Homework 1

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1 Proof by Contrapositive

We begin with some definitions.

Definition 1. An integer n is said to be *even* if there is an integer k such that $n = 2k$. Otherwise, n is said to be *odd*.

Remark: It follows that odd integers can be written uniquely as $n = 2k + 1$ for some integer k.

Definition 2. We say that x is a *rational number*, denoted $x \in \mathbb{Q}$, if there exist integers p, q such that $x = \dfrac{p}{q}$.

(Recommended) Problem 1. Suppose that $a, b \in \mathbb{R}$ such that $ab \notin \mathbb{Q}$. Then we have $a \notin \mathbb{Q}$ or $b \notin \mathbb{Q}$.

(Recommended) Problem 2. Suppose $x, y \in \mathbb{Z}$ such that xy is even. Then either x is even or y is even.

(Recommended) Problem 3. Suppose $x, y \in \mathbb{Z}$ such that $x + y \geq 15$. Then either $x \geq 8$ or $y \geq 8$.

(Recommended) Problem 4. Suppose $n \in \mathbb{Z}$ such that n^3 is even. Then n is even.