

Discrete Mathematics, 2016 Fall - HW 1

Due date: September 14, 2016

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To get full credit in all of the problems, use rigorous justification and unless otherwise indicated, make sure that your solution reads as a perfect English sentence. You should only assume integers, operations and order relations as given. If you use a statement or a definition from the textbook, make sure to indicate it.

Section 3

- 5) A rational number is a number formed by dividing two integers a/b where $b \neq 0$. The set of all rational numbers is denoted by \mathbb{Q} . Explain why every integer is a rational number, but not all rational numbers are integers.
- 6) Define what it means for an integer to be a perfect square. For example, the integers 0, 1, 4, 9 and 16 are perfect squares. Your definition should begin as: "An integer x is called a perfect square provided".
- 12) How many positive divisors does each of the following numbers have?
 - (a) 8,
 - (b) 32,
 - (c) (Optional, hand in only to check yourself) 2^n .

In the first two, you can just list the divisors as justification. For the third one however you need to justify your answer rigorously.

- 13) An integer n is called perfect provided it equals the sum of all of its divisors that are both positive and less than n . For example, 28 is perfect because the positive divisors of 28 are 1, 2, 4, 7, 14 and 28. Note that $1 + 2 + 4 + 7 + 14 = 28$.
 - (a) There is a perfect number smaller than 28. Find it.
 - (b) (Optional) Write computer code to find the next perfect number.