



## Gran Premio de México 2019

Primera Fecha

May 4th, 2019

Contest Session

This problem set contains 10 problems; pages are numbered from 1 to 11. Without considering this nor the General information page.

 $This\ problem\ set\ is\ used\ in\ simultaneous\ contests:$ 

Gran Premio de México 2019 – Primera Fecha Gran Premio de Centro América 2019 – Primera Fecha

## General information

Unless otherwise stated, the following conditions hold for all problems.

### Program name

1. Your solution must be called *codename.*c, *codename.*cpp, *codename.*java, *codename.*py2 or *codename.*py3, where *codename* is the capital letter which identifies the problem.

#### Input

- 1. The input must be read from standard input.
- 2. The input is described using a number of lines that depends on the problem. No extra data appear in the input.
- 3. When a line of data contains several values, they are separated by *single* spaces. No other spaces appear in the input. There are no empty lines.
- 4. The English alphabet is used. There are no letters with tildes, accents, diaereses or other diacritical marks (ñ, Ã, é, Ì, ô, Ü, ç, etcetera).
- 5. Every line, including the last one, has the usual end-of-line mark.

#### Output

- 1. The output must be written to standard output.
- 2. The result of the test case must appear in the output using a number of lines that depends on the problem. No extra data should appear in the output.
- 3. When a line of results contains several values, they must be separated by *single* spaces. No other spaces should appear in the output. There should be no empty lines.
- 4. The English alphabet must be used. There should be no letters with tildes, accents, diaereses or other diacritical marks  $(\tilde{n}, \tilde{A}, \acute{e}, \tilde{I}, \hat{o}, \ddot{U}, \varsigma, etcetera)$ .
- 5. Every line, including the last one, must have the usual end-of-line mark.

#### Development team

The following persons helped to develop the problem set by creating and improving statements, solutions, test cases and input and output checkers:

Juan Pablo Marín, UdeG CUCEI Moroni Silverio, FES Acatlán María Celeste Ramírez Trujillo Saraí Ramírez, FES Acatlán Lina Rosales Jesus Alejandro Rizo, ITESO Juan Felipe Baquero Jonathan Queiroz, UFBA

## Problem A - Add and substract

Author: Juan Pablo Marín

Little Santiago is learning how to add and substract numbers in the school. The teacher has created a game so that the students put their knowledge into practice and to verify that they are progressing in this topic.

Each of the N students in Santiago's class will take a sheet of his math notebook and write a number  $a_i$  between 1 and  $10^6$ . Next, each kid will sum one by one the difference between the number he has chosen with the number that each of the other students with greater list number have chosen only if this difference is not -1, 0 or 1. This is, the kid with list number i will sum each of the following numbers  $a_{i+1} - a_i$ ,  $a_{i+2} - a_i$ , ...,  $a_N - a_i$ , only if the number is not -1, 0 or 1. As you can see, the last kid will not have any difference to sum, that's why his task is to sum all the values that the other kids got.

Santiago has the list number N, help him get his sum right.

#### Input

The first line of input contains an integer N ( $1 \le N \le 10^6$ ), representing the number of students in the class. The second and last line in the input contains N integer numbers separated by space, where the i-th number represents the value  $a_i$ , the number the i-th kid wrote in his notebook.

#### Output

Output a single line with an integer indicating the result Santiago should get after performing his sums.

Sample output 1
-4

## Problem B - Box delivery

Author: Moroni Silverio

Jaime has a box transportation company and recently has been hired by a factory that needs K boxes to be moved to their new branch.

Jaime charges based on the number of trips N he does to deliver all the boxes. This charging schema allows Jaime to do some trips without carrying boxes, this way he can earn more money, but he can't do this all the time since eventually all the boxes should be delivered at the end of its N-th trip. The company that hired him is very serious and seems suspicious that Jaime charges by trip instead of by box delivered, that's why they have assigned a supervisor that will oversee Jaime's trips.

There are some days that the supervisor will be waiting for Jaime in the new branch to verify that he is bringing boxes, these days Jaime can't come without boxes to deliver. The worst is that some days the boss of the assigned supervisor will come to check the deliveries. He does not want Jaime to bring less than two boxes these days. If the supervisor or his boss find Jaime with less boxes than they expected in the branch in a given day, then Jaime risks his contract to be resigned.

Jaime has signed a contract where he states he will be doing one trip per day, also he already knows the days the supervisor will be in the new branch and the days the supervisors boss will be in the branch. Jaime is wondering in how many ways he can deliver the boxes without the risk that the supervisor or his boss resign his contract. Two delivery schedules differ if they differ on the number of boxes delivered in any given day.

### Input

The first line of input contains a single integer T ( $1 \le T \le 1000$ ), representing the number of test cases. Each of the following test cases start with a line with two numbers separated by a space N and K, ( $1 \le N, K \le 10^5$ ) representing the number of days Jaime will do trips to the new branch and the number of boxes he needs to deliver. The next line contains integer numbers separated by space representing the days the supervisor will be in the new branch waiting for Jaime to deliver boxes, the last line contains integer numbers separated by space representing the days the supervisors boss will be in the new branch waiting for Jaime to deliver boxes. All days are in the range [1, N]

#### Output

For each test case in the input, output a line containing the number of ways Jaime can schedule the trips. As the answer can be very big, print it modulo 986444689.

Sample input 1	Sample output 1	
1	1	
3 3		
1		
2		

# Problem C - Connecting cities

Author: Moroni Silverio

New government decided to build new roads to improve connections between the cities in the state. After some research, a set of roads were proposed and approved for construction.

Governor of the state wants to make possible to reach any city from another using only the new roads. Years after the construction, inhabitants started to complain that it was not possible to reach their destination without using old roads that are in a very bad shape. Worried about the complaints the inhabitants have brought, the governor decided to hire a new team to solve the problem, they suggested to build more roads. They assure that the new roads will help him to achieve his purpose of connecting all cities with new roads, but the governor is in doubt, the first team assured the same thing and today inhabitants are not happy. Governor has already spent a lot of money building roads and advertising them, so they have a low budget to build all the roads in the new plan. You were hired to help government to find what roads from the new team proposal should be built in order that all cities can be reached using the roads in the state spending the less amount of money.

#### Input

The first line of the input contains a single integer T ( $1 \le T \le 10$ ), the number of test cases. Each test case starts with a line that contains a single integer N ( $1 \le N \le 10^4$ ), representing the number of cities in the state. The next line contains a number C ( $1 \le C \le 10^5$ ), the number of roads that are currently built in the state. Each of the next C lines contain three integer numbers separated by space  $a_i$ ,  $b_i$ , and  $v_i$ , indicating that a road exists between cities  $a_i$  and  $b_i$  ( $1 \le a_i, b_i \le N, a_i \ne b_i$ ) and the cost when it was built was  $v_i$  ( $1 \le v_i \le 10^6$ ). The next line contains an integer P ( $1 \le P \le 10^5$ ), representing the number of roads in the new proposal, the next P lines contain three integer numbers separated by a space  $c_j$ ,  $d_j$ ,  $u_j$ , representing that there is a proposal to build a road between cities  $c_j$  and  $d_j$  ( $1 \le c_j, d_j \le N, c_j \ne d_j$ ) with a cost of  $u_j$  ( $1 \le u_j \le 10^6$ ).

### Output

For each test case in the input, print a line with a single integer, the amount of money that needs to be spent in order to connect all the cities if such way exists. If there is no way to connect all the cities with the given proposal, print a line with the text "You better hire someone else". If not a single road needs to be built to connect the cities print a line with the text "Thank you, Goodbye".

Sample input 1	Sample output 1
1	1
5	
4	
1 2 5	
2 3 5	
3 1 1	
4 5 1	
4	
5 1 1	
5 2 1	
3 4 1	
5 2 2	

# Problem D - Determining rally paths

Author: Moroni Silverio

During the celebration of the 50th anniversary of Jaime's school, they will be running a rally, in total E teams have signed to compete and there will be awesome prizes for each team that is able to finish the rally.

There are V stations in the rally and, when a team arrives at a station, the team should complete a challenge; once the challenge is completed, the rally staff will write a code  $C_i$  that demonstrates the team has completed the challenge in the station.

Before starting the rally, each team is assigned a starting and an ending station, then, when they start, they should complete the challenge in the first station and move to the next one, they repeat this until they finish the challenge in the ending station they have assigned. Once a team has finished all the challenges, the rally sheet of the team has all the codes of each challenge they completed concatenated, and they present the sheet to the rally staff to claim their prize. For example, if the codes in the stations a team visits are: 3, 2, 4, and 9 then the sheet will read 3249. To be fair, the rally has been planned in such way that there is always a unique path to go from one station to any other station in the rally.

As there are many teams, it is hard for the rally staff to determine what the sheet of each team should look like, that's why they are looking for help given the map of the rally and the initial and ending stations of each team to determine what number should be written in the sheet each team should provide to claim their rally prize.

#### Input

The first line of input contains two integer numbers separated by a space: V, and E, representing the number of stations in the rally and the number of teams that have signed to compete  $(2 \le V \le 10^5)$ ,  $1 \le E \le 10^5)$ . The next line contains V integer numbers separated by space, where the i-th number represents the code  $C_i$  that the i-th station will write in the team sheet when the challenge is completed  $(1 \le C_i < 1000)$ . Next V - 1 lines, contain two numbers separated by a space a and b meaning there is a path between stations a and b in the rally  $(1 \le a, b \le V)$ . Next E lines follow, each contains two numbers separated by a space  $e_s$ ,  $e_f$ , representing the starting and ending stations of every team.

#### Output

For each team in the input, output a line containing an integer number representing the number the team should have written in the paper to show they completed the rally and claim the prize. As the number can be very large, the staff wants the reminder of the number after dividing it by 188888881.

Sample input 1	Sample output 1
7 3	412
1 3 7 9 1 4 2	137
1 2	139
2 3	
5 2	
4 2	
5 6	
7 5	
6 7	
5 3	
5 4	

# Problem E - Egyptian binary system

Author: Juan Pablo Marín

Explorers in ancient Egypt have found evidence of what may be the first used binary system to encode data not so far from the Nile river.

A hieroglyphic S was found containing what looks like a binary system as it is conformed of a sequence of only two different shapes: the "eye" and the "spear". Linguists that arrived to the place to study the hieroglyphic are amazed since these shapes resemble the "0" and "1" that we use to represent binary symbols. Going further, they have found that there are some messages encoded as if this hieroglyphic contained data that old egyptians wanted us to discover. Some studies revealed that the hieroglyphic was made in different ages, and the data written from age to age always represented an odd number in its binary form if you took the "eye" as a 0 and the "spear" as a 1.

The data that revealed this amazing discovery is missing, right now there is no way to determine in what age a shape from the hieroglyphic was written, this is why scientists have asked for your help. They are aware that it is very difficult to obtain all the data from nothing, so they are only interested in determining in how many ways you can take a substring from the hieroglyphic such that the data you took may have been written in the same age; all what scientists know is that the data never started with an "eye" and it always represented an odd number in its binary form.

#### Input

The input consists of a single line that contains a string S representing the hieroglyphic. S contains only the '0' and '1' symbols. A '0' represent an "eye" in the hieroglyphic, and a '1' represents a "spear". S will contain no more than  $10^6$  characters.

### Output

Output a single line with an integer indicating the number of ways you can take the binary representation of an odd number based on the description above.

Sample input 1	Sample output 1
1	1
Sample input 2	Sample output 2
0	0
Sample input 3	Sample output 3
1010	3

## Problem F - Forecasting rock-paper-scissors

Author: Jonathan Queiroz

Roshamboland is a wonderful city, featuring an amazing riverfront, glamorous parks, and unmatched cultural attractions. Nevertheless, its inhabitants are mostly concerned with a single activity: playing rock-paper-scissors. Rock-paper-scissors is a two-player game in which each of the two players simultaneously forms one of three shapes (rock, paper or scissors) with their hands. Paper defeats rock, rock defeats scissors and scissors defeats paper. If both players form the same shape, the match ends in a draw.

A few years ago, the mayor of Roshamboland organized a huge rock-paper-scissors tournament, which was very well received by the city residents. Each day, two world-renowned players met in the Yan Ken Po Arena, the city's largest stadium, for an exciting televised match. Upon completion of each match, final results (win, loss or draw) were honorably recorded in the Official Gazette of Roshamboland.

The mayor's term is now coming to an end, and he would like to organize a new tournament as part of his (not quite legal) reelection campaign. As preparation for the upcoming tournament, he decided to hire a team of experienced Rosearchers to analyze the results of the previous edition. After months of investigation, they eventually reached the conclusion that the so-called world-renowned players always make the same moves. For example, a given player may always choose paper, while another may always choose scissors.

The mayor would like to use this groundbreaking discovery to make the next tournament more interesting. Based on logs from the Official Gazette, he needs you to figure out, for some pairs of players, who would win the match if they were to play against each other. Unfortunately the specific moves of players in the previous edition (rock, paper or scissors) were not recorded, and only match results are known (win, loss or draw). For each queried pair of players, you need to determine what would be the result of a match between them, or else, tell that there is not enough information to decide. Whenever determining the result of a match between two given players is feasible, you also need to inform the earliest moment in the tournament in which it became possible to do so. In other words, for each query involving players x and y, you also need to report the minimum integer k (if any) such that the results of the first k matches from the Official Gazette are sufficient to determine the result of a match between x and y.

### Input

The first line of the input contains two integers N and M, denoting the number of players in the tournament and the number of matches that took place, respectively  $(2 \le N \le 10^5, \ 0 \le M \le 10^5)$ . Each of the next M lines describes a match, and is composed by three integers x, y and z, where x and y denote the players and z denotes the result of the match  $(1 \le x, y \le N, -1 \le z \le 1)$  and  $x \ne y$ . The value z = 1 indicates that player x won against player y, while z = 0 indicates a tie and z = -1 indicates that player x lost. It is guaranteed that match results are consistent with the rules of rock-paper-scissors, and with the fact that any given player always chooses the same shape.

The next line contains a single integer Q, indicating the number of queries that follow  $(1 \le Q \le 3 \times 10^5)$ . Each of the following Q lines contains two integers x and y, denoting two players  $(1 \le x, y \le N, x \ne y)$ .

## Output

For each query between players x and y, you should print its answer in a single line. If the result of a match between the players cannot be predicted from the logs in the Official Gazette, simply print -1. Otherwise, print the earliest day in the tournament in which the match result could have been established with certainty, followed by the result itself (with 1 indicating a win for player x, 0 indicating a tie and -1 indicating a win for player y).

Sample input 1	Sample output 1	
5 6	2 1	
1 2 1	2 -1	
2 3 1	-1	
1 3 -1	1 1	
4 5 0	2 -1	
1 2 1	4 0	
5 4 0		
6		
2 3		
1 3		
3 5		
1 2		
3 2		
5 4		

Sample input 2	Sample output 2
10 7	3 1
6 10 1	3 -1
10 7 1	5 0
7 4 -1	5 0
7 5 1	4 0
4 8 -1	5 -1
5 2 -1	2 -1
1 3 -1	-1
11	-1
6 4	3 1
7 4	3 1
5 8	
5 8	
6 5	
8 7	
7 10	
8 9	
2 1	
4 7	
6 4	

## Problem G - Going to the world finals again

Author: Lina Rosales & Juan Felipe Baquero

Baker as you know, is a smart cat, he won regionals and classified to the next cat programming World Finals. Baker is planning his trip in which he has to travel a distance of exactly X kilometers, Baker wants to travel in several days (always more than one) so he can get to know and visit more places and also avoid jetlag.

For his trip, Baker decided that the first day he will travel for d kilometers, the second day he will travel for d+1, the next one d+2 and so on until he arrives at his destination traveling exactly X kilometers.

Baker's owner knows how annoying is to do a lot of short trips, she wants Baker to travel the longest possible distance in the first day, so that he can have a more enjoying trip to the World Finals.

Help Baker's owner know on how many different ways her beloved cat can do his trip.

## Input

The first line has an integer T ( $1 \le T \le 200$ ). The next T lines contain a single integer X ( $1 \le X \le 10^{15}$ ), representing the kilometers Baker should travel to get to the World Finals.

## Output

For each test case in the input, print "case y: s" where y is the number of test case starting in 1, and s the number of ways in which Baker can do the trip.

Sample input 1	Sample output 1
3 2012	case 1: 1 case 2: 3
30 16	case 3: 0

## Problem H - Husbands association

Author: Jesus Alejandro Rizo

Husbands association is a non profit association intended to help them to have a better relationship with their partners. They have been studying how their partners react to different problems and discussions that are brought to conversation, first step was to have a conversation in the first place! It seems like the partner has a level of tolerance T, and each problem reduces the tolerance of the partner in  $a_i$  units; if the tolerance of the partner reaches 0 then the problem is magnified and the husband will feel bad for the rest of the week. For example, let's say the tolerance of Mrs E. is 10, and Mr E. can bring five different problems, not doing the dishes takes 3 tolerance, doing the dishes takes 2, not taking trash out takes 5 tolerance, not dinning in a restaurant a night a week takes 1, and not bringing flowers in the anniversary takes 8.

Husbands association wants to build a software that helps husbands to determine the order of the problems to discuss without reaching the partner tolerance to 0, they have two strategies. The first one is to talk about the problem that takes more tolerance from the partner and keep doing this withouth reaching the partner tolerance to 0. The other approach is to talk about the problems that takes the lower number of tolerance so they can take more problems without bringing tolerance to 0.

Can you help the Husbands association to determine the maximum number of problems that can be discussed by the husband and his partner in both approaches?

#### Input

The first line of input contains two numbers separated by a space N and T ( $1 \le N \le 10^5$ ,  $1 \le T \le 10^6$ ), representing the number of problems and the tolerance of the partner, respectively. The next line contains N integer numbers between 1 and  $10^5$ , representing the tolerance each problem takes from the partner.

### Output

Output a single line with two integer numbers separated by a space, the first being the maximum number of problems the husband can discuss with his partner using the first approach described, the second is the maximum number of problems the husband can discuss with his partner using the second approach from the association.

Sample input 1	Sample output 1
5 10 3 2 4 1 8	2 3

# Problem I - Inspecting PIN numbers

Author: Juan Pablo Marín

John has came to you with a new idea to generate 5 digits PIN numbers for your bank applications. He says a secure method is applying the following algorithm:

- 1. Take a number K  $(1 \le K \le 10^5)$ .
- 2. Let S = K!.
- 3. Take last digit from S. If it is 0 delete it and repeat this step.
- 4. If S has less than 5 digits: add a 0 to its left and repeat this step.
- 5. Your PIN is the one formed with the last 5 digits from S.

It is simple, isn't it? But, the more you think about it, the more disturbing it becomes, it sounds like between all the values you can take for K there are values for the PIN that will repeat, for example if K = 9 and K = 10 the PIN is the same (36288).

You are willing to show John that his method is not so secure as he believes, that's why you have taken action and will create a program to count, for a given set of Q, queries how many PINs in the range x, y exist for the values of K between l and r.

#### Input

The first line of the input contains a single integer Q ( $1 \le Q \le 5*10^5$ ), representing the number of queries to answer. The next Q lines contain four values separated by a space l, r, x, y ( $1 \le l \le r \le 10^5$ ) and ( $1 \le x \le y \le 10^5$ )

### Output

For each query in the input, output a line containing a single integer, representing the number of values for K between l and r such that the PIN generated with that value lies in the range [x, y].

Sample input 1	Sample output 1
4	2
9 10 36288 36288	2
1 2 1 2	1
1 5 3 6	1
1 20 20 50	

## Problem J – Jaimina party invitations

Author: Saraí Ramírez

Jaimina is planning a party for her birthday. She will send physical invitations to all her friends, and to avoid wasting, she decided to make several invitations from each sheet of paper. For that, Jaimina makes cuts on the paper sheets satisfying three conditions:

- The cuts must be parallel to the sides of the sheets.
- All the resulting rectangles from a sheet must have the same dimensions.
- Jaimina will divide every sheet only by a number of rectangles that will be a power q of a prime number p.

Once she has the rectangles she will use for the invitations, she writes the message and decorates the perimeter of all the rectangles with colorful stripes.

Your task is helping her to know how many centimeters of stripes to buy. For this, you must consider that Jaimina will use as many sheets as different configurations are possible to create, so you must make sure that the stripes she buys will be enough to decorate all the resulting rectangles of all the different ways to convert the sheets into smaller rectangles with the mentioned conditions (each configuration is built exactly once).

We say two configurations are different if the dimensions of the resulting rectangles are distinct.

### Input

The first line of the input contains an integer T ( $1 \le T \le 1000$ ), representing the number of test cases that follows. Each of the next T lines describes a test case with four positive integers m, n, p and q separated by a space, where m and n represent the dimensions of the sheets of paper in centemeters ( $1 \le m \ne n \le 10^{16}$ ), p ( $1 \le p \le 10^{12}$ ) represents the prime number Jaimina has chosen to cut the sheets of paper and q ( $1 \le q \le 10^{18}$ ) is the power she wants to use (i.e Jaimina wants to create  $p^q$  invitations from every single sheet).

## Output

For each test case in the input, output a line containing an integer indicating the length of the stripes (in centimeters) that Jaimina needs. Because this number can be very large, output the remainder of dividing it by  $10^9 + 7$ .

Sample input 1	Sample output 1
4	264
10 12 5 1	98
2 5 2 2	234
1832	163939239
700 1050 7 49	