Zeitabhängige Wellenfunktion:

$$\psi(x,t) = \alpha e^{-itE_0/\hbar} |0\rangle + \beta e^{-itE_1/\hbar} |1\rangle + \cdots$$

Mit den Energien $E_n = \left(n + \frac{1}{2}\right) \omega\hbar$
Und den Eigenfunktionen $|n\rangle = \psi_n(x)$

0.1 Operatoren

Ortsoperator	$\hat{x} = x = i\hbar \frac{\partial}{\partial p}$
Impulsoperator	$\hat{p} = p = -i\hbar \frac{\partial}{\partial x}$
Abstiegsoperator	$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \hat{x} + \frac{i}{\sqrt{2\hbar m\omega}} \hat{p}$
Aufstiegsoperator	$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \hat{x} - \frac{i}{\sqrt{2\hbar m\omega}} \hat{p}$
Hamiltonoperator	$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 = \frac{\hat{p}^2}{2m} + V(\hat{x})$
Damit gilt	$\hat{x} = \frac{1}{2}\sqrt{\frac{2\hbar}{m\omega}}(\hat{a} + \hat{a}^\dagger)$
	$\hat{p} = \frac{1}{2i}\sqrt{2\hbar m\omega}(\hat{a} - \hat{a}^\dagger)$
	$\hat{a}\|n\rangle = \sqrt{n}\,\|n-1\rangle \qquad \hat{a}\|0\rangle = 0$
	$\hat{a}^\dagger \|n\rangle = \sqrt{n+1}\,\|n+1\rangle$
	$\|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}} \|0\rangle$

0.2 Mittelwerte

$$\begin{aligned}
\langle x \rangle &= \langle \psi | x | \psi \rangle \\
&= \int_{-\infty}^{\infty} x\, |\psi(x)|^2 \, dx \qquad \text{nur falls } x \text{ kein Operator!} \\
&= \int_{-\infty}^{\infty} \psi(x)^* \hat{x}\, \psi(x)\, dx \\
&= \frac{1}{2}\sqrt{\frac{2\hbar}{m\omega}} (\alpha^* \langle 0| + \beta^* \langle 1| + \dots)(\hat{a} + \hat{a}^\dagger)(\alpha |0\rangle + \beta |1\rangle + \dots) \\
\langle p \rangle &= \langle \psi | x | \psi \rangle \\
&= \int_{-\infty}^{\infty} \hat{p}\, |\psi(x)|^2 \, dx = \text{ analog zu } \langle \psi | x | \psi \rangle .
\end{aligned}$$

0.3 Pauli-Matrizen

$$\begin{aligned}
S_0 &= \frac{\hbar}{2}\sigma_0 = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
S_1 &= \frac{\hbar}{2}\sigma_1 = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
S_2 &= \frac{\hbar}{2}\sigma_2 = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\
S_3 &= \frac{\hbar}{2}\sigma_3 = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
\vec{S} &= \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}
\end{aligned}$$

$S_1 = S_x,\ \sigma_1 = \sigma_x,\ S_2 = S_y \dots$

$$\begin{aligned}
\gamma E_{\text{kin}} &= \frac{1}{2} m V^2 = \\
p &= \sqrt{2mE_{\text{kin}}}
\end{aligned}$$

0.4 De Broglie

$$\lambda = \frac{h}{p} = \frac{h}{m \cdot v}$$

0.5 Relativitätstheorie

Lorentzfaktor	$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
Bewegte Masse	$m = \gamma m_0 \qquad$ (Mit Ruhemasse m_0)
Impuls	$p = mv = \gamma m_0 v$
Energie	$E = mc^2 = \gamma m_0 c^2$
Kinetische Energie	$E_{\text{kin}} = \frac{1}{2} mv^2 = \frac{\gamma}{2} m_0 v^2 = mc^2 - m_0 c^2$
Photoeffekt	$E_\gamma = W_A + E_{\text{el,kin}}$

0.6 Konstanten

$$\begin{aligned}
m_e &= 9.10938291 \cdot 10^{-31} \text{ kg} \\
m_p &= 1.67262178 \cdot 10^{-27} \text{ kg} \\
m_n &= 1.67492735 \cdot 10^{-27} \text{ kg} \\
h &= 6.62606957 \cdot 10^{-34} \text{Js} \\
\hbar &= \frac{h}{2\pi} \\
E_\gamma &= h\nu \quad \text{(Energie des Photons)} \\
p_\gamma &= \frac{E_\gamma}{c} \quad \text{(Impuls des Photons)}
\end{aligned}$$

0.7 Kommutatoren

$$\begin{aligned}
\left[\hat{L}_i, \hat{x}_j\right] &= i\hbar\epsilon_{ijk}\hat{x}_k \\
\left[\hat{L}_i, \hat{p}_j\right] &= i\hbar\epsilon_{ijk}\hat{p}_k \\
\left[\hat{A}, \hat{B}\hat{C}\right] &= \left[\hat{A}, \hat{B}\right]\hat{C} + \hat{B}\left[\hat{A}, \hat{C}\right] \\
\left[\hat{A}\hat{B}, \hat{C}\right] &= \hat{A}\left[\hat{B}, \hat{C}\right] + \left[\hat{A}, \hat{C}\right]\hat{B}
\end{aligned}$$

0.8 Drehimpuls

$$\vec{r} = \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}, \quad \vec{p} = \begin{pmatrix} \hat{p}_x \\ \hat{p}_y \\ \hat{p}_z \end{pmatrix}, \quad \vec{L} = \vec{p} \times \vec{r}$$

$$\begin{aligned}
\text{Ł}_x &= yp_z - zp_y = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right) \\
\text{Ł}_y &= zp_x - xp_z = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right) \\
\text{Ł}_z &= xp_y - yp_x = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) \\
\left[L_i, \hat{f}\right] &= 0, \text{ für Skalaroperatoren } \hat{f} \\
\left[L_i, \hat{f}_k\right] &= -i\hbar\epsilon_{ijk}\hat{f}_k, \text{ für Vektoroperatoren } \hat{f} \\
\left[L_i, \hat{f}_{kl}\right] &= i\hbar(\epsilon_{ikp}\delta_{nl} + \epsilon_{iln}\delta_{kp})\hat{f}_{pn}
\end{aligned}$$

0.9 Potentialtopf

$$\begin{aligned}
k_1 = k_3 &= \sqrt{\frac{2mE}{\hbar^2}} \\
k_2 &= \sqrt{\frac{2m(E - V_0)}{\hbar^2}} \\
r &= A + B - 1 \\
\psi_{\text{I}}(x) &= e^{ik_1 x} + re^{-ik_1 x} \\
\psi_{\text{II}}(x) &= Ae^{ik_2 x} + Be^{ik_2 x} \\
\psi_{\text{III}}(x) &= te^{-ik_3(x-a)}
\end{aligned}$$

Dabei ist t bzw. r der Transmissions- bzw. Reflektionskoeffizent.