Discrete Mathematics, 2016 Fall - Worksheet 1

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In all of the above problems explain your answer in full English sentences.

1. Please determine which of the following are true and which are false using the definition of divisibility.

   (a) $3|99$ YES, because 33 is an integer such that $99 = 3 \cdot 33$.

   (b) $-3|3$ YES, ...

   (c) $-5|-5$ YES, ...

   (d) $-2|-7$ NO, the only number k such that $-7 = -2 * k$ is 3.5 which is not an integer.

   (e) $0|4$ NO, because for every integer k, we have $0 \cdot k = 0 \neq 4$.

   (f) $4|0$ YES, because for any integer k, we have $0 = 4 \cdot k$.

   (g) $0|0$ YES, because for any integer k, we have $0 = 0 \cdot k$.

2. Alternative definition to divisibility: We say that an integer a is divisible by an integer b provided $\frac{a}{b}$ is an integer. Explain why this alternative definition is different than the original one. Here different means different concepts. To answer this question you should find integers a and b such that a is divisible by b according to one definition but a is not divisible by b according to the other definition.

   Solution 1. *According to the definition,* 0 *divides* 0 *(see Problem 1 (g)). However,* $\frac{0}{0}$ *is not defined and therefore it cannot be an integer.*

3. Show using the definition of divisibility (the original one) that if a, b, c are integers and $c|a$ and $c|b$ then $c|a + b$.

   Solution 2. *Since* $c|a$ *and* $c|b$*, there are integers* k_1 *and* k_2 *such that* $a = k_1 c$ *and* $b = k_2 c$*. Then*

   $$a + b = k_1 c + k_2 c = (k_1 + k_2)c.$$

   As $k_1 + k_2$ *is an integer, this means that* $c|(a + b)$ *and the claim is proved.*

4. None of the following numbers is prime. Explain why they fail to satisfy the definition of a prime number. Which one of these is a composite number?

   (a) 21. Not a prime because for example 7 is a positive divisor that is not 1 and not 21.

   (b) 0. Not a prime because $0 \not> 1$.

   (c) π. Not a prime because it is not an integer.

   (d) -2. Not a prime because $-2 \not> 1$.

   (e) $1/2$. Not a prime because it is not an integer.

5. Formulate a definition of the following concepts:

   (a) Define what it means for a number to be the **square root** of another number.

   The number a is the square root of a number b provided $a \cdot a = b$.

   (b) Midpoint of a line segment. (Assume the notion of distance and line segment is given.)

   The midpoint of a line segment is the point on the line segment which is at equal distance from the two endpoints.

   (c) Teenager. (Assume the notions of age and human are given.)

   A teenager is a human whose age is between 10 and 19.

   (d) Grandmother. (Assume the notion of child and woman are given.)

   A grandmother is a woman who has a child and whose child has a child.