Gran Premio de México 2019

Segunda Fecha

May 11th, 2019

Contest Session

This problem set contains 12 problems; pages are numbered from 1 to 13. Without considering this nor the General information page.

This problem set is used in simultaneous contests:

Gran Premio de México 2019 – Segunda Fecha
Gran Premio de Centro América 2019 – Segunda Fecha

v1.7

General information

Unless otherwise stated, the following conditions hold for all problems.

Program name

1. Your solution must be called *codename*`.c`, *codename*`.cpp`, *codename*`.java`, *codename*`.py2` or *codename*`.py3`, where *codename* is the capital letter which identifies the problem.

Input

1. The input must be read from standard input.
2. The input is described using a number of lines that depends on the problem. No extra data appear in the input.
3. When a line of data contains several values, they are separated by *single* spaces. No other spaces appear in the input. There are no empty lines.
4. The English alphabet is used. There are no letters with tildes, accents, diaereses or other diacritical marks (ñ, Ã, é, Ì, ô, Ü, ç, etcetera).
5. Every line, including the last one, has the usual end-of-line mark.

Output

1. The output must be written to standard output.
2. The result of the test case must appear in the output using a number of lines that depends on the problem. No extra data should appear in the output.
3. When a line of results contains several values, they must be separated by *single* spaces. No other spaces should appear in the output. There should be no empty lines.
4. The English alphabet must be used. There should be no letters with tildes, accents, diaereses or other diacritical marks (ñ, Ã, é, Ì, ô, Ü, ç, etcetera).
5. Every line, including the last one, must have the usual end-of-line mark.

Development team

The following persons helped to develop the problem set by creating and improving statements, solutions, test cases and input and output checkers:

Juan Pablo Marín, UdeG CUCEI
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Problem A – Assigning company branches.

Author: Juan Pablo Marín

A very popular retail company with a total of N branches have decided to sell some of their business to their competitor, and they have made very clear that this merge is taking in consideration to always bring the best for their customers. The key point in the sale process is that the branches will be distributed between the two companies, a branch will be managed only by one company, and, the company that manages branch i, will get at the end of the year the annual profit p_i the branch produces. The annual profit each company makes is the sum of the profits of all the branches the company manages. Even when both companies have been competitors, they decided that to be fair, it is best if the absolute difference between their annual profits is minimum.

The distribution of the branches is becoming a very difficult process, it is hard to find a way to assign the branch managers with the given restrictions. In a meeting that has lasted for hours, maybe days, the teams of both companies have been looking to a map to see all the branches and to determine how to assign them. This map of the city is a plane where each branch is represented as a point in the plane. Jaime just came and heard the problem they have, and, being the problem solver he is, he just propossed what is considered the best solution: insted of dividing arbitary branches to each company, to draw a line in the map of the city and assign all branches in one side of the line to a company, and all branches in the other side to the other company. As Jaime found the solution, the companies asked him to find the best assignment possible. Given the map of the city and the annual profit of each branch, find the minimum difference between the annual profits of both companies when assigning the branches using Jaime's proposed solution.

Input

The first line contains a single integer N ($2 \leq N \leq 500$). Each of the next N lines contain three integer numbers separated by a space, x_i, y_i ($0 \leq x_i, y_i \leq 1000$) and p_i ($1 \leq p_i \leq 10^6$), representing the (x, y) coordinates of branch i and the annual profit p_i the branch produces.

Output

Output a single line with an integer indicating the minimum difference between the annual profits of both companies when asigning the branches using Jaime's proposed solution.

Sample input 1	**Sample output 1**
2 1 2 10 2 3 15	5

Sample input 2	**Sample output 2**
4 5 5 1 5 6 2 6 6 4 6 5 8	1

Problem B – Buggy visit counter.

Author: Moroni Silverio

Guinea pigs videos have become very popular on internet these days. The favourite internet site for people who love guinea pigs has a lot of videos of this funny animals to amuse them.
The director of this site have told the programmers to add a visit counter for each video, he wants, in the near future, to get more revenue from videos adding advertisements before they play, something that to this day nobody has done before. After some time, the visit counter has become a success and helped identify and capitalize a lot of money from the most popular videos in the site, however, someone noticed a visit counter with a negative number, meaning that, the visits counter is not as exact as everyone believes.
After hours of debugging, programmers found that the visits counter in the videos has a serious flaw, when the counter reaches some hardcoded value L, the next value it generates is a negative number; the first time the counter passes this limit the negative value is -1, and then it behaves normally, going from -1 to 0, from 0 to 1, and so on, until it reaches L again; the next number the counter generates is -2 and then it behaves normally, after passing L again it will generate -3, then -4 and so on.
The solution the programmers have proposed is to increase the value of L by one, but, soon the counter reached the new value of L and got to a negative number again, this time they decided to increase the curren value of L by 2 units, and they have kept doing this in such way that after the n-th time the counter has passed the stablished limit, they increase the limit value by n units. With all the changes to the value of L, nobody knows the number of visits a video has, they only know the initial value of L, and that the video counter is bugged.
Jaime has obtained the real number of visits C for a video and, in order to help the team, he is working on a program that given the values L and C, determines what is the visit count v the bugged counter should have produced for that video. For example, if $L = 3$ we can see the bugged visit counter will produce the values $1, 2, 3, -1, 0, 1, 2, 3, 4, -2, -1, 0, 1, ...$, if the value $C = 4$ then the bugged visit counter should show the value -1, if $C = 10$, the buggy counter shows -2.

Help the programmers know what is the value the bugger visit counter should show knowing the initial value for L and the number of visists C Jaime found for the video.

Input

The first line of the input contains a single integer T ($1 \leq T \leq 10^4$), representing the number of test cases. Each of the following T lines contain two integer numbers separated by a space L and C ($1 \leq L \leq 10^5$, $1 \leq C \leq 10^{15}$), the limit L the buggy counter initially had and the number of visits C the video has based on Jaime's log files.

Output

For each test case in the input, output a line with the value v, the number the buggy visit counter should show for a video with C visits.

Sample input 1	**Sample output 1**
2 3 10 3 4	-2 -1

Problem C – Credit card PIN number.

Author: Saraí Ramírez

Febo has a new credit card and must choose a PIN. To create his new PIN, it is necessary to satisfy two conditions:

1. The number must have exactly K digits.
2. Two (or more) equal digits should not be next to each other.

For example, if the PIN should have $K = 4$ digits, then Febo could select the PIN 1234, but, not the PIN 1123 as two digits 1 are next to each other.
Can you count the number of valid PINs from which Febo can choose the PIN for his new credit card?

Input

The first line will have a natural number T such that $1 \leq T \leq 1000$, and denotes the number of cases you will receive. Each of the next T lines contains an integer K, the number of digits Febo's PIN should have ($2 \leq K \leq 10^6$).

Output

For each test case in the input, print a line with a single integer, the number of valid PINs from which Febo can choose the PIN for his new credit card. Since the answer could be very big, print it modulo $10^9 + 7$.

Sample input 1	**Sample output 1**
3 4 11 400200	7290 867843772 57223964

Problem D – Delivery schedules.

Author: Moroni Silverio

Jaime's delivery box company has expanded to several cities now. He has hired people that helps him do the delivery, they receive boxes in some cities and deliver in other cities.
To save time and money, Jaime encourages his employees to search the routes that allow them to receive and deliver boxes in the same trip. The employees do it, but at the way they want. Employees communicate a lot them and they talk about the problems they face delivering boxes in certain cities; that's why some employees have decided to avoid going to some cities during their trips unless it is strictly necessary to go (they have to receive or deliver boxes in that city), this means if an employee does not want to go through city x, when delivering boxes from city a to city b, he will not use any path that goes through x in the delivery unless $a = x$ or $b = x$.
Jaime is not happy with this, he is a bussiness man now and he sees that when an employee avoids cities in the trip, they usually take more time for the trip which means also more cost for the company to operate, however, since the employees are efficient and always follow the shortest path to do the delivery even when they will not go through certain cities, he decided he prefers to change the employees delivery schedule instead of the way they work.
Each morning, Jaime assigns to each of his employees a list of deliveries the employee has to do, in order to make Jaime's customers happy, it is important that the employee performs the deliveries in the order Jaime gave them. The delivery trucks has capacity only to carry the packages for one delivery, so the employees always reload the truck before starting a new delivery.
Jaime wants to know the total time each of his employees will need to complete all the deliveries in the schedule. He has given you a map that lists the roads that can be used to do the delivery and the time it takes for an employee to drive that road. He also gave you the list of cities each employee avoids, Jaime has sorted this list in such way that if the j-th employee does not want to go to city c, then all employees listed after him will also avoid going through city c. He also provided you with the delivery schedule of each employee.

Help Jaime find the time each employee will take to complete the assigned delivery schedule. For this, consider the time it takes to the employee to go from the starting city G to the city where the employee receives the boxes of the first delivery, the time it takes for the employee in each delivery to go from the city where the boxes were received to the city where the boxes should be delivered, the time it takes to go from the city where boxes were delivered in the previous delivery to the city where the boxes needs to be received for the next delivery, and the time it takes to go from the city where boxes are delivered in the last delivery to the starting city G.

Input

The first line of the input contains an integer number N ($1 \leq N \leq 200$), the number of cities in the map, the next line contains an integer number M ($1 \leq M \leq N^2$), the number of roads in the map. The next M lines contain three integer numbers each a_i, b_i, t_i ($1 \leq a_i, b_i \leq N, 1 \leq t_i \leq 10^8$), representing the i-th road connects cities a_i and b_i, which can be traversed in any direction and that takes t_i time to drive. The next line contains an integer number E ($1 \leq E \leq 10^4$), the number of employees in Jaime's company. Each of the following E lines start with a number x_j, representing the number of cities the j-th employee and all employes listed after him will avoid, followed by x_j integer numbers representing the x_j cities the $j - th$ employee and all employees listed after him will avoid in their trips unless it is strictly necessary to go. Each of the next E lines describes the schedule for an employee, each line contains an even number of integers separated by a space, each pair of consecutive integers represent a delivery job for the employee, where the first city of the delivery job represent the city where the employee needs to receive the boxes for that delivery and the second city represents the city where the employee needs to deliver the boxes, there will be at most 500 jobs in an employee delivery schedule. The last line of input contains an integer G, the city where all employees start the delivery day.

Output

For each employee in the input, print a line with a single integer, representing the total time it takes for the employee to complete his delivery schedule. If it is not possible for an employee to complete the schedule, print a line with the text "Assign a better schedule".

Sample input 1	**Sample output 1**
7 7 1 2 3 1 3 7 2 3 2 2 5 1 1 4 2 4 6 1 1 7 1 3 1 2 1 6 1 7 1 3 1 5 7 6 4 1 1	14 Assign a better schedule 4

Sample input 2	**Sample output 2**
4 4 1 4 10 1 3 7 4 3 2 3 2 1 2 1 2 1 3 1 4 1 4 1	18 20

Problem E – Exponential points game.

Adapted by: Juan Pablo Marín

The exponential points game is a two person board game. The board consists of N tiles numbered from 1 to N arranged over a straight line. Before the game begins, both players agree in a set of K tiles to be marked so that none of the players will take any of those tiles in the game.
The two players will alternate turns. On each turn, the player marks 1 or more unmarked tiles, but if the player marks more than one tile, all tiles marked in the turn should share a side (tile i is adjacent to tile $i-1$ and to tile $i+1$). In turn k, if the player marked m_k tiles then he scores $m_k \times 2^{k-1}$ points. The game continues until all the boxes are marked. As you can guess, the player with higher score wins the game.
Santiago is a fan of this game so he has challenged you to play. Since he is more experienced in the game, he says he will take the first turn, and also, he won't take any points from that turn to give you a chance to win. Then in this game, the first turn, Santiago will get $m_1 \times 0$ points, in the second turn you get $m_2 \times 1$ points, the third turn Santiago gets $m_3 \times 2$ points, the fourth turn you get $m_4 \times 4$ points, etc.
What Santiago does not know is that you are an expert in the game as well, so, you suspect Santiago is not giving a chance to win taking the first turn, instead, he is ensuring you will lose. You know both of you will play the game optimally, can you determine the minimum number of tiles Santiago has to mark in the first turn so that you don't have a chance to win?

Input

The first line of the input contains two numbers separated by a space N ($1 \leq N \leq 1000$) and K ($0 \leq K \leq N$), representing the number of tiles in the board and the number of tiles marked before the game starts, respectively. The following K lines contain each a number between 1 and N, inclusive representing the tiles that are marked before the game starts. No tile is marked twice in the input.

Output

Output a single line with an integer indicating the minimum number of tiles to take in the first turn so that Santiago wins the game. If no such number exists, print -1.

Sample input 1	**Sample output 1**
`4 0`	`2`

Sample input 2	**Sample output 2**

`5 3`
`1`
`2`
`3`
	`-1`

Sample input 3	**Sample output 3**

`8 3`
`1`
`5`
`7`
	`1`

Problem F – Forgotten PIN number.

Author: Saraí Ramírez

Febo is a person so distracted that he forgot the PIN of his new credit card a few hours later he received it. The only thing he remembers is all the digits in the PIN are distinct. To create a new PIN, he must follow some procedures in the bank: An executive helps him get the forgotten PIN back and, for obvious security reasons, this has to be changed.
Since Febo knows himself and his terrible memory, decided his new PIN will keep exactly the same digits used in the last one, with this condition: Two characters that were together in the forgotten PIN must not be side by side (in the same order) again.
The bank is so sophisticated that to make their PINs more secure, they use a numeric system with 10^7 digits for their PINs, nevertheless, we were not able to print them and instead, we identify them with integer numbers from 1 to 10^7.
Given the forgotten PIN, how many options (with the same digits in Febo's forgotten PIN) are excluded for creating the new one?

Input

The first line of input contains a single integer number T such that $1 \leq T \leq 100$, and denotes the number of cases you will receive. Each of the next T lines contains a forgotten PIN, represented as a list of numbers p_i separated by space ($1 \leq p_i \leq 10^7$), where p_i is the i^{th} digit in Febo's forgotten PIN.

Output

For each test case in the input, print a line with a single integer, representing the number of invalid PINs that can be generated with the given forgotten PIN. Since the answer could be very big, print it modulo $10^9 + 7$.

Sample input 1	**Sample output 1**
3 1 2 3 3 8 11 4 25 40 400 30 1 4 1000000	3 13 2921

Problem G – Gato currency converter.

Author: Moroni Silverio

Jaime's country has been invited to join a new group of countries which was recently formed (and they have accepted to join them). They call themselves the "International Competitive Programming Countries" (ICPC). This group has brought many new things to Jaime's country, among them, a new currency called gato.
Some people have had difficulties adopting this new currency, including Jaime. He must change all the prices of his products in his little shop. The thing is, he is not that good at currency conversion.
It is a well known fact that five pesos (peso is Jaime's country old currency) equal one gato, or at least that's what Jaime remembers. However, he does not know how to convert quantities less than 5 pesos to gato. That's why he has decided that, after converting a price to gato, according to the rule he remembers, if there are pesos remaining, he will convert them to one gato, and he will continue doing so until he remembers how to do conversion between pesos and gato.
Help Jaime determine the new prices of some of the products he is selling in his shop.

Input

The first line of the input contains an integer C ($1 \leq C \leq 10^5$), representing the number of products in Jaime's shopping mall (yes, it may not be that "little"). The second line contains C integers separated by a space C_i ($1 \leq C_i \leq 10^6$, $1 \leq i \leq C$), representing the prices in pesos of the products Jaime's shopping mall has for sale. The next line contains an integer Q ($1 \leq Q \leq 10^5$), representing the number of queries Jaime is about to ask concerning his products price conversion. Each of the last Q lines contain a single integer number q_j ($1 \leq q_j \leq C$), representing that Jaime needs your help with the conversion of the price of the q_j^{th} product to gato currency.

Output

For each query in the input, output a line containing the price in gato of the product in Jaime's store queried.

Sample input 1	**Sample output 1**
4 1 5 6 25 4 1 4 3 2	1 5 2 1

Problem H – Hidden number.

Author: María Celeste Ramírez

As you know, Santiago is an expert summing numbers, this time he is summing numbers from a list L of N positive integer numbers, not necessarily different. He likes to perform sums so much that he selects an integer X and looks for a subsequence of S from L such that the sum of the elements of S equals X. For example, if the list is $(10, 3, 1, 2, 2, 4)$, and $X = 14$, then, Santiago can take the subsequences $(10, 3, 1)$, $(10, 4)$, or $(10, 2, 2)$ since $10 + 3 + 1 = 10 + 4 = 10 + 2 + 2 = 14$.
Santiago has noticed there may be some values for X for which he can choose multiple different subsequences S, however, you have pointed out that there also exist some values for X for which no possible subsequence S exists. Santiago does not believe you, thats why he wants you to find the smallest possible value for X for which no subsequence S exists in his list.

Input

The first line of the input contains an integer N ($1 \leq N \leq 10^6$), representing the number of elements in the list. The second and last line in the input contains N numbers separated by a space between, the numbers in the list, each number will have a value between 1 and 10^6.

Output

Output a single line with an integer indicating the smallest positive integer value for X such that Santiago can not find a subsequence S.

Sample input 1	**Sample output 1**
2 1 1	3

Sample input 2	**Sample output 2**
5 3 2 5 4 10	1

Problem I – ICPC training.

Author: Juan Pablo Marín

Training for the ICPC is hard, but you really want to go to the world finals that is why this vacations you will train as hard as you can. You have identified that each day you train hard you get t units of tiredness, however, if you rest a day your tiredness is reduced in r units but its never below 0, this is, if your current tiredness is 3 and $r = 4$ and you decided to rest this day then your tiredness will be 0. Tiredness is accumulative so if you train for d consecutive days your tiredness will increase in $d * t$ units. The same happens when you rest, if you rest for d consecutive days your tiredness will decrease in $d * r$ units.

You are planning your training for this vacations, in the plan you will decide for each of the N days in your vacations if you train or rest, also, you don't want to have more than T units of tiredness in any moment of the vacations, and want to train in exactly M days (these M days don't need to be consecutive).

As you are in vacations you decided your optimal training plan is that in which you train the M days in the less number of days possible from your vacations, so you can take the rest of the days to enjoy with your family. Can you find the number of days in your optimal plan?

Input

The input consists of a single line that contains five integer numbers separated by a space N ($1 \leq N \leq 10^6$), M ($1 \leq M \leq N$), T, t and r ($1 \leq T, t, r \leq 1000$), representing the number of days in your vacations, the minimum numbers of days you want to train, the maximum tiredness you want to have in any moment in your vacations, the tiredness that you get if you train a day, and the tiredness you lose if you rest a day respectively.

Output

Output a single line with an integer indicating the minimum number of days in the plan so that you have trained M days, if there is no plan where you can train M days then print -1.

Sample input 1	**Sample output 1**
`10 5 50 25 15`	`10`

Sample input 2	**Sample output 2**
`5 1 10 11 5`	`-1`

Problem J – Jaime's multiplications.

Author: Moroni Silverio

One reason why Jaime loves so much small prime numbers, those less than 30, is that he can perform operations over them with his mind without taking much effort.
This evening, Jaime has created a list of N positive integer numbers multiplying only the prime numbers he loves, and all the multiplications were done with his mind. After a quick look to the clock, Jaime determined it was time to take a rest. Jaime went to bed to take a nap, but he was unable to fall asleep, something is bothering his mind. He can not stop thinking in the list of numbers he created, he believes he can take a subsequence of the list such that when multiplying all the integer numbers of the subsequence, he will get a perfect square number.
Jaime is really tired, so you decided to help him find such subsequence, however, you know the next thing will bother Jaime is he will want to find the longest possible subsequence, so you want to find that one before Jaime asks for it, that way he will be able to have a good rest.
Given the list of numbers Jaime created, find the size of the longest subsequence from Jaime's list such that if you multiply all the elements of the subsequence, the resulting integer number is a perfect square number.

Input

The first line of input contains a single integer N ($1 \leq N \leq 10^5$), the size of the list Jaime created. The second and last line in the input contains N integer numbers separated by space, representing each of the x_i numbers in Jaime's list ($2 \leq x_i \leq 10^{18}$). It is guaranteed that each of the numbers in Jaime's list are generated multiplying only prime numbers less than 30.

Output

Output a single line with an integer indicating the size of the longest subsequence whose product is a perfect square number.

Sample input 1	**Sample output 1**
3 12 3 7	2

Problem K – Keep it healthy.

Author: Moroni Silverio

Jaime loves cakes. If it was possible, he would eat cake every day, nevertheless he cares about his health status and that is why he doesn not eat cake every single day. However, when he has the oportunity (he is invited to a birthday party or a wedding), he eats as much as he can.
Today is a special ocassion and Jaime found a big rectangular shaped cake with a grid drawn with buttercream frosting with r rows, and c columns, which divide the cake into $r \times c$ square sections. Each position in the cake may have some raisins. Jaime's doctor advised him that he must eat only a piece of cake, no more.
The doctor never specified any size for the piece of cake Jaime can eat, so he could eat the whole cake, but, there is a little problem: Jaime does not like raisins (who does?), that is why he wants to eat the biggest piece of cake that does not contain raisins on it. Jaime will decide the piece of cake using the following conditions:

- It must have rectangular shape.
- it must contain complete sections of the grid.
- It must not contain any raisins.
- It must have the biggest perimeter among the slices with no raisins.

Eating this piece of cake, Jaime will trick his body to think he is eating a lot of cake, and at the same time, he will be taking care of his health. Help Jaime find his piece of cake.

Input

The first line of the input contains two integers r, c ($1 \leq r, c \leq 300$), representing the height, and width of the cake, respectively. Each of the next r lines have c numbers separated by a space, representing the number p_{ij} of raisins the cake has in that position ($0 \leq p_{ij} \leq 9$).

Output

Output a single line with an integer, representing the perimeter of the piece of cake Jaime will eat.

Sample input 1	**Sample output 1**
4 5 5 0 3 1 1 0 0 0 9 1 0 0 0 1 0 3 2 9 1 0	10

Problem L – Land distribution.

Author: Félix Arreola

As you may know, some country groups like 'ICPC' hand over his territories to the best programmers in the world. For every problem solved, they award one square of land. Every programmer in the ICPC group solves easily two problems. If they do not do it, they are expelled from the group.

The ICPC land has a rectangular shape with W width and H height, and is divided into $W \times H$ square territories distributed among the N programmers of the group. Each programmer has his own house that already occupies one square of land and the house is immovable.

The results from the last contest were just published, and the land needs to be redistributed based on these results. The rules to reorder the land are simple:

- The land owned by a programmer must have a rectangular shape.
- The land owned by a programmer must contain his home.
- The area of the land owned by a programmer must be equal to the number of problems the programmer solved in the contest.
- A territory of the land must belong to only one programmer.

As the ICPC head of territory distribution, your task is to find a way to divide the land for the N programmers in the group, following the mentioned rules.

Input

The first line of input contains two integers separated by a space W, H ($1 \leq W, H \leq 12$), representing the width and height of the land to be distributed. The next line contains an integer N ($1 \leq N \leq 24$), the number of programmers in the group. Each of the next N lines contains four integer numbers separated by a space C_i, X_i, Y_i, S_i, where C_i is an upper case letter from the english alphabet that identifies programmer i, X_i and Y_i are the coordinates for the i-th programmer house ($1 \leq X_i \leq W$, $1 \leq Y_i \leq H$), and S_i represents the number of problems solved by the i-th programmer ($2 \leq S_i \leq 20$).

Output

For each test case in the input, print H lines with W characters each, representing the distribution of the land, the character in the i-th line and j-th column should be the upper case letter that identifies the programmer who owns that territory. If you can not distribute the lands according by the rules, print a line with the string "NO MAP". It is guaranteed that if a solution exists, it is unique.

Sample input 1	**Sample output 1**
5 5 4 A 1 1 9 B 1 5 2 C 5 1 6 D 5 5 8	AAABB AAADD AAADD CCCDD CCCDD