Oscillating ball driven by an initial kinetic energy

Yongxing Wang

We consider a 3D oscillating ball which is an extension of the 2D disc in [1, 2]. We use this example to test stability of the proposed approach by investigating the evolution of total energy:

$$E_{total}(t_n) = \frac{\rho^f}{2}\int_{\Omega}|\mathbf{u}_n|^2 + \frac{\rho^\delta}{2}\int_{\Omega_n^s}|\mathbf{u}_n|^2 + \frac{\Delta t\mu^f}{2}\sum_{k=1}^{n}\int_{\Omega}\mathrm{D}\mathbf{u}_k : \mathrm{D}\mathbf{u}_k + \frac{c_1}{2}\int_{\Omega_{\mathbf{X}}^s}\left(tr_{\mathbf{F}_n\mathbf{F}_n^T} - d\right). \tag{1}$$

The four different energy contributions/terms in the above equation have the following meanings successively: Kinetic energy of fluid plus fictitious fluid, kinetic energy of solid minus fictitious fluid, viscous dissipation and potential energy of the solid.

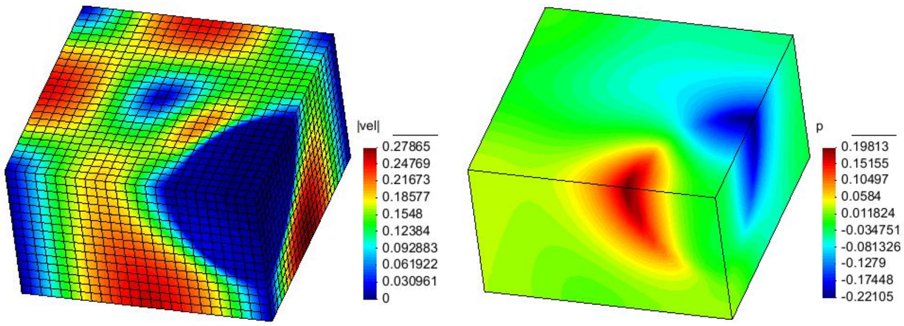
The ball is initially located at the centre of $\Omega = [0,1]\times[0,1]\times[0,0.6]$ with a radius of 0.2. Using the property of symmetry this computation is carried out on 1/8 of domain Ω: $[0,0.5]\times[0,0.5]\times[0,0.3]$. The initial velocities of x and y components are the same as that used in [1, 2], which are prescribed by the stream function

$$\Phi = \Phi_0 \sin(ax)\sin(by), \tag{2}$$

with $\Phi_0 = 5.0\times10^{-2}$ and $a = b = 2\pi$. The z component is initially set to be 0. In this test, $\rho^f = 1$, $\mu^f = 0.01$, $\rho^s = 1.5$ and $c_1 = 1$. In order to visualise the mesh and deformation of the solid, a snapshot of fluid velocity and pressure are presented in Figure 1, and the corresponding deformed solid is displayed in Figure 2 (a). Figure 2 (b) indicates an excellent agreement between deformations of the 2D disc and the 3D ball, which is expected because this 3D test is a symmetric extension of the 2D disc. It can be seen from Figure 3 that the total energy is nonincreasing, which is an indication of stability. In addition, the total energy converges to the initial system energy as we reduce the size of the time step, which shows the desired energy conservation property of the proposed scheme.

References

[1] Y. Wang, P. K. Jimack, M. A. Walkley, A one-field monolithic fictitious domain method for fluid–structure interactions, Computer Methods in Applied Mechanics and Engineering 317 (2017) 1146–1168.

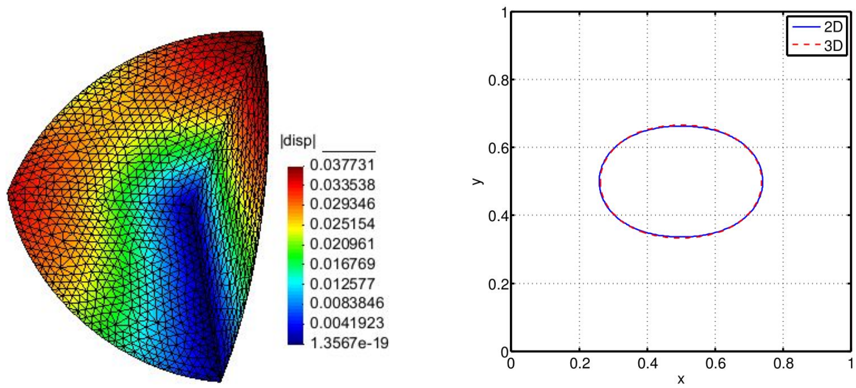


(a) Velocity.

(b) Pressure.

Figure 1: Distributiong of velocity norm and pressure on the fluid mesh at $t = 0.21$ (the ball is maximally stretched), $\Delta t = 5.0 \times 10^{-3}$.

[2] H. Zhao, J. B. Freund, R. D. Moser, A fixed-mesh method for incompressible flow–structure systems with finite solid deformations, Journal of Computational Physics 227 (6) (2008) 3114–3140.



(a) Displacement on the solid mesh. (b) Compare with the corresponding 2D problem.

Figure 2: Solid deformation corresponding to Figure 1 and comparison between 2D and 3D cases. The data of the 1/8 ball has been symmetrically extended to a whole ball when plotting the above figure in (b).

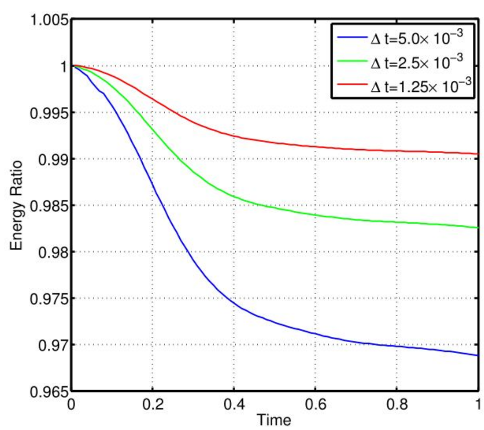


Figure 3: Evolution of the energy ratio $E_{total}(t_n)/E_{total}(t_0)$.