Rocket Motor Calculations
Optimisation of a solid fuel motor using RNX-57V propellant
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Operating Parameters

$p\_chamber := 1 \times 10^{6}$ * Design pressure

$p\_exit := 101325$ * Ambient pressure

$T\_exit := 1016 + 273$ * Burn temperature at ambient pressure
(see 'Burn Temperature Measurement' report)

Casing Geometry

$t\_casing := 0.0016$ * Tube wall thickness

$\phi\_i\_casing := 0.0605$ * Tube inner diameter

$\phi\_segment := \phi\_i\_casing - 0.002$ * Propellant OD

$\phi\_screw := 0.005$ * Cap screw diameter

$N\_screws := 10$ *

$L\_segment := 0.1$ * Segment length

$t\_segment := 0.02$ * Segment wall thickness

$N\_segments := 4$ * # propellant segments

Material Properties

$\sigma\_yield\_casing := 250 \cdot 10^{6}$ * ϕ2.5" 304 stainless handrail tube purchased from Little Metals Co.

$\tau\_screws\_yield := 500 \cdot 10^{6}$ * Standard black finish cap screws

Aerodynamic Parameters

$Cd := 0.75$ * Drag coeff from CFD simulation

$\phi\_airframe := .087$

$$Area := \pi \cdot \frac{\phi\_airframe^{2}}{4}$$ * Compatible with Cd

$m\_dry := 6$ * Allowing 1.2 kg more than 1st rocket for payload and electronics, assuming some mass will be lost from tube

$\rho\_air := 1.2$ * Assume 25 deg. C, Patmos

Propellant Parameters

$A\_RNX := 400.9$ * From Nakka's charts, adjusted after static test

$n\_RNX := 0.641$ * From Nakka's charts

$k := 1.055$ * Determined from Nakka's charts

$R\_RNX := 287$ * Universal gas constant, will not be accurate with high temperatures

$\rho\_RNX := 1664$ * Measured by volumetric displacement, agrees with literature

Burn Surface Area

$$A\_ends(d\_burn) := \frac{N\_segments \cdot 2 \cdot \pi}{4} \cdot \left[\phi\_segment^2 - (\phi\_segment - 2 \cdot t\_segment + 2 \cdot d\_burn)^2\right]$$

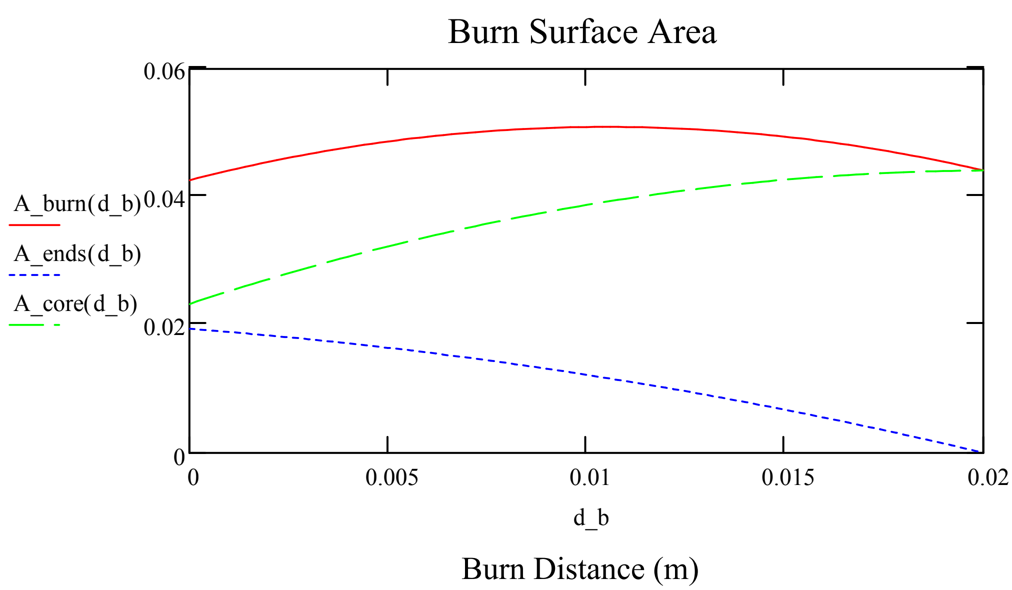
$$A\_core(d\_burn) := N\_segments \cdot \pi \cdot (\phi\_segment - 2 \cdot t\_segment + 2 \cdot d\_burn) \cdot (L\_segment - 2 \cdot d\_burn)$$

$$A\_burn(d\_burn) := A\_core(d\_burn) + A\_ends(d\_burn)$$

$$N\_points := 100$$

$$d\_b := 0, \frac{t\_segment}{N\_points} .. t\_segment$$

Array of burn distance for plotting against burn surface area



Solve for the maximum burn area

$$Grad(d) := \frac{d}{dd} A\_burn(d) \rightarrow 8 \cdot \pi \cdot (.1 - 2 \cdot d) - 8 \cdot \pi \cdot (.185e\text{-}1 + 2 \cdot d) + 2 \cdot \pi \cdot [(-.740e\text{-}1) - 8 \cdot d]$$

$$guess := 0$$

Initail guess for the solver to work with

$$d\_A\_max := root(Grad(guess), guess)$$

$$A\_burn\_max := A\_core(d\_A\_max) + A\_ends(d\_A\_max)$$

Propellant mass assuming square end faces

$$m\_propellant := N\_segments \cdot \rho\_RNX \cdot \frac{\pi}{4} \cdot \left[\phi\_segment^2 - (\phi\_segment - 2 \cdot t\_segment)^2\right] \cdot L\_segment$$

Motor Operating Conditions

Potter and Wiggert Equation 9.3.12:

$$\text{Mach\_exit} := \left[\left(\frac{2}{k-1}\right)\cdot\left(\frac{\text{p\_chamber}}{\text{p\_exit}}\right)^{\frac{k-1}{k}} - \frac{2}{k-1}\right]^{\frac{1}{2}} \quad *$$

$$\text{Mach\_exit} = 2.147$$

Combining Potter and Wiggert Equations 9.3.12 and 9.3.13:

$$\text{T\_chamber} := \text{T\_exit}\cdot\left[1 + \left(\frac{k-1}{2}\right)\cdot\text{Mach\_exit}^2\right] \quad *$$

$$\text{T\_chamber} = 1.452 \times 10^3$$

Casing Stresses

$$\sigma\text{\_hoop} := \text{p\_chamber}\cdot\frac{\phi\text{\_i\_casing}}{2\cdot\text{t\_casing}} \quad *$$

$$\sigma\text{\_hoop} = 1.891 \times 10^7$$

$$\sigma\text{\_axial} := \text{p\_chamber}\cdot\frac{\phi\text{\_i\_casing}^2}{(\phi\text{\_i\_casing} + 2\cdot\text{t\_casing})^2 - \phi\text{\_i\_casing}^2} \quad *$$

$$\sigma\text{\_axial} = 9.21 \times 10^6$$

$$\sigma\text{\_von\_Mises} := \left[\frac{1}{2}\cdot\left[(\sigma\text{\_hoop} - \sigma\text{\_axial})^2 + (\sigma\text{\_axial} - \text{p\_chamber})^2 + (\text{p\_chamber} - \sigma\text{\_hoop})^2\right]\right]^{\frac{1}{2}} \quad *$$

$$\text{Safety\_factor\_casing} := \frac{\sigma\text{\_yield\_casing}}{\sigma\text{\_von\_Mises}} \quad *$$

$$\sigma\text{\_von\_Mises} = 1.553 \times 10^7$$

$$\text{Safety\_factor\_casing} = 16.103$$

$$\tau\text{\_screws} := \text{p\_chamber}\cdot\frac{\phi\text{\_i\_casing}^2}{\phi\text{\_screw}^2\cdot\text{N\_screws}} \quad *$$

(Assumes pure shear)

$$\tau\text{\_screws} = 1.464 \times 10^7$$

$$\text{Safety\_factor\_screws} := \frac{\tau\text{\_screws\_yield}}{\tau\text{\_screws}} \quad *$$

$$\text{Safety\_factor\_screws} = 34.151$$

$$\sigma\text{\_holes} := \text{p\_chamber}\cdot\frac{\frac{\pi}{4}\phi\text{\_i\_casing}^2}{\phi\text{\_screw}\cdot\text{t\_casing}\cdot\text{N\_screws}} \quad *$$

(Assumes pressure acts uniformly over a rectangle of dimensions ϕ_hole x t_wall)

$$\text{Safety\_factor\_holes} := \frac{\sigma\text{\_yield\_casing}}{\sigma\text{\_holes}} \quad *$$

$$\sigma\text{ holes} = 3.593 \times 10^7$$

$$\text{Safety\_factor\_holes} = 6.957$$

Nozzle Geometry at Design Condition

$$K\_n := A\_RNX \cdot \left(\frac{p\_chamber}{10^6}\right)^{n\_RNX} *$$

(Ratio of burn surface area to throat area, p conv. to MPa) **Note:** For p_chamber = 1 MPa, n_RNX has no effect on K_n. A_RNX only was adjusted to correct chamber pressure from previous iteration

Nakka Equation 14 - Nozzle Theory

$$K\_n = 400.9$$

$$Expansion\_ratio := \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \cdot \left(\frac{p\_exit}{p\_chamber}\right)^{\frac{1}{k}} \cdot \left[\frac{k+1}{k-1} \cdot \left[1 - \left(\frac{p\_exit}{p\_chamber}\right)^{\frac{k-1}{k}}\right]\right]^{\frac{1}{2}} *$$

$$Expansion\_ratio = 0.383$$

$$A\_star := \frac{A\_burn\_max}{K\_n} * \qquad A\_star = 1.27 \times 10^{-4}$$

$$A\_exit := \frac{A\_star}{Expansion\_ratio} * \qquad A\_exit = 3.313 \times 10^{-4}$$

$$\phi\_throat := \sqrt{4 \cdot \frac{A\_star}{\pi}} * \qquad \phi\_throat = 0.013$$

$$\phi\_exit := \sqrt{\frac{4 \cdot A\_exit}{\pi}} * \qquad \phi\_exit = 0.021$$

Perfromance Metrics

Nakka Equation 3 - Solid Rocket Motor Thrust Calculations:

$$Thrust := A\_star \cdot p\_chamber \cdot \left[\frac{2 \cdot k^2}{k-1} \cdot \left[\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \cdot \left[1 - \left(\frac{p\_exit}{p\_chamber}\right)^{\frac{k-1}{k}}\right]\right]\right]^{\frac{1}{2}} * \qquad Thrust = 163.252 *$$

Potter and Wigert Equation ???:

$$m\_dot := p\_chamber \cdot A\_star \cdot \left(\frac{k}{R\_RNX \cdot T\_chamber}\right)^{\frac{1}{2}} \cdot \left(\frac{k+1}{2}\right)^{\frac{k+1}{2 \cdot (1-k)}} *$$

$$Burn\_time := \frac{m\_propellant}{m\_dot} *$$ (Assumes constant burn rate throughout) $$Burn\_time = 13.229 *$$

Trajectory Approximation by Numerical Method

$t\_inc := 0.1 \quad t\_sim := 70 \quad N\_sim := floor\left(\frac{t\_sim}{t\_inc}\right) \quad tim := 0, t\_inc .. t\_sim$

$$thrust(tim) := \begin{cases} Thrust & \text{if } tim < Burn\_time \\ 0 & \text{otherwise} \end{cases}$$

$$mass(tim) := \begin{cases} m\_propellant + m\_dry - \frac{m\_propellant}{Burn\_time} \cdot tim & \text{if } tim < Burn\_time \\ m\_dry & \text{otherwise} \end{cases}$$

$$U := \left| \begin{array}{l} U_0 \leftarrow 1 \cdot 10^{-12} \\ \text{for } i \in 1 .. N\_sim - 1 \\ \quad \left| \begin{array}{l} Fd_i \leftarrow \frac{1}{2} \cdot Cd \cdot \rho\_air \cdot Area \cdot \left(U_{i-1}\right)^2 \cdot \frac{U_{i-1}}{\left|U_{i-1}\right|} \\ F\_total_i \leftarrow thrust(i \cdot t\_inc) - Fd_i - 9.81 \cdot mass(i \cdot t\_inc) \\ U_i \leftarrow U_{i-1} + \frac{F\_total_i}{mass(i \cdot t\_inc)} \cdot t\_inc \end{array} \right. \\ \text{return } U \end{array} \right.$$

$$h := \left| \begin{array}{l} h_0 \leftarrow 0 \\ \text{for } i \in 0 .. N\_sim - 1 \\ \quad h_{i+1} \leftarrow h_i + U_i \cdot t\_inc \\ \text{return } h \end{array} \right.$$

$h\_max := max(h) \qquad h\_max = 1.778 \times 10^3$

$U\_max := max(U) \qquad U\_max = 145.954$

Results Summary

m_propellant = 1.61 *

Mach_exit = 2.147 *

T_chamber = 1.452×10^{3} *

Safety_factor_casing = 16.103 *

Safety_factor_screws = 34.151 *

Safety_factor_holes = 6.957 *

ϕ_throat = 0.013 *

ϕ_exit = 0.021 *

Thrust = 163.252 *

Burn_time = 13.229 *

h_max = 1.778×10^{3}

U_max = 145.954