

Ex 3

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1 Q1

Given a robot moving in a circle with radius R around $(0,0)$ with a constant speed of ω . We want to know at each moment the location of the robot in x, y coordinate, and track his angular speed θ . The state vector is:

$$\vec{x} = \begin{bmatrix} P_x \\ P_y \\ \theta \\ \omega \end{bmatrix}$$

* The angles are in **radians**

1. Write the functions that transform the current state to the next state, with Δt as the time difference.
2. Write the **Jacobian** matrix
3. Same as 1-2, but the state vector is as follows:

$$\vec{x} = \begin{bmatrix} R \\ \theta \\ \omega \end{bmatrix}$$

1.1 A1

1.

$$\begin{aligned} \vec{x} &= \begin{bmatrix} P_x \\ P_y \\ \theta \\ \omega \end{bmatrix} = \begin{bmatrix} R \cos(\theta) \\ R \sin(\theta) \\ \theta \\ \omega \end{bmatrix} \\ \vec{x}^{t+1} &= \begin{bmatrix} R \cos(\theta + \Delta t \cdot \omega) \\ R \sin(\theta + \Delta t \cdot \omega) \\ \theta + \Delta t \cdot \omega \\ \omega \end{bmatrix} = \begin{bmatrix} R (\cos(\theta) \cos(\Delta t \cdot \omega) - \sin(\theta) \sin(\Delta t \cdot \omega)) \\ R (\sin(\theta) \cos(\Delta t \cdot \omega) + \sin(\Delta t \cdot \omega) \cos(\theta)) \\ \theta + \Delta t \cdot \omega \\ \omega \end{bmatrix} = \\ &\quad \begin{bmatrix} P_x \cos(\Delta t \cdot \omega) - P_y \sin(\Delta t \cdot \omega) \\ P_y \cos(\Delta t \cdot \omega) + P_x \sin(\Delta t \cdot \omega) \\ \theta + \Delta t \cdot \omega \\ \omega \end{bmatrix} \end{aligned}$$

2.

$$\begin{aligned} J &= \begin{bmatrix} \frac{\partial(P_x \cos(\Delta t \cdot \omega) - P_y \sin(\Delta t \cdot \omega))}{\partial P_x} & \frac{\partial(P_x \cos(\Delta t \cdot \omega) - P_y \sin(\Delta t \cdot \omega))}{\partial P_y} & \frac{\partial(P_x \cos(\Delta t \cdot \omega) - P_y \sin(\Delta t \cdot \omega))}{\partial \theta} & \frac{\partial(P_x \cos(\Delta t \cdot \omega) - P_y \sin(\Delta t \cdot \omega))}{\partial \omega} \\ \frac{\partial(P_y \cos(\Delta t \cdot \omega) + P_x \sin(\Delta t \cdot \omega))}{\partial P_x} & \frac{\partial(P_y \cos(\Delta t \cdot \omega) + P_x \sin(\Delta t \cdot \omega))}{\partial P_y} & \frac{\partial(P_y \cos(\Delta t \cdot \omega) + P_x \sin(\Delta t \cdot \omega))}{\partial \theta} & \frac{\partial(P_y \cos(\Delta t \cdot \omega) + P_x \sin(\Delta t \cdot \omega))}{\partial \omega} \\ \frac{\partial(\theta + \Delta t \cdot \omega)}{\partial P_x} & \frac{\partial(\theta + \Delta t \cdot \omega)}{\partial P_y} & \frac{\partial(\theta + \Delta t \cdot \omega)}{\partial \theta} & \frac{\partial(\theta + \Delta t \cdot \omega)}{\partial \omega} \\ \frac{\partial(\omega)}{\partial P_x} & \frac{\partial(\omega)}{\partial P_y} & \frac{\partial(\omega)}{\partial \theta} & \frac{\partial(\omega)}{\partial \omega} \end{bmatrix} = \\ &\quad \begin{bmatrix} \cos(\Delta t \cdot \omega) & -\sin(\Delta t \cdot \omega) & 0 & 0 \\ \sin(\Delta t \cdot \omega) & \cos(\Delta t \cdot \omega) & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

3.

$$F = \begin{bmatrix} R \\ \theta + \Delta t \cdot \omega \\ \omega \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial R}{\partial R} & \frac{\partial(\theta + \Delta t \cdot \omega)}{\partial R} & \frac{\partial \omega}{\partial R} \\ \frac{\partial R}{\partial \theta} & \frac{\partial(\theta + \Delta t \cdot \omega)}{\partial \theta} & \frac{\partial \omega}{\partial \theta} \\ \frac{\partial R}{\partial \omega} & \frac{\partial(\theta + \Delta t \cdot \omega)}{\partial \omega} & \frac{\partial \omega}{\partial \omega} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \Delta t & 1 \end{bmatrix}$$

2 Q2

Assuming that the state vector is as follows:

$$\vec{x} = \begin{bmatrix} P_x \\ P_y \\ \theta \\ \omega \end{bmatrix}$$

and we have a sensor that can measure the distance from the $(0,0)$ point to the robot times 2, and the angle in degrees, calculate the Jacobian matrix.

2.1 A2

$$H = \begin{bmatrix} \frac{\partial(2\sqrt{P_x^2 + P_y^2})}{\frac{\partial P_x}{\partial(180\theta/\pi)}} & \frac{\partial(2\sqrt{P_x^2 + P_y^2})}{\frac{\partial P_y}{\partial(180\theta/\pi)}} & \frac{\partial(2\sqrt{P_x^2 + P_y^2})}{\frac{\partial \theta}{\partial(180\theta/\pi)}} & \frac{\partial(2\sqrt{P_x^2 + P_y^2})}{\frac{\partial \omega}{\partial(180\theta/\pi)}} \end{bmatrix} = \begin{bmatrix} \frac{2P_x}{P_x^2 + P_y^2} & \frac{2P_y}{P_x^2 + P_y^2} & 0 & 0 \\ 0 & 0 & \frac{180}{\pi} & 0 \end{bmatrix}$$

3 Q3

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