Ex 3

Shai Aharon/301206967

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1 Q1

Given a robot moving in a circle with radius R around $(0, 0)$ with a constant speed of ω. We want to know at each moment the location of the robot in x, y coordinate, and track his angular speed θ. The state vector is:

$$\vec{x} = \begin{bmatrix} P_x \\ P_y \\ \theta \\ \omega \end{bmatrix}$$

* The angles are in **radians**

1. Write the functions that transform the current state to the next state, with Δt as the time difference.
2. Write the **Jacobian** matrix
3. Same as 1-2, but the state vector is as follows:

$$\vec{x} = \begin{bmatrix} R \\ \theta \\ \omega \end{bmatrix}$$

1.1 A1

1.

$$\vec{x} = \begin{bmatrix} P_x \\ P_y \\ \theta \\ \omega \end{bmatrix} = \begin{bmatrix} R\cos(\theta) \\ R\sin(\theta) \\ \theta \\ \omega \end{bmatrix}$$

$$\vec{x}t + 1 = \begin{bmatrix} R\cos(\theta + \Delta t \cdot \omega) \\ R\sin(\theta + \Delta t \cdot \omega) \\ \theta + \Delta t \cdot \omega \\ \omega \end{bmatrix} = \begin{bmatrix} R\left(\cos(\theta)\cos(\Delta t \cdot \omega) - \sin(\theta)\sin(\Delta t \cdot \omega)\right) \\ R\left(\sin(\theta)\cos(\Delta t \cdot \omega) + \sin(\Delta t \cdot \omega)\cos(\theta)\right) \\ \theta + \Delta t \cdot \omega \\ \omega \end{bmatrix} =$$

$$\begin{bmatrix} P_x\cos(\Delta t \cdot \omega) - P_y\sin(\Delta t \cdot \omega) \\ P_y\cos(\Delta t \cdot \omega) + P_x\sin(\Delta t \cdot \omega) \\ \theta + \Delta t \cdot \omega \\ \omega \end{bmatrix}$$

2.

$$J = \begin{bmatrix} \frac{\partial(P_x\cos(\Delta t\cdot\omega)-P_y\sin(\Delta t\cdot\omega))}{\partial P_x} & \frac{\partial(P_y\cos(\Delta t\cdot\omega)+P_x\sin(\Delta t\cdot\omega)}{\partial P_x} & \frac{\partial(\theta+\Delta t\cdot\omega)}{\partial P_x} & \frac{\partial(\omega)}{\partial P_x} \\ \frac{\partial(P_x\cos(\Delta t\cdot\omega)-P_y\sin(\Delta t\cdot\omega))}{\partial P_y} & \frac{\partial(P_y\cos(\Delta t\cdot\omega)+P_x\sin(\Delta t\cdot\omega)}{\partial P_y} & \frac{\partial(\theta+\Delta t\cdot\omega)}{\partial P_y} & \frac{\partial(\omega)}{\partial P_y} \\ \frac{\partial(P_x\cos(\Delta t\cdot\omega)-P_y\sin(\Delta t\cdot\omega))}{\partial\theta} & \frac{\partial(P_y\cos(\Delta t\cdot\omega)+P_x\sin(\Delta t\cdot\omega)}{\partial\theta} & \frac{\partial(\theta+\Delta t\cdot\omega)}{\partial\theta} & \frac{\partial(\omega)}{\partial\theta} \\ \frac{\partial(P_x\cos(\Delta t\cdot\omega)-P_y\sin(\Delta t\cdot\omega))}{\partial\omega} & \frac{\partial(P_y\cos(\Delta t\cdot\omega)+P_x\sin(\Delta t\cdot\omega)}{\partial\omega} & \frac{\partial(\theta+\Delta t\cdot\omega)}{\partial\omega} & \frac{\partial(\omega)}{\partial\omega} \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\Delta t\cdot\omega) & \sin(\Delta t\cdot\omega) & 0 & 0 \\ \sin(\Delta t\cdot\omega) & \cos(\Delta t\cdot\omega) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \Delta t & 1 \end{bmatrix} =$$

3.

$$F = \begin{bmatrix} R \\ \theta+\Delta t\cdot\omega \\ \omega \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial R}{\partial R} & \frac{\partial(\theta+\Delta t\cdot\omega)}{\partial R} & \frac{\partial\omega}{\partial R} \\ \frac{\partial R}{\partial\theta} & \frac{\partial(\theta+\Delta t\cdot\omega)}{\partial\theta} & \frac{\partial\omega}{\partial\theta} \\ \frac{\partial R}{\partial\omega} & \frac{\partial(\theta+\Delta t\cdot\omega)}{\partial\omega} & \frac{\partial\omega}{\partial\omega} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \Delta t & 1 \end{bmatrix}$$

2 Q2

Assuming that the state vector is as follows:

$$\vec{x} = \begin{bmatrix} P_x \\ P_y \\ \theta \\ \omega \end{bmatrix}$$

and we have a sensor that can measure the distance from the $(0,0)$ point to the robot times 2, and the angle in degrees, calculate the Jacobian matrix.

2.1 A2

$$F = \begin{bmatrix} 2\sqrt{P_x^2+P_y^2} \\ \frac{180\cdot\theta}{\pi} \end{bmatrix}$$

$$H = \begin{bmatrix} \frac{\partial\left(2\sqrt{P_x^2+P_y^2}\right)}{\partial P_x} & \frac{\partial(180\theta/\pi)}{\partial P_x} \\ \frac{\partial\left(2\sqrt{P_x^2+P_y^2}\right)}{\partial P_y} & \frac{\partial(180\theta/\pi)}{\partial P_y} \\ \frac{\partial\left(2\sqrt{P_x^2+P_y^2}\right)}{\partial\theta} & \frac{\partial(180\theta/\pi)}{\partial\theta} \\ \frac{\partial\left(2\sqrt{P_x^2+P_y^2}\right)}{\partial\omega} & \frac{\partial(180\theta/\pi)}{\partial\omega} \end{bmatrix} = \begin{bmatrix} \frac{2P_x}{P_x^2+P_y^2} & 0 \\ \frac{2P_y}{P_x^2+P_y^2} & 0 \\ 0 & \frac{180}{\pi} \\ 0 & 0 \end{bmatrix}$$

3 Q3

TBA