Zeitabhngige Wellenfunktion:

$$\psi(x,t) = \alpha e^{-itE_0/\hbar} |0\rangle + \beta e^{-itE_1/\hbar} |1\rangle + \cdots$$

Mit den Energien $E_n = \left(n + \frac{1}{2}\right) \omega\hbar$
Und den Eigenfunktionen $|n\rangle = \psi_n(x)$

0.1 Operatoren

Ortsoperator	$\hat{x} = x = i\hbar\frac{\partial}{\partial p}$
Impulsoperator	$\hat{p} = p = -i\hbar\frac{\partial}{\partial x}$
Abstiegsoperator	$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} + \frac{i}{\sqrt{2\hbar m\omega}}\hat{p}$
Aufstiegsoperator	$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} - \frac{i}{\sqrt{2\hbar m\omega}}\hat{p}$
Hamiltonoperator	$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 = \frac{\hat{p}^2}{2m} + V(\hat{x})$
Damit gilt	$\hat{x} = \frac{1}{2}\sqrt{\frac{2\hbar}{m\omega}}(\hat{a} + \hat{a}^\dagger) + \sqrt{\frac{\hbar}{2m\omega}}(\hat{a} + \hat{a}^\dagger)$
	$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle \qquad \hat{a}\,|0\rangle = 0$
	$\hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n\rangle$
	$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}\,|0\rangle$

0.2 Mittelwerte

$$\langle x\rangle = \langle\psi|x|\psi\rangle = \int_{-\infty}^{\infty} x\,|\psi(x)|^2\,dx \qquad \text{nur falls } x \text{ kein Operator!}$$

$$= \int_{-\infty}^{\infty} \psi(x)^*\hat{x}\,\psi(x)\,dx$$

$$= \frac{1}{2}\sqrt{\frac{2\hbar}{m\omega}}(\alpha^*\,\langle 0| + \beta^*\,\langle 1| + \ldots)(\hat{a} + \hat{a}^\dagger)(\alpha\,|0\rangle + \beta\,|1\rangle + \ldots)$$

$$\langle p\rangle = \langle\psi|x|\psi\rangle = \int_{-\infty}^{\infty} \hat{p}\,|\psi(x)|^2\,dx = \text{ analog zu } \langle\psi|x|\psi\rangle\,.$$