Alpha leg

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Abstract—

Index Terms—Hopping robot

1 INTRODUCTION

Bla bla

2 MASS-SPRING-DAMPER MODEL

When the leg is in contact with the ground, the equation of motion is:

$$m\ddot{y} + c\dot{y} + ky = -mg \tag{1}$$

The general solution for $y(t)$ and is:

$$y(t) = e^{\left(-\frac{c}{2m} - iw_d\right)t}C_1 + e^{\left(-\frac{c}{2m} + iw_d\right)t}C_2 - \frac{gm}{k} \tag{2}$$

where C_1 and C_2 are constants depending on initial conditions and damped natural frequency w_d is defined as:

$$w_d = \frac{1}{2m}\sqrt{4km - c^2} \tag{3}$$

When the leg is in contact with the ground the initial conditions are:

$$\begin{aligned} y(t) &= 0 \\ \dot{y}(t) &= -v_0 \end{aligned} \tag{4}$$

where v_0 is the velocity of the ball just prior to contact with the ground. Follows the constant values:

$$\begin{aligned} C_1 &= \tfrac{gmw_d}{2kw_d} + i\tfrac{cg-2kv_0}{4kw_d} \\ C_2 &= \tfrac{gmw_d}{2kw_d} - i\tfrac{cg-2kv_0}{4kw_d} \end{aligned} \tag{5}$$

And the final solution:

$$y(t) = \left[\frac{cg - 2kv_0}{2kw_d}sin(w_dt) + \frac{mg}{k}cos(w_dt)\right]e^{-\frac{c}{2m}t} - \frac{mg}{k} \tag{6}$$

2.1 Calculation of total jumping period

The total jumping period consist of time when the leg is in contact with the ground Tc and flight time Tf.

The contact time Tc can be obtained from **??** by finding the first solution of the equation $y(0) = 0$. In order to solve it analytically, the equation **??** is rearranged as:

$$\begin{aligned} y(t) = &-\frac{v0}{w_d}e^{-\frac{c}{2m}t} \cdot sin(w_dt) \\ &+ \frac{mg}{k} \cdot \left[e^{-\frac{c}{2m}t}(cos(w_dt) + \frac{c}{2mw_d}sin(w_dt) - 1\right] \end{aligned} \tag{7}$$

Assuming $\frac{mg}{k} << 1$m, which is acceptable for our spring model(see section II), the equation **??** is approximated as:

$$y(t) = -\frac{v0}{w_d}e^{-\frac{c}{2m}t} \cdot sin(w_dt) \tag{8}$$

and the minimum non zero solution which represents the contact time Tc is:

$$Tc = \frac{\pi}{w_d} \tag{9}$$

The flight time Tf is defined as:

$$Tf = \frac{2v_1}{g} \tag{10}$$

where

$$v_1 = \dot{y}(Tc) = v_0e^{-\frac{c\pi}{2mw_d}} \tag{11}$$

2.2 Energy loss

The loss of energy coused by damping factor c can be obtained from difference of kinetic energy v_0 and v_1:

$$\Delta EKIN = EKINv_1 - EKINv_0 = \frac{mv_0^2}{2}\left(e^{\frac{-c\pi}{mw_d}} - 1\right) \tag{12}$$

The same energy loss can be obtained with:

$$E_{Closs} = \int_0^{T_c} c\dot{y}^2 dy \tag{13}$$

$$E_{Closs} = \frac{v_0^2 \left(4m^2 + \frac{e^{\frac{-cT_c}{m}\left(-c^2 - 4m^2 w_d^2 + c^2 Cos[2T_c w_d] + 2cmw_d Sin[2T_c w_d]]\right)}}{w_d^2} \right)}{8m} \tag{14}$$

3 Conclusion

References

[1] L. Righetti and A.J. Ijspeert,Design Methodologies for Central Pattern Generators: An Application to Crawling Humanoids, in Proc. Robotics: Science and Systems, 2006.

[2] Chen et al., Locomotion control of quadruped robots based on CPG-inspired workspace trajectory generation, ICRA 2011, pp. 1250-1255, 2011.

[3] Wang et al., Kinematics Analysis and Motion Simulation of a Quadruped Walking Robot with Parallel Leg Mechanism, The Open Mechanical Engineering Journal, vol. 4, pp. 77-85, 2010.