

## Discrete Mathematics, 2016 Fall - Worksheet 18

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In all of the above problems explain your answer in full English sentences.

1. For the given integers  $a, b$ , find the integers  $q$  and  $r$  such that  $a = qb + r$  and  $0 \leq r < b$ .
  - (a)  $a = 100, b = 3$
  - (b)  $a = -100, b = 3$
2. For the given integers  $a, b$ , compute  $a \operatorname{div} b$  and  $a \operatorname{mod} b$ .
  - (a)  $a = 99, b = 3$ .
  - (b)  $a = -99, b = 3$ .
  - (c)  $a = 10, b = 3$ .
3. Please calculate:
  - (a)  $\gcd(20, 25)$
  - (b)  $\gcd(-89, -98)$ .
4. For each pair of integers  $a, b$  in the previous problem, find integers  $x$  and  $y$  such that  $ax + by = \gcd(a, b)$ .
5. Let  $a$  and  $b$  be positive integers. Prove that  $2^a$  and  $2^b - 1$  are relatively prime.
6. Decide if the following diophantine equations have a solution or not and if yes find a solution:
  - $3x + 4x = 2$
  - $6x - 2y = 4$