@LEG

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Abstract—

Index Terms—Hopping robot

1 INTRODUCTION

Bla bla

2 MASS-SPRING-DAMPER MODEL

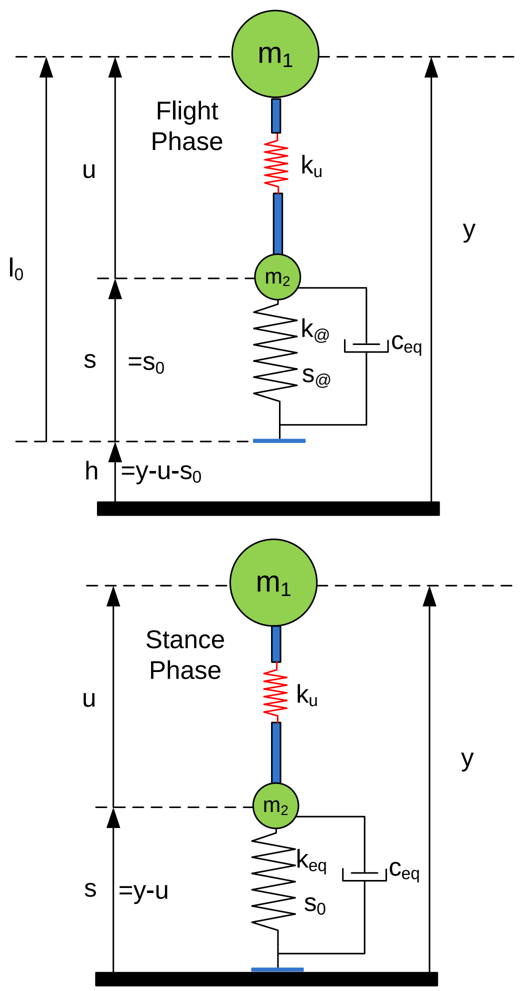

Fig. 1: MassSpringDamperActuation,flight-stance phase

Introduction to Lagrange Euler modeling approach.Virtual work.

The Lagrangian function:

$$L\left(q_i, \dot{q}_i\right) = T(q_i, \dot{q}_i) - V(y) \tag{1}$$

The Lagrangian equation:

$$\frac{d}{dt}\frac{\partial}{\partial \dot{q}_i}L(q_i, \dot{q}_i) - \frac{\partial}{\partial q_i}L(q_i, \dot{q}_i) = Q_i \tag{2}$$

Q_i - generalized nonconservative force derived by using virtual work principle

Virual work principle assumes infinitesimal virtual displacement on the system which results the virtual work done on the system.

$$\delta W_j = Q_j \cdot \delta q_j \tag{3}$$

2.1 Flight Phase

The potential energy of @LEG (Fig.1) is defined as:

$$V_{flight} = m_1 \cdot g \cdot y + m_2 \cdot g(y-u) + \frac{1}{2}k_u(u_0-u)^2 \tag{4}$$

where u_0 is motor reference position.

The kinetic energy of @LEG (Fig.1) is defined as:

$$T_{flight} = \frac{1}{2}m_1 \cdot \dot{y}^2 + \frac{1}{2}m_2(\dot{y}-\dot{u})^2 \tag{5}$$

The Lagrangian function:

$$\begin{aligned} L = T_{flight} - V_{flight} = \frac{1}{2}m_1 \cdot \dot{y}^2 + \frac{1}{2}m_2(\dot{y}-\dot{u})^2 \\ -m_1 \cdot g \cdot y - m_2 \cdot g(y-u) - \frac{1}{2}k_u(u_0-u)^2 \end{aligned} \tag{6}$$

The @LEG contains one generalized forces, the spring damping force Q_c. Because there is no mass m_3 at the end of the leg($s = s_0$), the flight phase does not sense any virtual work.

Two cases of virtual displacement are observed separately.

CASE 1:Suppose there is virtual displacement $\delta y \neq 0$ while $\delta u = 0$(by using the (2) follows):

$$(m_1+m_2)\ddot{y} - m_2\ddot{u} + (m_1+m_2)g = 0 \tag{7}$$

CASE 2:Suppose there is virtual displacement $\delta u \neq 0$ while $\delta y = 0$(by using the (2) follows):

$$m_2(\ddot{u} - \ddot{y}) - m_2 g - k_u(u_0 - u) = 0 \tag{8}$$

2.2 Stance Phase

The potential energy of @LEG (Fig.1) is defined as:

$$V_{stance} = m_1 \cdot g \cdot y + m_2 \cdot g(y-u) + \frac{1}{2}k_{eq}(s_0 - y + u)^2 + \frac{1}{2}k_u(u_0 - u)^2 \tag{9}$$

The kinetic energy of @LEG (Fig.1) is defined as:

$$T_{stance} = \frac{1}{2}m_1 \cdot \dot{y}^2 + \frac{1}{2}m_2(\dot{y} - \dot{u})^2 \tag{10}$$

The Lagrangian function:

$$\begin{aligned} L = T_{stance} - V_{stance} = \frac{1}{2}m_1 \cdot \dot{y}^2 + \frac{1}{2}m_2(\dot{y} - \dot{u})^2 \\ -m_1 \cdot g \cdot y - m_2 \cdot g(y-u) - \frac{1}{2}k_{eq}(s_0 - y + u)^2 \\ -\frac{1}{2}k_u(u_0 - u)^2 \end{aligned} \tag{11}$$

Again two virtual displacement are observed separately, δy and δu. Suppose there is virtual displacement $\delta y \neq 0$ while $\delta u = 0$.

$$\begin{aligned} W_{yS} = Q_{yS} \cdot \delta y = -c(\dot{y} - \dot{u})\delta s \\ s = y - u \Rightarrow \dot{s} = \dot{y} - \dot{u} \\ \delta s = \delta y, \delta u = 0 \end{aligned} \tag{12}$$

Finally:

$$Q_{yS} = -c \cdot (\dot{y} - \dot{u}) \tag{13}$$

Virtual displacement $\delta u \neq 0$ while $\delta y = 0$ provides:

$$\begin{aligned} W_{uS} = Q_{uS} \cdot \delta u = c(\dot{y} - \dot{u})\delta s \\ s = y - u \Rightarrow \dot{s} = \dot{y} - \dot{u} \\ \delta s = -\delta u, \delta y = 0 \end{aligned} \tag{14}$$

Finally:

$$Q_{uS} = c \cdot (\dot{y} - \dot{u}) \tag{15}$$

By using the lagrangian Equation (2) and Equations (13) and (15) follows:

$$(m_1+m_2)\ddot{y} - m_2\ddot{u} + (m_1+m_2)g - k_{eq}(s_0 - y + u) = -c(\dot{y} - \dot{u}) \tag{16}$$

$$m_2\ddot{u} - m_2\ddot{y} - m_2 g + k_{eq}(s_0 - y + u) - k_u(u_0 - u) = c(\dot{y} - \dot{u}) \tag{17}$$

The potential energy of the mass m is defined as:

$$E_{pot} = mg \cdot y(t) + \frac{k \cdot (y(t) - u(t) - l_0)^2}{2} \tag{18}$$

The kinetic energy of the mass m is defined as:

$$E_{kin} = \frac{m \cdot \dot{y}^2}{2} \tag{19}$$

By using the Lagrangian function, the equation of motion can be obtained as:

where F_c is damping force defined as:

$$F_c = -c \cdot [\dot{y}(t) - \dot{u}(t))] \tag{20}$$

When the leg is in contact with the ground, the final equation of motion is:

$$m\ddot{y} + c[\dot{y}(t) - \dot{u}(t)] + k[y(t) - u(t) - l_0] = -mg \tag{21}$$

The general solution for $y(t)$ and is:

$$y(t) = e^{\left(-\frac{c}{2m} - iw_d\right)t}C_1 + e^{\left(-\frac{c}{2m} + iw_d\right)t}C_2 - \frac{gm}{k} \tag{22}$$

where C_1 and C_2 are constants depending on initial conditions and damped natural frequency w_d is defined as:

$$w_d = \frac{1}{2m}\sqrt{4km - c^2} \tag{23}$$

When the leg is in contact with the ground the initial conditions are:

$$\begin{aligned} y(t) = 0 \\ \dot{y}(t) = -v_0 \end{aligned} \tag{24}$$

where v_0 is the velocity of the ball just prior to contact with the ground. Follows the constant values:

$$\begin{aligned} C_1 = \tfrac{gmw_d}{2kw_d} + i\tfrac{cg - 2kv_0}{4kw_d} \\ C_2 = \tfrac{gmw_d}{2kw_d} - i\tfrac{cg - 2kv_0}{4kw_d} \end{aligned} \tag{25}$$

And the final solution:

$$y(t) = \left[\frac{cg - 2kv_0}{2kw_d} sin(w_d t) + \frac{mg}{k} cos(w_d t)\right] e^{-\frac{c}{2m}t} - \frac{mg}{k} \tag{26}$$

2.3 Calculation of total jumping period

The total jumping period consist of time when the leg is in contact with the ground Tc and flight time Tf.

The contact time Tc can be obtained from 26 by finding the first solution of the equation $y(0) = 0$. In order to solve it analytically, the equation 26 is rearranged as:

$$\begin{aligned} y(t) = -\frac{v0}{w_d} e^{-\frac{c}{2m}t} \cdot sin(w_d t) \\ + \frac{mg}{k} \cdot \left[e^{-\frac{c}{2m}t}(cos(w_d t) + \frac{c}{2mw_d} sin(w_d t) - 1\right] \end{aligned} \tag{27}$$

Assuming $\frac{mg}{k}$ << 1m, which is acceptable for our spring model(see section II), the equation 27 is approximated as:

$$y(t) = -\frac{v0}{w_d} e^{-\frac{c}{2m}t} \cdot sin(w_d t) \tag{28}$$

and the minimum non zero solution which represents the contact time Tc is:

$$Tc = \frac{\pi}{w_d} \tag{29}$$

The flight time Tf is defined as:

$$Tf = \frac{2v_1}{g} \tag{30}$$

where

$$v_1 = \dot{y}(Tc) = v_0 e^{-\frac{c\pi}{2mw_d}} \tag{31}$$

2.4 Energy loss

The loss of energy coused by damping factor c can be obtained from difference of kinetic energy v_0 and v_1:

$$\Delta EKIN = EKINv_1 - EKINv_0 = \frac{mv_0^2}{2}\left(e^{\frac{-c\pi}{mw_d}} - 1\right) \tag{32}$$

The same energy loss can be obtained with:

$$E_{Closs} = \int Fd \cdot dy = \int (c \cdot \frac{dy}{dt})\frac{dy}{dt} dt = \int_0^{T_c} c\dot{y}^2 dy \tag{33}$$

$$E_{Closs} = \frac{v_0^2 \left(4m^2 + \frac{e^{\frac{-cT_c}{m}}\left(-c^2 - 4m^2 w_d^2 + c^2 Cos[2T_c w_d] + 2cmw_d Sin[2T_c w_d]]\right)}{w_d^2}\right)}{8m} \tag{34}$$

3 Conclusion

References

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[3] Wang et al., Kinematics Analysis and Motion Simulation of a Quadruped Walking Robot with Parallel Leg Mechanism, The Open Mechanical Engineering Journal, vol. 4, pp. 77-85, 2010.