Rocket Motor Calculations R3.6
Optimisation of a solid fuel motor using RNX-N180 propellant
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Recalculated A and n for Nuplex 180 system epoxy from static tests of R3.3 and R3.5.

Operating Parameters

$p\_chamber := 4.5 \times 10^{6}$ Design pressure

$p\_exit := 101325$ Ambient pressure

$T\_exit := 1016 + 273$ Burn temperature at ambient pressure (see 'Burn Temperature Measurement' report)

Casing Geometry

$t\_casing := 0.002$ Tube wall thickness

$\phi\_i\_casing := 0.053$ Tube inner diameter

$\phi\_segment := \phi\_i\_casing - 0.002$ Propellant OD

$\phi\_screw := 0.006$ Cap screw diameter

$N\_screws := 8$

$L\_segment := 0.08$ Segment length

$t\_segment := 0.020$ Segment wall thicknes

$N\_segments := 3$ # propellant segments

Material Properties

$\sigma\_yield\_casing := 250 \cdot 10^{6}$ Supplier (Atlas Steels) spec for mild steel tubing

$\tau\_screws\_yield := 500 \cdot 10^{6}$ Standard black finish cap screws

Aerodynamic Parameters

$Cd := 0.75$ Drag coeff from CFD simulation

$\phi\_airframe := .087$

$$Area := \pi \cdot \frac{\phi\_airframe^{2}}{4}$$

Compatible with Cd

$m\_dry := 6$ Allowing 1.2 kg more than 1st rocket for payload and electronics, assuming some mass will be lost from tube

$\rho\_air := 1.2$ Assume 25 deg. C, Patmos

Propellant Parameters

$A\_RNX := 512.0$ From Nakka's charts, adjusted after static test

$n\_RNX := 0.2580$ From Nakka's charts

$k := 1.055$ Determined from Nakka's charts

$R\_RNX := 287$ Universal gas constant, will not be accurate with high temperatures

$\rho\_RNX := 1664$ Measured by volumetric displacement, agrees with literature

Burn Surface Area

$$A\_ends(d\_burn) := \frac{N\_segments \cdot 2 \cdot \pi}{4} \cdot \left[\phi\_segment^2 - (\phi\_segment - 2 \cdot t\_segment + 2 \cdot d\_burn)^2\right]$$

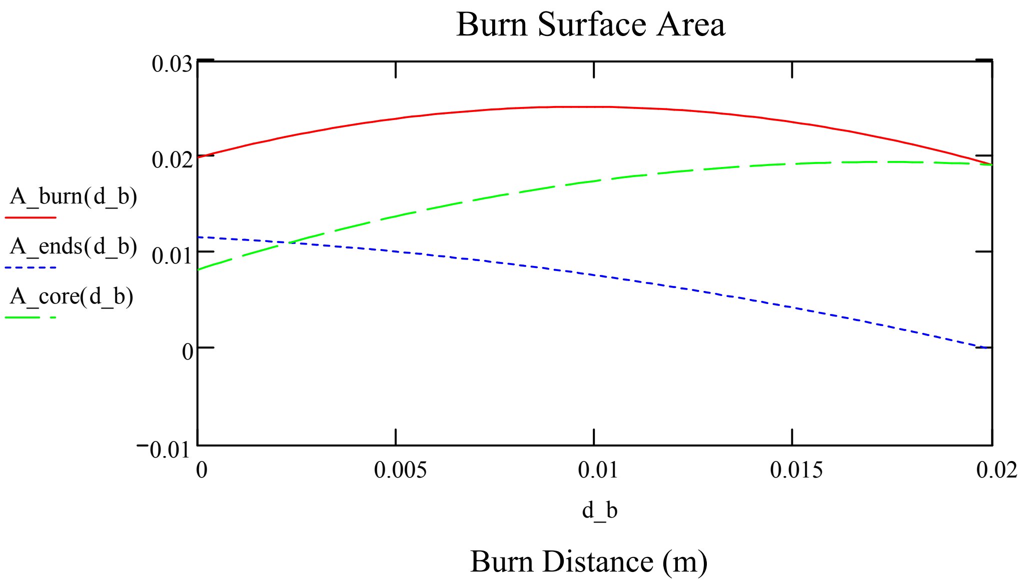
$$A\_core(d\_burn) := N\_segments \cdot \pi \cdot (\phi\_segment - 2 \cdot t\_segment + 2 \cdot d\_burn) \cdot (L\_segment - 2 \cdot d\_burn)$$

$$A\_burn(d\_burn) := A\_core(d\_burn) + A\_ends(d\_burn)$$

$$N\_points := 100$$

$$d\_b := 0, \frac{t\_segment}{N\_points} .. t\_segment$$

Array of burn distance for plotting against burn surface area



Solve for the maximum burn area

$$Grad(d) := \frac{d}{dd} A\_burn(d) \rightarrow 6 \cdot \pi \cdot (.8e\text{-}1 - 2 \cdot d) - 6 \cdot \pi \cdot (.11e\text{-}1 + 2 \cdot d) + \frac{3}{2} \cdot \pi \cdot [(-.44e\text{-}1) - 8 \cdot d]$$

$$guess := 0$$

Initail guess for the solver to work with

$$d\_A\_max := root(Grad(guess), guess)$$

$$A\_burn\_max := A\_core(d\_A\_max) + A\_ends(d\_A\_max)$$

Propellant mass assuming square end faces

$$m\_propellant := N\_segments \cdot \rho\_RNX \cdot \frac{\pi}{4} \cdot \left[\phi\_segment^2 - (\phi\_segment - 2 \cdot t\_segment)^2\right] \cdot L\_segment$$

Motor Operating Conditions

Potter and Wiggert Equation 9.3.12:

$$\text{Mach\_exit} := \left[\left(\frac{2}{k-1}\right)\cdot\left(\frac{\text{p\_chamber}}{\text{p\_exit}}\right)^{\frac{k-1}{k}} - \frac{2}{k-1}\right]^{\frac{1}{2}} \quad *$$

$\text{Mach\_exit} = 2.8$

Combining Potter and Wiggert Equations 9.3.12 and 9.3.13:

$$\text{T\_chamber} := \text{T\_exit}\cdot\left[1 + \left(\frac{k-1}{2}\right)\cdot\text{Mach\_exit}^2\right] \quad *$$

$\text{T\_chamber} = 1.571 \times 10$

Casing Stresses

$$\sigma\text{\_hoop} := \text{p\_chamber}\cdot\frac{\phi\text{\_i\_casing}}{2\cdot\text{t\_casing}} \quad *$$

$\sigma\text{\_hoop} = 5.963 \times 10$

$$\sigma\text{\_axial} := \text{p\_chamber}\cdot\frac{\phi\text{\_i\_casing}^2}{(\phi\text{\_i\_casing} + 2\cdot\text{t\_casing})^2 - \phi\text{\_i\_casing}^2} \quad *$$

$\sigma\text{\_axial} = 2.873 \times 1$

$$\sigma\text{\_von\_Mises} := \left[\frac{1}{2}\cdot\left[(\sigma\text{\_hoop} - \sigma\text{\_axial})^2 + (\sigma\text{\_axial} - \text{p\_chamber})^2 + (\text{p\_chamber} - \sigma\text{\_hoop})^2\right]\right]^{\frac{1}{2}} \quad *$$

$$\text{Safety\_factor\_casing} := \frac{\sigma\text{\_yield\_casing}}{\sigma\text{\_von\_Mises}} \quad *$$

$\sigma\text{\_von\_Mises} = 4.786 \times 10^7$

$\text{Safety\_factor\_casing} = 5.224$

$$\tau\text{\_screws} := \text{p\_chamber}\cdot\frac{\phi\text{\_i\_casing}^2}{\phi\text{\_screw}^2\cdot\text{N\_screws}} \quad *$$

(Assumes pure shear)

$\tau\text{\_screws} = 4.389 \times 10$

$$\text{Safety\_factor\_screws} := \frac{\tau\text{\_screws\_yield}}{\tau\text{\_screws}} \quad *$$

$\text{Safety\_factor\_screws} = 11.392$

$$\sigma\text{\_holes} := \text{p\_chamber}\cdot\frac{\frac{\pi}{4}\phi\text{\_i\_casing}^2}{\phi\text{\_screw}\cdot\text{t\_casing}\cdot\text{N\_screws}} \quad *$$

(Assumes pressure acts uniformly over a rectangle of dimensions ϕ_hole x t_wall)

$$\text{Safety\_factor\_holes} := \frac{\sigma\text{\_yield\_casing}}{\sigma\text{\_holes}} \quad *$$

$\sigma\text{\_holes} = 1.034 \times 10$

$\text{Safety\_factor\_holes} = 2.41$

Nozzle Geometry at Design Condition

$$K\_n := A\_RNX \cdot \left(\frac{p\_chamber}{10^6}\right)^{n\_RNX}$$ *

(Ratio of burn surface area to throat area, p conv. to MP
Note: For p_chamber = 1 MPa, n_RNX has no effect on K_n. A_RNX only was adjusted to correct chamber pressure from previous iteration

Nakka Equation 14 - Nozzle Theory

$K\_n = 754.742$

$$Expansion\_ratio := \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \cdot \left(\frac{p\_exit}{p\_chamber}\right)^{\frac{1}{k}} \cdot \left[\frac{k+1}{k-1} \cdot \left[1 - \left(\frac{p\_exit}{p\_chamber}\right)^{\frac{k-1}{k}}\right]\right]^{\frac{1}{2}}$$ *

$Expansion\_ratio = 0.116$

$$A\_star := \frac{A\_burn\_max}{K\_n}$$ *

$A\_star = 3.347 \times 10^{-}$

$$A\_exit := \frac{A\_star}{Expansion\_ratio}$$ *

$A\_exit = 2.877 \times 10^{-4}$

$$\phi\_throat := \sqrt{4 \cdot \frac{A\_star}{\pi}}$$ *

$\phi\_throat = 6.528 \times 10^{-}$

$$\phi\_exit := \sqrt{\frac{4 \cdot A\_exit}{\pi}}$$ *

$\phi\_exit = 0.019$

Perfromance Metrics

Nakka Equation 3 - Solid Rocket Motor Thrust Calculations:

$$Thrust := A\_star \cdot p\_chamber \cdot \left[\frac{2 \cdot k^2}{k-1} \cdot \left[\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \cdot \left[1 - \left(\frac{p\_exit}{p\_chamber}\right)^{\frac{k-1}{k}}\right]\right]\right]^{\frac{1}{2}}$$ *

$Thrust = 244.548$

Potter and Wigert Equation ???:

$$m\_dot := p\_chamber \cdot A\_star \cdot \left(\frac{k}{R\_RNX \cdot T\_chamber}\right)^{\frac{1}{2}} \cdot \left(\frac{k+1}{2}\right)^{\frac{k+1}{2 \cdot (1-k)}}$$ *

$$Burn\_time := \frac{m\_propellant}{m\_dot}$$ *

(Assumes constant burn rate throughout)

$Burn\_time = 5.60$

Trajectory Approximation by Numerical Method

$t\_inc := 0.1$ $t\_sim := 70$ $N\_sim := \mathrm{floor}\left(\frac{t\_sim}{t\_inc}\right)$ $tim := 0, t\_inc .. t\_sim$

$$thrust(tim) := \begin{cases} Thrust & \text{if } tim < Burn\_time \\ 0 & \text{otherwise} \end{cases}$$

$$mass(tim) := \begin{cases} m\_propellant + m\_dry - \frac{m\_propellant}{Burn\_time} \cdot tim & \text{if } tim < Burn\_time \\ m\_dry & \text{otherwise} \end{cases}$$

```
U := | U_0 ← 1·10^(-12)
     | for i ∈ 1..N_sim - 1
     |   | Fd_i ← (1/2)·Cd·ρ_air·Area·(U_(i-1))^2·U_(i-1)/|U_(i-1)|
     |   | F_total_i ← thrust(i·t_inc) - Fd_i - 9.81·mass(i·t_inc)
     |   | U_i ← U_(i-1) + F_total_i/mass(i·t_inc)·t_inc
     | return U
```

```
h := | h_0 ← 0
     | for i ∈ 0..N_sim - 1
     |   h_(i+1) ← h_i + U_i·t_inc
     | return h
```

$h\_max := \max(h)$ $h\_max = 1.146 \times 1$

$U\_max := \max(U)$ $U\_max = 143.0$

CAD Geometry

$L\_igniter := 0.01$ $Shoulder := 0.022$ $L\_overlap := 0.008$ $Clearance := 0.00005$ (Radial Clearance

$L\_tube := L\_segment \cdot N\_segments + L\_igniter + Shoulder \cdot 2 + L\_overlap$

$$Hole\_offset := \frac{Shoulder - 0.01}{2} + \frac{L\_overlap}{2}$$

Results Summary

$m\_propellant = 0.778$ *

$Mach\_exit = 2.82$ *

$T\_chamber = 1.571 \times 10^{3}$ *

$Safety\_factor\_casing = 5.224$ *

$Safety\_factor\_screws = 11.392$ *

$Safety\_factor\_holes = 2.417$ *

$\phi\_throat = 6.528 \times 10^{-3}$ *

$\phi\_exit = 0.019$ *

$Thrust = 244.548$ *

$Burn\_time = 5.604$ *

$h\_max = 1.146 \times 10^{3}$

$U\_max = 143.013$

Propellant Proportions

$Excess := 1.7$ Mix excess propellant to allow for spillage, machining etc.

$n\_Hardener := 0.034$ $n\_Resin := 0.206$ $n\_Rust := 0.08$ $n\_Nitrate := 0.68$

$Mass := m\_propellant \cdot Excess$ $Mass = 1.322$

$Hardener := n\_Hardener \cdot Mass$ $Resin := n\_Resin \cdot Mass$ $Rust := n\_Rust \cdot Mass$ $Nitrate := n\_Nitrate \cdot Ma$

$Hardener = 0.045$

$Resin = 0.272$

$Rust = 0.106$

$Nitrate = 0.899$

$Mass = 1.322$

Output to Design Tables

Nozzle							
Revision	Dimension	Throat	Exit	ID tube	Shoulder	Clearance	Cap Screw
R3.4	Comment	*Radius*	*Radius*	*Radius*			
		Throat@Sket	it@Sketcl	ID@Sketch1	lder@Ske	Clearance(	Diameter@S
	Default	3.264244	9.569476	26.5	22	0.05	6

$$\left(\frac{\phi\_throat}{2} \cdot 1000 \quad \frac{\phi\_exit}{2} \cdot 1000 \quad \frac{\phi\_i\_casing}{2} \cdot 1000 \quad Shoulder \cdot 1000 \quad Clearance \cdot 1000 \quad \phi\_screw \cdot 1000 \right)$$

Tube					
Revision	Dimension	ID tube	L tube	Cap Screw	Hole offset
R3.4	Comment				
		ID@Sketch1	ngth@Extrud	Diameter@Sketc	Offset@Sketch2
	Default	53	302	6	10

$$\left(\phi\_i\_casing \cdot 1000 \quad L\_tube \cdot 1000 \quad \phi\_screw \cdot 1000 \quad Hole\_offset \cdot 1000 \right)$$

Balkhead					
Revision	Dimension	ID tube	Shoulder	Clearance	Cap Screw
R3.4	Comment	*Radius*			
		ID@Sketch1	Shoulder@Sketch1	Clearance@SI	Diameter@S
	Default	26.5	22	0.05	6

$$\left(\frac{\phi\_i\_casing}{2} \cdot 1000 \quad Shoulder \cdot 1000 \quad Clearance \cdot 1000 \quad \phi\_screw \cdot 1000 \right)$$

Cap Screw		
Revision	Dimension	Diameter
R3.4	Comment	
		Diameter@Sketch1
	Default	6

$$\phi\_screw \cdot 1000$$

Propellant Grain				
Revision	Dimension	OD	t	L
R3.4	Comment			
		OD@Sketch1	t@Sketch1	L@Extrude
	Default	51	20	80

$$\left(\phi\_segment \cdot 1000 \quad t\_segment \cdot 1000 \quad L\_segment \cdot 1000 \right)$$

Noge: To update, right click on each table and select 'Save As'. **Delete previous version before saving or Mathcad will crash.**