Discrete Mathematics, 2016 Fall - Worksheet 12

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In all of the above problems explain your answer in full English sentences.

1. Calculate the first six terms of the sequence (that is, a_0 through a_5)

$$a_n = 2a_{n-1} + 2, \qquad a_0 = 1$$

2. Let $e_0 = 1$, $e_1 = 4$ and, for $n > 1$, let $e_n = 4(e_{n-1} - e_{n-2})$. What are the first five terms of the sequence $e_0, e_1, e_2, \ldots$? Prove $e_n = (n+1)2^n$ for every natural n.

3. Solve the following first order recurrence relations.

   (a) $a_n = \frac{2}{3}a_{n-1}, a_0 = 4$.

   (b) $a_n = 2a_{n-1} + 2, a_0 = 2$.

4. Solve the following second order recurrence relations.

   (a) $a_n = 3a_{n-1} + 4a_{n-2}, a_0 = 3, a_1 = 2$.

   (b) $a_n = -6a_{n-1} - 9a_{n-2}, a_0 = 3, a_1 = 6$.

5. What can go wrong with the technique we used to solve the non-homogeneous equation on the slides? Try to solve

   - $a_n = 4a_{n-1} + 5a_{n-2} + 4, a_0 = 2, a_1 = 3$.
   - $a_n = 3a_{n-1} - 2a_{n-2} + 5, a_0 = a_1 = 3$.
   - $a_n = 6a_{n-1} - 9a_{n-2} - 2, a_0 = -1, a_1 = 4$.
   - $a_n = 2a_{n-1} - a_{n-2} + 2, a_0 = 4, a_1 = 2$.