.

Section 4

2) Below you will find pairs of statements A and B. For each pair, please indicate which of the following three sentences are true and which are false:

   - If A, then B.
   - If B, then A.
   - A if and only if B.

   You may just list the true statements.

   (a) A: Polygon $PQRS$ is a rectangle. B: Polygon $PQRS$ is a parallelogram.

   (b) A: Ellen resides in Los Angeles. B: Ellen resides in California.

   (c) A: This year is divisible by 4. B This year is a leap year.

   (d) A: Lines l_1 and l_2 are parallel. B: Lines l_1 and l_2 are perpendicular.

Section 5

15) Let x be an integer. Prove that $0|x$ if and only if $x = 0$.

18) Prove that the difference between consecutive perfect squares is odd.

Section 6

8) An integer is a palindrome if it reads the same forwards and backwards when expressed in base-10. For example, 1331 is a palindrome. Disprove that all palindromes with two or more digits are divisible by 11.

Section 7

11-12) A **tautology** is a Boolean expression that evaluates to *True* for all possible values of its variables (e.g. $x \vee \neg x$). Use either a truth table or the properties listed in Theorem 7.2 in the textbook together with the fact that $x \to y$ is logically equivalent to $(\neg x) \vee y$ to prove that the following statements are tautologies. Use each way at least once.

(a) $(x \vee y) \vee (x \vee \neg y)$.

(b) $(x \wedge (x \to y)) \to y$.

(c) $(\neg(\neg x)) \leftrightarrow x$.

13) A **contradiction** is a Boolean expression that evaluates to *False* for all possible values of its variables (e.g. $x \wedge \neg x$). Prove that the following are contradictions. You can use your favorite method whether it's truth tables or properties.

(a) $(x \vee y) \wedge (x \wedge \neg y) \wedge \neg x$.

(b) $x \wedge (x \to y) \wedge (\neg y)$.

(c) $(x \to y) \wedge ((\neg x) \to y) \wedge \neg y$.

Section 8

12) A U.S. social security number is a nine-digit number. The first digit may be 0 just like all the others.

(a) How many SSN-s are available?

(b) How many have none of their digits equal to 8?

(c) How many have at least one digit equal to 8?

(d) How many have exactly one 8?

(e) How many are there that do not have two consecutive digits the same?

13) Let n be a positive integer. Prove that $n^2 = (n)_2 + n$ in two different ways:

(a) First, show that this equation is true algebraically.

(b) Second, interpret the terms n^2, $(n)_2$ and n in the context of list counting and use that to argue why the equation must be true.