The University of Saskatchewan

Saskatoon, Canada

Department of Computer Science

CMPT 280– Intermediate Data Structures and Algoirthms

Assignment 2

Date Due: January 24, 2017, 10:00pm

Total Marks: 53

1 Submission Instructions

- Assignments must be submitted using Moodle.
- Responses to written (non-programming) questions must be submitted in a PDF file, plain text file (`.txt`), Rich Text file (`.rtf`), or MS Word's `.doc` or `.docx` files. Digital images of handwritten pages are also acceptable, provided that they are **clearly** legible, and in a common format such as JPEG or PNG.
- Programs must be written in Java.
- If you are using Eclipse (or similar development environment), do not submit the workspace (project). Hand in only those files identified in Section 5. Export your `.java` source files from the workspace and submit only the `.java` files.
- No late assignments will be accepted. See the course syllabus for the full late assignment policy for this class.

2 Background

2.1 Timing Analysis

Questions 1 through 8 in Section 3 concern timing analysis. These questions are not programming questions and should be submitted in one of the acceptable document formats listed above.

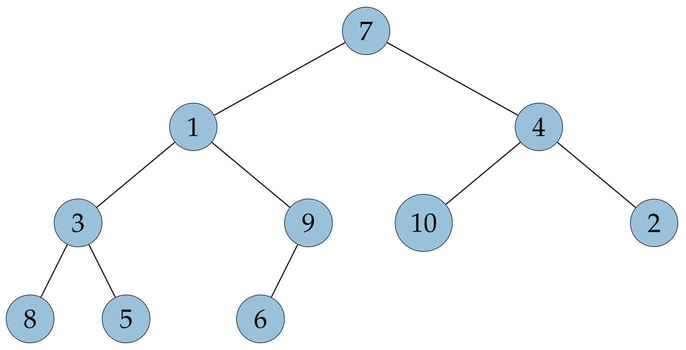
2.2 Arrayed Trees

Question 9 is about a bounded binary tree implementation. You should remember binary trees from CMPT 115 (or similar course) – they are trees in which each node has at most two children. What you probably didn't know is that binary trees can be stored using an array, rather than a linked structure. In such an array, the contents of the root node are stored in offset 1 of the array (offset 0 is unused). The contents of the children of the node whose contents are stored at offset i are stored at offset $2i$ and $2i+1$, respectively. Thus, the left child of the root is at offset $2 \times 1 = 2$, the right child of the root is at offset $2 \times 1 + 1 = 3$, the left child of the left child of the root is at offset $2 \times 2 = 4$, and so on. The parent of the node whose contents are at offset i, is at offset $i/2$ (integer division). Thus, the parent of node at offset 7 is at offset 3.

Example 1
Here is the array representation of a tree storing the elements 1 through 10, in no particular order. The array

-	7	1	4	3	9	10	2	8	5	6

is a representation of the tree:



Note that we do not allow any unused entries in the array between used ones. All the data items in the array are stored contiguously. This means that we can represent only a particular subset of binary trees with this representation. Namely, it is the set of trees where all levels except possibly the last level are complete (full) and the nodes in the bottom level are all as far to the left as possible. You might be thinking that this is too restrictive and not very useful because we can't represent **all** binary trees with this data structure. However, as we will see on future assignments, this array-based tree data structure is highly useful and efficient as a basis for implementing certain other important data structures.

Also note that if we read off the items from left to right in each level of the tree, starting from the top level, we get the items in the same order as they appear in the array.

3 Your Tasks

Question 1 ():

Suppose the exact time required for an algorithm A is given by the function

$$T_A(n) = 280n + 21n^{2.5}(\log_2 n) + 12n^3 + 101 + 42\sqrt{n}$$

3, 4(a) (2 points) Which of the following statements are true?

1. Algorithm A is $O(\log n)$
2. Algoirthm A is $O(n)$
3. Algoirthm A is $O(n^3)$
4. Algoirthm A is $O(2^n)$

(b) (1 point) Give the correct time complexity for A in big-Θ notation. O(n^3)

Question 2 ():

For each of the following functions, give the tightest upper bound chosen from among the usual simple functions listed in Section 4.5 of the course readings. Answers should be expressed in big-O notation.

(a) (1 point) $f_1(n) = n\log_2 n + n^4 \log_2 n + \frac{2^n}{4200}$ 2^n

(b) (1 point) $f_3(n) = 0.8n^3 + n^2 \log_2 n^2 + \log_2(2^n)$ n^2 log2 n^2

(c) (1 point) $f_2(n) = 4n^{0.5} + \frac{\log_2 n}{n} + 1234$ n^0.5

Question 3 ():

If possible, simplify the following expressions. *Hint: See slide 11 of topic 4 of the lecture slides!*

(a) (1 point) $O(n^2) + O(\log n) + O(n \log n)$ O(n^2)

(b) (1 point) $O(2^n) \cdot O(n^2)$ O(2^n * n^2)

(c) (1 point) $42O(n \log n) + 18O(n^3)$ O(n^3)

(d) (1 point) $O(n^2 \log_2 n^2) + O(m)$ *(yes, that's an 'm', not a typo; note that m is independent of n)*
O(n^2log(2)n^2)

Question 4 (5 points):

Consider the function $f(n) = 2n^3 + 5n^2 + 42$. Use the definition of big-O to prove that $f(n) \in O(n^3)$.

Question 5 (5 points):

Consider the function $g(n) = 12n^2 \log n^2 + 6n + 42$. Use the definition of big-O to prove that $g(n) \in O(n^2 \log n^2)$.

Question 6 (3 points):

Consider again the function $g(n) = 12n^2 \log n^2 + 6n + 42$. Use the definition of big-O to prove that $g(n)$ is **not** in $O(n)$.

Question 7 ():

Consider the following Java code fragment:

```
// Print out all ordered pairs of numbers between 1 and n
for(i = 1; i <= n; i++) {
   for(j = 1; j <= n; j++) {
      System.out.println( i + ",␣" + j) ;
   }
}
```

(a) (3 points) Determine the exact number of statements (i.e. the *statement counting approach*) that are executed when we run this code fragment as a function of n. Show all of your calculations.

i->n + 1(overlimit)
j->n + 1(overlimit)
print-> n
((n+1)+n)*n + 1
=2n^2 + n + 1

(b) (1 point) Express the function you obtained in part a) in big-Θ notation. n*n = O(n^2)

Question 8 ():

Consider the following pseudocode:

```
Algorithm roundRobinTournament(a)
This algorithm generates the list of matches that must be
played in a round-robin pirate-dueling tournament (a tournament where
each pirate duels each other pirate exactly once).

a is an array of strings containing names of entrants into an tournament

n = a.length
for i = 0 to n-1
   for j = i+1 to n-1
      print a[i] + "␣duels␣" + a[j] + ",␣Yarrr!"
```

(a) (5 points) Determine the exact number of statements (i.e. use the *statement counting approach* that are executed by the above algorithm. Express your answer as a function of `n`. Show all your calculations. (2*(n^2-n))/2 + (n-1) + n + 1 = n^2 + n

(b) (1 point) Express the function you obtained in part a) in big-Θ notation. O(n^2)

Question 9 (20 points):

Your task is to write a Java class called `ArrayedBinaryTreeWithCursors280<I>` which extends and implements the abstract class `ArrayedBinaryTree280<I>` (provided in the `lib280-asn2.tree` package as part of `lib280-asn2`). This week's lab will also talk more about array-based trees.

Methods to be Implemented

Some of the work of implementing `ArrayedBinaryTreeWithCursors280<I>` has already been done, but there is more to do.

Firstly, there are several methods in `ArrayedBinaryTreeWithCursors280<I>` which are defined but not implemented; these are marked with `//TODO` comments. You must implement these methods.

Secondly, since `ArrayedBinaryTreeWithCursors280<I>` implements the interfaces `Dict280<I>` and `Cursor280<I>`, there are several missing methods required by these interfaces that also needed to be implemented. The headers for these methods are not yet present in `ArrayedBinaryTreeWithCursors280<I>` – you must add them. The interfaces `Dict280<I>` and `Cursor280<I>` and their ancestors (yes, they have ancestor interfaces!) document what these methods are supposed to do. Until the missing methods are added, the compiler will complain on line 15 that there are unimplemented abstract methods inherited from the interfaces. This is normal, and will disappear once all required methods have been added. *Hint: You can automatically generate the missing method headers in IntelliJ by pressing CTRL-i.*

Implementation Requirements and Hints

You may not modify any of the existing code in the provided `ArrayedBinaryTreeWithCursors280.java` **file**, but you can add to it. **You may also not modify any other files within** `lib280-asn2`.

There is already a regression test included in `ArrayedBinaryTreeWithCursors280<I>`. Your completed implementation of the arrayed binary tree should pass the given regression test. If all the regression tests are successful, the only output should be: `Regression test complete`.

Hint: You don't need to declare an array instance variable to hold the data items in `ArrayedBinaryTreeWithCursors280<I>`*. There is already one inherited from* `ArrayedBinaryTree280<I>`*. You should look at the other inherited instance variables too!*

Hint: one of your first major decisions after adding the appropriate method headers for the inherited abstract methods will be to start implementing the `insert` *method and decide where the new element should be inserted. If you think about it, there's really only one place it can go...*

Hint: The algorithm for deleting an element is to replace the element to be deleted by the right-most element in the bottom level of the tree, then delete the right-most element in the bottom level of the tree.

Reminder: the elements of the `items` *array (defined in the abstract class* `ArrayedBinaryTree280<I>` *) represent the nodes of the tree. You are storing the* ***contents*** *of nodes in the array. There is no node class. It is very important that the contents of the root are stored in offset 1 and we don't use offset 0 of the array, otherwise, the given formulae for finding the child or parent of a node at offset i will not work correctly.*

4 Files Provided

lib280-asn2: A copy of lib280 which includes solutions to assignment 1, the `ArrayedBinaryTree280<I>` abstract class, and the partially completed `ArrayedBinaryTreeWithCursors280<I>` class for Question 11. The `ArrayedTree280<I>` interface can be found in the `lib280-asn2.tree` package.

5 What to Hand In

You must submit the following files:

Q1-8.doc/docx/rtf/pdf/txt - your answers to questions 1 through 10. Digital images of handwritten pages are also acceptable, provided that they are **clearly** legible.

ArrayedBinaryTreeWithCursors280.java - your arrayed binary tree implementation and regression test.