Discrete Mathematics, 2016 Spring - HW 5

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To get full credit in all of the problems, use rigorous justification and unless otherwise indicated, make sure that your solution reads as a perfect English sentence. You should only assume integers, operations and order relations as given. If you use a statement or a definition from the textbook, make sure to indicate it.

Section 17

8) Fifty runners compete in a $10K$ race. How many outcomes are possible if

   (a) We want to know in what place every runner finished.

   (b) The race is a qualifying race and we just want to pick the 10 fastest runners.

   (c) The race is an Olympic final event, and we care only about who gets the gold, silver, and bronze medals.

17) Let $n \geq k \geq m \geq 0$ be integers. Consider the following formula:

$$\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n-m}{k-m}$$

Give two different proofs. One proof should use the factorial formula, the other should be combinatorial.

30) (a) What is the coefficient of x^3y^5 in $(x+y+4)^{10}$?

   (b) Prove that $\binom{n}{a\ b\ c} = \binom{n}{a}\binom{n-a}{b}$. Here a, b, c are natural numbers with $a+b+c=n$.

Section 18

8) Express $\left(\!\binom{n}{k}\!\right)$ using factorial notation.

N/A) 8 identical prizes are given out to chosen students in a class of size 32.

   (a) How many ways can this be done if one student can only get one prize?

   (b) How many ways can this be done if any student can get any number of prizes?

7,11) (a) Calculate $\left(\!\binom{8}{4}\!\right)$ and $\left(\!\binom{4}{8}\!\right)$. Notice anything interesting?

(b) Show that for any positive integer a,

$$\left(\!\!\binom{2a}{a}\!\!\right) = 2\left(\!\!\binom{a}{2a}\!\!\right).$$

Section 19

3) How many integers between 1 and $1,000,000$ (inclusive) are not divisible by 2, 3, or 5?

8) How many lattice paths through the grid in the figure avoids both locations A and B?