Harmonischer Oszillator

$$\psi(x,t) = \alpha e^{-itE_0/\hbar}\,|0\rangle + \beta e^{-itE_1/\hbar}\,|1\rangle + \cdots$$

$$V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$$

Mit den Energien $E_n = \left(n+\frac{1}{2}\right)\omega\hbar$
Und den Eigenfunktionen $|n\rangle = \psi_n(x)$
Schrödingergleichung:

$$\mathrm{i}\hbar\frac{\partial}{\partial t}|\,\psi(t)\rangle = \hat{H}|\,\psi(t)\rangle$$

Operatoren

Ortsoperator	$\hat{x} = x = i\hbar\frac{\partial}{\partial p}$
Impulsoperator	$\hat{p} = p = -i\hbar\frac{\partial}{\partial x}$
Abstiegsoperator	$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} + \frac{i}{\sqrt{2\hbar m\omega}}\hat{p}$
Aufstiegsoperator	$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} - \frac{i}{\sqrt{2\hbar m\omega}}\hat{p}$
Hamiltonoperator	$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 = \frac{\hat{p}^2}{2m} + V(\hat{x})$
Damit gilt	$\hat{x} = \frac{1}{2}\sqrt{\frac{2\hbar}{m\omega}}(\hat{a}+\hat{a}^\dagger)$
	$\hat{p} = \frac{1}{2i}\sqrt{2\hbar m\omega}(\hat{a}-\hat{a}^\dagger)$
	$\hat{a}\,\lvert n\rangle = \sqrt{n}\,\lvert n-1\rangle \qquad \hat{a}\,\lvert 0\rangle = 0$
	$\hat{a}^\dagger\,\lvert n\rangle = \sqrt{n+1}\,\lvert n+1\rangle$
	$\lvert n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}\,\lvert 0\rangle$

Mittelwerte

$$\begin{aligned}
\langle x\rangle &= \langle\psi|x|\psi\rangle \\
&= \int_{-\infty}^{\infty} x\,|\psi(x)|^2\,dx \qquad \text{nur falls } x \text{ kein Operator!} \\
&= \int_{-\infty}^{\infty} \psi(x)^*\hat{x}\,\psi(x)\,dx \\
&= \frac{1}{2}\sqrt{\frac{2\hbar}{m\omega}}(\alpha^*\langle 0| + \beta^*\langle 1| + \ldots)(\hat{a}+\hat{a}^\dagger)(\alpha\,|0\rangle + \beta\,|1\rangle + \ldots) \\
\langle p\rangle &= \langle\psi|x|\psi\rangle \\
&= \int_{-\infty}^{\infty} \hat{p}\,|\psi(x)|^2\,dx = \text{ analog zu } \langle\psi|x|\psi\rangle\,.
\end{aligned}$$

Pauli-Matrizen

$$S_0 = \frac{\hbar}{2}\sigma_0 = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$S_x = S_1 = \frac{\hbar}{2}\sigma_1 = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$S_y = S_2 = \frac{\hbar}{2}\sigma_2 = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

$$S_z = S_3 = \frac{\hbar}{2}\sigma_3 = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\vec{S} = \begin{pmatrix}S_x\\ S_y\\ S_z\end{pmatrix}$$

$S_1 = S_x,\ \sigma_1 = \sigma_x,\ S_2 = S_y\ldots$

De Broglie

$$\lambda = \frac{h}{p} = \frac{h}{m\cdot v}$$

Relativitätstheorie

Lorentzfaktor	$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
Bewegte Masse	$m = \gamma m_0$ (Mit Ruhemasse m_0)
Impuls	$p = mv = \gamma m_0 v$
Energie	$E = mc^2 = \gamma m_0 c^2$
Kinetische Energie	$E_{\text{kin}} = \frac{1}{2}mv^2 = \frac{\gamma}{2}m_0 v^2 = mc^2 - m_0c^2$
Photoeffekt	$E_\gamma = W_A + E_{\text{el,kin}}$
$v_1 \oplus v_2$	$= \frac{v_1+v_2}{1+\frac{v_1 v_2}{c^2}}$

Konstanten

$$\begin{aligned}
m_e &= 9.10938291\cdot 10^{-31}\ \text{kg} \\
m_p &= 1.67262178\cdot 10^{-27}\ \text{kg} \\
m_n &= 1.67492735\cdot 10^{-27}\ \text{kg} \\
h &= 6.62606957\cdot 10^{-34}\,\text{Js} \\
\hbar &= \frac{h}{2\pi} \\
E_\gamma &= h\nu \qquad \text{(Energie des Photons)} \\
p_\gamma &= \frac{E_\gamma}{c} \qquad \text{(Impuls des Photons)}
\end{aligned}$$

Kommutatoren

$$\left[\hat{L}_i, \hat{x}_j\right] = i\hbar\epsilon_{ijk}\hat{x}_k$$

$$\left[\hat{L}_i, \hat{p}_j\right] = i\hbar\epsilon_{ijk}\hat{p}_k$$

$$\left[\hat{A}, \hat{B}\hat{C}\right] = \left[\hat{A}, \hat{B}\right]\hat{C} + \hat{B}\left[\hat{A}, \hat{C}\right]$$

$$\left[\hat{A}\hat{B}, \hat{C}\right] = \hat{A}\left[\hat{B}, \hat{C}\right] + \left[\hat{A}, \hat{C}\right]\hat{B}$$

Drehimpuls

$$\vec{r} = \begin{pmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{pmatrix}, \quad \vec{p} = \begin{pmatrix}\hat{p}_x\\ \hat{p}_y\\ \hat{p}_z\end{pmatrix}, \quad \vec{L} = \vec{p}\times\vec{r}$$

$$Ł_x = yp_z - zp_y = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$

$$Ł_y = zp_x - xp_z = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$$

$$Ł_z = xp_y - yp_x = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

$$\left[L_i, \hat{f}\right] = 0, \text{ für Skalaroperatoren } \hat{f}$$

$$\left[L_i, \hat{f}_k\right] = -i\hbar\epsilon_{ijk}\hat{f}_k, \text{ für Vektoroperatoren } \hat{f}$$

$$\left[L_i, \hat{f}_{kl}\right] = i\hbar(\epsilon_{ikp}\delta_{nl} + \epsilon_{iln}\delta_{kp})\hat{f}_{pn}$$

Potentialtopf

$$k_1 = k_3 = \sqrt{\frac{2mE}{\hbar^2}}, \qquad k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

$$r = A + B - 1, \qquad \psi_{\mathrm{I}}(x) = e^{ik_1x} + re^{-ik_1x}$$

$$\psi_{\mathrm{II}}(x) = Ae^{ik_2x} + Be^{-ik_2x}, \qquad \psi_{\mathrm{III}}(x) = te^{-ik_3(x-a)}$$

Dabei ist t bzw. r der Transmissions- bzw. Reflektionskoeffizent.

Allgemein

$$\sin(x\pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x\pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\operatorname{div}\vec{X} = \vec{\nabla}\vec{X} = \begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{pmatrix} X = \begin{pmatrix}\frac{\partial\vec{X}}{\partial x}\\ \frac{\partial\vec{X}}{\partial y}\\ \frac{\partial\vec{X}}{\partial z}\end{pmatrix}$$

$$\operatorname{rot}\vec{F} = \vec{\nabla}\times\vec{F} = \begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{pmatrix}\times\begin{pmatrix}F_x\\ F_y\\ F_z\end{pmatrix} = \begin{pmatrix}\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\\ \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\\ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\end{pmatrix}$$

Maxwell Elektrisches Feld $\vec{E}$, magnetisches Feld $\vec{B}$, Ladungsdichte ρ, elektrische Stromdichte $\vec{j}$

$$\operatorname{div}\vec{E} = \vec{\nabla}\vec{D} = \frac{\rho}{\varepsilon_0}$$

$$\operatorname{div}\vec{B} = \vec{\nabla}\vec{B} = 0$$

$$\operatorname{rot}\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\operatorname{rot}\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$