

Zeitabhngige Wellenfunktion:

$$\psi(x,t) = \alpha e^{-itE_0/\hbar} |0\rangle + \beta e^{-itE_1/\hbar} |1\rangle + \dots$$

Mit den Energien  $E_n = \left(n + \frac{1}{2}\right) \omega \hbar$   
Und den Eigenfunktionen  $|n\rangle = \psi_n(x)$

0.1 Operatoren

Ortsoperator	$\hat{x} = x = i\hbar \frac{\partial}{\partial p}$
Impulsoperator	$\hat{p} = p = -i\hbar \frac{\partial}{\partial x}$
Abstiegsoperator	$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \hat{x} + \frac{i}{\sqrt{2\hbar m\omega}} \hat{p}$
Aufstiegsoperator	$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \hat{x} - \frac{i}{\sqrt{2\hbar m\omega}} \hat{p}$
Hamiltonoperator	$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 = \frac{\hat{p}^2}{2m} + V(\hat{x})$
Damit gilt	$\hat{x} = \frac{1}{2} \sqrt{\frac{2\hbar}{m\omega}} (\hat{a} + \hat{a}^\dagger)$ $\hat{p} = \frac{1}{2i} \sqrt{2\hbar m\omega} (\hat{a} - \hat{a}^\dagger)$ $\hat{a}  n\rangle = \sqrt{n}  n-1\rangle \quad \hat{a}  0\rangle = 0$ $\hat{a}^\dagger  n\rangle = \sqrt{n+1}  n+1\rangle$ $ n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}  0\rangle$

0.2 Mittelwerte

$$\begin{aligned} \langle x \rangle &= \langle \psi | x | \psi \rangle = \int_{-\infty}^{\infty} x |\psi(x)|^2 dx \quad \text{nur falls } x \text{ kein Operator!} \\ &= \int_{-\infty}^{\infty} \psi(x)^* \hat{x} \psi(x) dx \\ &= \frac{1}{2} \sqrt{\frac{2\hbar}{m\omega}} (\alpha^* \langle 0| + \beta^* \langle 1| + \dots) (\hat{a} + \hat{a}^\dagger) (\alpha |0\rangle + \beta |1\rangle + \dots) \\ \langle p \rangle &= \langle \psi | x | \psi \rangle = \int_{-\infty}^{\infty} \hat{p} |\psi(x)|^2 dx = \text{analog zu } \langle \psi | x | \psi \rangle. \end{aligned}$$

0.3 Pauli-Matrizen

$$\begin{aligned} S_0 &= \frac{\hbar}{2} \sigma_0 = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ S_1 &= \frac{\hbar}{2} \sigma_1 = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ S_2 &= \frac{\hbar}{2} \sigma_2 = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ S_3 &= \frac{\hbar}{2} \sigma_3 = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned}$$

$$S_1 = S_x, \sigma_1 = \sigma_x, S_2 = S_y \dots$$