Lab - 5

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CS-302, Modeling and Simulation

In this lab we have analyzed the basic SIR model and complex 9-compartment SARS model for the epidemic spread. We have also analyzed some parameters that affects the model. We have also analyzed some real life situations like vaccination, quarantine, lockdown and effects of these on the epidemic spread.

I. INTRODUCTION

We can make estimation and prediction about the spread of disease using various mathematical modeling. One of the basic model is SIR (Susceptible, Infected, Recovered) model, there are also some more complex and advanced models like SARS model etc. In this lab, we have analyzed both SIR as well as SARS model with specific different conditions like, vaccination, lockdown, quarantined etc. in the time of epidemic spread.

II. MATHEMATICAL MODELING OF SIR

This SIR model is very basic in order to model the spread of epidemic. Here, we have main 3 compartments S (susceptible), I (Infected), R (Recovered). Suppose, N is the total population such that $N = S + I + R$.

At the starting of epidemic, all the population falls under the susceptible compartment. As time increases, number of infected people starts increasing. As plotted in fig.(1) the number of infected people reach to certain maximum value and then starts decreasing also leads to increase in number of recovered people.

There are some assumptions in this model:

- The all population remains constant.
- The other parameters like recovery rate, infection rate, contact rate are same for all the populations.
- If person gets infected then he/she starts spreading virus immediately.
- If person gets recovered then he/she will not get infected again.

The differential equations for this model are as follows.

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \alpha I$$

$$\frac{dR}{dt} = \alpha I$$

β = Infection rate per day
α = Recovery rate per day

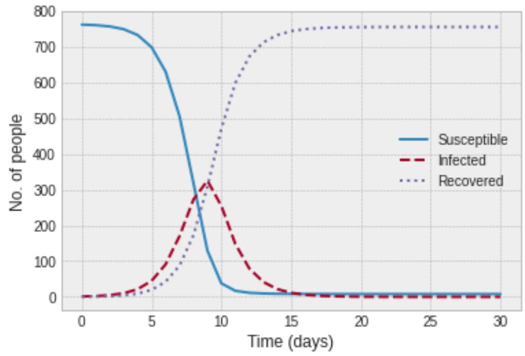

FIG. 1: Graphs of Susceptibles (S), Infecteds (I), and Recovereds (R) versus time (t) in days. $S_0 = 762, I_0 = 1, \beta = 0.00218, \alpha = 0.5, N = 763$

III. RESULT AND ANALYSIS FOR DIFFERENT PARAMETERS IN SIR MODEL

A. Effect of R_0 (Reproduction Number) on SIR Model

This Reproduction Number R_0 denotes the expected no. of people get infected by an infected person.

$$R_0 = \frac{\beta}{\alpha}$$

Higher the value of R_0, higher the spread of disease due to infection.

- $R_0 > 1$
  - One infected person infects more than one susceptible persons. Which leads to epidemic. From fig.(2), we can say that as R_0 increasing then we get higher peak.

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- $R_0 \leq 1$
  - In this case we would not have epidemic as we can see from fig.(2), that the number of infected people starts decreasing from beginning.

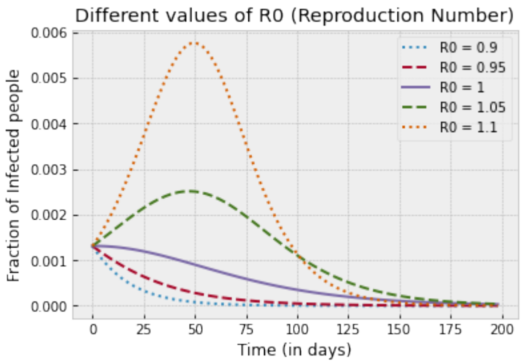

FIG. 2: Effect of R_0 on infected people

B. Effect of S_0 (Initial Susceptibles) on SIR Model

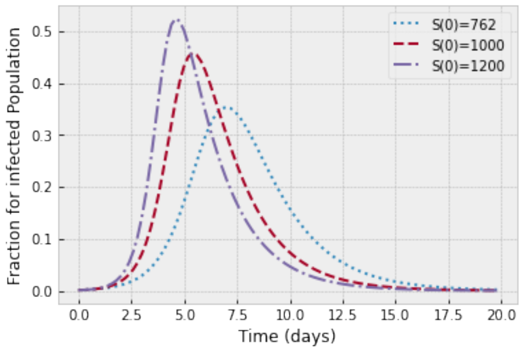

FIG. 3: Effect of S_0 on infected people

The spread of the epidemic is also depends on the initial number of susceptibles S_0. From the fig.(3) we can see that as the S_0 increases, the maximum value of the infected people also increases.

Here, For higher number of susceptibles, epidemic spreads very quickly in less time because of the large population, it leads to spread very quickly. For small population (S_0), epidemic spreads slowly and peak of the infected people is also decreases.

C. Role of Vaccination

Case 1: Vaccination when immunization begins immediately

Here, we have analyzed the case where the immunization begins immediately after taking the dose of vaccination. From fig.(4), we can see that the graph of infected people becomes small if we compare with the fig.(1).

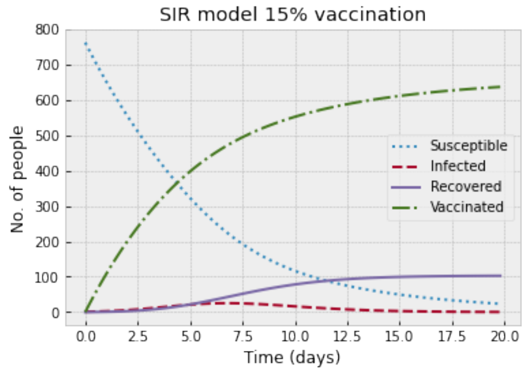

FIG. 4: Vaccination Rate = 15%, $S_0 = 762, I_0 = 1, \beta = 0.00218, \alpha = 0.5, N = 763$

Case 2: Vaccination when immunization begins after some time period

Here, we are assuming that the immunization begins at some days after taking the dose of vaccination. So, it is possible that though someone has taken the vaccine, he/she could be infected by the disease.

This leads to the point that although the vaccination rate is same, the no. of people get immune through vaccination will not be the same as we compare with the previous case. So, we would get slightly similar kind of behaviour as we got in basic SIR model without any vaccination.

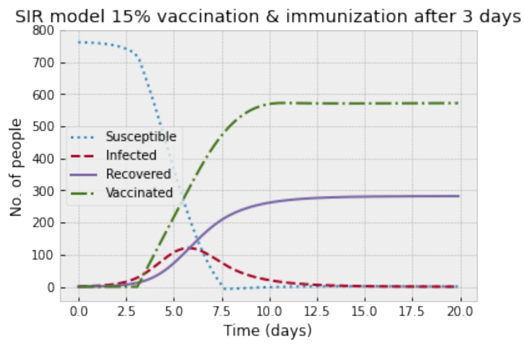

FIG. 5: Vaccination Rate = 15%, $S_0 = 762, I_0 = 1, \beta = 0.00218, \alpha = 0.5, N = 763$. Immunization begins after 3 days

Case 3: All the people are vaccinated and an infected person comes at 2^{nd} Day, Immunization begins after 4 days.

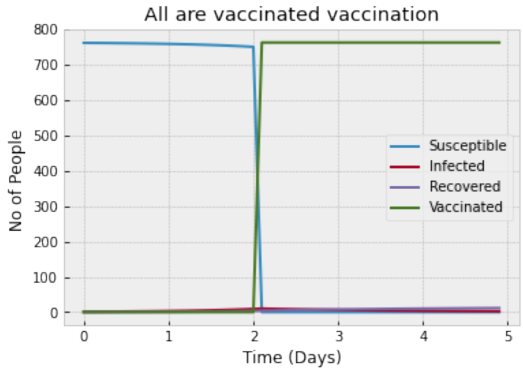

FIG. 6: 100% vaccination before any case arises. At t = 0 one infected person detected and immunization begins at t = 2.

Here, all the students get vaccinated before any infected person arises but immunization begins after 4 days of vaccination. At 2^{nd} day one student get infected but immunization begins at 4^{th} day so, this person will infect other person in next 2 days. So, the number of infected student will be very less. It means size of the epidemic will be very less.

Case 4: Vaccination when immunization begins immediately and Vaccination is partially effective

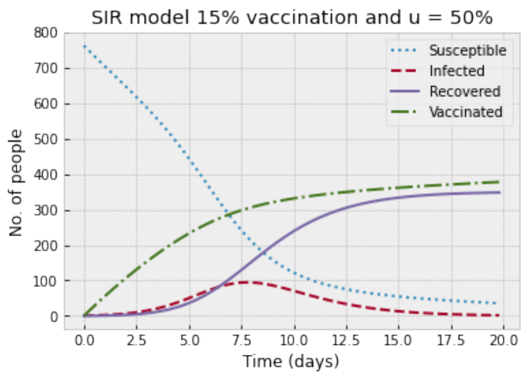

FIG. 7: Vaccination Rate = 15%, $S_0 = 762, I_0 = 1, \beta = 0.00218, \alpha = 0.5, N = 763, \mu = 50\%$

In real life scenario, vaccines don't work out for all the people and some Vaccinated people becomes susceptible again. Let's take individuals becomes susceptible again at a rate of $\mu = 0.5$.

From the fig.(7) we can see that the peak for the infected people rises as compare to the fig(4). We can say that, this all happens because now the effectiveness of the vaccination decreases by μ factor.

D. Effects of Lockdown on Epidemic Spreads

Case 1: Constant Lockdown Effect

The main reason behind the spread of the epidemic is the contact with an infected person. So, applying lockdown we can reduces the number of contacts between infected people and susceptible people. Also, the rate at which the epidemic spreads will also decreases.

From the fig.(9) we can observe the lockdown effect between days $T_1 = 5$ to $T_2 = 20$. At the T_1 time, we can see the peak in infected population and during the lockdown period, the infected population decreases also susceptible population and recovered population becomes slightly constant during the period of lockdown.

Also, we can notice that the maximum value of infected population also decreases if we compare the results with non-lockdown condition. Lockdown effect is a temporary solution to do preparation on public health center and make people aware about the epidemic.

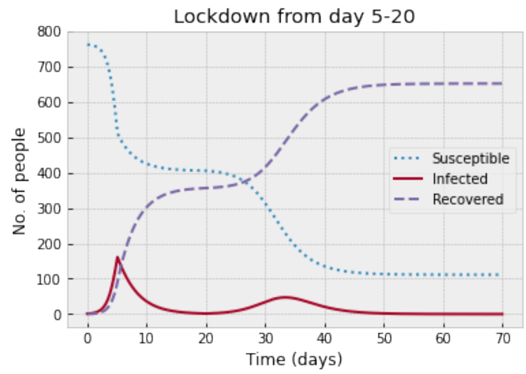

FIG. 8: Constant Lockdown Effect between Time $T_1 = 5$ days to $T_2 = 20$ days

Case 2: Lockdown Effect based on people behaviour

In the previous case we have taken lockdown effect strict but in real life it is not the case. Lockdown effect will take time to be effective and same is the case with the unlocking. So, if we compare results with the constant lockdown, we can see from fig.(9) that the infection rate goes down a bit slowly compared to constant lockdown case.

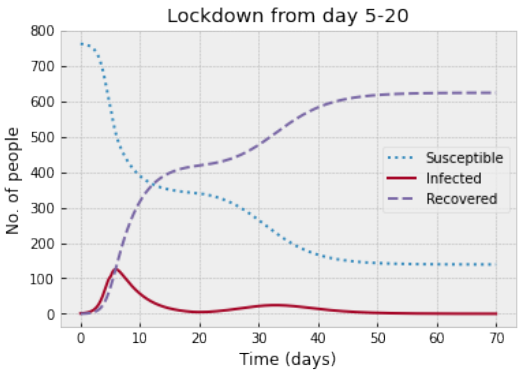

FIG. 9: Lockdown Effect between Time $T_1 = 5$ days to $T_2 = 20$ days

IV. CONCLUSION

We have seen the basic SIR model. It gives useful insights about the population in susceptible, infected and recovered stages with respect to time. This model helps in determining how the population get infected and how epidemic spreads with respect to time. There are also some parameters that affects this model like, Reproduction Number, Initial Susceptibles etc.

With the help of vaccination and lockdown, we can reduce the loss due to epidemic. Also there is a human behaviour is attached with the lockdown phase, that affects the curve of infected population. Lockdown effect is a temporary solution to do preparation on public health center and make people aware about the epidemic.

With some small vaccination rate, we get great results. But, in real life, the immunization due to vaccination start affecting positively after some delay of few days, that also affect the curves of this model.

V. MATHEMATICAL MODELING OF SARS

This model is an extension to the above SIR model and it contains additional exposed compartment compare to the SIR Model. The exposed compartment contains the population which came in contact with an infected population. This model also captures the death, quarantine and isolation effect on the epidemic spread. The SARS Model contains 9 different Compartments.

There are some assumptions in this model:

- There are no births.
- The only deaths are because of epidemic.
- Quarantine and isolation are completely effective. Someone who has the disease and is in quarantine or isolation cannot spread the disease.
- For susceptible individuals with exposure to the disease, the quarantine proportion (q) is the same for non-infected as for infected people.
- The number of contacts of an infected individual with a susceptible person is constant and does not depend on the population density.

Equations

From the above assumptions, we got 9 different compartments and below are the differential equations:

$$\frac{dS}{dt} = uS_q - \frac{kbSI_u(1-q)}{N} - \frac{kqSI_u(1-b)}{N} - \frac{kbqSI_u}{N}$$

$$\frac{dS_q}{dt} = \frac{kqSI_u(1-b)}{N} - uS_q$$

$$\frac{dE}{dt} = \frac{kbSI_u(1-b)}{N} - pE$$

$$\frac{dE_q}{dt} = \frac{kbqSI_u}{N} - pE_q$$

$$\frac{dI_u}{dt} = pE - (v+m+w)I_u$$

$$\frac{dI_q}{dt} = pE_q - (v+m+w)I_q$$

$$\frac{dI_d}{dt} = w(I_u + I_q) - (v+m)I_d$$

$$\frac{dR}{dt} = v(I_u + I_q + I_d)$$

$$\frac{dD}{dt} = m(I_u + I_q + I_d)$$

$$\frac{dN}{dt} = \frac{-dD}{dt}$$

Where,

- N: Total Population
- m: per capita death rate
- v: per capita recovery rate
- b: the probability that the contact between S and I_U results in transmission of the disease
- k: the mean number of contacts
- p: the fraction per day of E who become infectious
- q: the fraction per day of S who go into quarantine
- u: the fraction per day of S_q who leave quarantine

VI. RESULTS AND ANALYSIS OF DIFFERENT PARAMETERS IN SARS MODEL

For the the analysis part, we have taken S and S_Q compartments as one group; I_U, I_Q, I_D, E, E_Q compartments as one group; D and R also as an individual separate group.

we have taken constant's value as : $k = 10/day$, $b = 0.06$, $1/p = 5days$, $v = 0.04$, $m = 0.0975$, and $w = 0.0625$ and $N_0 = 10M$ people

Below is the figure for the SARS Model for the given constant.

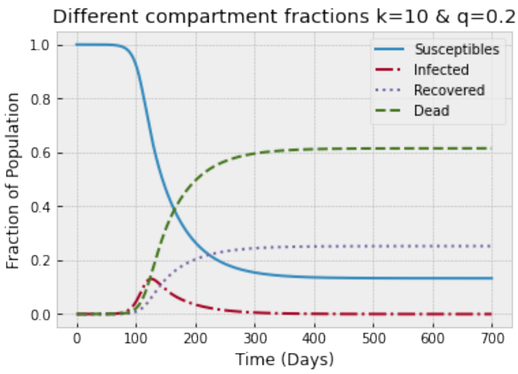

FIG. 10: SARS Model. Parameters : b = 0.06, u = 1/10, p = 1/5, m = 0.0975, w = 0.0625, v = 0.04, k = 10

A. Variation of q in SARS Model

Here, q means the fraction per day of S who goes into quarantine. So, in the case of susceptible population, the saturation value of change in susceptibles changes as the q value changes.

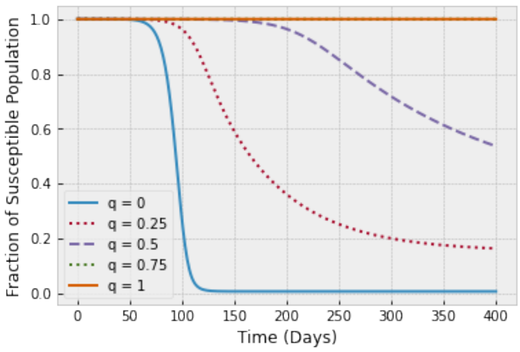

FIG. 11: Fraction of Susceptible Population with Different q. Parameters : b = 0.06, u = 1/10, p = 1/5, m = 0.0975, w = 0.0625, v = 0.04, k = 10

From the fig.(11) we can observe that as the q value increases, the saturation level of susceptible population also increases and time taken to reach saturation is also increases as q increases. That means as quarantine increases, the susceptible population increases means lesser the infected population.

In case of an Infected population, lower maximum value attained as the value of q increases.

That means as q increases more people will be quarantined so fraction of infected people decreases and less number of the susceptible who will be infected. We can observe that from the fig.(12).

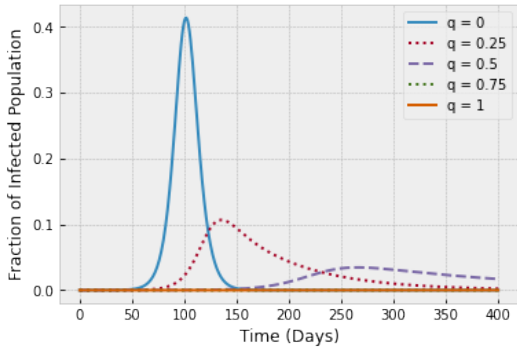

FIG. 12: Fraction of Infected Population with Different q. Parameters : b = 0.06, u = 1/10, p = 1/5, m = 0.0975, w = 0.0625, v = 0.04, k = 10

In case of a recovered population, value of fraction of population comes down as the q increases.

From the fig.(13) we can observe that as the q increases the less individuals who gets infected and more people will be quarantined. Hence, the number of recovered population decrease as the q increases.

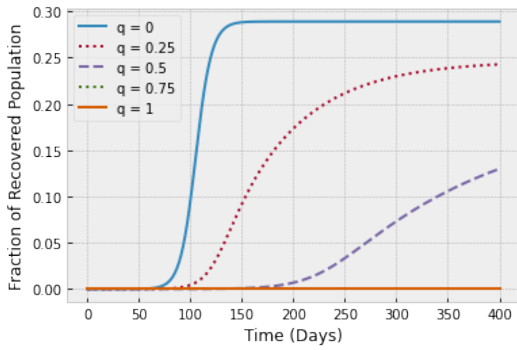

FIG. 13: Fraction of Recovered Population with Different q. Parameters : b = 0.06, u = 1/10, p = 1/5, m = 0.0975, w = 0.0625, v = 0.04, k = 10

In the case of a fraction of death individuals, same trend can be observed in fig.(14). As the number of q increases the less individuals who gets infected and more people will be quarantined hence the number of deaths will decreases.

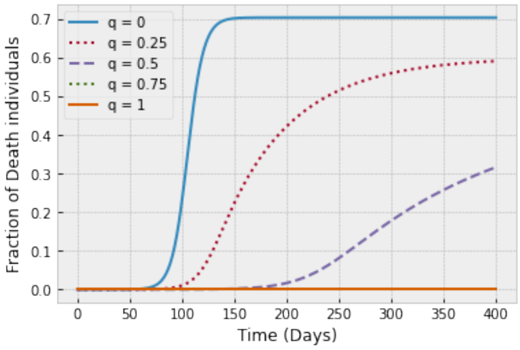

FIG. 14: Fraction of Death Individuals with Different q. Parameters : b = 0.06, u = 1/10, p = 1/5, m = 0.0975, w = 0.0625, v = 0.04, k = 10

B. Variation of k in SARS Model

Here, k represents mean number of contacts per day someone from infectious_undetected (I_U) has with someone in susceptible (S).

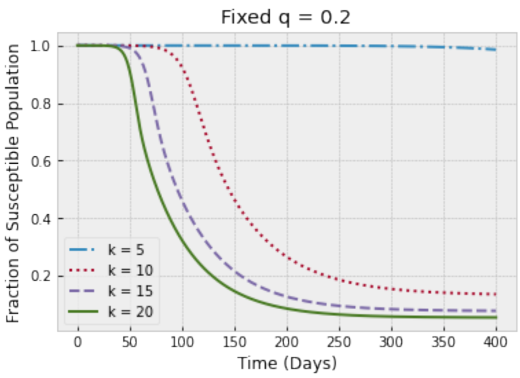

FIG. 15: Fraction of Susceptible Population with Different k. Parameters : b = 0.06, u = 1/10, p = 1/5, m = 0.0975, w = 0.0625, v = 0.04, q = 0.2

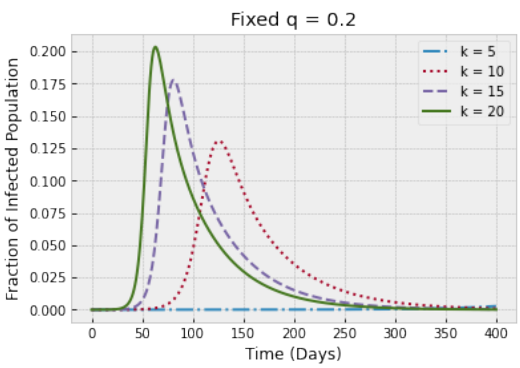

FIG. 16: Fraction of Infected Population with Different k. Parameters : b = 0.06, u = 1/10, p = 1/5, m = 0.0975, w = 0.0625, v = 0.04, q = 0.2

So, From the fig.(15), for the fixed quarantined factor, as k increases, the saturation level of susceptible population decreases. This means that as number of contacts increases then more individuals get infected. Also time taken to achieve the saturation level also decreases.

From the fig.(16), as k increases, the population of infected individuals increases quickly.

From the fig.(17), as k increases, the population of recovered individuals also increases. This is because, more people got infected as k increases and therefore we can see more deaths with increase in contacts (fig.(18)).

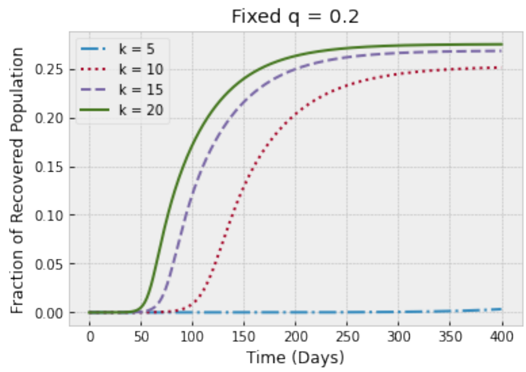

FIG. 17: Fraction of Recovered Population with Different k. Parameters : b = 0.06, u = 1/10, p = 1/5, m = 0.0975, w = 0.0625, v = 0.04, q = 0.2

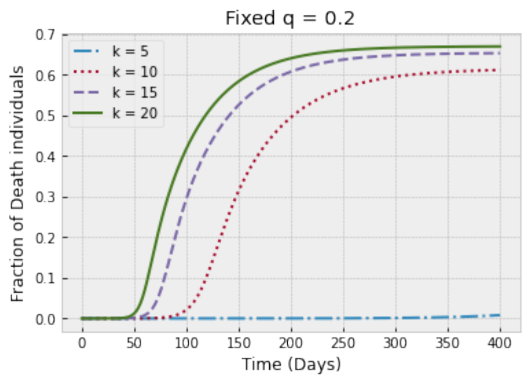

FIG. 18: Fraction of Death Individuals with Different k. Parameters : b = 0.06, u = 1/10, p = 1/5, m = 0.0975, w = 0.0625, v = 0.04, q = 0.2

C. Different values of $1/(w + m + v)$

Here the value $1/(v+m+w)$ shows that the individual is staying in the Infected Undetected (I_u) compartment for 1/(v+m+w) days.

From fig.(19), we can say that as the value of $1/(w + m + v)$ increases then susceptible population decreases rapidly. Thus, epidemic will spread faster.

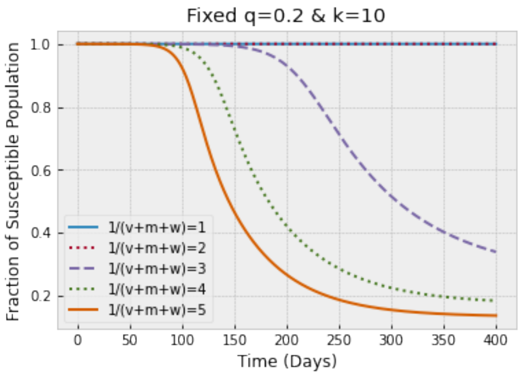

FIG. 19: Fraction of Susceptible Population with Different (w + m + v). Parameters : b = 0.06, u = 1/10, p = 1/5, k = 10, q = 0.2

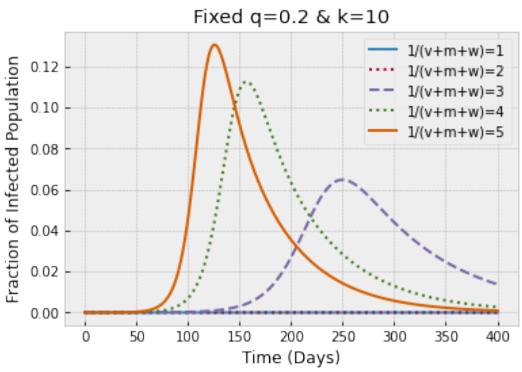

FIG. 20: Fraction of Infected Population with Different (w + m + v). Parameters : b = 0.06, u = 1/10, p = 1/5, k = 10, q = 0.2

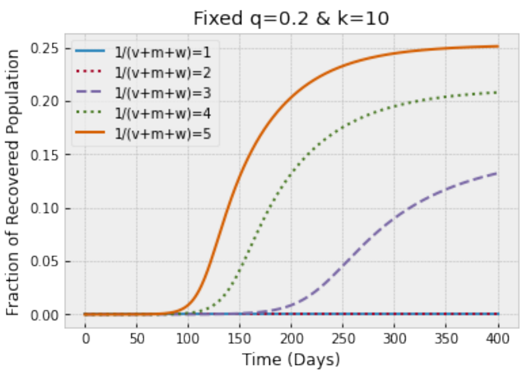

FIG. 21: Fraction of Recovered Population with Different (w + m + v). Parameters : b = 0.06, u = 1/10, p = 1/5, k = 10, q = 0.2

Fig.(20) shows that if the value of $1/(w + m + v)$ increases then the height of a peek of infected population will also increase. This means more people will get infected.

Fig.(21) shows that if the value of $1/(w + m + v)$ increases then the fraction of recovered population also increases.

Fig.(22) shows that if the value of $1/(w + m + v)$ increases then the fraction of death individuals also increases.

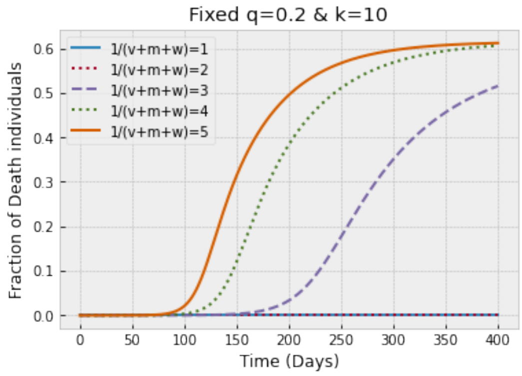

FIG. 22: Fraction of Death Individuals with Different (w + m + v). Parameters : b = 0.06, u = 1/10, p = 1/5, k = 10, q = 0.2

D. Variation on Applying Quarantine

Now, let say we want to see the effect of quarantine timing on the epidemic. In real life quarantine will be applied after the spread of the diseases but if we introduce quarantine early before too much spread of the diseases then we can control the epidemic.

In case of the susceptible population, we can observe from the fig.(23) that as the quarantine delay increases the saturation level of the susceptible population decreases.

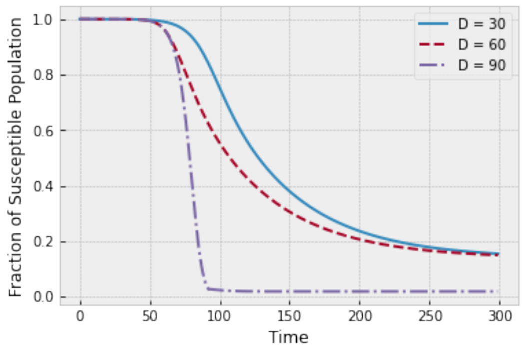

FIG. 23: Fraction of Susceptible Populations with Different Quarantine Delay. Parameters : b = 0.06, u = 1/10, p = 1/5, k = 10, q = 0.2, m = 0.0975, w = 0.0625, v = 0.04

In case of the Infected population, we can observe from the fig.(24) that as the quarantine delay increases the maximum infected people increases. It means delayed quarantine causes more spread of the diseases.

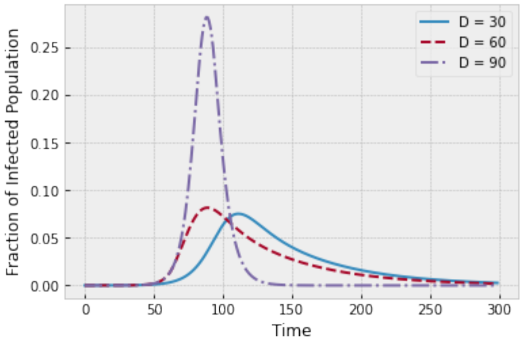

FIG. 24: Fraction of Infected Populations with Different Quarantine Delay. Parameters : b = 0.06, u = 1/10, p = 1/5, k = 10, q = 0.2, m = 0.0975, w = 0.0625, v = 0.04

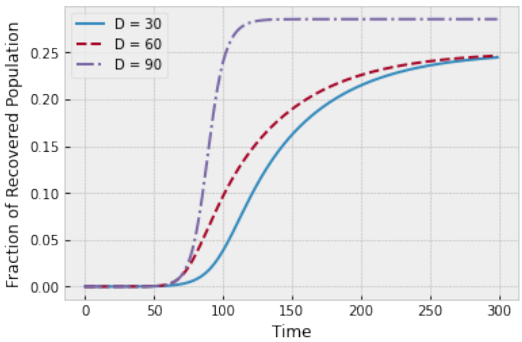

FIG. 25: Fraction of Recovered Populations with Different Quarantine Delay. Parameters : b = 0.06, u = 1/10, p = 1/5, k = 10, q = 0.2, m = 0.0975, w = 0.0625, v = 0.04

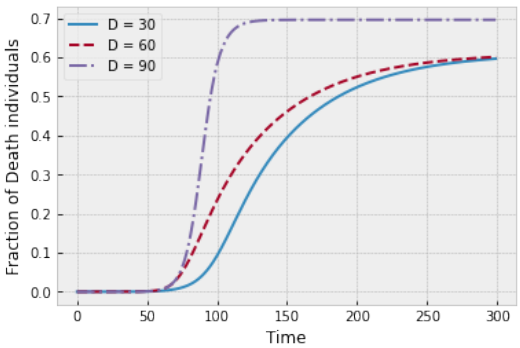

FIG. 26: Fraction of Death Individuals with Different Quarantine Delay. Parameters : b = 0.06, u = 1/10, p = 1/5, k = 10, q = 0.2, m = 0.0975, w = 0.0625, v = 0.04

When the more delay in starting quarantine then fraction of infected people will be more so, the value of the recovered population increases. In simple terms as more the quarantine delay number of infected people increases so, the number of recovered people is also increases. We can observe that from the fig.(25)

We can observe from the fig.(26) that When the more delay in starting quarantine then the fraction of death individuals also increases. If we introduce quarantine before the fraction of death becomes too high then we can control the epidemic.

E. Reproduction Number in Interventions

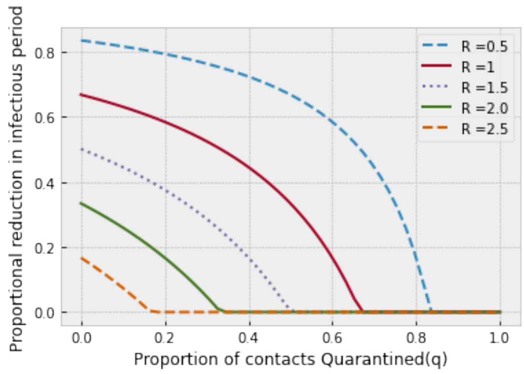

FIG. 27: Contour plot showing values of the reproduction number with interventions(R_0), as a function of the proportion of contacts effectively quarantined (q) and the reduction in infectiousness achieved by infection control and isolation

We know that In epidemic, quarantines impacts on the reproduction number (R). Let say R is the reproduction number without any interventions and R_{int} is the new reproduction number with the effects of interventions.

$$R_{int} = R(1-q)\frac{D_{int}}{D}$$

Where, D is the mean duration of infectiousness in absence of interventions and D_{int} is the mean duration of infectiousness in presence of interventions.

From the above equation we can say that as the value of q increases R_{int} decreases. Here, if D_{int} is reducing then R_{int} is also reduces.

This means that if the quarantine increases then the mean duration of infectiousness in presence of quarantine decreases and the mean duration of infectiousness in absence of any interventions (in this case quarantine) is either increasing or somewhat constant. Therefore the ratio of D_int and D decreases as q increases.

VII. CONCLUSION

As k (mean number of contacts per day someone from infectious undetected (I_U) has with someone in susceptible (S).) increases, the infected population increases, so in order to prevent that, we need to make less contacts.

Also we have seen that as q (fraction per day of individuals in susceptible (S) who have had exposure to disease that go into quarantine) increases, then the death of population decreases. So, in order to reduce deaths because of epidemic, quarantining the susceptible population will eventually reduce spread and death of people.

If we take precautions late and impose the lockdown and quarantine late then there will be more infections and more deaths due to the epidemic. Also these late actions can create pressure on public health sector as well.

So, using mathematical modeling we can predict the similar kind of epidemic spread with appropriate assumptions and appropriate parameters, so that we can reduce the death rate and losses that occur because of the epidemic.

[1] A. Shiflet and G. Shiflet, *Introduction to Computational Science: Modeling an Simulation for the Sciences*, Princeton University Press.3, 276 (2006).

[2] Lipsitch *et. al.*, *Transmission Dynamics and Control of Severe Acute Respiratory Syndrome*, SCIENCE, VOL 300, 20 JUNE 2003