Computer Systems 1
Lecture 3

Logic Gates

Dr. John T. O'Donnell
School of Computing Science
University of Glasgow

Topics

Lab and Quiz

- There is a weekly 2-hour lab, starting this week
- Boyd Orr 715
- Go to your scheduled lab and be sure to sit in the right section (identified as red, blue, etc.)
- There is a weekly lab sheet on Moodle — study it in advance
- Each lab will have some paper & pencil problems, and some problems using the computers
- Please try the problems before your lab

Hardware: Digital Circuits

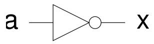
- We will start with primitive components
- Study their behaviour when they are connected
- Build up a family of useful circuits
- Gradually build larger and more powerful families of circuits, using the ones already defined.
- We'll end up with some circuits that can do real computing.

Logic gates

- Digital circuits are built from a very small number of primitive little machines
- Only a few are needed!
  - Logic gates — we'll use just four: inv, and2, or2, xor2
  - Flip flop — just one is needed: dff
- So we need to
  - Learn what these five components do
  - Learn how to connect them into *digital circuits*

The Inverter

- A basic component
- Takes an input bit a and produces an output bit x
- The bits are carried on wires
- The technical name for a wire is signal
- The output is the logical opposite of the input.

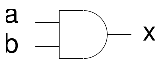


x = inv a

a	x
0	1
1	0

The 2-input and gate

The output is 1 if all inputs are 1.

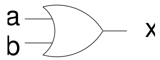


x = and2 a b

a	b	x
0	0	0
0	1	0
1	0	0
1	1	1

The 2-input inclusive or gate

The output is 1 if any input is 1.

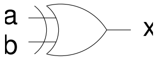


x = or2 a b

a	b	x
0	0	0
0	1	1
1	0	1
1	1	1

The 2-input exclusive or gate

The output is 1 if either input is 1, but not both.



x = xor2 a b

a	b	x
0	0	0
0	1	1
1	0	1
1	1	0

Gate delay

- A logic gate is a primitive component
- It takes a small number of input bits, and produces a result according to a fixed truth table
- As long as the inputs remain stable, the output remains stable
- If an input changes at a point in time, it will take the logic gate a small amount of time to bring the output to the new value: the gate delay
- Gate delays are on the order of 0.01 ns (ns = nanosecond, there are 10^9 ns per second).
- May be faster or slower, depending on the technology.

Combinational circuits

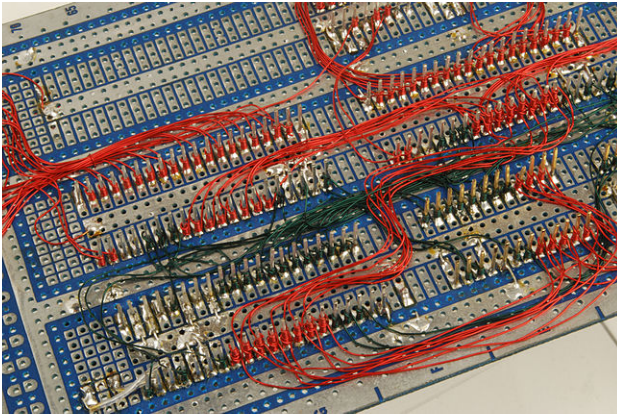
A combinational circuit...

- Consists of logic gates connected together
- The circuit has some inputs and produces some outputs
- It contains no feedback loops
- The outputs depend on the current input values (i.e. the circuit has no memory).

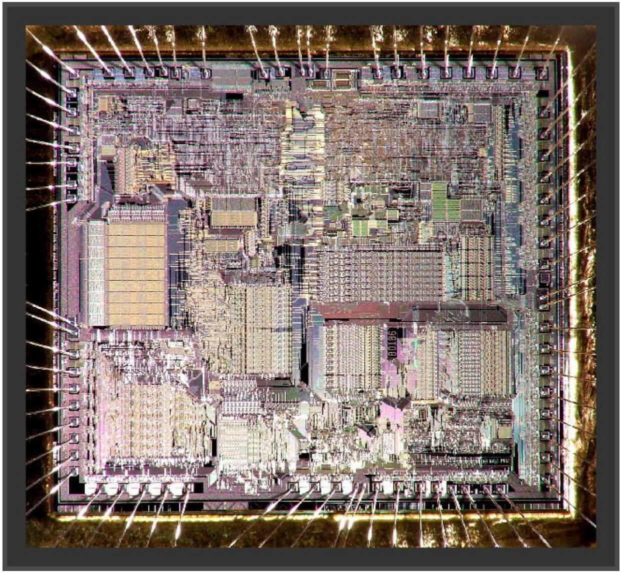
Circuit simulation

- When you learn to program, you learn how to "execute a program by hand".
- This is an essential skill:
  - It helps to understand how the computer executes the program
  - It enables you to understand bugs
- For a digital circuit, the equivalent skill is *circuit simulation*
  - You're given the inputs to a circuit
  - You calculate what its outputs should be, by doing the same calculations the hardware would make

Connecting components with wires



An integrated circuit: Intel A80186 CPU

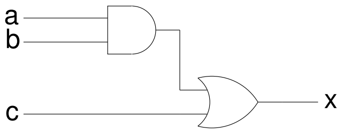


How to simulate a combinational circuit

- You're given the inputs to the circuit. The other signal values are unknown.
- Find an output of a logic gate where all the inputs are known, and write down the output value (using the truth table for the logic gate).
- Repeat until all signals are known.

Example of circuit simulation

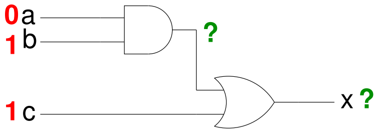
- Inputs are a, b, c (each is a bit)
- Output is x (a bit)



Given specific values of a, b, c, what is the output x?

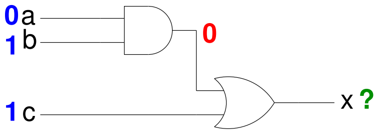
Example of circuit simulation

First, we're given the inputs to the circuit. Other signals are unknown.



After one gate delay

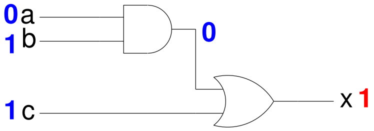
Look for a component where all the inputs are known, and note the value of its output sigmal.



In this example, we can't simulate the or2 gate, because we don't yet know its top input. But we can simulate the and2 gate, and its output is 0.

After two gate delays

Look for a component where all the inputs are known, and note the value of its output sigmal.



Continue the process until all signals are known.

Building block circuits

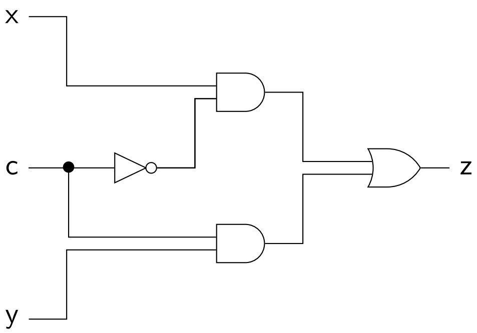
- To do Useful Stuff a circuit usually needs to be fairly big
- But it's hard to understand or to design a really big circuit
- The solution is abstraction
    - We will put several components together to build a black box circuit
    - Then several of those will combine to make a bigger one
    - And then a still-larger one. . .
- We're going to look at several example circuits that are really important — they are critically important in computers

Multiplexer

The multiplexer is the fundamental circuit used to make decisions

- A multiplexer is a hardware version of the if-then-else expression
- The idea is to choose between two values (x, y) based on another value (c)
    - Given three inputs: c, x, y
    - Produces one output
        - If c = 0 the output is x
        - If c = 1 the output is y
- In digital hardware circuits, there are no statements
    - So, there are no "if statements" to determine whether other statements should be executed
    - Instead, we have signals whose values depend on other signals

Multiplexer: z = (if c=0 then x else y)



Simulating the multiplexer

$\boxed{\text{To find out what the circuit does, simulate it!}}$

- Given an set of input values, you can calculate the corresponding output.
- Example: Suppose $c = 1, x = 0, y = 1$. Then the inverter outputs 0, the first and gate has inputs 0 and 0 so it outputs 0; the second and gate has inputs 1 and 1 so it outputs 1; the or2 gate has inputs 0 and 1 so it outputs 1. This is the output of the entire circuit.
- By doing this for all 8 sets of input values, you can fill in the truth table for the circuit, which fully describes its behaviour.

Operators in Boolean Algebra

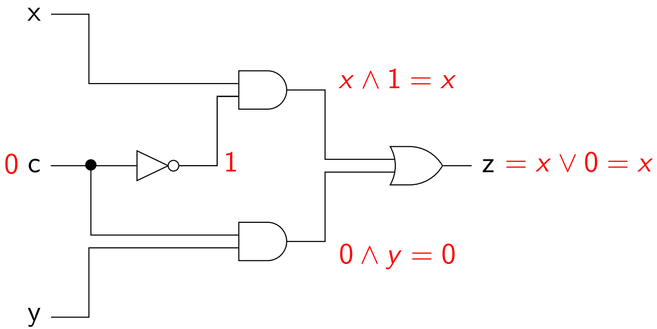
x	$\neg x$
0	1
1	0

x	y	$x \wedge y$	$x \vee y$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

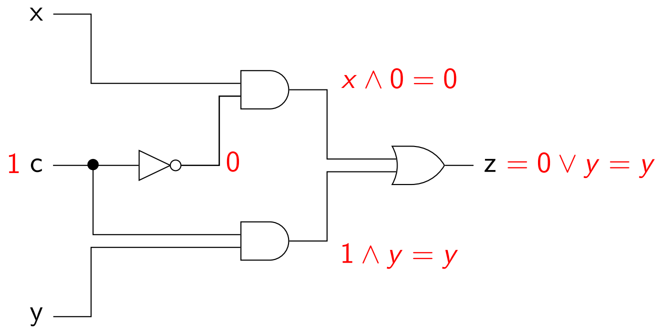
These are the same as the definitions of the logic gates (except we don't have a Boolean operator for the "exclusive or" gate)

Multiplexer when c=0: z = (if c=0 then x else y) = x

We can calculate the output z using Boolean algebra



Multiplexer when c=1: z = (if c=0 then x else y) = y



Truth table for mux1

c	x	y	out
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Meaning of the multiplexer

```
mux1 c x y = if c is zero then x else y
```

c	x	y	out
0	**0**	0	**0**
0	**0**	1	**0**
0	**1**	0	**1**
0	**1**	1	**1**
1	0	**0**	**0**
1	0	**1**	**1**
1	1	**0**	**0**
1	1	**1**	**1**

Multiplexers are important!

- Multiplexers are a central black box which we will use again and again.
- One way to understand the circuit is to calculate its truth table by simulating it for each set of input values.
- Most conditional operations in digital circuits are implemented, at the lowest level, by a multiplexer.
- In a circuit, we don't "execute if-then-else statements". There are no statements! Instead, we define signals using if-then-else *expressions*, implemented with multiplexers

Emergent behaviour

- Some systems have these characteristics:
  - They comprise parts with relatively *simple behaviour*
  - They whole system has *complicated behaviour*
  - This complex behaviour emerges from the interaction of the simple parts
- Examples in science, sociology, politics, philosophy, . . .

Complex computing from simple logic gates

- Each of our five primitive devices has extremely simple behaviour
- You can understand completely what each of them does
- None of them does anything remotely like computing:
  - They can't do arithmetic
  - They can't execute statements
  - They can't make decisions
  - They can't do much at all!
- So how can a computer work?
  - All the interesting things that a computer can do are emergent behaviours
  - *Emergent* means that a system whose components are all simple can exhibit complex behaviour

Examples of emergent behaviour

- (Chemistry) The ideal gas law
- (Biology) Motion of a flock of birds
- (Computing) Addition of binary numbers

Decisions

- Ultimately, all decisions become a choice of two alternative values on a signal (wire)
- All of these choices are made by the multiplexer circuit
  - Its behaviour is clearly conditional: the output is x if $c = 0$ but otherwise it's y
  - Yet the implementation of the multiplexer just uses a few logic gates
- This is the fundamental point where conditional behaviour emerges from a lower level behaviour of logic gates!

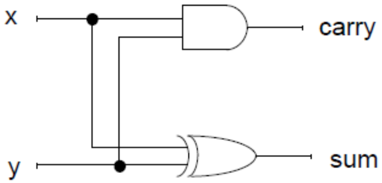
Every decision that a computer system makes ultimately comes down to a multiplexer.

The half adder: adding two bits

- A half adder adds two bits. Since the result could be 0, 1, or 2, we need a two-bit representation of the sum, called (carry, sum).
- Specify the circuit abstractly by writing the complete addition table. Then we recognise that the carry function is just and2, and the sum function is just xor2.

x	y	result	carry	sum
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	2	1	0

The half adder



x	y	result	carry	sum
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	2	1	0

The full adder: adding three bits

- To add two binary numbers, we must add three bits for each column:
  - The two data bits (one from each word)
  - The carry input from the column to the left
- Solution: the full adder
- A full adder adds three bits x, y, z and outputs a carry c and sum s

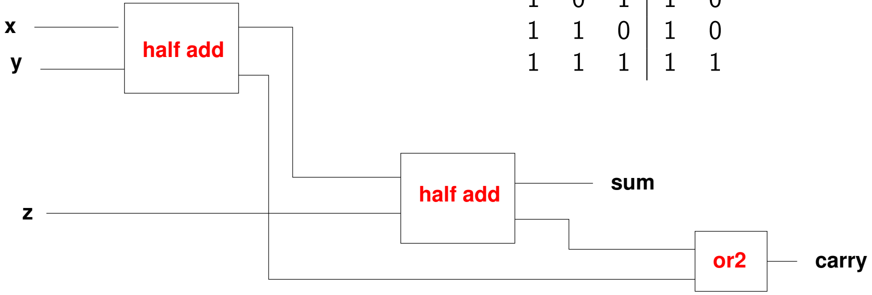
Truth table of full adder

x	y	z	c	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

- The sum is 1 if an odd number of inputs are 1
- The carry is 1 if two or more inputs are 1
- Think of carry, sum as a 2-bit binary number giving the result

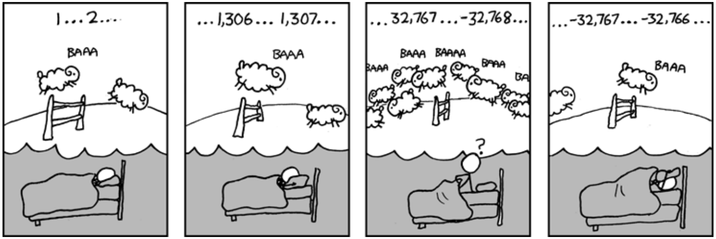
The full adder circuit

x	y	z	c	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



To do

- Revise the lecture slides
- Solve the lab exercises (they are on Moodle)
- Go to your lab and discuss the exercises
- Ask questions about any of the lecture material



https://xkcd.com/571/