Algorithms and Data Structures 2

Recap Lectures 15-16

Dr Michele Sevegnani

School of Computing Science
University of Glasgow

michele.sevegnani@glasgow.ac.uk

Topics we covered so far

- **Red-black trees**
  - Definition
  - Properties
  - Insertion
- **B-trees**
  - Definition
  - Motivation
  - Properties
  - Search
  - Insertion
- **Variants**

Question 1

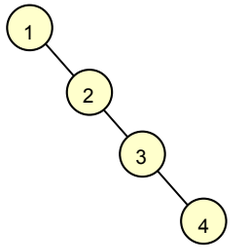
- **How many different BSTs are there containing the elements 1,2,3,4?**

- **How many different BSTs are there containing the elements 1,2,3,4,5, for which the root contains 5?**

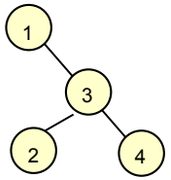
Question 1: solution

- **There are at most 4! = 24 (one per order of insertion)**
- **Not all of these will be unique**
  - There are 14 unique BSTs (1-6)

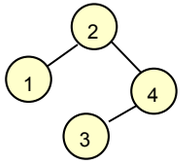
1,2,3,4 →



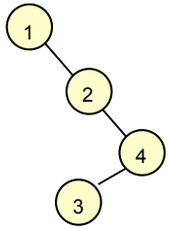
1,3,2,4 →
1,3,4,2



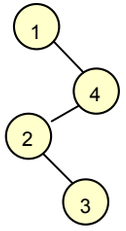
2,1,4,3 →
2,4,1,3
2,4,3,1



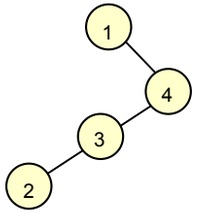
1,2,4,3 →



1,4,2,3 →



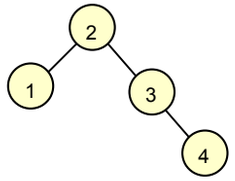
1,4,3,2 →



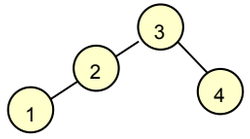
Question 1: solution (cont.)

- **There are at most 4! (one per order of insertion)**
- **Not all of these will be unique**
  - There are 14 unique BSTs (7-12)

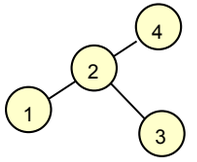
2,1,3,4 →
2,3,1,4
2,3,4,1



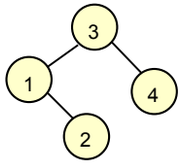
3,2,1,4 →
3,2,4,1
3,4,2,1



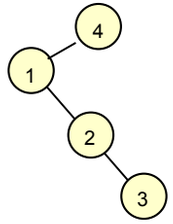
4,2,1,3 →
4,2,3,1



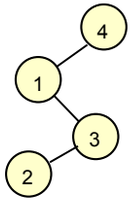
3,1,2,4 →
3,4,1,2
3,1,4,2



4,1,2,3 →



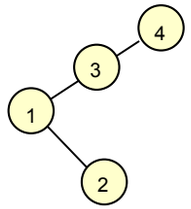
4,1,3,2 →



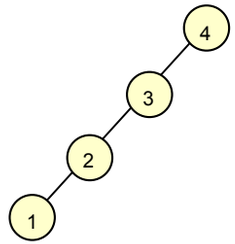
Question 1: solution (cont.)

- **There are at most 4! (one per order of insertion)**
- **Not all of these will be unique**
  - There are 14 unique BSTs (13-14)

4,3,1,2 →



4,3,2,1 →



Question 1: solution (cont.)

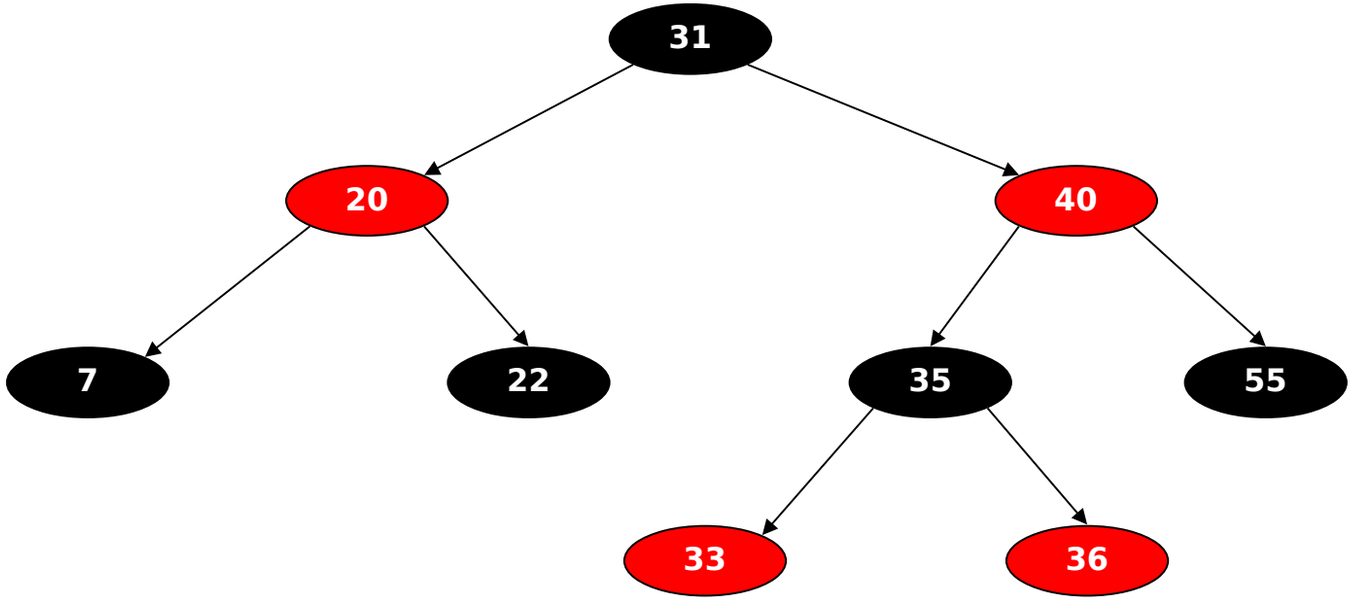
- Every BST on 1,2,3,4,5 with root 5 must result from some permutation of 1,..,5 in which 5 is the first element

- All these trees have 5 as root and a left subtree formed by the different permutations on 1..4

- From the above there are 14 possible left subtrees, so again the answer is 14

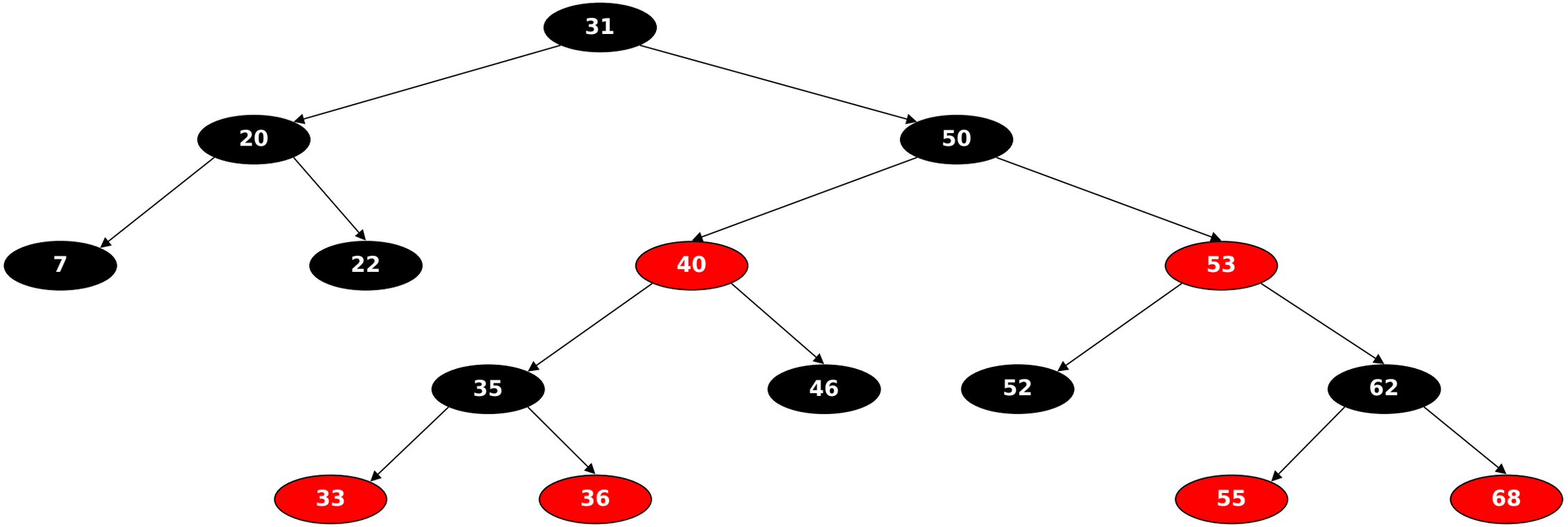
Question 2

- **What is a red-black tree?**
- **Show that the red-black tree formed by inserting the numbers 55,40,31,20,7,22,35, 33,36 in order is**



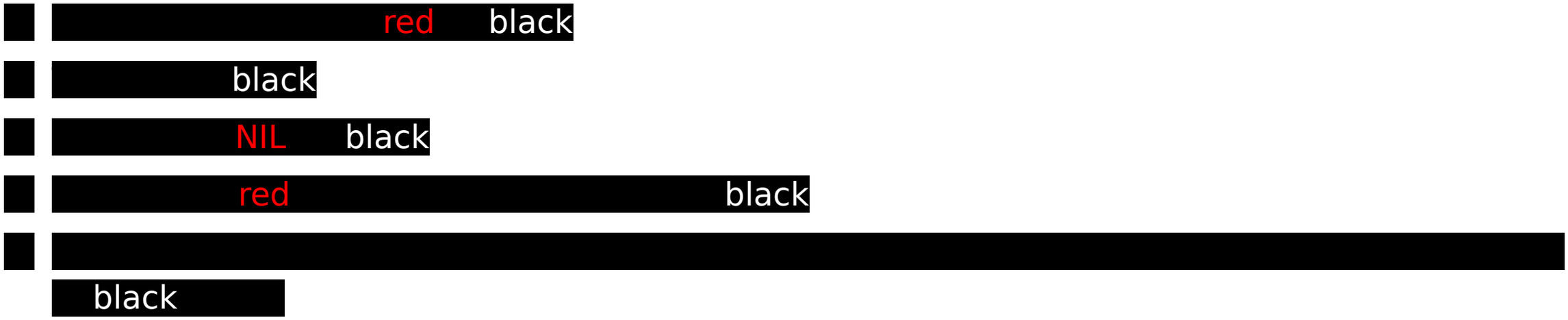
Question 2 (cont.)

- **Show that, by further inserting the numbers 50,46,53,52,68,62 the red-black tree becomes**



Question 2: solution

- **A red-black tree is a binary search tree with an extra attribute colour, which can be either RED or BLACK and satisfies the red-black properties**



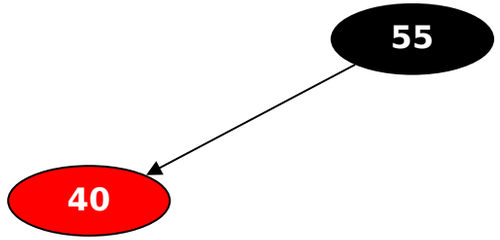
Question 2: solution (cont.)

- **Show that the red-black tree formed by inserting the numbers 55,40,31,20,7,22,35, 33,36 in order is**

Restore root

Question 2: solution (cont.)

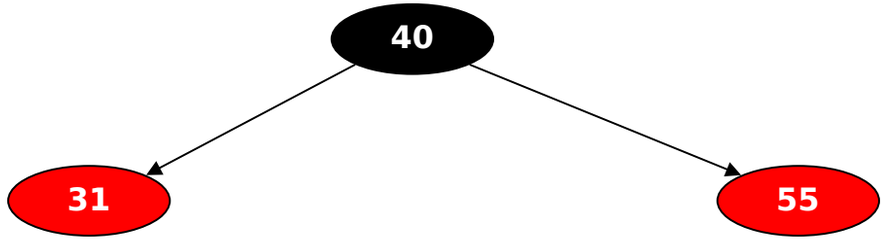
- **Show that the red-black tree formed by inserting the numbers 55,40,31,20,7,22,35, 33,36 in order is**



Question 2: solution (cont.)

- **Show that the red-black tree formed by inserting the numbers 55,40,31,20,7,22,35, 33,36 in order is**

Right rotation



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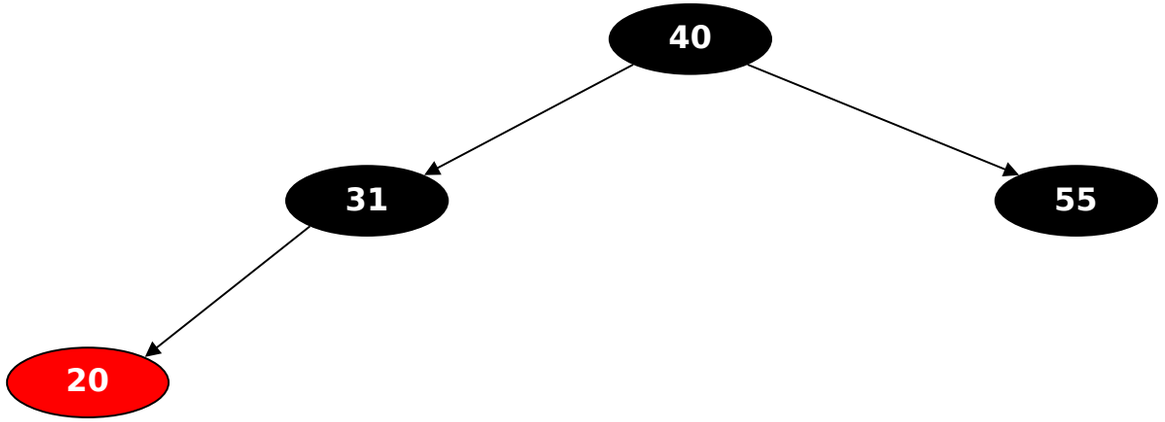

Push down blackness

Question 2: solution (cont.)

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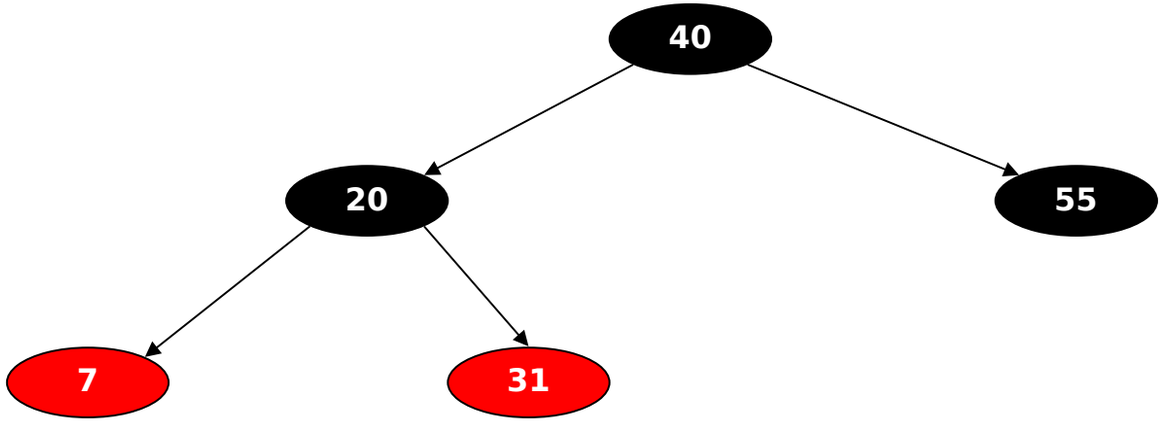

Right rotation

Question 2: solution (cont.)

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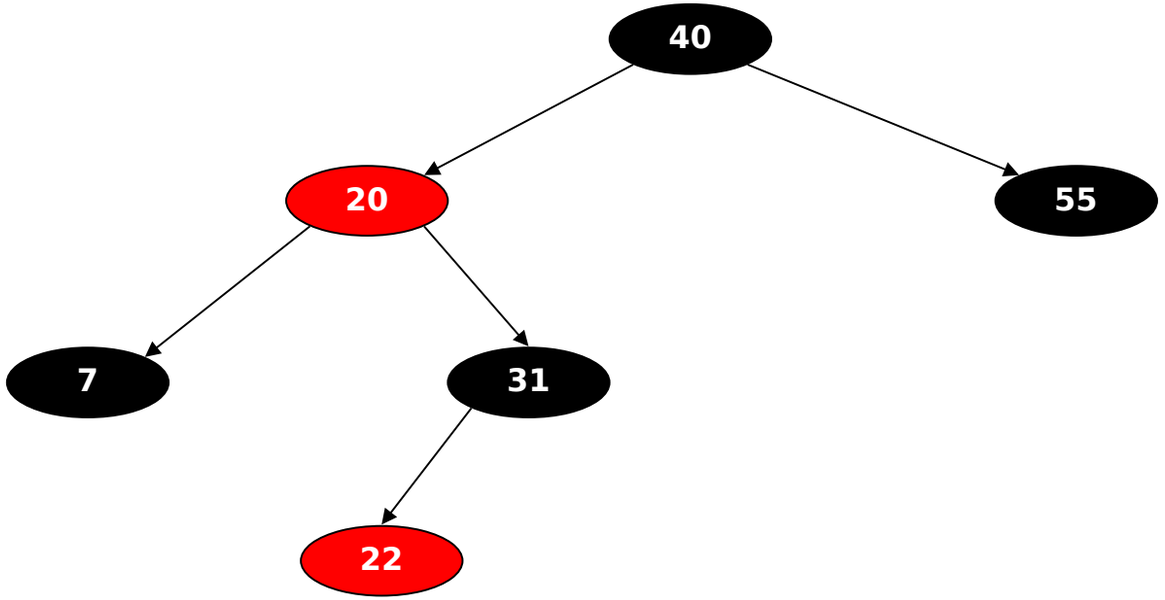

Push down blackness

Question 2: solution (cont.)

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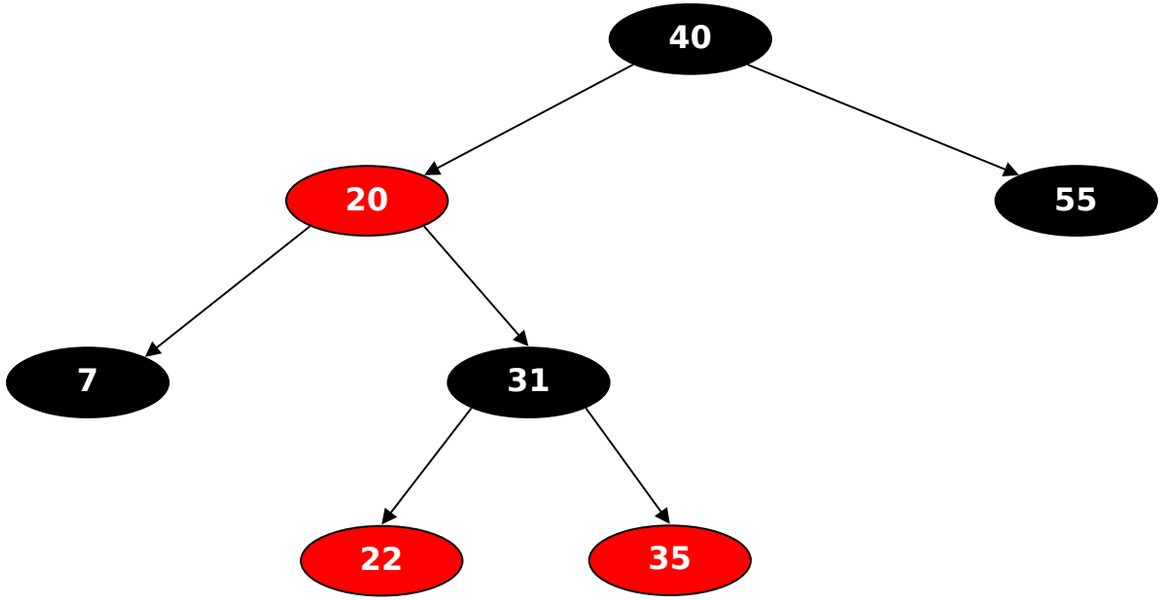

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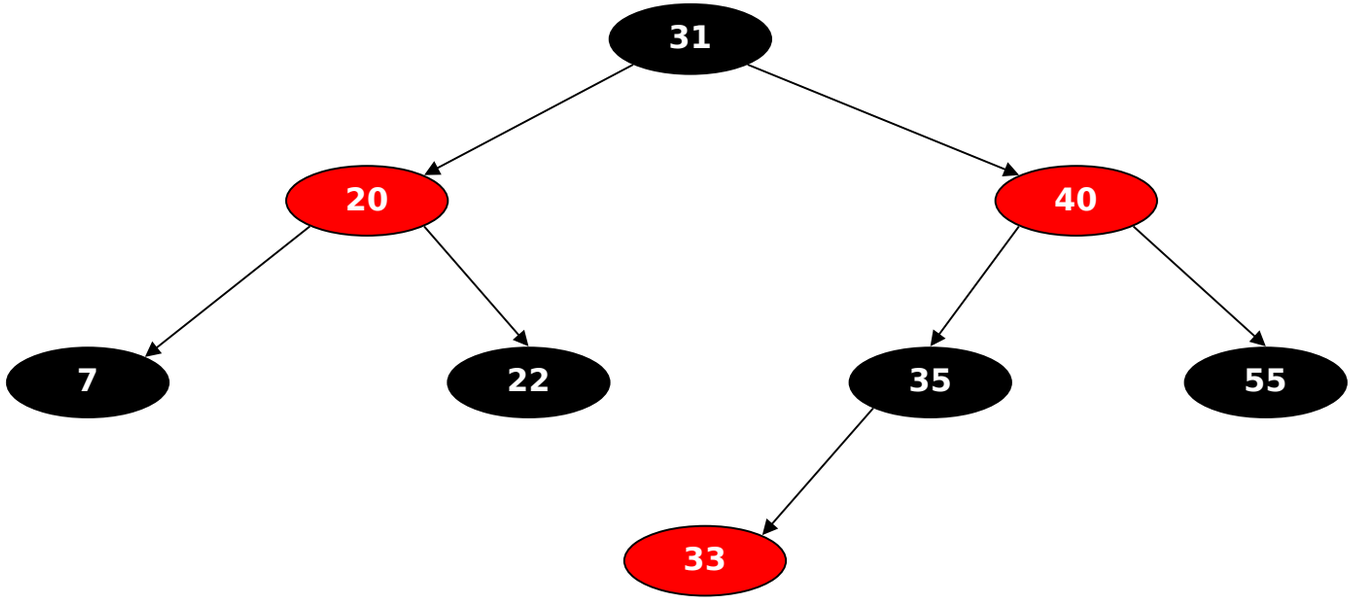

Uncle is red, push down blackness
Left rotation on 20
Right rotation on 40

Question 2: solution (cont.)

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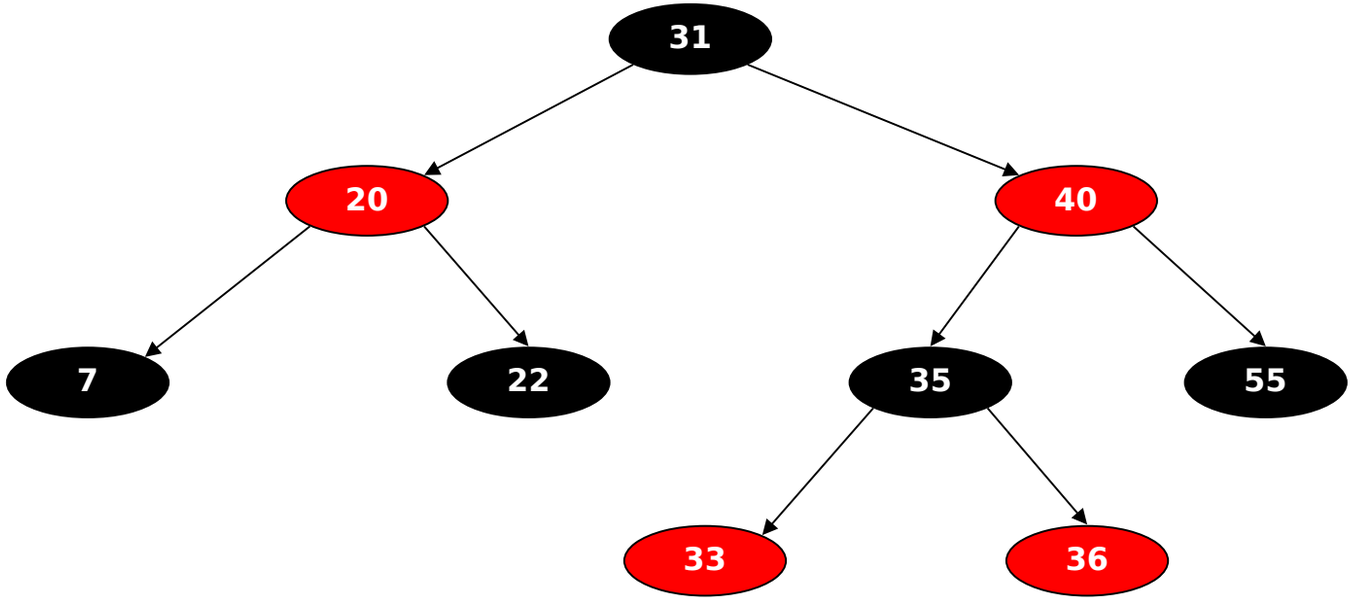

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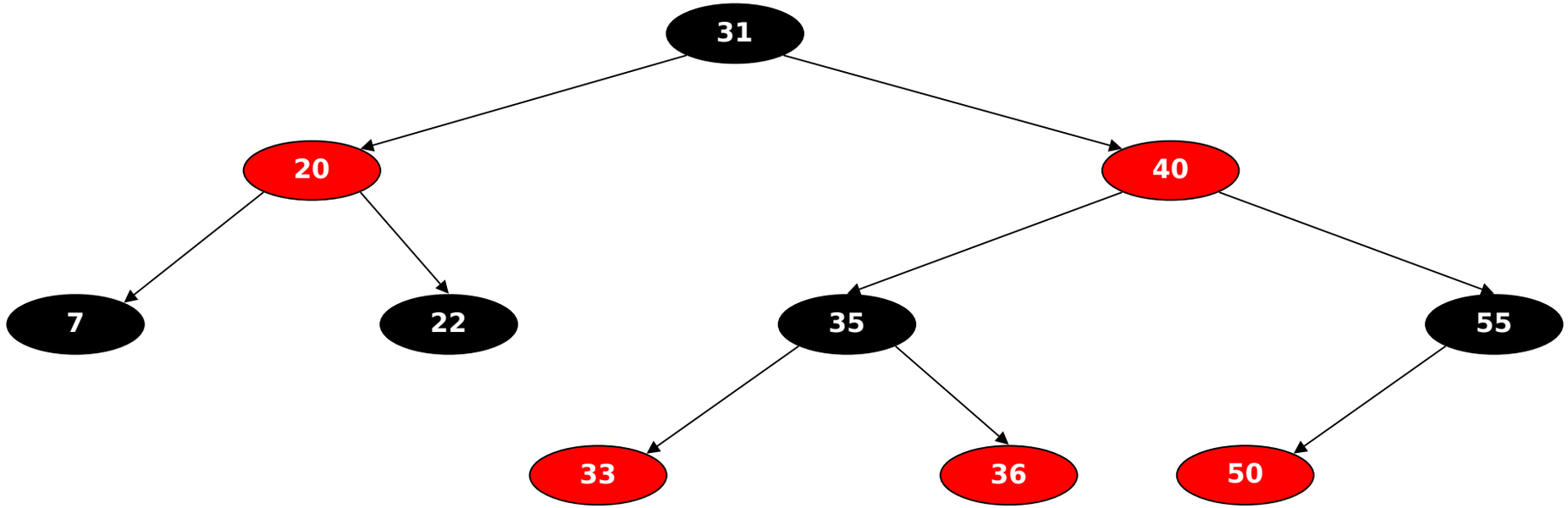

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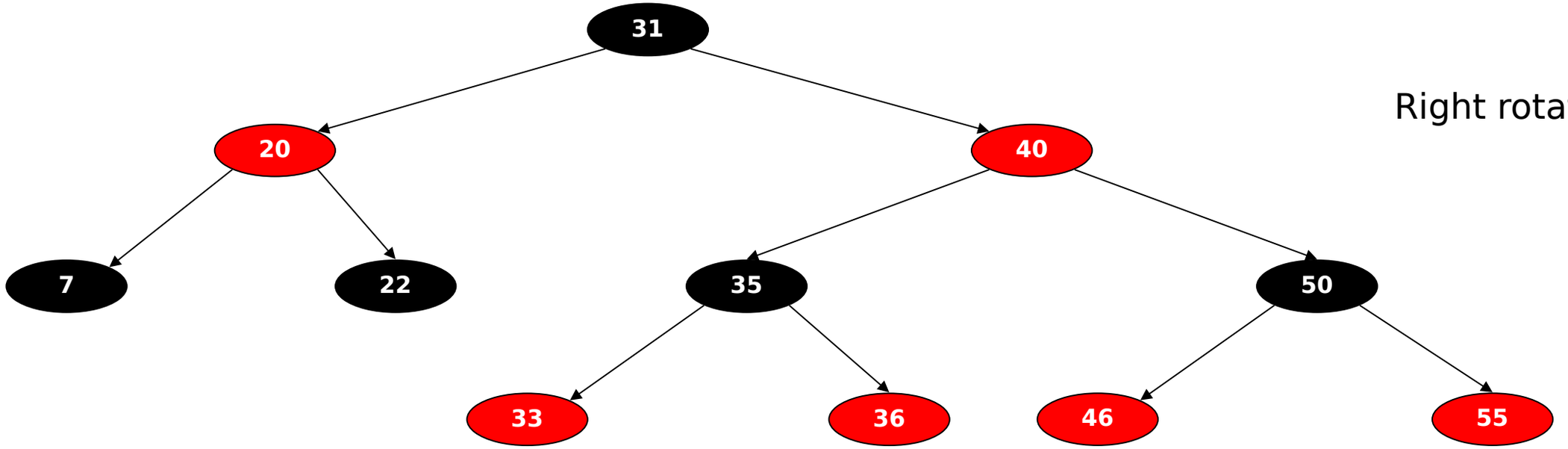

Right rotation

Question 2: solution (cont.)

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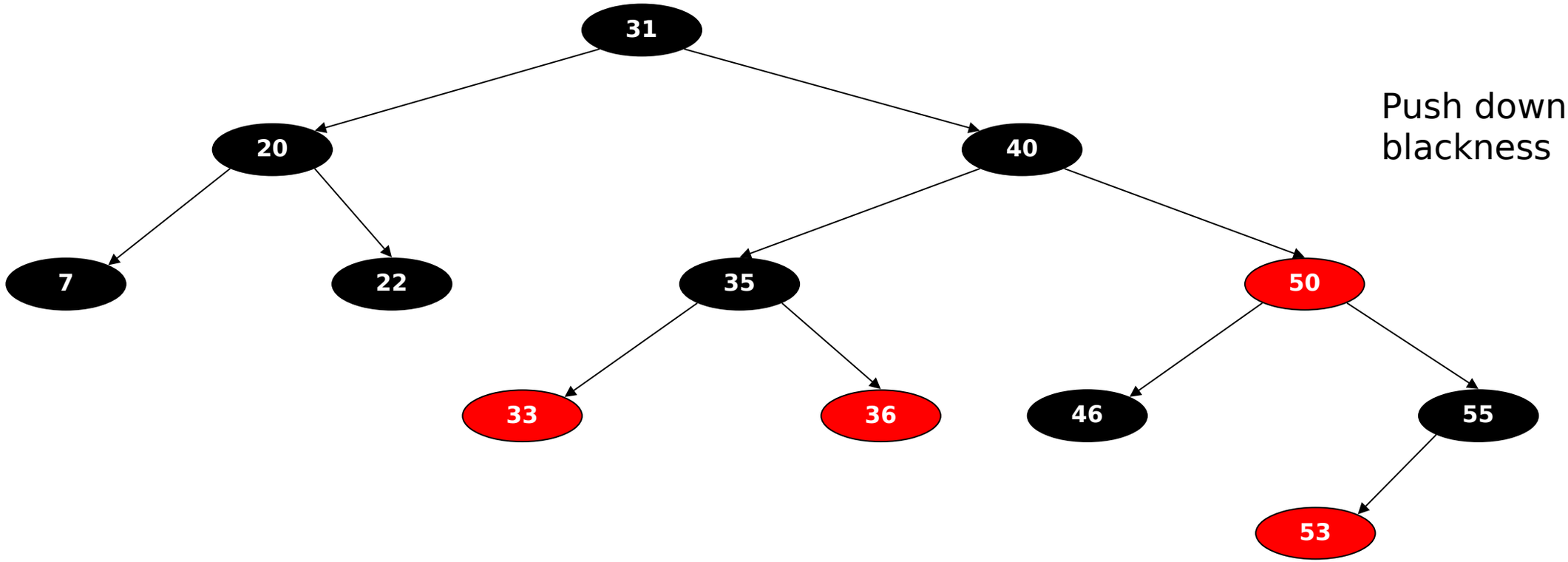

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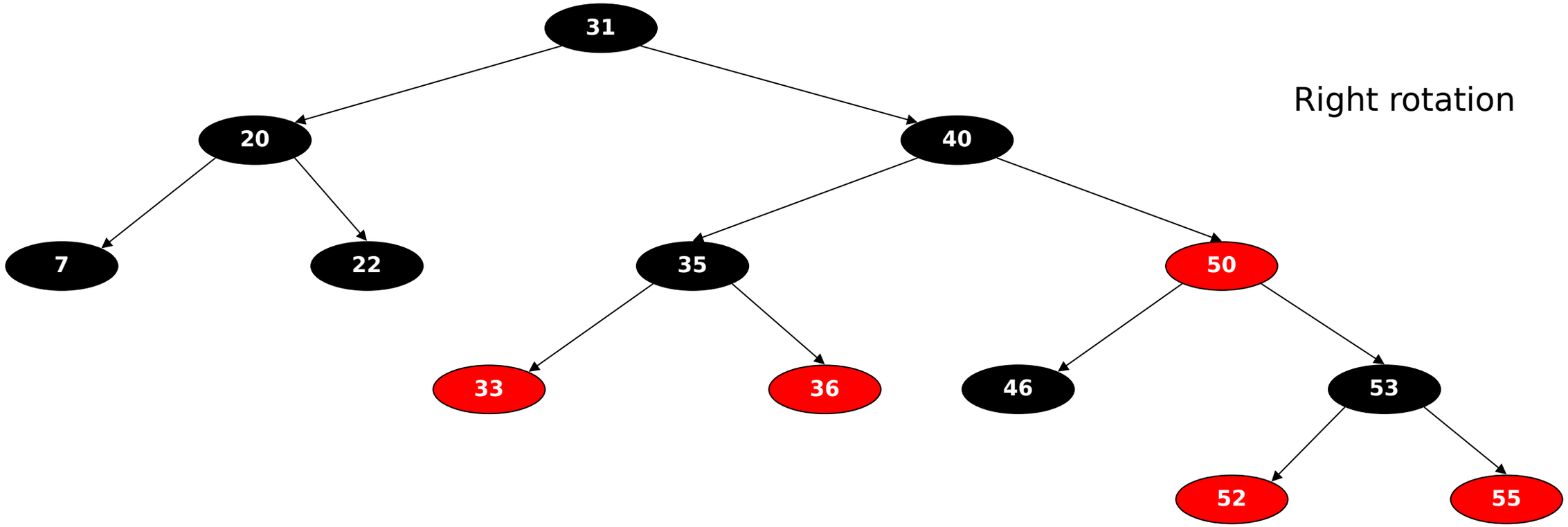

Right rotation

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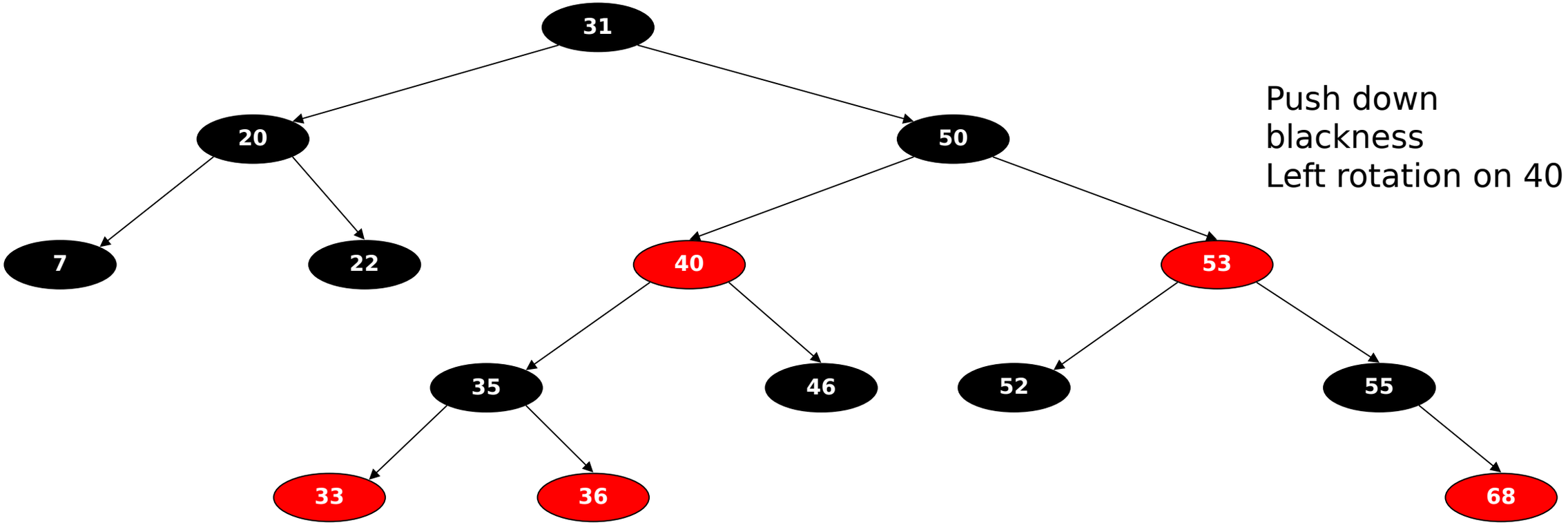

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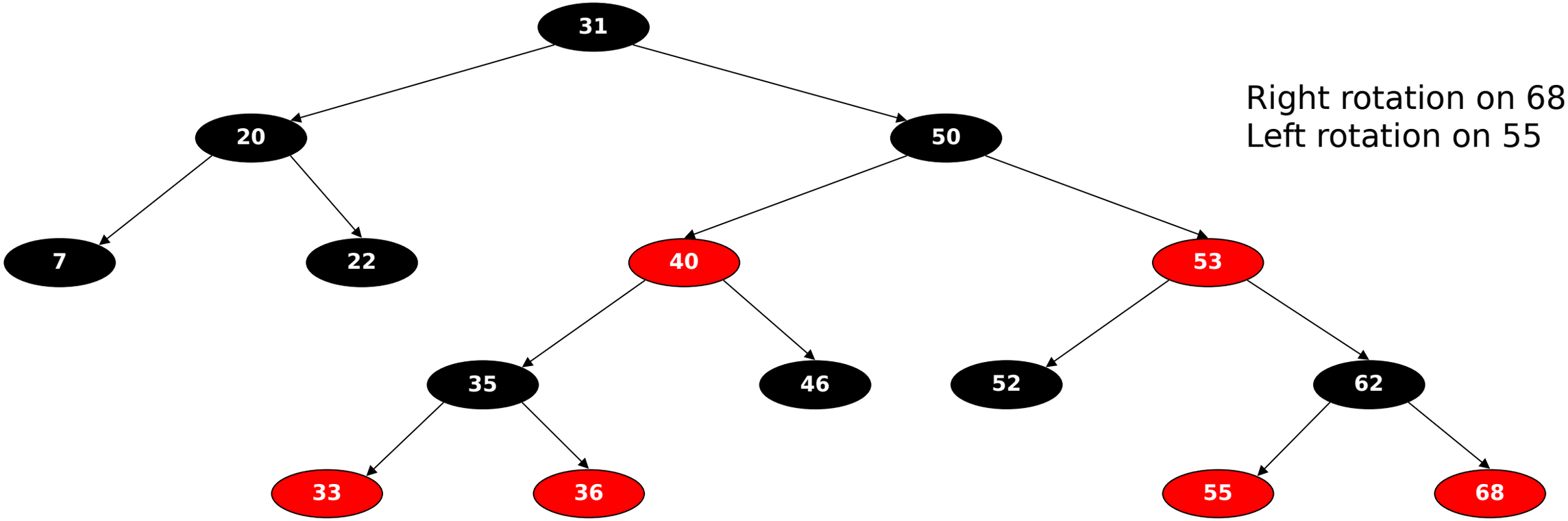


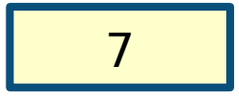
Right rotation on 68
Left rotation on 55

Question 3

- **Add the sequence of keys given below to an empty B-tree with t = 2**
  - 7,12,4,3,5,8,10,6,9,2
  - Each node can store at most 2t - 1 = 3 keys

Question 3: solution

- **Add the sequence of keys given below to an empty B-tree with t = 2**
  - 7,12,4,3,5,8,10,6,9,2



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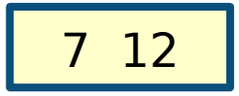

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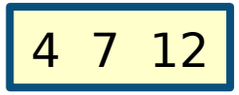

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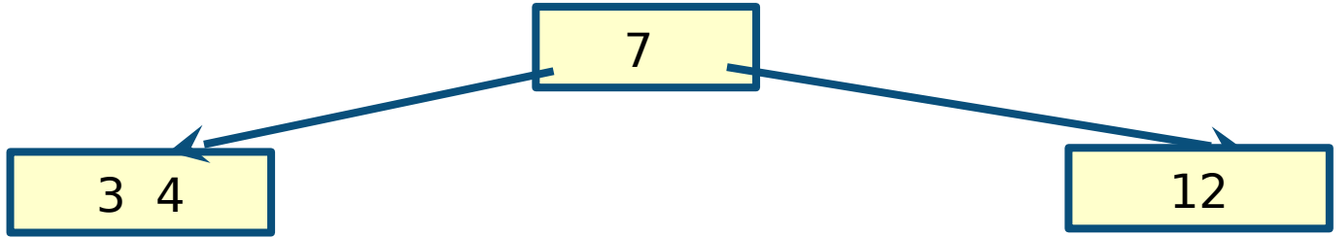

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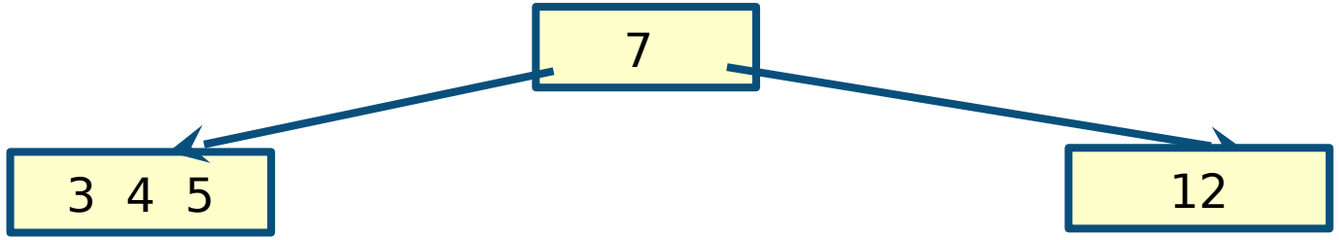

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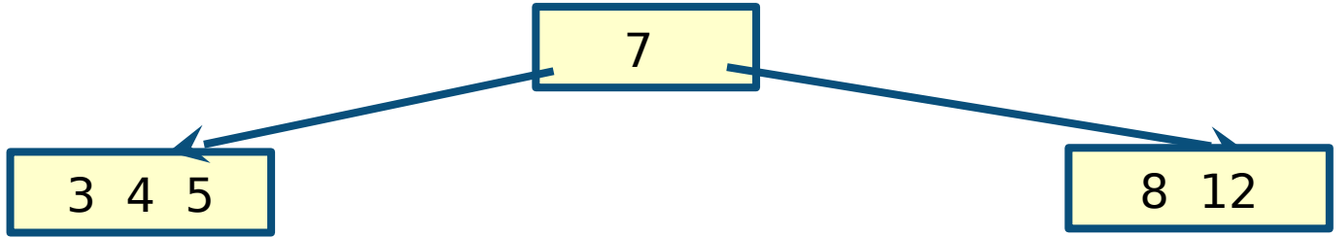

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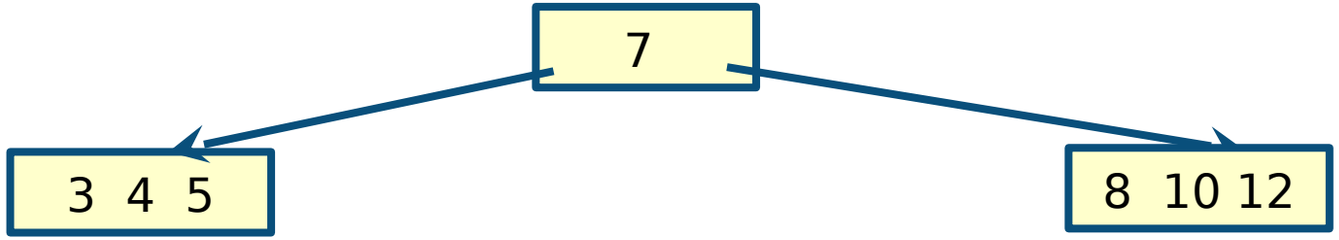

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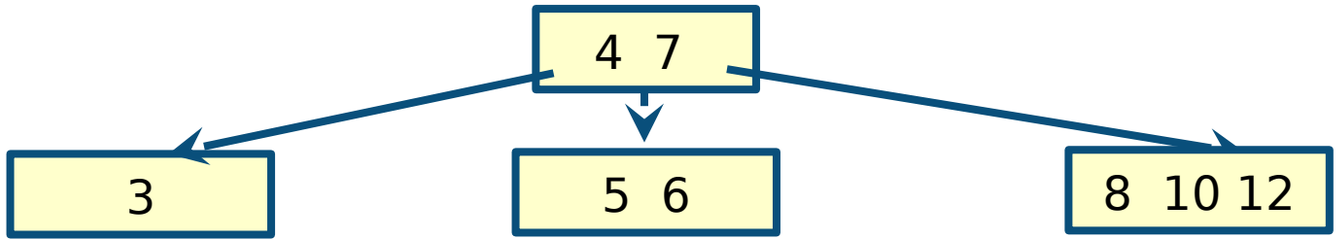

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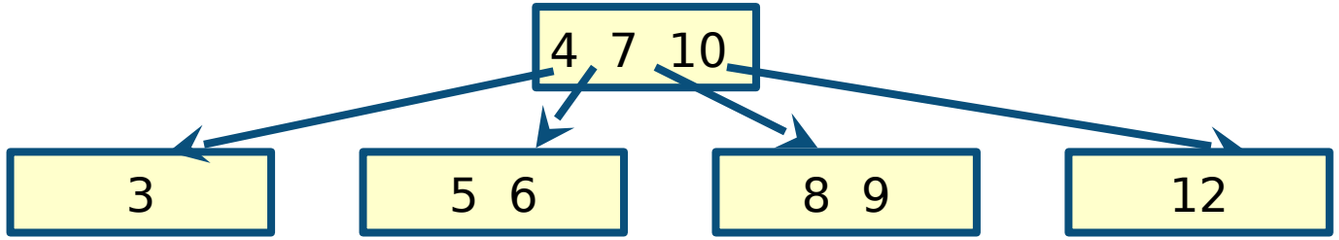

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