Ex 2

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1 Q1

Given an agent with varying location and speed, where both are in one dimension only. The movement model is a constant speed model. The initial location error is 2 meters, and the initial speed error is 1.2 feet(1 feet = 0.3048 meter). The sensor measures only the location, and its accuracy is a Gaussian distribution with a variance of 0.5 feet. The initial state is $x = 8, v = 5\frac{m}{s}$ $\Delta t = 1$ sec

1. Write the following matrices: H, P, F and the Kalman Gain (K)
2. Assuming the sensor measures that the agent is at 43 feet (13.1054 meter), calculate the status vectors (P, X) and the new K after the update.

1.1 A1

1. $P = \begin{bmatrix} 4 & 0 \\ 0 & 0.36576^2 \end{bmatrix}$
   $F = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
   $H = \begin{bmatrix} 1 & 0 \end{bmatrix}$
   $K = HP_0H^T(HP_0H^T - R_0)^{-1} = 4 \cdot (4 + 0.1524^2)^{-1} = \frac{4}{4.0232} = 0.9942$
2. Measurement $(\mu_1, \Sigma_1) = (Z_1, R_1) = (13.1054, 0.1524^2)$

$\hat{x} = x_1 + K(\mu_1 - \mu_0) = 8 + K(13.1054 - 8) = 8 + 0.9942(5.1054) = 8 + 5.076 = 13.076$

$$\hat{P} = F \cdot P \cdot F^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 0.13378 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0.13378 & 0.13378 \end{bmatrix} = \begin{bmatrix} 4.13378038 & 0.13378038 \\ 0.13378038 & 0.13378038 \end{bmatrix}$$

$$K_1 = HP_kH_T(H_KP_KH_K^T + R_K)^{-1} =$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 4.13378038 & 0.13378038 \\ 0.13378038 & 0.13378038 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 4.13378038 & 0.13378038 \\ 0.13378038 & 0.13378038 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0.15242^2 \right)^{-1} =$$

$$\frac{4.13378}{4.13378 + 0.02323} = 0.9467$$

2 Q2

Assume that the sensor measures both location(feet) and speed ($\frac{m}{s}$). The variance of the sensor location is 0.5 $feet$ and the variance for the sensor speed measurement is $4\frac{m}{s}$. 1. Repeat [Q1] , what will be the shape of the Kalman Gain matrix? 2. Assuming that the sensor measured at Δt a location of 43 $feet$ and a speed of $4\frac{m}{s}$, compute the status vector (P, X), and the Kalman gain after the **Update**.

2.1 A2

1. $F = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$P_0 = \begin{bmatrix} 4 & 0 \\ 0 & 0.36576^2 \end{bmatrix}$

$P' = FP_0F^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 0.13378 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 0.13378 \end{bmatrix}$

$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$K = HP'H^T(HP'H^T - R_0)^{-1} =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 0.13378 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 0.13378 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.1524^2 & 0 \\ 0 & 4^2 \end{bmatrix} \right)^{-1}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 0.13378 \end{bmatrix} \left(\begin{bmatrix} 4 & 0 \\ 0 & 0.13378 \end{bmatrix} + \begin{bmatrix} 0.1524^2 & 0 \\ 0 & 4^2 \end{bmatrix} \right)^{-1} =$$

$$\begin{bmatrix} 4 & 0 \\ 0 & 0.13378 \end{bmatrix} \begin{bmatrix} 5.62004576 & 0 \\ 0 & 16.13378 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 9.942 \cdot 10^{-1} & 4.894 \cdot 10^{-3} \\ 4.894 \cdot 10^{-3} & 8.479 \cdot 10^{-1} \end{bmatrix}$$

Thus, K is a 2×2 shape matrix.

2. Measurement $\begin{bmatrix} 13.1064 \\ 4 \end{bmatrix}$

$\hat{x}^- = Fx_0 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 5 \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$

$$\hat{x} = \begin{bmatrix} 13 \\ 5 \end{bmatrix} + K(\begin{bmatrix} 13.1064 \\ 4 \end{bmatrix} - \begin{bmatrix} 13 \\ 5 \end{bmatrix}) =$$

$$\begin{bmatrix} 13 \\ 5 \end{bmatrix} + \begin{bmatrix} 9.942 \cdot 10^{-1} & 4.894 \cdot 10^{-3} \\ 4.894 \cdot 10^{-3} & 8.479 \cdot 10^{-1} \end{bmatrix} \begin{bmatrix} 0.1064 \\ -1 \end{bmatrix} =$$

$$\begin{bmatrix} 13 \\ 5 \end{bmatrix} + \begin{bmatrix} 0.10089 \\ -0.8473 \end{bmatrix} = \begin{bmatrix} 13.10089 \\ 4.1526 \end{bmatrix}$$

$$
\begin{aligned}
P_1 &= (I - KH)P' \\
&= \left(I - \begin{bmatrix} 9.942 \cdot 10^{-1} & 4.894 \cdot 10^{-3} \\ 4.894 \cdot 10^{-3} & 8.479 \cdot 10^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \begin{bmatrix} 4 & 0 \\ 0 & 0.13378 \end{bmatrix} \\
&= \begin{bmatrix} 0.00574466 & 0.99510514 \\ 0.99510514 & 0.15209976 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 0.13378 \end{bmatrix} \\
&= \begin{bmatrix} [0.15687271 & 0.13389406] \\ 4.13389406 & 0.15347351 \end{bmatrix}
\end{aligned}
$$

3 Q3

Write a program in python that performs Kalman filter as described this question (1-D location and speed), and returns $P \ and \ X$