Lab -1

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CS-302, Modeling and Simulation

In this lab we modeling the radioactive chain of three elements and analyze how this chain works.

I. INTRODUCTION

In this lab, we model the situation where one radioactive substance decays into another radioactive substance, forming a chain of such substances.

II. MODEL

Radioactive substance A, decays into radio active substance B, and this radio active substance B also decays into substance C, and we have a chain of a substances. the decay rate of A is a and decay rate of B is b.we can write difference equation of A and B as given below.

$$\Delta A = -aA\Delta t \tag{1}$$

$$\Delta B = -(aA - bB)\Delta t \tag{2}$$

III. EXPERIMENTS

A Develop a model for a radioactive chain of three elements A,B,C.

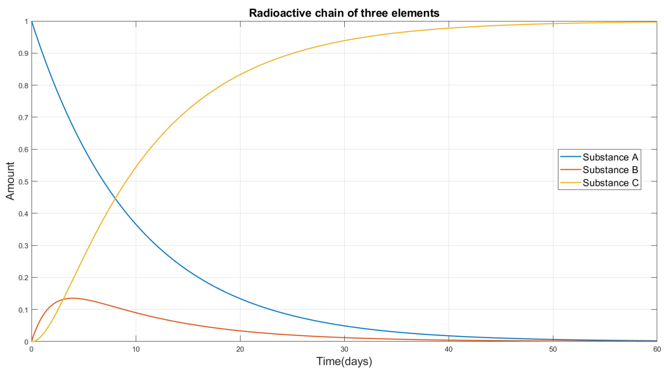

FIG. 1: Amount of elements A,B,C vs Time.

Here,we take A_0=1, a=0.1, b=0.5, days=60; As time passes substance A will decay and produce substance B with rate a and similarly, B will decay and produce substance C with rate b.

B Explain the shape of graphs.

We can clearly see that from the figure 1 that substance A will have decaying cure nature always because it's initial amount is fixed and decay rate too. For substance B will have bell shape curve as initially amount of A will generate B and as time passes A will reduce and B will increases, at some point of time A is too small to add something significant in B and in other hand B will continuously generating C from that we can say that at some particular time B will have a peak this theory we will prove analytically in future. Now substance C is increases as B decreases so C will have increasing nature of curve and since it's amount becoming from B and B from A so Amount of C cant be more than initial amount of A so as time passes curve of C will converges to A_0.

C Find max disintegration day for different rate of changes a.

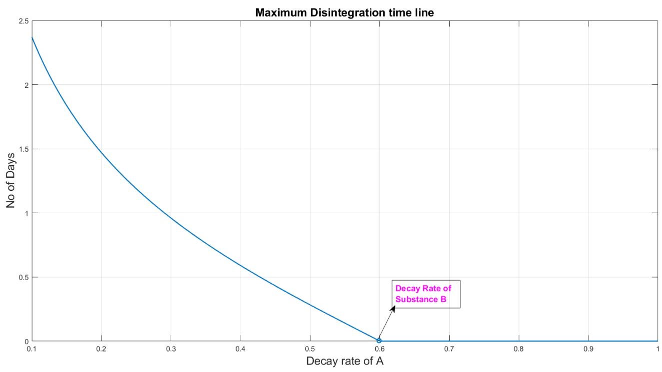

FIG. 2: No. of days to get max total radioactivity vs decay rate of substance A (a).

b=0.6, days=100;, a=0.1 to 1 with step size 0.001

Conclusion: The quantity $a * A + b * B$ it's giving change in A + change in B at a particular moment we need to find time at which maximum disintegration occurs so as we can clearly see there is

(a) $a < b$
Here, b is greater than a so b*B term dom-

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inates in the total contribution so when a change in B is maximum then overall sum.

(b) $a \geq b$
if a is greater than b then a*A will dominate in final answer and a*A will be maximum at first step so for this condition it's answer always be the constant which is t=0

D With b being the decay rate of B, in several cases where $a < b$, observe that eventually we have the following approximation:

$$\frac{B}{A} = \frac{a}{b-a}$$

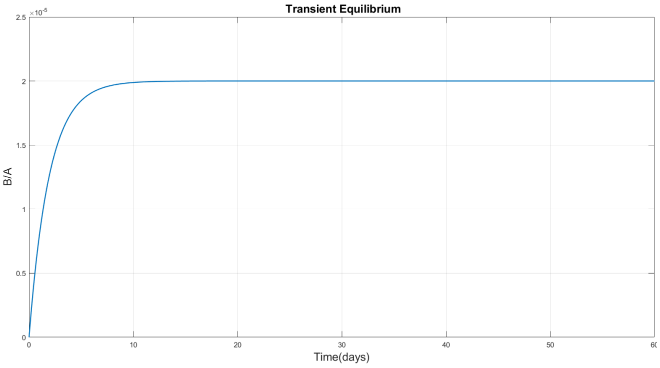

FIG. 3: B/A vs days when $a < b$

a=0.00001, b=0.5, days=60

- Analytic solution:

$A = A_0 e^{-at}$
$B = \frac{aA_0}{b-a}\,(e^{-at} - e^{-bt})$
given that $a < b$ so, $e^{-bt} \to 0$

$\therefore \frac{B}{A} \approx \frac{a}{b-a}(TransientEqualibrium)$

E Here we are repeating above part with condition $a > b$

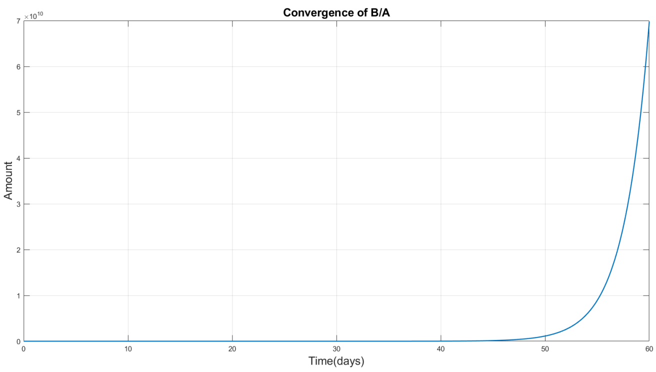

FIG. 4: B/A vs days when $a > b$

a=0.5, b=0.1, days=60

F Verify the observation from part (E) analytically using work similar to that in part (D).
$A = A_0 e^{-at}$
$B = \frac{aA_0}{b-a}\,(e^{-at} - e^{-bt})$
given that $a > b$ so, $e^{-at} \to 0$
$\therefore \frac{B}{A} \approx \frac{(-a)(A_0 e^{-bt})}{(b-a)(A_0 e^{-at})}$
$since$,e$^{-at} \to 0$
$\therefore \frac{B}{A} \to \infty$

G If a is much smaller than b, we have $A \approx A_0$ and $B\frac{aA_0}{b-a}$. With the two amounts being almost constant, we have a situation called secular equilibrium.

$$Ra^{226} \to Rn^{222} \to Po^{218}$$

Decay rate of Ra^{226} is $a = 0.00000117/da$ and the Decay rate of Rn^{222} is $b = 0.181/da$.

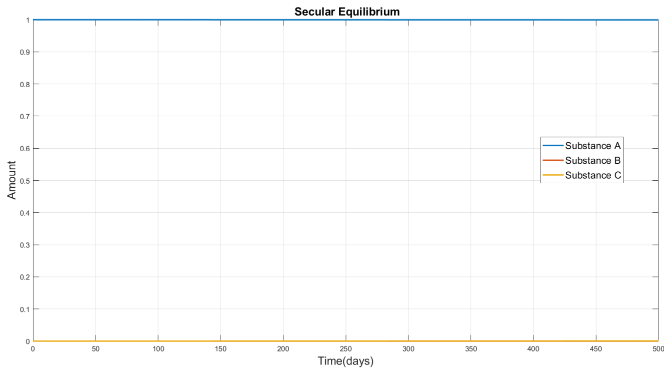

FIG. 5: Amount of elements A,B,C vs Time. when $a << b$

conclusion:
as we can see the decay rate of A is too small so that there is no decay at all but if we simulate for some 10 lac years then we can see some decay.

H Show analytically that the approximations from Part (G) hold.
$A = A_0 e^{-at}$
$A = A_0(1 + (-at) + \frac{(-at)^2}{2!} + ...)$
as $a << b$ and $0 < a < 1$ and $0 < b < 1$
$therefore A \approx A_0$

now, $B = \frac{aA_0}{b-a}\,(e^{-at} - e^{-bt})$
$a << b$ and $0 < a < 1$ and $0 < b < 1$
$therefore e^{-bt} \to 1$
$\therefore B \approx \frac{aA_0}{b-a}(e^{-bt})$
$\therefore B \approx \frac{aA_0}{b-a}$

I In the radioactive chain

$$Bi^{210} \to Po^{210} \to Pb^{206}$$

(bismuth-210 to polonium-210 to lead-206), the decay rate of Bi^{210} , a, is 0.0137/da and the decay rate of Po^{210}, b, is 0.0051/da. Assuming the initial mass of Bi^{210} is 10^{-8} g and using your model from part(A), find, approximately, the maximum mass of Po^{210} and when the maximum occurs.

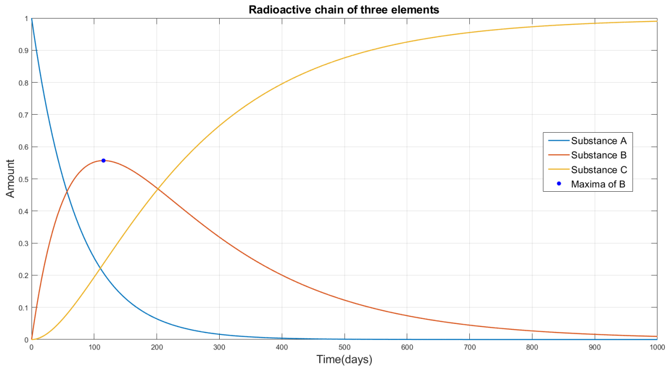

FIG. 6: Amount of elements A,B,C vs Time.

max amount=0.5568, days=114.900, a=0.0137, b=0.0051

J In Part (D), we verified that $B = \frac{aA_0}{b-a}(e^{-at} - e^{-bt})$.Using this result, find analytically the maximum of mass of substance B and when this maximum occurs.
$B = \frac{aA_0}{b-a}(e^{-at} - e^{-bt})$
$\therefore \frac{dB}{dt} = \frac{(aA_0)}{(b-a)} \frac{d}{dt}(\mathrm{e}^{-at} - e^{-bt}) = 0$
$\therefore -ae^{-at} + be^{-bt} = 0$
$\therefore ae^{-at} = b^e{-bt}$
$\therefore e^{(-b+a)t} = \frac{a}{b}$
$(-b+a)t = \log_e(a/b)$
$t = \frac{\log_e(a/b)}{a-b}$
max Quantity of B during decay is
$B(t_1) = \frac{aA_0}{b-a}(e^{-at_1} - e^{-bt_1})$
where, $t_1 = \frac{\log_e(a/b)}{a-b}$
we can put all values from Que. and get value around $B(t_1) = 0.55656$ and time($t_1 = 114.9018$)

K Check your approximations of Part (I) using your solution to Part (J).

- so, experimental answer (0.5568) and analytical answer(0.55656) are almost same
- Time for reaching B at maximum is ($t_1 = 114.9018$) analytically and in simulation we got ($t_1 = 114.9$).

L For the chain in Part (G), use your solution to Part (J) to find when the largest mass of Rn^{222} occurs. t=66 days

M For the chain in Part g, use your simulation of Part a to approximate the time when the largest mass of Rn^{222} occurs. How does your approximation compare with the analytical solution of Part (I)?

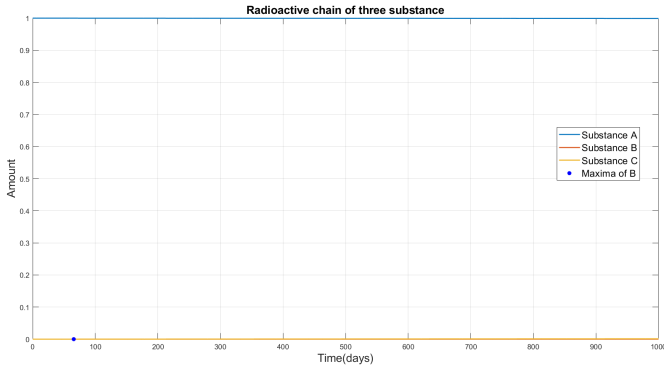

FIG. 7: Amount of elements A,B,C vs Time.

By simulation, we found the value t=65.6 days for part (g), which is pretty close to the result obtained in part (l)