Zeitabhngige Wellenfunktion:

$$\psi(x,t) = \alpha e^{-itE_0/\hbar}\left|0\right\rangle + \beta e^{-itE_1/\hbar}\left|1\right\rangle + \cdots$$

Mit den Energien $E_n = \left(n+\frac{1}{2}\right)\omega\hbar$
Und den Eigenfunktionen $\left|n\right\rangle = \psi_n(x)$

0.1 Operatoren

Ortsoperator	$\hat{x} = x = i\hbar\frac{\partial}{\partial p}$
Impulsoperator	$\hat{p} = p = -i\hbar\frac{\partial}{\partial x}$
Abstiegsoperator	$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} + \frac{i}{\sqrt{2\hbar m\omega}}\hat{p}$
Aufstiegsoperator	$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} - \frac{i}{\sqrt{2\hbar m\omega}}\hat{p}$
Hamiltonoperator	$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 = \frac{\hat{p}^2}{2m} + V(\hat{x})$
Damit gilt	$\hat{x} = \frac{1}{2}\sqrt{\frac{2\hbar}{m\omega}}(\hat{a}+\hat{a}^\dagger)$
	$\hat{p} = \frac{1}{2i}\sqrt{2\hbar m\omega}(\hat{a}-\hat{a}^\dagger)$
	$\hat{a}\left|n\right\rangle = \sqrt{n}\left|n-1\right\rangle \qquad \hat{a}\left|0\right\rangle = 0$
	$\hat{a}^\dagger\left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle$
	$\left|n\right\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}\left|0\right\rangle$

0.2 Mittelwerte

$$\begin{aligned}
\langle x\rangle = \langle\psi|x|\psi\rangle &= \int_{-\infty}^{\infty} x\,|\psi(x)|^2\,dx \qquad \text{nur falls } x \text{ kein Operator!}\\
&= \int_{-\infty}^{\infty} \psi(x)^*\hat{x}\,\psi(x)\,dx\\
&= \frac{1}{2}\sqrt{\frac{2\hbar}{m\omega}}(\alpha^*\left\langle 0\right| + \beta^*\left\langle 1\right| + \dots)(\hat{a}+\hat{a}^\dagger)(\alpha\left|0\right\rangle + \beta\left|1\right\rangle + \dots)
\end{aligned}$$

$$\langle p\rangle = \langle\psi|x|\psi\rangle = \int_{-\infty}^{\infty} \hat{p}\,|\psi(x)|^2\,dx = \text{ analog zu } \langle\psi|x|\psi\rangle\,.$$

0.3 Pauli-Matrizen

$$S_0 = \frac{\hbar}{2}\sigma_0 = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$S_1 = \frac{\hbar}{2}\sigma_1 = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$S_2 = \frac{\hbar}{2}\sigma_2 = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

$$S_3 = \frac{\hbar}{2}\sigma_3 = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$S_1 = S_x,\ \sigma_1 = \sigma_x,\ S_2 = S_y \dots$