Discrete Mathematics, 2016 Fall - Worksheet 13

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In all of the above problems explain your answer in full English sentences.

1. Prove the following proposition.

   Proposition 1. *Let a, b, and c be sequences of numbers and let s be a number.*

   - *If, for all n, $c_n = a_n + b_n$ then $\Delta c_n = \Delta a_n + \Delta b_n$.*
   - *If, for all n, $b_n = sa_n$, then $\Delta b_n = s\Delta a_n$*

   Proof. Textbook page 159. □

2. Each of the following sequences is generated by a polynomial expression. For each, find the polynomial expression that gives a_n.

   - $1, 6, 17, 34, 57, 86, 121, 162, 209, 262, \ldots$

     Solution. *By writing out the differences, we see that $k = 2$, while*

     $$a_0 = 1, \qquad \Delta a_0 = 5, \qquad \Delta^2 a_0 = 6$$

     and thus

     $$a_n = 1 \cdot \binom{n}{0} + 5\binom{n}{1} + 6\binom{n}{2} = 1 + 5n + 6\frac{n(n-1)}{2} = 3n^2 + 2n + 1$$

   - $6, 5, 6, 9, 14, 21, 30, 41, 54, 69, \ldots$.

     Solution. *Once again, $k = 2$ and*

     $$a_0 = 6, \qquad \Delta a_0 = -1, \qquad \Delta^2 a_0 = 2$$

     and therefore

     $$a_n = 6 \cdot \binom{n}{0} - \binom{n}{1} + 2\binom{n}{2} = 6 - n + n(n-1) = n^2 - 2n + 6$$

3. Find a polynomial formula for $1^2 + 2^2 + 3^2 + \ldots n^2$.

   Solution. *Textbook page 162.*