

Harmonischer Oszillator

$\psi(x,t) = \alpha e^{-itE_0/\hbar} |0\rangle + \beta e^{-itE_1/\hbar} |1\rangle + \dots$

$V(x) = \frac{1}{2} k x^2 = \frac{1}{2} m \omega^2 x^2$

Mit den Energien $E_n = (n + \frac{1}{2}) \omega \hbar$
Und den Eigenfunktionen $|n\rangle = \psi_n(x)$
Schrödingergleichung:

$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$

Operatoren

Ortsoperator $\hat{x} = x = i\hbar \frac{\partial}{\partial p}$
Impulsoperator $\hat{p} = p = -i\hbar \frac{\partial}{\partial x}$
Abstiegsoperator $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \hat{x} + \frac{i}{\sqrt{2\hbar m\omega}} \hat{p}$
Aufstiegsoperator $\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \hat{x} - \frac{i}{\sqrt{2\hbar m\omega}} \hat{p}$
Hamiltonoperator $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega^2 \hat{x}^2 = \frac{\hat{p}^2}{2m} + V(\hat{x})$
Damit gilt $\hat{x} = \frac{1}{2} \sqrt{\frac{2\hbar}{m\omega}} (\hat{a} + \hat{a}^\dagger)$
 $\hat{p} = \frac{1}{2i} \sqrt{2\hbar m\omega} (\hat{a} - \hat{a}^\dagger)$
 $\hat{a} |n\rangle = \sqrt{n} |n-1\rangle \qquad \hat{a} |0\rangle = 0$
 $\hat{a}^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$
 $|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}} |0\rangle$

Mittelwerte

$\langle x \rangle = \langle \psi | x | \psi \rangle$

$= \int_{-\infty}^{\infty} x |\psi(x)|^2 dx$ nur falls x kein Operator!

$= \int_{-\infty}^{\infty} \psi(x)^* \hat{x} \psi(x) dx$

$= \frac{1}{2} \sqrt{\frac{2\hbar}{m\omega}} (\alpha^* \langle 0 | + \beta^* \langle 1 | + \dots) (\hat{a} + \hat{a}^\dagger) (\alpha | 0 \rangle + \beta | 1 \rangle + \dots)$

$\langle p \rangle = \langle \psi | p | \psi \rangle$

$= \int_{-\infty}^{\infty} \hat{p} |\psi(x)|^2 dx =$ analog zu $\langle \psi | x | \psi \rangle$.

Pauli-Matrizen

$S_0 = \frac{\hbar}{2} \sigma_0 = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$S_x = S_1 = \frac{\hbar}{2} \sigma_1 = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$S_y = S_2 = \frac{\hbar}{2} \sigma_2 = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$S_z = S_3 = \frac{\hbar}{2} \sigma_3 = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\vec{S} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$

$S_1 = S_x, \sigma_1 = \sigma_x, S_2 = S_y \dots$

De Broglie

$\lambda = \frac{h}{p} = \frac{h}{m \cdot v}$

Relativitätstheorie

Lorentzfaktor $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Bewegte Masse $m = \gamma m_0$ (Mit Ruhemasse m_0)

Impuls $p = mv = \gamma m_0 v$

Energie $E = mc^2 = \gamma m_0 c^2$

Kinetische Energie $E_{\text{kin}} = \frac{1}{2} mv^2 = \frac{\gamma}{2} m_0 v^2 = mc^2 - m_0 c^2$

Photoeffekt $E_\gamma = W_A + E_{\text{el,kin}}$

$v_1 \oplus v_2 = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$

Konstanten

$m_e = 9.10938291 \cdot 10^{-31} \text{ kg}$

$m_p = 1.67262178 \cdot 10^{-27} \text{ kg}$

$m_n = 1.67492735 \cdot 10^{-27} \text{ kg}$

$h = 6.62606957 \cdot 10^{-34} \text{ Js}$

$\hbar = \frac{h}{2\pi}$

$E_\gamma = h\nu$ (Energie des Photons)

$p_\gamma = \frac{E_\gamma}{c}$ (Impuls des Photons)

Kommutatoren

$[\hat{L}_i, \hat{x}_j] = i\hbar \epsilon_{ijk} \hat{x}_k$

$[\hat{L}_i, \hat{p}_j] = i\hbar \epsilon_{ijk} \hat{p}_k$

$[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}] \hat{C} + \hat{B} [\hat{A}, \hat{C}]$

$[\hat{A}\hat{B}, \hat{C}] = \hat{A} [\hat{B}, \hat{C}] + [\hat{A}, \hat{C}] \hat{B}$

Drehimpuls

$\vec{r} = \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}, \quad \vec{p} = \begin{pmatrix} \hat{p}_x \\ \hat{p}_y \\ \hat{p}_z \end{pmatrix}, \quad \vec{L} = \vec{p} \times \vec{r}$

$L_x = yp_z - zp_y = -i\hbar \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right)$

$L_y = zp_x - xp_z = -i\hbar \left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right)$

$L_z = xp_y - yp_x = -i\hbar \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right)$

$[L_i, \hat{f}] = 0$, für Skalaroperatoren \hat{f}

$[L_i, \hat{f}_k] = -i\hbar \epsilon_{ijk} \hat{f}_k$, für Vektoroperatoren \hat{f}

$[L_i, \hat{f}_{kl}] = i\hbar (\epsilon_{ikp} \delta_{nl} + \epsilon_{iln} \delta_{kp}) \hat{f}_{pn}$

Potentialtopf

$k_1 = k_3 = \sqrt{\frac{2mE}{\hbar^2}}, \quad k_2 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$

$r = A + B - 1, \quad \psi_I(x) = e^{ik_1 x} + r e^{-ik_1 x}$

$\psi_{II}(x) = A e^{ik_2 x} + B e^{-ik_2 x}, \quad \psi_{III}(x) = t e^{-ik_3(x-a)}$

Dabei ist t bzw. r der Transmissions- bzw. Reflektionskoeffizient.
Allgemein

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\text{div } \vec{X} = \vec{\nabla} \cdot \vec{X} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} X_x \\ X_y \\ X_z \end{pmatrix} = \begin{pmatrix} \frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + \frac{\partial X_z}{\partial z} \end{pmatrix}$

$\text{rot } \vec{F} = \vec{\nabla} \times \vec{F} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \\ \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \\ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \end{pmatrix}$

$\frac{\partial^2}{\partial x^2} e^{-ax^2} = 2ae^{-ax^2} (2ax^2 - 1)$

Maxwell

Elektrisches Feld \vec{E} , magnetisches Feld \vec{B} , Ladungsdichte ρ , elektrische Stromdichte \vec{j}

$\text{div } \vec{E} = \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

$\text{div } \vec{B} = \vec{\nabla} \cdot \vec{B} = 0$

$\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\text{rot } \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$