Zeitabhängige Wellenfunktion:

$$\psi(x,t) = \alpha e^{-itE_0/\hbar} |0\rangle + \beta e^{-itE_1/\hbar} |1\rangle + \cdots$$

Mit den Energien $E_n = \left(n + \frac{1}{2}\right)\omega\hbar$
Und den Eigenfunktionen $|n\rangle = \psi_n(x)$

## 0.1 Operatoren

| | |
|---|---|
| Ortsoperator | $\hat{x} = x = i\hbar\frac{\partial}{\partial p}$ |
| Impulsoperator | $\hat{p} = p = -i\hbar\frac{\partial}{\partial x}$ |
| Abstiegsoperator | $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} + \frac{i}{\sqrt{2\hbar m\omega}}\hat{p}$ |
| Aufstiegsoperator | $\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} - \frac{i}{\sqrt{2\hbar m\omega}}\hat{p}$ |
| Hamiltonoperator | $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 = \frac{\hat{p}^2}{2m} + V(\hat{x})$ |
| Damit gilt | $\hat{x} = \frac{1}{2}\sqrt{\frac{2\hbar}{m\omega}}(\hat{a} + \hat{a}^\dagger)$ |
| | $\hat{p} = \frac{1}{2i}\sqrt{2\hbar m\omega}(\hat{a} - \hat{a}^\dagger)$ |
| | $\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle \qquad \hat{a}\,|0\rangle = 0$ |
| | $\hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$ |
| | $|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}\,|0\rangle$ |

## 0.2 Mittelwerte

$$\begin{aligned}
\langle x\rangle &= \langle\psi|x|\psi\rangle \\
&= \int_{-\infty}^{\infty} x\,|\psi(x)|^2\,dx \qquad \text{nur falls } x \text{ kein Operator!} \\
&= \int_{-\infty}^{\infty} \psi(x)^*\hat{x}\,\psi(x)\,dx \\
&= \frac{1}{2}\sqrt{\frac{2\hbar}{m\omega}}(\alpha^*\,\langle 0| + \beta^*\,\langle 1| + \ldots)(\hat{a} + \hat{a}^\dagger)(\alpha\,|0\rangle + \beta\,|1\rangle + \ldots) \\
\langle p\rangle &= \langle\psi|x|\psi\rangle \\
&= \int_{-\infty}^{\infty} \hat{p}\,|\psi(x)|^2\,dx = \text{ analog zu } \langle\psi|x|\psi\rangle\,.
\end{aligned}$$

## 0.3 Pauli-Matrizen

$$\begin{aligned}
S_0 &= \frac{\hbar}{2}\sigma_0 = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \\
S_1 &= \frac{\hbar}{2}\sigma_1 = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \\
S_2 &= \frac{\hbar}{2}\sigma_2 = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \\
S_3 &= \frac{\hbar}{2}\sigma_3 = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \\
\vec{S} &= \begin{pmatrix}S_x\\ S_y\\ S_z\end{pmatrix}
\end{aligned}$$

$S_1 = S_x,\ \sigma_1 = \sigma_x,\ S_2 = S_y \ldots$

$$\begin{aligned}
\gamma E_{\text{kin}} &= \frac{1}{2}mV^2 = \\
p &= \sqrt{2mE_{\text{kin}}}
\end{aligned}$$

## 0.4 De Broglie

$$\lambda = \frac{h}{p} = \frac{h}{m\cdot v}$$

## 0.5 Relativitätstheorie

| | |
|---|---|
| Lorentzfaktor | $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ |
| Bewegte Masse | $m = \gamma m_0$ (Mit Ruhemasse $m_0$) |
| Impuls | $p = mv = \gamma m_0 v$ |
| Energie | $E = mc^2 = \gamma m_0 c^2$ |
| Kinetische Energie | $E_{\text{kin}} = \frac{1}{2}mv^2 = \frac{\gamma}{2}m_0v^2 = mc^2 - m_0c^2$ |

## 0.6 Konstanten

$$\begin{aligned}
m_e &= 9.10938291\cdot 10^{-31}\ \text{kg} \\
m_p &= 1.67262178\cdot 10^{-27}\ \text{kg} \\
m_n &= 1.67492735\cdot 10^{-27}\ \text{kg} \\
h &= 6.62606957\cdot 10^{-34}\text{Js} \\
\hbar &= \frac{h}{2\pi} \\
E_\gamma &= h\nu \qquad \text{(Energie des Photons)} \\
p_\gamma &= \frac{E_\gamma}{c} \qquad \text{(Impuls des Photons)}
\end{aligned}$$

## 0.7 Kommutatoren

$$\begin{aligned}
\left[\hat{L}_i, \hat{x}_j\right] &= i\hbar\epsilon_{ijk}\hat{x}_k \\
\left[\hat{L}_i, \hat{p}_j\right] &= i\hbar\epsilon_{ijk}\hat{p}_k \\
\left[\hat{A}, \hat{B}\hat{C}\right] &= \left[\hat{A}, \hat{B}\right]\hat{C} + \hat{B}\left[\hat{A}, \hat{C}\right] \\
\left[\hat{A}\hat{B}, \hat{C}\right] &= \hat{A}\left[\hat{B}, \hat{C}\right] + \left[\hat{A}, \hat{C}\right]\hat{B}
\end{aligned}$$

## 0.8 Drehimpuls

$$\vec{r} = \begin{pmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{pmatrix}, \quad \vec{p} = \begin{pmatrix}\hat{p}_x\\ \hat{p}_y\\ \hat{p}_z\end{pmatrix}, \quad \vec{L} = \vec{p}\times\vec{r}$$

$$\begin{aligned}
Ł_x &= yp_z - zp_y = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right) \\
Ł_y &= zp_x - xp_z = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right) \\
Ł_z &= xp_y - yp_x = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) \\
\left[L_i, \hat{f}\right] &= 0, \text{ für Skalaroperatoren } \hat{f} \\
\left[L_i, \hat{f}_k\right] &= -i\hbar\epsilon_{ijk}\hat{f}_k, \text{ für Vektoroperatoren } \hat{f} \\
\left[L_i, \hat{f}_{kl}\right] 23 &= i\hbar(\epsilon_{ikp}\delta_{nl} + \epsilon_{iln}\delta_{kp})\hat{f}_{pn}
\end{aligned}$$