The Optimal Design of State-Run Lotteries

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Abstract

People have long debated whether state-run lotteries exploit the poor or are a win-win that generates enjoyment and government revenues. We study socially optimal lottery design in an optimal taxation framework with biased consumers and estimate sufficient statistics for optimal policy. Lottery sales respond more to changes in jackpot expected values than to changes in price or lower prizes, consistent with a specific type of probability weighting. In our survey, bias proxies such as innumeracy decline with income and explain 43 percent of lottery spending. In our model, current multi-state lottery designs increase welfare but may harm heavy-spending low-income people.

JEL Codes: D12, D61, D91, H21, H42, H71.

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"[A lottery] preys upon the hard earnings of the poor; it plunders the ignorant and simple."

– U.S. Supreme Court, in Phalen v. Virginia (1850)

"In our stressful world, the ability to dream is well worth the price of a lottery ticket ... The lottery is simply a form of entertainment that happens to benefit your state."

– National Association of State and Provincial Lotteries (2021b)

"Since this regressive, addictive, partially hidden tax is here to stay, might a little improvement still be conceivable? ... Here's a modest suggestion: States should consider reducing their skim of the wagers."

– Purdue University president and former Indiana governor Mitch Daniels (2019)

People have long debated whether states should run lotteries. Opponents argue that lotteries are a regressive tax on people who are bad at math. Proponents argue that lotteries are a win-win, generating both consumer surplus and government revenues. If states do run lotteries, there are further debates, such as the optimal "implicit tax"—the share of revenues that is allocated to the government instead of returned to prize winners. Economists are divided: a recent survey of the University of Chicago IGM experts panel found that 23 percent of leading economists believe that state-run lotteries increase social welfare, 28 percent disagree, and 45 percent were uncertain or had no opinion.[1]

These debates matter. Americans spent a remarkable $679 per household (or $87 billion in total) on lottery tickets in 2019, generating $25 billion in state government revenues. This was more than the revenue raised by federal estate or tobacco taxes, and just less than the revenue raised by the federal gas tax (Internal Revenue Service 2021). Americans spend more on lottery tickets than they do on cigarettes, and more than they do on music, sports tickets, movie tickets, books, and video games combined (Isidore 2015).

Embedded within these debates is a series of fundamental empirical and theoretical questions. How much of lottery consumption is driven by innumeracy or other behavioral biases, as opposed to entertainment and other normatively respectable preferences? Do lower-income people really spend more on lotteries, and is this good (because it reflects consumer surplus for people with higher marginal utility) or bad (because it reflects exploitation of behavioral biases)? How should states design lotteries to maximize welfare, accounting for consumer surplus, possible behavioral biases, concerns about regressivity, and the value of public funds?

This paper considers state-run lotteries in an optimal taxation framework, extended to allow possible behavioral biases. We present new empirical data and analyses that identify a set of parameters that determine optimal policy. We then use the empirical parameters to simulate the welfare effects of lotteries and different design choices. In doing so, we provide the first welfare analysis of state-run lotteries that theoretically and quantitatively considers the alleged behavioral biases at the heart of the policy discussion.

[1] See www.igmchicago.org/surveys/state-run-lotteries.

In our theoretical model, heterogeneous individuals choose labor supply, lottery purchases, and numeraire good consumption to maximize their perceived utility. In the model, a rational consumer might play the lottery because of entertainment value, anticipatory utility, the consumption utility from spending possible prize money, or other reasons. However, lottery consumption might be affected by behavioral biases, such as misperceiving small probabilities, overconfidence, and self-control problems, which the policymaker views as driving a wedge between individuals' perceived utility and "normative utility." The policymaker sets a nonlinear income tax and the lottery's price and attributes, such as prize amounts and probabilities, to maximize normative utility subject to a revenue-raising constraint. The planner is inequality averse, placing higher welfare weights on people with lower earning ability.

Our model can be applied to goods other than lotteries. It generalizes the "optimal sin tax" model of Allcott, Lockwood, and Taubinsky (2019) to settings where the government can also regulate or directly control a good's attributes—for example, cigarette nicotine content or lightbulb energy efficiency. In our setting, the policymaker has multiple instruments that determine the implicit tax on lotteries, including raising the price, reducing the jackpot or other prizes, and reducing the prize probabilities. Optimal policy takes into account how these instruments can both *counteract* bias (the usual corrective taxation logic) and *affect* bias—for example, if a change in jackpot probability affects the misperception of that probability.

We derive formulas that characterize optimal lottery prices and attributes as functions of several sufficient statistics. Stronger behavioral biases imply a higher implicit tax, i.e. higher prices and/or lower payouts to winners. Relatively more behavioral bias among lower-income people implies a higher implicit tax, because the corrective benefits of the implicit tax accrue to the poor. However, relatively more consumption among lower-income people implies a lower implicit tax, because the mechanical utility losses from taxation accrue to the poor. The relative importance of the corrective benefits versus the mechanical losses depends on the demand slope: more responsive demand means that the corrective benefits of demand changes outweigh the mechanical losses, whereas less responsive demand means that the corrective benefits will be relatively small. Importantly, different means of adjusting the implicit tax affect different types of individuals. Optimal multidimensional policy thus involves striking a balance between instruments best targeted to counteract bias and instruments with the best distributional properties.

We then estimate the sufficient statistics for optimal policy. To estimate the aggregate semi-elasticities of demand with respect to prizes and price, we exploit natural experiments built into the two large multi-state lotteries, Mega Millions and Powerball. When nobody wins the jackpot, the jackpot prize money is "rolled over" to the next drawing. Since the winning numbers are randomly selected, rollovers are random conditional on ticket sales, generating conditionally random variation in the jackpot over time. In California, lower prizes also roll over if they are not won, and the second prize is unlikely enough that it rolls over regularly. We identify the effect of ticket prices on demand from event studies when Mega Millions and Powerball separately increased their prices from $1 to $2.

The semi-elasticities of demand with respect to the jackpot expected value, second prize expected value, and price are about 1.0, statistically zero, and -0.5, respectively. Strikingly, this means that ticket sales increase twice as much if the jackpot expected value rises by \$1 than if the price decreases by \$1. This is the opposite of what would be expected for risk-averse consumers, and it is not explained by substitution across games or time. Of course, this is qualitatively consistent with the long literature on probability weighting.[2] However, the jackpot elasticity is so large relative to the second prize elasticity that it cannot be fit by three standard functional forms from the literature (Goldstein and Einhorn 1987; Tversky and Kahneman 1992; Prelec 1998), although it can be fit by the neo-additive form formalized by Chateauneuf, Eichberger, and Grant (2007).

We estimate additional key statistics using a new nationally-representative survey on the AmeriSpeak panel, a high-quality probability-based sample that includes households that might not participate in cheaper opt-in surveys. The spending distribution is highly skewed, generating some imprecision in the estimated means, but point estimates suggest that lottery spending declines moderately with income. People with household income under \$50,000 spend 29 percent more on the lottery than people with household income above \$100,000.

The survey provides novel evidence on the relationship between lottery spending and proxies for behavioral bias. Measures of perceived self-control problems, financial illiteracy, statistical mistakes (such as the Gambler's Fallacy, non-belief in the Law of Large Numbers, and difficulty calculating expected values), and incorrect beliefs about expected returns from lottery play are highly statistically significantly associated with more lottery spending, even after controlling for demographics, risk aversion, and questions measuring how much people enjoy playing the lottery. Interestingly, not all of these relationships suggest that bias increases consumption: the average person actually underestimates the share of lottery revenues that is returned to winners. We measure and correct for imperfect test-retest reliability by resampling the same survey respondents one year later, building on other resampling designs such as Beauchamp, Cesarini, and Johannesson (2017), Gillen, Snowberg, and Yariv (2019), Chapman et al. (2020), and Stango and Zinman (2021).

Regression predictions suggest that Americans would spend 43 percent less on lotteries if they had perfect self control, had the financial literacy and statistical ability of the highest-scoring people in our sample, and had correct beliefs about expected returns. Lower-income people score lower on financial literacy and statistical ability, so the point estimates suggest that a larger share of their spending is explained by bias. Although we control for a rich array of demographics and preference measures, a key caveat is that these regression predictions are not the causal effects of behavioral biases. With that important caveat, these results are consistent with concerns that behavioral biases play a role in lottery spending.

In the last part of our paper, we use the empirical moments to calibrate a structural model of lottery demand and to study optimal lottery design. Empirical implementations of our sufficient statistics formulas and our fully structural model both suggest that the optimal implicit tax rate on

[2] See, for example, Kahneman and Tversky (1979), Tversky and Kahneman (1992), Prelec (1998), Gonzalez and Wu (1999), Wakker (2010), Filiz-Ozbay et al. (2015), and Bernheim and Sprenger (2020).

lotteries is slightly higher than the current Mega Millions and Powerball designs. This suggests that recent trends in those two games toward higher implicit taxes may have increased welfare. In our model, actual consumer surplus is much smaller than consumers' perceived surplus at our baseline behavioral bias estimates, but the current Mega Millions and Powerball designs increase welfare and deliver close to maximal welfare levels. The gains are unequally distributed: below-median spenders benefit primarily from redistributed government revenues, while low-income people with above-median spending receive approximately zero or possibly negative net surplus (even accounting for their share of redistributed revenues), because they are more biased. These results hinge on the magnitude of behavioral bias, highlighting the importance of the evidence from our survey. For example, if we alternatively assume zero bias, then low-income heavy spenders derive the most surplus. If bias is twice as large as our baseline estimate, lotteries would reduce overall welfare.

Our paper contributes a series of new reduced-form results to the literature on lottery demand and is the first, to our knowledge, to collect the full set of empirical moments necessary to calibrate a structural model of lottery demand with behavioral bias. To our knowledge, our paper is also the first to formalize and empirically implement a framework for studying optimal lottery policy design with behavioral bias and redistributive concerns.

We build on four distinguished literatures. The first is a reduced-form empirical literature on state-run lotteries; see Clotfelter and Cook (1989, 1990), Kearney (2005a), and Grote and Matheson (2011) for overviews. Within that literature, one set of papers studies how lottery spending varies by income and other demographics.[3] While lotteries have changed substantially in the past two decades, to our knowledge our AmeriSpeak survey is the first to measure individual-level lottery spending using a nationwide probability sample since Clotfelter et al. (1999).[4] A second set of papers studies aggregate demand patterns and substitution across games.[5] We extend that work by (i) using instrumental variables to address simultaneity bias in the relationship between jackpots and sales, (ii) using new data to estimate the elasticity with respect to the second prize, which we find to be very different than the jackpot elasticity, and (iii) exploiting recent Mega Millions and Powerball price changes to estimate the price elasticity of demand for those games. Both (ii) and (iii) are necessary for structural estimates of the probability weighting function.

The second literature we build on studies behavioral biases that might affect gambling and lottery demand.[6] While much of this literature studies one or two biases in isolation, our survey extends this literature by offering a more comprehensive measure of the many different biases that could affect lottery demand. The third literature studies the use of lotteries to encourage beneficial behaviors such as saving money (Kearney et al. 2011), charitable giving (Landry et al. 2006), and

[3] See, for example, Clotfelter and Cook (1987), Clotfelter et al. (1999), Farrell and Walker (1999), Price and Novak (1999, 2000), and Oster (2004).

[4] Lottery spending is "drastically underreported" in the Consumer Expenditure Survey (Kearney 2005b).

[5] See, for example, Clotfelter and Cook (1989), Cook and Clotfelter (1993), Farrell, Morgenroth, and Walker (1999), Farrell et al. (2000), Kearney (2005b), Grote and Matheson (2006), and Guryan and Kearney (2010).

[6] Studies outside of laboratory settings include Clotfelter and Cook (1993), Haisley, Mostafa, and Loewenstein (2008), Guryan and Kearney (2008), Post et al. (2008), Snowberg and Wolfers (2010), and Suetens, Galbo-Jørgensen, and Tyran (2016). A separate large literature, e.g. Kahneman and Tversky (1979) and subsequent work, studies related biases in laboratory settings.

completing health assessments (Haisley et al. 2012). These behavior change-focused lotteries are distinct from the state-run lotteries that we study. The fourth literature is the work in *behavioral public economics* studying other settings where behavioral biases affect optimal policy design.[7]

Sections 1–6 present the background, model, data, reduced-form empirical results, model calibration, and optimal policy simulations, respectively.

1 Background on Lotteries in the United States

From 1995 to 2019, real lottery spending grew from $540 per household ($53 billion in total) to $679 per household ($87 billion in total); see Appendix Figure A1. There are two major types of lottery games: instant (or "scratch-off") games and draw games, where players choose numbers and win if their numbers match those selected in the next drawing. The largest draw games are two multi-state lotteries, Mega Millions and Powerball. From 1995 to 2019, instant games grew from 38 to 65 percent of sales, Mega Millions and Powerball grew from 3 to 9 percent, and all other games dropped from 59 to 26 percent.

Of the $87 billion in 2019 sales, 60 percent was returned in prizes, 10 percent was overhead (6.5 percent commissions and 3.9 percent administrative costs), and the remaining 29 percent represented state government proceeds; see Appendix Figure A2. This 29 percent implicit tax has decreased from 34 percent since 1995, a trend that is associated with the growing market share of instant games, which return a larger share to winners. Lottery prizes are taxed as income, so an additional share of prize money returns to governments through the income tax.

Our aggregate demand estimation focuses on Mega Millions and Powerball from June 2010 through February 2020. In Mega Millions, players select five numbers from 1 to 70 and one Mega Ball number from 1 to 25; this is the "5/70 + 1/25 format." Powerball uses a 5/69 + 1/26 format. Players win the jackpot if their numbers match all six balls selected in the next semi-weekly drawing; players can also win lower prizes from $2 to $1,000,000 by choosing one or more numbers correctly. Table 1 presents ticket price and prize information for the formats in place during our sample period. For both games, tickets now cost $2, and the jackpot odds are about 1/300,000,000. The ratios of ticket expected value to price are 0.41 and 0.45, so accounting for 10 percent overhead gives implicit tax rates of 49 and 45 percent, respectively.

Jackpot amounts are determined on a parimutuel basis: the jackpot prize pool depends on ticket sales, and the jackpot is split equally among all winners. Jackpot winners can choose to receive 30 annual installments that increase by five percent per year or the discounted present value of that annuity at current interest rates; most choose the latter. Before each drawing, Mega Millions and Powerball advertise an "estimated jackpot," which is the undiscounted value of the annuity

[7]See, e.g., Bernheim and Rangel (2004, 2009), Mullainathan, Schwartzstein, and Congdon (2012), Allcott, Mullainathan, and Taubinsky (2014), Allcott and Taubinsky (2015), Baicker, Mullainathan, and Schwartzstein (2015), Bernheim, Fradkin, and Popov (2015), Handel and Kolstad (2015), Ambuehl, Bernheim, and Lusardi (2017), Spinnewijn (2017), Taubinsky and Rees-Jones (2018), Allcott, Lockwood, and Taubinsky (2019), Handel, Kolstad, and Spinnewijn (2019), Beshears et al. (2020), Farhi and Gabaix (2020), Goldin and Reck (2020), and Rees-Jones and Taubinsky (2020); see Bernheim and Taubinsky (2018) for a review.

based on projected ticket sales. If no one wins the jackpot, the jackpot prize pool is rolled over to the next drawing. If someone wins the jackpot, the next drawing's jackpot returns to a reset value, which was $40 million for both Mega Millions and Powerball as of the end of our sample in February 2020.[8]

In most states, the lower prize amounts are fixed. However, the California Supreme Court ruled in 1996 that the California State Lottery Act allows parimutuel games where players play against other players, but not games where players play against the house. As a result, both Mega Millions and Powerball have California-specific parimutuel prize pools for each lower prize that roll over to the next draw if they are not won. The second prize odds are about 1/12,000,000 for both games, so there are many drawings when no one wins the second prize and it thus rolls over. The third and lower prizes have high enough odds that they generally don't roll over.

Powerball began in 1992 with 15 states, and Mega Millions began in 2002 with 9 different states. Both games replaced earlier multi-state games with different names, and both have gradually added states over time. After a cross-selling agreement was reached, individual states began to offer both games in 2010. By June 2010, 42 states had joined Mega Millions and 41 had joined Powerball. From September 2014 through the end of our sample, both games were available through all 44 state lotteries and the D.C. lottery.[9]

Mega Millions and Powerball have both adjusted their pricing and formats several times since 2010, as shown in Table 1. There are five key trends. First, both games increased prices from $1 to $2. Second, the jackpot odds have been reduced, increasing rollovers: the jackpot was won every 7.4 drawings in 2010–2011 but only every 15.4 drawings in 2018–2019. Third, the prize structure is being hollowed out: there is less expected value in the middle prizes and more in the jackpot and lowest prizes. Fourth, as a result of the previous two trends, average jackpots have grown: the average jackpot was $63 million in 2010–2011 and $174 million in 2018–2019. Fifth, the ratio of expected value to price is decreasing, i.e. winners receive a smaller share of the revenues. This pushes against the overall trend of lower implicit taxes described above.

2 Theoretical Framework

In this section, we extend a standard static optimal taxation model to the case where a government controls both the price and attributes of a good. In our application, this good is the lottery, but the model could also apply to other goods whose attributes are directly or indirectly controlled by the government, such as cigarettes, alcohol, energy-using appliances, museums, public media, and health insurance. We make a number of simplifying assumptions for the body of the paper, but we relax these in Appendix C, and provide a summary in Section 2.2.4.

[8]In March 2020, lottery sales dropped substantially due to the coronavirus pandemic, and both games temporarily lowered their reset values to $20 million at the beginning of April. We say "current design" to refer to the design in place just before the pandemic.

[9]Alabama, Alaska, Hawaii, Nevada, and Utah do not have state lotteries. Mississippi established its lottery at the end of 2019 and joined Mega Millions and Powerball in January 2020.

2.1 A Static Model of Lottery Demand

Individuals choose labor supply (and thus earnings z), numeraire consumption c, and whether to buy a lottery ticket $x_t \in \{0,1\}$ in each of many choice occasions indexed by t. We disaggregate demand into a series of binary decisions on different choice occasions because probability weighting (Kahneman and Tversky 1979; Tversky and Kahneman 1992) most plausibly applies at the level of each individual ticket, rather than at the level of an aggregate portfolio. This is consistent with the evidence on narrow bracketing (Tversky and Kahneman 1981; Rabin and Weizsacker 2009), particularly as it applies to lottery tickets (Haisley, Mostafa, and Loewenstein 2008). To simplify exposition in the body of the paper, we assume that there is a single choice occasion; the more general case with multiple choice occasions is characterized in Appendix C and produces identical formulas under the assumption of no income effects on lottery expenditures, which we maintain in the body of the paper.

Earnings are subject to a nonlinear income tax $T(z)$. The price of the numeraire is normalized to one, and the price of a lottery ticket is p. We define $\boldsymbol{a}$ as a vector of lottery attributes, such as prize amounts, probabilities, and advertising.

Individuals have heterogeneous types θ capturing income-earning ability, preferences, and behavioral biases. On each choice occasion, individuals receive a lottery taste shock ε drawn from a continuous distribution. We think of ε as representing the transaction cost of buying a lottery ticket. We let $F(\theta, \varepsilon)$ denote the joint distribution over types, and we let F_θ denote the marginal distribution over θ. We define $u(\boldsymbol{a}; \theta, \varepsilon)$ as the perceived subutility from purchasing a lottery ticket, and we define $\psi(z;\theta)$ as the labor disutility of generating earnings z. Individuals maximize "perceived utility"

$$U = G(c + x \cdot u(\boldsymbol{a}; \theta, \varepsilon) - \psi(z;\theta)), \tag{1}$$

subject to budget constraint $z - T(z) = c + px$.

Individuals buy a lottery ticket if the utility exceeds the price: $u(\boldsymbol{a}; \theta, \varepsilon) > p$. The type-$\theta$ demand function is then

$$s(p, \boldsymbol{a}; \theta) := \Pr\left(u(\boldsymbol{a}; \theta, \varepsilon) > p\right). \tag{2}$$

We define $\bar{s}(p, \boldsymbol{a}) := \int s(p, \boldsymbol{a}; \theta) dF_\theta(\theta)$ as population-level aggregate demand. When no ambiguity arises, we sometimes suppress arguments in the demand functions.

The function u can reflect any number of possible motives for playing the lottery in addition to the expected utility from possible prizes. For example, it can include any entertainment derived from playing the lottery (Conlisk 1993; Kearney 2005b), or anticipatory utility from thinking about a chance of winning (e.g., Loewenstein 1987; Caplin and Leahy 2001; Brunnermeier and Parker 2005; Gottlieb 2014). We also allow for perceptual distortions, such as over- or under-estimating the likelihood of winning or imperfect processing of small probabilities (Woodford 2012; Steiner and Stewart 2016), or biases induced by salience or focusing effects (Bordalo, Gennaioli, and Shleifer 2013; Koszegi and Szeidl 2013; Bushong, Schwartzstein, and Rabin 2021). Probability weighting could be generated either by normatively relevant preferences such as anticipatory utility, or by perceptual distortions. To formalize the possibility of mistakes, we draw a distinction between perceived utility, which individuals maximize, and "normative utility," which enters the planner's objective function:

$$V = G(c + x \cdot v(\boldsymbol{a}; \theta, \varepsilon) - \psi(z; \theta)). \tag{3}$$

We define $s^V(p, \boldsymbol{a}; \theta) := \Pr(v(\boldsymbol{a}; \theta, \varepsilon) > p)$ as the type-θ demand function that would obtain if individuals maximized normative utility.

Bias is the wedge between perceived utility and normative utility from lottery consumption: $\gamma(\boldsymbol{a}; \theta, \varepsilon) := u(\boldsymbol{a}; \theta, \varepsilon) - v(\boldsymbol{a}; \theta, \varepsilon)$. Under the assumption that ε is additively separable, so that γ does not depend on ε, γ is equal to the price reduction that produces the same change in demand as bias does, so $s^V(p - \gamma, \boldsymbol{a}; \theta) = s(p, \boldsymbol{a}; \theta)$. This representation of bias mirrors Allcott, Lockwood, and Taubinsky (2019, henceforth "ALT").

We define $C(\boldsymbol{a}, \bar{s})$ as the cost to the government of selling $\bar{s}$ tickets with attributes $\boldsymbol{a}$; this includes the expected prize payout plus administration, marketing, and any other costs. We refer to the percent markup above average cost, $\frac{p\bar{s} - C(\boldsymbol{a}, \bar{s})}{p\bar{s}}$, as the "implicit tax rate."

Two features of the utility specification in equation (1), separability and quasilinearity in numeraire consumption, imply that financial windfalls do not affect either lottery expenditures or labor supply. The negligible labor supply income effects assumption is supported by Gruber and Saez (2002) and Saez, Slemrod, and Giertz (2012).[10] The more general model in Appendix C allows for income effects on lottery consumption, and we discuss the implications of that below.

2.1.1 Examples

Suppose that the lottery offers a single large prize of amount w with a probability π, so that $\boldsymbol{a} = \{w, \pi\}$, and consider demand from a given type of agent (with index θ suppressed for simplicity). Further suppose that utility is linear in the numeraire and the perceived subutility from a lottery ticket is $u(\boldsymbol{a}; \varepsilon) = (1 + \phi)\pi m(w) - \varepsilon$, where $m(w)$ is the utility gain from winning a prize of size w, and $(1+\phi)\pi$ is the decision weight that the individual applies to that utility. If $\phi = 0$, the individual is an expected utility maximizer. The random transaction cost shock ε determines whether the individual purchases a ticket on this occasion. We now provide examples of the types of biases that can be accommodated by this framework.

Misperceived probability of winning. Suppose the individual misperceives the probability of winning as $\tilde{\pi} \neq \pi$, and the normative decision weight (with no misperception) is $\left(1 + \phi^V\right)\pi$, with $\phi^V = 0$ for expected utility decisionmakers and $\phi^V > 0$ for individuals who experience anticipatory utility. Then normative utility from lottery consumption is $v(\boldsymbol{a}; \varepsilon) = \left(1 + \phi^V\right)\pi m(w) - \varepsilon$, perceived utility is $u(\boldsymbol{a}; \varepsilon) = \left(1 + \phi^V\right)\tilde{\pi} m(w) - \varepsilon$, and bias is $\gamma(\boldsymbol{a}) = u(\boldsymbol{a}; \varepsilon) - v(\boldsymbol{a}; \varepsilon) = (1 + \phi^V)(\tilde{\pi} - \pi)m(w)$.

Present focus (and addiction). Suppose that $\phi^V \pi m(w)$ is the immediate hedonic gain from anticipatory utility and joy of playing. Suppose further that individuals are quasi-hyperbolic

[10] Saez, Slemrod, and Giertz (2012) review the empirical literature on labor supply elasticities and argue that "in the absence of compelling evidence about significant income effects in the case of overall reported income, it seems reasonable to consider the case with no income effects."

discounters who discount all consumption other than the immediate utility $\phi^V \pi m(w)$ by a factor $\beta < 1$. Finally, suppose that normative utility corresponds to long-run preferences that set $\beta = 1$. Then normative and perceived utility can be written, respectively, as $v(\boldsymbol{a}; \varepsilon) = \left(1 + \phi^V\right) \pi m(w) - \varepsilon$ and $u(\boldsymbol{a}; \varepsilon) = \left(1 + \phi^V/\beta\right) \pi m(w) - \varepsilon$. Bias is then $\gamma(\boldsymbol{a}) = (1/\beta - 1)\phi^V \pi m(w)$.

Our framework can also accommodate the interaction between present focus and addiction, as studied by, e.g., Gruber and Kőszegi (2001) in the context of smoking. Suppose that buying a lottery ticket today imposes expected costs of d on one's future self because prior experience with gambling makes it more painful not to gamble, and possibly also makes future gambling less enjoyable. If these expected costs are downweighted by present focus β (but additional anticipatory utility $\phi^V \pi m$ is not), then $u(\boldsymbol{a}; \varepsilon) = (1 + \phi^V)\pi m(w) - \beta d - \varepsilon$, $v(\boldsymbol{a}; \varepsilon) = (1 + \phi^V)\pi m(w) - d - \varepsilon$, and $\gamma(\boldsymbol{a}) = (1 - \beta)d$.

Misforecasted happiness. Suppose individuals overestimate the happiness they would gain from winning the prize w by a factor of b. Then $u(\boldsymbol{a}; \varepsilon) = (1 + \phi)\pi(1 + b)m(w) - \varepsilon$, while $v(\boldsymbol{a}; \varepsilon) = (1 + \phi)\pi m(w) - \varepsilon$, and thus $\gamma(\boldsymbol{a}) = (1 + \phi)\pi b m(w)$.

2.2 Optimal Lottery Design: a Sufficient Statistics Approach

2.2.1 Government Objective Function

The policymaker selects T, p, and $\boldsymbol{a}$ to maximize normative utility, aggregated across individuals using type-specific Pareto weights $\mu(\theta, \varepsilon)$:

$$\max_{T,p,\boldsymbol{a}} \left[\int_{\theta,\varepsilon} \mu(\theta, \varepsilon) V(c, s, \boldsymbol{a}, z; \theta, \varepsilon) dF(\theta, \varepsilon) \right], \tag{4}$$

subject to a government budget constraint,

$$\int_{\theta} (ps(\theta) + T(z(\theta))) dF_{\theta}(\theta) - C(\boldsymbol{a}, \bar{s}) \geq R, \tag{5}$$

and to individuals' maximization of U.

We let λ denote the marginal value of public funds (i.e., the multiplier on the government budget constraint in equation (5) at the optimum), and we define $g(\theta, \varepsilon) := \mu(\theta, \varepsilon) U'_c$ to denote the weighted marginal utility from consumption for type (θ, ε). Following Saez (2002a) and others, we assume that this "social marginal welfare weight" $g(\theta, \varepsilon)$ is equal for all individuals with a given level of earnings (and thus, all who have a common type θ) under the optimal tax system. We thus use $g(z)$ to denote social marginal welfare weights as a function of income. Under the assumption of negligible income effects on labor supply, $\mathbb{E}[g(z)] = 1$.

2.2.2 Elasticity Concepts and Sufficient Statistics

We proceed by characterizing the first-order conditions for the optimal choice of price and each (continuously variable) attribute $a \in \mathbb{R}$ from the attribute vector $\boldsymbol{a}$; for the remainder of this section we therefore write all expressions as functions of a, implicitly holding fixed attributes other than a.

The socially optimal choices of attributes and price depend on three types of sufficient statistics: elasticities, a money-metric measure of bias, and the "progressivity of bias correction." These statistics are endogenous to the policy choices, though we suppress those arguments for notational simplicity. Because we assume that social marginal welfare weights depend only on income, the relevant statistics are functions of income z, rather than of types θ. We define $s(z)$ as the lottery demand aggregated across all z-earners.

We define $\zeta_p(z) := \frac{d\ln s(z)}{dp}$ as the semi-elasticity of demand with respect to ticket price, and we define $\zeta_a(z) := \frac{d\ln s(z)}{da}$ as the semi-elasticity of demand with respect to attribute a. We define $\bar{\zeta}_p$ and $\bar{\zeta}_a$ as population-wide semi-elasticities.

We define $\kappa(\theta)$ and $\kappa(z)$, respectively, to be the average willingness-to-pay (WTP) for a marginal increase in a of θ-types and z-earners. If $u'_a(a;\theta,\varepsilon)$ does not depend on ε, as is the case with additively separable shocks, then $\kappa(\theta) = -\zeta_a(\theta)/\zeta_p(\theta) \cdot s(\theta)$. We define $\bar{\kappa} := \mathbb{E}[\kappa(z)]$ as the average WTP of all buyers of the lottery ticket.

Following ALT, and with some abuse of notation, we define $\gamma_p(z) := \frac{\mathbb{E}\left[\gamma(\theta)\frac{ds(\theta)}{dp}|z(\theta)=z\right]}{\mathbb{E}\left[\frac{ds(\theta)}{dp}|z(\theta)=z\right]}$ and $\bar{\gamma}_p := \frac{\mathbb{E}\left[\gamma(\theta)\frac{ds(\theta)}{dp}\right]}{\mathbb{E}\left[\frac{ds(\theta)}{dp}\right]}$ to be the average bias of z-earners and all individuals, respectively, who are marginal to a price change. We define $\gamma_a(z)$ and $\bar{\gamma}_a$ analogously over individuals marginal to an attribute change.

Because bias can depend on the attribute a, individuals' perceived utility from a change in a may be biased. The bias in z-earners' valuation of a marginal increase in a is $\rho(z) := \mathbb{E}\left[\frac{d}{da}\gamma(a;\theta)\cdot s(\theta)|z(\theta)=z\right]$. We define $\bar{\rho} := \mathbb{E}\left[\rho(z)\right]$ as the population average. By definition, $\bar{\kappa} - \bar{\rho}$ is the average impact on normative utility of increasing a, measured in dollars.

Because our framework includes redistributive motives, the welfare effects of a change in consumption depend on whose consumption is changed. All else equal, the welfare change is more positive when more of the benefits of correcting bias accrue low-income individuals. To capture these redistributive concerns, we define a statistic $\sigma_p := Cov\left[g(z), \frac{\bar{\gamma}_p(z)}{\bar{\gamma}_p}\zeta_p(z)s(z)\right]/\zeta_p(z)$, which we refer to as the "progressivity of bias correction" from a price change. We define an analogous statistic for an attribute change: $\sigma_a := Cov\left[g(z), \frac{\bar{\gamma}_a(z)}{\bar{\gamma}_a}\zeta_a(z)s(z)\right]/\zeta_a(z)$. These statistics quantify the extent to which the benefits of bias correction accrue to low-income individuals, per unit change in $\bar{s}$.

2.2.3 Optimal Policy

Our main result can be stated in terms of intuitive mark-up formulas. The result is a special case of Proposition C1 in Appendix C.

Proposition 1. *If p and a are interior optima, then*

$$p - \overbrace{\frac{\partial C}{\partial \bar{s}}}^{\textit{Mark-up above MC}} = \overbrace{\bar{\gamma}_p(1+\sigma_p)}^{\textit{Bias correction}} - \frac{\overbrace{Cov\left[s(z), g(z)\right]}^{\substack{\textit{Regressivity of} \\ \textit{increasing } p}}}{|\bar{\zeta}_p|\bar{s}} \tag{6}$$

$$p - \overbrace{\frac{\partial C}{\partial \bar{s}}}^{\textit{Mark-up above MC}} = \overbrace{\bar{\gamma}_a(1+\sigma_a)}^{\textit{Bias correction}} - \frac{\overbrace{\bar{\kappa} - \bar{\rho} - \frac{\partial C}{\partial a}}^{\substack{\textit{Mechanical effect on} \\ \textit{consumer surplus} \\ \textit{and revenues}}} + \overbrace{Cov[\kappa(z) - \rho(z), g(z)]}^{\substack{\textit{Regressivity of} \\ \textit{increasing } a}}}{\bar{\zeta}_a \bar{s}} \tag{7}$$

For intuition, notice first that equation (6) is a special case of equation (7) if we think of price p as one of the attributes. Because we assume that people do not misperceive prices, $\rho(z) \equiv 0$. Moreover, $\kappa(z) = s(z)$, as the mechanical effect on consumer surplus of lowering the price by dp is simply $dps(z)$. Finally, $\frac{\partial C}{\partial p} = \bar{s}$, as the mechanical revenue effect of lowering price by dp is simply a decrease $dp\bar{s}$ in revenue. Thus, $\bar{\kappa} - \bar{\rho} - \frac{\partial C}{\partial a} = 0$ when $a = p$, and equation (7) reduces to equation (6).

Equation (7) is a generalization of Ramsey-style optimal commodity tax formulas. The wedge $p - \frac{\partial C}{\partial \bar{s}}$ is analogous to a per-unit tax on a lottery ticket. This "tax" must equal the difference between two terms. The first term, $\bar{\gamma}_a(1+\sigma_a)$, is a Pigouvian correction that corresponds to the social marginal benefits of decreasing lottery consumption. This term is increasing in σ_a, the progressivity of bias correction. The second term is the mechanical effect on consumer welfare net of revenues; it consists of two parts. The first part, $\bar{\kappa} - \bar{\rho} - \frac{\partial C}{\partial a}$, is the direct effect on consumer surplus net of revenues. The second part is the extent to which the effects on consumer surplus are distributed in a regressive or progressive way. In the case of a price change, this is simply the extent to which lower-income individuals buy more lottery tickets, and are thus more impacted by the price change. In the case of an attribute change more generally, this is the extent to which lower-income individuals have a higher (normative) WTP for this attribute change.

Appendix B presents special cases of Proposition 1, including (i) no bias and homogeneous preferences; (ii) no bias and heterogeneous preferences; (iii) homogeneous bias and preferences; and (iv) the revenue-maximizing lottery structure.

2.2.4 Generalizations and Connection to the Atkinson-Stiglitz Theorem

Atkinson-Stiglitz. The generalization in Proposition C1 in Appendix C allows income effects on lottery consumption. Of particular note is the connection to the theorem of Atkinson and Stiglitz (1976) and related work, which states that under homogeneous preferences and weak separability between consumption and labor, it is efficient to address redistributive goals through income taxation, not through differential commodity taxes. The logic of the Atkinson-Stiglitz Theorem does not apply to the Proposition 1, because our utility specification implies that all variation in $s(z)$ across the income distribution is driven by how tastes for gambling vary with earnings ability.

At the same time, a version of the Atkinson-Stiglitz Theorem does hold in our setting under assumptions that parallel theirs. A corollary of Proposition C1 is that when all variation in $s(z)$ across z is driven by causal income effects, rather than heterogeneous preferences, the optimal values of p and a are determined only by $\bar{\gamma}_p(1+\sigma_p)$ and $\bar{\gamma}_a(1+\sigma_a)$, respectively. Under more general assumptions that allow both preference heterogeneity and causal income effects on lottery consumption, Proposition C1 shows that Proposition 1 remains mostly in force, except with $s(z)$ and $\kappa(z)$ replaced by $s_{pref}(z)$ and $\kappa_{pref}(z)$, which we define as the amount of cross-sectional variation in s and κ that cannot be explained by causal income effects.

Substitution to other gambling. Our baseline formulas assume that when individuals substitute away from the lottery in question, they do not substitute to other goods over which they might misoptimize, such as other lotteries or casino gambling. This simplifying assumption is motivated by our empirical result in Section 3 that there is limited substitution between state-run lottery games, and by Kearney's (2005b) result that the introduction of state-run lotteries has no detectable effect on non-state-sponsored gambling. That said, introducing substitution to other lotteries in Proposition 1 is easy. Substitution to other state-run lotteries subject to the same bias would not affect Proposition 1, provided the elasticities reflect total state-run lottery demand. Building on ALT's formulas, a diversion ratio r to other (non-state-run) gambling subject to the same bias implies that $\bar{\gamma}$ should be replaced by $(1-r)\bar{\gamma}$ in Proposition 1.

3 Aggregate Lottery Demand

This section presents estimates of the aggregate semi-elasticities of demand $\bar{\zeta}_a$ and $\bar{\zeta}_p$ for the two major multi-state lotteries, Mega Millions and Powerball. We assume a static demand function with constant semi-elasticity with respect to price and the expected value of each prize; we see below that our data are consistent with these assumptions. k, j, and t index prizes, games, and drawing dates, respectively, π_{kjt} is the win probability from Table 1, and w_{kjt} is the prize amount.[11] Aggregate demand is

$$\ln \bar{s}_{jt} = \sum_k \bar{\zeta}_k \pi_{kjt} w_{kjt} + \bar{\zeta}_p p_{jt} + \varepsilon_{jt}. \tag{8}$$

For reasons described below, we do not directly estimate equation (8). Instead, we separately estimate the prize elasticities $\bar{\zeta}_k$ in Section 3.2 and the price elasticity $\bar{\zeta}_p$ in Section 3.3.

3.1 Data: Lottery Prizes and Aggregate Sales

Our primary analyses use draw-level sales and prize data for Mega Millions and Powerball from June 2010 through February 2020. We scraped jackpot and sales data from the website LottoReport.com, and we scraped California second prize amounts from the California Lottery website. Jackpots

[11] The jackpot expected value also depends on the probability of sharing the jackpot with a second winner, which in turn depends on sales. However, there are rarely two jackpot winners, so we assume for simplicity that the jackpot is never shared.

are advertised jackpot amounts, and ticket sales exclude the Just the Jackpot, Power Play, and Megaplier add-ons.

Table 2 presents descriptive statistics. There are 2,035 observations: two draws per week for almost ten years, for both Mega Millions and Powerball. The California sample is slightly smaller, because California did not join Powerball until April 2013. The average jackpot was $116.3 million, and the average California second prize was $893,400. The average draw sold 23 million tickets nationwide and 3.3 million in California.

We run 36-month event study analyses around the four format changes described in Table 1. To maintain a consistent set of states in the national sales data as states join Mega Millions or Powerball, we construct separate national sales series for each event window that include total sales from only the states that participated for the full 36-month period. For example, California, Florida, Louisiana, and the Virgin Islands joined Mega Millions and/or Powerball during the 36-month window around the Powerball price increase on January 15, 2012, so we exclude them from the national sales totals used for that event study.

We also use data on lottery sales by game, state, and week from 1995 through February 2020 purchased from La Fleur's, a standard data provider. To study substitution across games, we limit to a balanced panel including instant games plus the 85 other games that were offered continuously and for which we have complete data from June 2010–February 2020 in the 41 states that had Mega Millions and Powerball over that period, representing 61 percent of total lottery sales reported in the Census of Governments over the sample period.

Because prize amounts are typically round numbers in nominal dollars, we use nominal dollars throughout the paper except for Appendix Figures A1 and A2.

3.2 Prize Semi-Elasticities

3.2.1 Estimation Strategy

We identify the jackpot elasticity $\bar{\zeta}_1$ off of jackpot variation generated by randomness in whether someone won the jackpot in the previous draw, and we identify the second prize elasticity $\bar{\zeta}_2$ off of analogous variation in California second prizes. As an example, Figure 1 illustrates the identifying variation for Powerball in 2014. Over that year, the jackpot varied from $40 million to $400 million. In most drawings, nobody wins the jackpot, so the prize pool rolls over to the next drawing. Each of the 11 times that someone won the jackpot in 2014, it returned to its $40 million reset value. The odds of winning the jackpot were about 1/200,000,000 in 2014, so the expected value of the jackpot varied from roughly $0.20 to $2.50. The California second prize varied from $183,000 to $7.7 million, and the second prize odds were about 1/5,000,000, so the second prize expected value varied from roughly $0.04 to $1.50. This illustrates that the California second prize expected value can be material (and perhaps even larger than the jackpot), although the jackpot expected value is about 4.9 times larger on average during our sample.

The figure also plots California ticket sales against the right axis. Sales and jackpots move together, and sales rise especially sharply when jackpots are unusually high. However, sales do not seem to respond to second prize variation, suggesting that $\bar{\zeta}_2$ is close to zero.

While there is some randomness in prize amounts, there are two reasons why we do not directly estimate equation (8). First, the draw t advertised jackpot is directly determined by forecasted ticket sales for draw t, potentially generating simultaneity bias. Second, due to rollovers, the draw t prize pool is also directly affected by previous draws' sales, which may be correlated with draw t demand if demand shocks are serially correlated.

To isolate the random variation in prize amounts, we instrument for prize k's expected value $\pi_{kjt}w_{kjt}$ with a forecast $\pi_{kjt}Z_{kjt}$ based on whether or not the prize rolled over from the previous period. Define r_{kjt} as an indicator for whether prize k rolled over from $t-1$ into t. A rollover from $t-1$ is less likely when $t-1$ ticket sales $\bar{s}_{j,t-1}$ are higher, but the realization of r_{kjt} is random conditional on $\bar{s}_{j,t-1}$. Let $\iota_{kjf(t)}$ denote the observed average percent increase in prize k in game j conditional on a rollover when format f is in effect, and let $\underline{w}_{kjf(t)}$ denote prize k's reset value in game j.[12] The prize forecast is

$$Z_{kjt} = r_{kjt} \cdot \left(1 + \iota_{kjf(t)}\right) \cdot w_{kj,t-1} + (1 - r_{kjt}) \cdot \underline{w}_{kjf(t)} \tag{9}$$

Rollovers r_{kjt} and past prizes $w_{kj,t-1}$ both depend on previous demand, and demand shocks may be serially correlated. We address this by including a vector of controls $\boldsymbol{H}_{j,t-1}$ for demand in prior periods. In our primary specifications, $\boldsymbol{H}_{j,t-1}$ is the vector of logged sales $\ln \bar{s}_{jl}$ and jackpot expected values $\pi_{1jt}w_{1jl}$ (and for California sales, second prize expected values $\pi_{2jt}w_{2jl}$) in previous periods $l \in \{t-1, t-2, t-3, t-4\}$, as well as the squares of those variables. Thus, after controlling for $\boldsymbol{H}_{j,t-1}$, we identify only off of conditionally random variation in the prize pool forecast Z_{kjt} delivered by randomness in whether the prize rolled over.[13]

We also add a vector of fixed effects collectively denoted ξ_{jt}: game-format fixed effects to soak up any changes in demand caused by changes in lower prize amounts and probabilities, game-regional coverage fixed effects to soak up changes in demand when new states join, game-quarter of sample indicators to soak up changes in demand over time, and game-weekend fixed effects to soak up higher demand for Friday and Saturday draws. We do not include time fixed effects that are common across both games. We do not want time fixed effects because they could introduce bias if consumers substitute across games, and we do not need time fixed effects because the lagged sales in $\boldsymbol{H}_{j,t-1}$ soaks up time-varying demand patterns.

Incorporating these modifications into equation (8), our estimating equation is

$$\ln \bar{s}_{jt} = \bar{\zeta}_1 \pi_{1jt} w_{1jt} \left(+\bar{\zeta}_2 \pi_{2jt} w_{2jt}\right) + \boldsymbol{\beta}_H \boldsymbol{H}_{j,t-1} + \xi_{jt} + \epsilon_{jt}. \tag{10}$$

The second prize elasticity term in parentheses is included only in our California-specific estimates.

[12] The second prize pool in California does not have a fixed reset value: it is allocated a pre-determined share of revenues from each draw. We set $\underline{w}_{2jf(t)}$ to the average second prize amount in the first draw after the second prize is won during format f.

[13] This regression is related to a dynamic panel regression in the sense that $\boldsymbol{H}_{j,t-1}$ includes lags of the dependent variable. While serial correlation would bias our estimate of the regression coefficient on $\boldsymbol{H}_{j,t-1}$, we do not use or interpret that coefficient.

We instrument for $\bar{\zeta}_1 \pi_{1jt} w_{1jt}$ (and $\bar{\zeta}_2 \pi_{2jt} w_{2jt}$) with the jackpot forecast $\pi_{1jt} Z_{1jt}$ (and the second prize forecast $\pi_{2jt} Z_{2jt}$).

3.2.2 Estimation Results

Figure 2 presents binned scatter plots of the variation that identifies the prize semi-elasticities $\bar{\zeta}_1$ and $\bar{\zeta}_2$, conditional on the controls in equation (10). Panel (a) shows that national ticket sales are highly responsive to the jackpot expected value fitted values $\widehat{\pi_{1jt} w_{1jt}}$ from the first stage of equation (10). The relationship is very close to linear, and the slope is about 1.0, meaning that a \$0.10 increase in the jackpot expected value increases sales by about 10 log points. However, Panel (b) shows that California ticket sales are not responsive to the California second prize expected value first-stage fitted values.

Table 3 presents estimates of equation (10). The first stages are very strong; see Appendix Table A1. In all estimates in this section, we use Newey-West standard errors with up to ten lags. Our standard errors do not change much if we allow more or fewer Newey-West lags or cluster standard errors at the intersection of game and month, quarter, or half year; see Appendix Table A2.

Panel (a) presents estimates of the jackpot elasticity $\bar{\zeta}_1$ using the nationwide sales data, while Panel (b) presents estimates of the jackpot and second prize elasticities $\bar{\zeta}_1$ and $\bar{\zeta}_2$ using California sales only. In both panels, column 1 presents the OLS estimates without any $\boldsymbol{H}$ controls, column 2 presents estimates with only one lag in $\boldsymbol{H}_{j,t-1}$ and no quadratic terms, and column 3 presents our primary IV estimates with four lags and quadratic terms.

The OLS estimates are slightly larger, consistent with slight simultaneity bias. The estimates match the binned scatter plots in Figure 2. In both panels of Table 3, the jackpot semi-elasticity is around 1.0. In Panel (b), the second prize semi-elasticity is statistically indistinguishable from zero, and the 95 percent confidence intervals in column 1 exclude values larger than about 0.2.

If consumers were risk-neutral and responded only to the lottery's expected value, these two semi-elasticities would be the same: sales would respond equally to variation in expected value coming from the jackpot versus the second prize. If consumers were risk-averse and weighed the potential utility from each prize by its objective probability, the jackpot elasticity would be smaller than the second prize elasticity. The much larger jackpot elasticity could arise for several reasons. First, individuals might underappreciate differences between the small second prize probability and the very small jackpot probability. Second, individuals could derive larger anticipatory utility from larger prizes, and anticipatory utility might be insensitive to probabilities. Either of these explanations could be consistent with prior work on probability weighting. Third, because the second prize expected value doesn't vary much (its standard deviation is only one-third of the jackpot's), individuals might be inattentive to the second prize, as in Gabaix (2014) and related theories of focusing and salience (Bordalo, Gennaioli, and Shleifer 2013; Koszegi and Szeidl 2013; Bushong, Schwartzstein, and Rabin 2021). Column 4 of Panel (b) of Table 3 provides suggestive evidence that attention might play a role: the second prize semi-elasticity is statistically significantly larger (and marginally insignificantly positive with p-value ≈ 0.2) when jackpots are relatively low. Fourth, while jackpot amounts are covered in the media and heavily promoted on billboards and the landing pages of lottery websites, the estimated second prize for the next draw can be inferred but is not directly reported anywhere.[14] Of course, the second prize might not be promoted precisely because individuals would be unresponsive.

Substitution. As described in Clotfelter and Cook (1990), Grote and Matheson (2011), and the literature cited therein, it is ambiguous whether draw games such as Mega Millions and Powerball are substitutes or complements for other lottery games. While their similarity suggests that they might be substitutes, the attention to high jackpots could also spill over to other games. Furthermore, high jackpots bring additional consumers into lottery outlets, where they can immediately buy other games. It could also be that high jackpots primarily bring in new consumers who wouldn't otherwise buy lottery tickets.

Appendix D.1 shows that Mega Millions or Powerball jackpots have tightly estimated zero effects on sales of other games. Our confidence intervals rule out diversion ratios of more than about three percent to the other multi-state game, major state-level draw games, instant games, and other state-level games. This limited substitution is consistent with our "narrow bracketing" model assumption under which people consider each lottery in isolation rather than forming a utility-maximizing portfolio from a combination of different lotteries.

Short-run versus long-run responses. As in many other studies, we have a well-identified short-run elasticity, but our policy analysis requires a long-run elasticity. To address this concern, Appendix D.2 shows that the jackpot elasticity is very similar when we aggregate over time and that lagged jackpot amounts have positive but relatively small effects on current sales. While not dispositive, this is consistent with the idea that the long-run elasticity is not much different from the short-run elasticity.

3.3 Price Elasticity

3.3.1 Estimation Strategy

We identify the price semi-elasticity $\bar{\zeta}_p$ from the change in ticket sales when Mega Millions and Powerball raised their prices from \$1 to \$2. To use these event studies, we must also consider the simultaneous format changes described in Table 1. Both games substantially increased their jackpot amounts and expected values. However, as shown in Table 1, the expected return (i.e., expected value per dollar spent) from all lower prizes per did not change very much. Because of this limited change, and because we saw insignificant elasticity with respect to second prize variation in Table 3, we assume that the lower prize changes did not affect demand.

Define W_{jt} as an indicator for the 12-month period around game j's price change, six months before and six months after. Our estimating equation modifies equation (8) into an event study

[14] Individuals can indirectly infer the California upcoming draw's second prize amount by looking on the California lottery website for the previous second prize amount and whether it was won in California, and then adding a guess about the increase in the prize pool from the upcoming draw's sales.

estimator that identifies the change in sales within a 12-month event window around the price change:

$$\ln \bar{s}_{jt} = \bar{\zeta}_p p_{jt} W_{jt} + \beta_1 W_{jt} + \beta_2 \pi_{1jt} w_{1jt} + \xi_j + \epsilon_{jt}, \tag{11}$$

where ξ_j is now simply an event fixed effect. The jackpot expected value $\pi_{1jt} w_{1jt}$ control increases precision and controls for the simultaneous format change. Because lagged sales and jackpots are also affected by the price change, we do not control for $\boldsymbol{H}_{j,t-1}$, and we do not instrument for the jackpot. Since our OLS and IV results were quite similar in Table 3, this is unlikely to affect our results. To identify the coefficients on the controls with the most relevant data, we limit the regression sample to the 36-month period around the price change.

3.3.2 Estimation Results

The red lines on Figure 3 present ticket sales residual of the jackpot expected value, i.e. $\ln \bar{s}_{jt} - \hat{\beta}_2 \pi_{1jt} w_{1jt}$, in the 36-month event study window for the game whose price changed. Panel (a) presents the Powerball price change in January 2012, while Panel (b) presents the Mega Millions price change in November 2017. We recenter so that the average residual equals zero before the price change when the jackpot is within \$10 million of the reset value. In both event studies, sales drop by about 50 log points immediately after the price increase.

The other lines on Figure 3 present ticket sales for the other multi-state game, major state draw games, and all other games, respectively. To reduce noise, the other multi-state game's sales are residual of that game's jackpot expected value control. Some long-run trends become visible over the full 36 month period: both Mega Millions and Powerball sales gradually decline after the other game's price increases, and other game sales (mostly instant games) gradually increase. However, the effect on own-game sales is much larger and more immediate than these gradual trends for other games, suggesting that the other games' trends are unrelated to the price change.

Table 4 presents estimates of equation (11). Column 1 presents our primary estimates pooling the two event studies, while columns 2 and 3 consider each event study separately. Consistent with Figure 3, the estimates suggest that the price changes decreased demand by 43 to 56 log points.

While we model demand as a function of the jackpot expected value, it could be that individuals pay attention only to the jackpot amount and not the probability (Cook and Clotfelter 1993). From Table 1, we can calculate that the Mega Millions jackpot probability decreased by 14 percent when the price increased, while the Powerball jackpot probability increased by 12 percent. If demand responds to the jackpot amount instead of the expected value, our price elasticity estimates would be biased, although in opposite directions for the two games. Column 4 shows that the pooled estimate changes little when we control for the jackpot amount.

Substitution. Consistent with the graphical evidence in Figure 3, Appendix Table A6 shows statistically zero substitution to other games after Mega Millions and Powerball increase prices. However, the estimates are not precise enough to rule out economically significant diversion ratios.

Placebo tests. As shown in Table 1, there were also two format changes during our sample period that did not involve price changes. We can use these as placebo tests: if our price elasticity estimates are unbiased, these format changes should not affect demand. Appendix Table A9 presents estimates of equation (11) for these two other format changes, where we substitute p_{jt} for a post-format change indicator variable and again limit to the 36 months around the format change. These format changes have statistically zero effect on demand after controlling for the jackpot level, and the confidence intervals suggest that any confounding effects are small relative to the price change coefficients in Table 4.[15]

Summary. Lottery purchases are (i) highly elastic to jackpot variation, (ii) unresponsive to variation in smaller prizes, and (iii) less responsive to a \$1 price change than to a \$1 jackpot expected value change. The third result suggests that jointly raising price and jackpot expected value increases demand, consistent with the trends toward larger jackpots and higher ticket prices. Taken together, these three results are consistent with a probability weighting function that weights the jackpot higher than its objective probability and places little weight on smaller prizes.

4 Individual-Level Demand

4.1 AmeriSpeak Survey

This section provides evidence on the remaining statistics required for policy analysis: consumption by income $s(z)$, average bias $\bar{\gamma}$, and the progressivities of bias correction σ_p and σ_a. We use a new survey that we designed to measure lottery spending and proxies for preferences and biases potentially related to lottery purchases. The survey was fielded on AmeriSpeak, a high-quality survey panel managed by the National Opinion Research Corporation (NORC). Unlike internet panels that allow anyone to opt in, AmeriSpeak is a probability sample that includes only U.S. households that had been randomly selected (and heavily incentivized) to participate. This helps to reduce sample selection biases that can make surveys unrepresentative on unobserved characteristics. The average spending estimates in Section 4.2 are weighted for national representativeness on age, sex, race, education, geography, and other key characteristics using sample weights provided by NORC. The bias proxy regressions in Section 4.3 are left unweighted to maximize precision, although the results with sample weights are similar.

The survey was fielded in April 2020, and a follow-up survey was fielded in April 2021. 3,013 people completed the 2020 survey, of whom 2,879 passed basic data quality checks. Table 5 presents descriptive statistics. Panel (a) presents panelist demographics, Panel (b) presents survey questions on spending and income effects, and Panel (c) presents questions that proxy for preferences and bias. Sample sizes differ on individual questions due to item non-response. Appendix E.1 presents

[15] In both of these format changes, the jackpot probabilities decreased substantially—much more than the jackpot probabilities changed when the games increased prices. This means that controlling for jackpot level vs. jackpot expected value matters more. Appendix Table A9 shows that controlling for the jackpot expected value (instead of level) does not explain the demand increases after the format changes. This is consistent with a model in which individuals respond to the jackpot level but are not responsive to the jackpot probability.

the text of the survey questions, which we summarize here.

Spending and income effects. *Monthly lottery spending* is the response to the question, "How many dollars did you spend in total on lottery tickets in an average month in 2019?" We asked panelists to "please include Mega Millions, Powerball, and other lotto/prize drawings, instant/scratch-off games, and any other lottery game offered by your state lottery." To ensure that large expenditures were correctly reported, any person who reported more than $500 monthly lottery spending was asked to explicitly confirm or update their response.

To measure income effects, the survey asked people to report the percent change in their household income and lottery spending in 2019 compared to 2018 (*income change* and *spending change*), as well as how much they think their lottery spending would change "if you got a raise and your income doubled" (*self-reported income effect*).

Preference proxies. We construct three proxies of preferences for playing the lottery. First, we proxy for risk aversion using two questions: "In general, how willing or unwilling are you to take risks?" and a second question measuring aversion to financial risks when saving or investing money. Our *risk aversion* preference proxy is the average of these two measures after standardizing each to have a standard deviation equal to one. Second, our *lottery seems fun* preference proxy is the extent to which people agree or disagree that "For me, playing the lottery seems fun." Third, our *enjoy thinking about winning* preference proxy, which is intended to measure anticipatory utility, is the extent to which people agree or disagree that "I enjoy thinking about how life would be if I won the lottery."

We designed the survey questions to allow us to construct proxies for six biases that might be related to lottery purchases.

Self-control problems. Self-control problems might affect lottery purchases if the enjoyment of playing is in the present, while the cost of buying the ticket (reduced consumption of other goods) is incurred later. To measure perceived self-control problems, the survey said, "It can be hard to exercise self-control, and some people feel that there are things they do too much or too little – for example, exercise, save money, or eat junk food. Do you feel like you play the lottery too little, too much, or the right amount?" Our *self-control problems* bias proxy is the response to that question, on a seven-point scale from "far too little" (coded as -3) to "the right amount" (coded as 0) to "far too much" (coded as +3).

Financial illiteracy. Financial illiteracy and innumeracy might affect lottery purchases by making it harder to evaluate risky prospects and correctly perceive small probabilities. *Financial literacy* is the share of correct answers on five standard questions from Lusardi and Mitchell (2014), and *financial numeracy* is the share of correct answers on three numeracy questions from Banks and Oldfield (2007). Our *financial illiteracy* bias proxy is the share of incorrect answers across all eight questions.

Statistical mistakes. Poor statistical reasoning might similarly make it harder to evaluate risky prospects and correctly perceive small probabilities. The survey included three measures of statistical reasoning. First, we measured the Gambler's Fallacy (Jarvik 1951; Tversky and

Kahneman 1971; Rabin 2002) by eliciting beliefs about the probability that an unbiased coin lands heads after three different sequences of heads and tails. The true probability is of course 50 percent. *Gambler's Fallacy* is the share of answers that differ from 50 percent; Table 5 reports that people gave some other answer 29 percent of the time. Second, we measured non-belief in the Law of Large Numbers (Benjamin, Rabin, and Raymond 2016; Benjamin, Moore, and Rabin 2018) by asking the probability that out of 1000 coin flips, the number of heads would be between 481 and 519 (correct answer = 0.78), 450 and 550 (correct answer = 0.9986), and 400 and 600 (correct answer = 0.9999). *Non-belief in the Law of Large Numbers* is the average absolute deviation from the correct answer. Third, we asked people to calculate the expected value of four simple example lotteries. *Expected value miscalculation* is the share of answers that are incorrect. To construct our *statistical mistakes* bias proxy, we standardize each of these three measures to have a standard deviation equal to one in the 2020 sample, take the average, standardize the average to have a standard deviation equal to one in 2020, and recenter so that zero is the best score in 2020.

Overconfidence. Overconfidence could increase lottery purchases by increasing people's perception of the chance of winning. The survey said, "Imagine **you** could keep buying whatever lottery tickets you want, over and over for a very long time. For every $1000 you spend, how much do you think you would win back in prizes, on average?" The survey also asked people to report how much "the **average** lottery player" would win back. *Overconfidence* is the difference between own and average person expected winnings per $1 spent.

Expected returns. Misunderstanding the expected returns for the average person might also affect lottery purchases. *Expected returns* is the response to the question, "Think about the total amount of money spent on lottery tickets nationwide. What percent do you think is given out in prizes?"

Predicted life satisfaction. As argued by Kahneman et al. (2006) and others, people may overestimate the effect of wealth on happiness, and such a bias would cause people to overestimate the utility gains from winning a lottery. Using random variation in lottery winnings conditional on winning some amount, Lindqvist, Ostling, and Cesarini (2020) estimate that the effect of an additional $100,000 on life satisfaction (measured on a 0–10 scale) is 0.071 points. The survey told panelists about the Lindqvist, Ostling, and Cesarini (2020) study, informed them that the sample average life satisfaction was 7.21 out of 10, and asked them to predict the effect of an additional $100,000 on life satisfaction. *Predicted life satisfaction* is the response to that question.

2021 follow-up survey. We fielded the April 2021 follow-up survey to help address measurement error. NORC invited everyone who had completed the 2020 survey to participate. 2,186 people responded, representing a normal follow-up response rate for AmeriSpeak, of whom 2,124 passed our data quality checks. The 2021 survey asked the same preference and bias proxy questions as in 2020. It also re-elicited *monthly lottery spending* in 2019 for 104 people who reported outlying values in 2020; we use this to confirm or winsorize the outlying responses.[16]

[16]Specifically, we resampled the 104 people who had reported spending more than $150 per month or more than 10 percent of their income on lottery tickets. If the 2021 report was within 50 percent of the 2020 report, we use the average. Otherwise, we use the minimum. We then regress this modified monthly spending on the 2020 report and

4.2 Lottery Spending by Income

The spending distribution is highly skewed: in our survey data, the top 10 percent of spenders account for 56 percent of spending, while over 40 percent of people report no spending at all; see Appendix Figure A4. This skewness reduces the precision of our estimates and underscores the importance of our efforts to validate outlying self-reports of monthly spending. The average spending of $15 per month multiplied by 255 million American adults gives $47 billion, which is smaller than the $87 billion total nationwide sales reported in Figure A1. Clotfelter et al. (1999, Table 6) also found that survey responses understate total nationwide sales. This understatement and other forms of measurement error would bias our conclusions only if correlated with income or bias proxies. As a suggestive test, we find no evidence that income or bias proxies are correlated with the change in 2021 vs. 2020 reports of *monthly lottery spending*; see Appendix Table A14.

Figure 4 presents average monthly lottery spending by income. People with household income above $100,000 spend an average of $13 per month on the lottery, while people with household income under $50,000 spend an average of $17, or 29 percent more.[17] The cross-sectional income elasticity of lottery spending (from a regression of ln(1 + spending) on ln(income)) is -0.111. The share of people with non-zero spending also declines slightly with income; see Appendix Figure A5. Clotfelter et al. (1999, Table 10) also found that lower-income households spend more on lotteries, although their point estimates suggest a steeper decline in spending by income as of 1998.

Proposition C1 in Appendix C distinguishes two reasons why lottery spending might vary with income: the causal effect of income and preference heterogeneity that is correlated with income. The survey offers two ways to measure causal income effects. First, regressing *spending change* on *income change* suggests a causal income elasticity of 0.194; see Appendix Table A12 for formal regression results. This should be interpreted cautiously because changes in life circumstances correlated with income changes might also change lottery consumption preferences. Second, the average of *self-reported income effect* (the amount by which people thought their lottery spending would change if their income doubled) is -1.4 percent, suggesting a causal income elasticity of $d\ln s/d\ln z \approx -0.014/\ln(2/1) \approx -0.02$. This should be interpreted cautiously because the question was hypothetical.

While the exact point estimates differ, both of these strategies are consistent in suggesting limited income effects, and they are both statistically less negative than the cross-sectional income elasticity of -0.111 illustrated in Figure 4.

4.3 Association Between Bias Proxies and Lottery Spending

This section presents the relationships between bias proxies and lottery spending. Although we control for confounders such as measures of preferences, caution is warranted in interpreting these

use the prediction for the 25 out of the 104 people who did not take the 2021 survey.

[17]If we do not winsorize outliers using the second elicitation in 2021, average monthly spending increases by about $9. However, our policy analyses use only the *differences* in spending across income groups, and those differences are not significantly affected by winsorization; see Appendix Figure A6.

relationships as causal.

Define s_i as person i's monthly lottery spending, define b_{ik} as person i's value of bias proxy k, and define b_k^V as the benchmark value of b_k for an unbiased consumer who does not have self-control problems, has high financial literacy and statistical reasoning ability, is not overconfident, and has correct beliefs about expected returns and the effect of lottery winnings on life satisfaction. Define the standardized bias proxy $\tilde{b}_{ik} := \frac{b_{ik}-b_k^V}{SD(b_{ik})}$ as the difference between person i's proxy b_{ik} and the unbiased value b_k^V in 2020 standard deviation units, and define $\tilde{\boldsymbol{b}}_i$ as a vector of the six standardized bias proxies. All bias proxies are signed so that a more positive value should cause more lottery consumption. Finally, define $\boldsymbol{x}_i$ as a vector of controls, including the three preference proxies (*risk aversion*, *lottery seems fun*, and *enjoy thinking about winning*), the demographic characteristics presented in Panel (a) of Table 5, and state fixed effects.

Test-retest reliability. To summarize test-retest reliability, Figure 5 presents binned scatter plots of (unstandardized) bias proxies b_{ik} elicited in 2021 vs. 2020. The area of each circle is proportional to the share of observations in each bin. For *overconfidence*, there is close to zero correlation between 2020 and 2021 responses. The large mass at zero reflects the fact that 65 percent of people expect to win the same amount as the average lottery player. Any reported optimism or pessimism about one's potential winnings is not persistent across years. For the other five bias proxies, however, the correlations range from 0.34 to 0.75.

For comparison, Stango and Zinman (2021, Table 3) find within-person rank correlations of 0.04 to 0.59 for bias proxies similar to ours in surveys separated by three years, and Chapman et al. (2020, Table 2) find correlations of 0.30 to 0.96 between "econographic" preference measures elicited twice on the same survey.

Descriptive correlations. Figure 6 presents binned scatter plots of (unstandardized) bias proxies b_{ik} against the natural log of 1 + *monthly lottery spending*, using the 2020 survey data. We use natural logs because spending is skewed and it is natural to think of bias entering multiplicatively, as in the examples from Section 2.1.1, and we add 1 to spending to be able to include zero-spending observations. Our results in Table 6 below are similar when using spending levels or the inverse hyperbolic sine of spending. In each of the six panels, a vertical line indicates the unbiased benchmark b_k^V.

For *self-control problems*, the unbiased benchmark b_k^V is playing the lottery "the right amount" instead of "too little" or "too much." The relationship between lottery spending and *self-control problems* is not as close to linear as with the other five bias proxies. To the left of zero, a stronger feeling that one plays the lottery "too little" is not clearly associated with spending. We thus recode negative values as 0 for the regressions and predictions described below. To the right of zero, a stronger feeling that one plays "too much" is positively associated with spending, suggesting that self-control problems might contribute to overconsumption. This contribution may be limited, however, because the circle sizes indicate that 71 percent of people report playing "the right amount," and more people report playing "too little" than "too much."

For *financial illiteracy* and *statistical mistakes*, we define the unbiased benchmarks b_k^V as the best scores in the sample. For *financial illiteracy*, this is answering all eight questions correctly, and *statistical mistakes* is constructed so that 0 is the best score in the sample. The best fit lines' upward slopes mean that people with higher financial illiteracy and more statistical mistakes spend more on lotteries. The figure suggests that this might contribute to overconsumption: many people score relatively poorly on these two scales, and people with the worst scores spend 100 log points more on the lottery than people with the best scores.

For *overconfidence*, we define the unbiased benchmark b_k^V as predicting no difference between one's own lottery winnings and the average person's lottery winnings. The figure suggests that overconfidence may not contribute much to overconsumption: there is little relationship between overconfidence and spending, and 65 percent of people reported the same expected earnings for themselves and the average player. The lack of relationship between *overconfidence* and spending is unsurprising given the low test-retest reliability.

For *expected returns*, the unbiased benchmark b_k^V is 60 percent: the true share of state lottery ticket sales that are given out as prizes, using data reported in Appendix Figure A2. The best fit line's upward slope means that people who think the expected returns are higher spend more on lotteries. Most of the mass is to the left of b_k^V, and the average person believes that only 29 percent of lottery spending is returned to winners. This suggests that people might play *more* if they didn't underestimate the expected returns.

Finally, the unbiased benchmark b_k^V for *predicted life satisfaction* is the actual effect from Lindqvist, Ostling, and Cesarini (2020): an additional \$100,000 of lottery winnings increased life satisfaction by 0.071 points on the 0–10 scale. The slope of the best fit line suggests that predicting that additional winnings increase life satisfaction by 1 additional point on the 0–10 scale is associated with spending about 4.3 log points more on lottery tickets.

Regression estimates. To test whether these relationships survive controls for demographics and preferences, we estimate the following regression:

$$\ln(s_i + 1) = \boldsymbol{\tau}\tilde{\boldsymbol{b}}_i + \boldsymbol{\beta}\boldsymbol{x}_i + \epsilon_i. \tag{12}$$

Table 6 presents results. Column 1 presents the full model, column 2 presents the regression without any controls, and columns 3 and 4 progressively add controls. Column 1 shows that four of the unconditional relationships from Figure 6 survive controls: *self-control problems*, *financial illiteracy*, *statistical mistakes*, and *expected returns* are all strongly conditionally associated with lottery spending, while *overconfidence* and *predicted life satisfaction* are not. All the estimated $\hat{\tau}$ coefficients are positive.

Comparing columns 2 and 3 shows that the preference proxies explain a large share of the variation in lottery spending, increasing the R^2 from 0.16 to 0.36. These controls also materially attenuate the $\boldsymbol{\tau}$ coefficients, which underscores the importance of having collected the preference control variables. Adding the demographic controls in column 4 increases the R^2 slightly and has limited effects on the $\boldsymbol{\tau}$ coefficients.

Measurement error can attenuate the relationships in columns 1–4. To address this, we use the Obviously Related Instrumental Variables (ORIV) approach of Gillen, Snowberg, and Yariv (2019): we estimate equation (12) in a stacked dataset with the 2021 $\tilde{\boldsymbol{b}}_i$ and $\boldsymbol{x}_i$ below the 2020 $\tilde{\boldsymbol{b}}_i$ and $\boldsymbol{x}_i$, instrumenting for the 2020 variables with their 2021 values and vice versa, while clustering standard errors by i. The ORIV approach is more efficient than an unstacked IV approach that instruments the 2020 variables with their 2021 values *or* vice versa, and it avoids the ambiguity that arises if those two unstacked estimates are different.

We drop *overconfidence* because the low test-retest reliability causes a weak instruments problem. After that drop, the first stage regressions involve highly statistically significant relationships between the resampled values of the same variable and limited correlations with other variables; see Appendix Table A13. The one exception is that *financial illiteracy* and *statistical mistakes* are moderately correlated.

Column 5 of Table 6 presents OLS estimates in the subsample that also responded in 2021; the coefficients change little relative to column 1. Column 6 presents the ORIV estimates using the same sample as column 5. The coefficient on *self-control problems* grows substantially, consistent with its lower test-retest reliability. The *financial literacy* coefficient also grows, helping make *statistical mistakes* insignificant. *Expected returns* and *predicted life satisfaction* also become statistically insignificant. Interestingly, the coefficient on education shrinks substantially and becomes statistically insignificant, meaning that after correcting for measurement error, our bias proxies explain why higher-education people spend less on the lottery.

Share of consumption explained by bias. We can use the regression results to predict what lottery spending would be without systematic bias, i.e., if all individuals' bias proxies b_{ik} equaled the unbiased benchmarks b_k^V. Define $\hat{s}^V$ as predicted consumption with $\tilde{b}_{ik} = 0$. Equation (12) implies that $\ln(s_i + 1) - \ln(\hat{s}_i^V + 1) = \hat{\boldsymbol{\tau}}\tilde{\boldsymbol{b}}_i$, and thus $\hat{s}_i^V = \frac{s_i+1}{\exp(\hat{\boldsymbol{\tau}}\tilde{\boldsymbol{b}}_i)} - 1$. We winsorize at $\hat{s}_i^V \geq 0$, and we fix $\hat{s}_i^V = 0$ for people with zero spending ($s_i = 0$). Using the OLS $\hat{\boldsymbol{\tau}}$ in column 1 of Table 6, the share of consumption statistically explained by bias is $\frac{\sum_i \left(s_i - \hat{s}_i^V\right)}{\sum_i s_i} \approx 43$ percent.

We can also compute the share of consumption explained by each individual bias k by constructing $\hat{s}_i^V$ with $\exp(\hat{\tau}_k \tilde{b}_{ik})$ instead of $\exp(\hat{\boldsymbol{\tau}}\tilde{\boldsymbol{b}}_i)$. Figure 7 presents estimates using the OLS $\hat{\tau}_k$. For *self-control problems*, *financial literacy*, and *statistical mistakes*, the average $\tilde{b}_{ik} > 0$ and $\hat{\tau}_k$ is positive, so Figure 7 correspondingly suggests that these biases increase lottery spending. The $\hat{\tau}_k$ coefficients for *overconfidence* and *predicted life satisfaction* are both very close to zero, so the figure suggests little effect on spending. Since the average person underestimates expected lottery returns, the average $\tilde{b}_{ik} < 0$ for *expected returns*, and the figure suggests that this reduces spending by about 10 percent.

As shown in Section 2, optimal policy depends on whether lower-income people are more or less biased. Figure 8 presents binned scatter plots of each bias proxy by income. *Self-control problems* is the only bias proxy that becomes more positive with income: higher-income people are more likely to report that they play the lottery "too much." *Financial illiteracy* declines strongly with income: people with household incomes under \$20,000 incorrectly answered 44 percent of our eight financial literacy and financial numeracy questions, while people with household incomes over \$100,000 incorrectly answered only 17 percent. Similarly, people with household incomes under \$20,000 scored about 0.7 standard deviations worse on our statistical mistakes questions than people with household incomes over \$100,000. *Overconfidence*, *expected returns*, and *predicted life satisfaction* differ little across income groups.

Using these differences in bias proxies by income, we construct the share of consumption explained by bias separately for each income group. Figure 9 shows that this share declines moderately by income, from 46 percent for people with household incomes under \$50,000 to 40 percent for people with household incomes over \$100,000.

We can also adjust these two figures for measurement error by using the instrumental variables estimates from column 6 of Table 6; see Appendix Figures A7 and A8. Consistent with the results in Table 6, the shares of consumption explained by *self-control problems* and *financial illiteracy* grow, while the shares explained by the other variables attenuate. The estimates are less precise, but they continue to suggest that a larger share of consumption is explained by bias for lower-income people. On average across all incomes, the share of consumption explained by bias increases slightly to 47 percent.

5 Structural Model

In this section, we use the reduced-form moments from Sections 3 and 4 to calibrate a model of lottery demand and bias. In Section 6, we use this model for policy evaluation.

5.1 Setup

We assume a commonly used functional form for the perceived subutility from a lottery ticket:

$$u(\boldsymbol{a};\theta,\varepsilon_t)=\sum_k \Phi_k(\theta)m\left(w_k;\theta\right)-\varepsilon_t. \tag{13}$$

where w_k continues to denote the amount of prize k. The term Φ_k represents the decision weight applied to each potential outcome (which may depend on the prize probability π_k); $m(w_k)$ denotes the value function, representing the agent's utility gain from winning each potential prize; and $\varepsilon_t > 0$ is the taste shock representing transaction costs, with marginal distribution F_ε. We normalize this potential utility gain by the agent's local marginal utility of consumption, so that $\sum_k \Phi_k(\theta)m\left(w_k;\theta\right)$ can be interpreted in dollars as the certainty equivalent of the lottery, absent transaction costs.[18] This representation nests expected utility (the special case in which $\Phi_k = \pi_k$) as well as many modifications proposed in the literature on prospect theory and cumulative prospect theory.[19]

Type θ's demand function is the aggregation across many binary purchase decisions:

[18] Formally, if $\mathcal{U}(W)$ is the utility function over continuation wealth W and y denotes the agent's (net) continuation wealth absent any prize, then $m(w)=\frac{\mathcal{U}(y+w)-\mathcal{U}(y)}{\mathcal{U}'(y)}$.

[19] See, e.g., Kahneman and Tversky (1979), Prelec (1998), Gonzalez and Wu (1999), Wakker (2010), Filiz-Ozbay et al. (2015), and Bernheim and Sprenger (2020).

$$s(p, \boldsymbol{a}; \theta) = \Pr\left(\sum_k \Phi_k(\theta) m\left(w_k; \theta\right) - \varepsilon_t > p\right) \tag{14}$$

$$= F_\varepsilon\left[\sum_k \Phi_k(\theta) m\left(w_k; \theta\right) - p\right]. \tag{15}$$

As in Section 2, the assumption of quasilinearity in price implies no causal effects of income on demand. In Section 6.1, we show that incorporating either of our estimates of causal income effects from Section 4.2 has very little effect on optimal policy.

In Section 2.1.1, we showed how misperceived probabilities, present focus, and some other biases entered utility by multiplying $m\left(w\right)$. In line with those examples, we assume that a share χ of the difference $\Phi_k - \pi_k$ between the decision weight and the objective probability is due to bias, and the remaining share is due to normative factors such as anticipatory utility. Thus, normative utility is

$$v(\boldsymbol{a}; \theta, \varepsilon_t) = u(\boldsymbol{a}; \theta, \varepsilon_t) - \underbrace{\chi \sum_k \left(\Phi_k - \pi_k\right) m\left(w_k\right)}_{\gamma(\boldsymbol{a}; \theta, \varepsilon_t)}. \tag{16}$$

We now describe our functional form choices for the components of equations (14) and (16). For legibility, we sometimes suppress dependence on θ.

Utility of prizes. We assume that $m(w_k)$ arises from a concave constant relative risk aversion (CRRA) utility function over continuation wealth.[20] Appendix F.2 presents the formal derivation of m arising from continuation utility with a constant coefficient of relative risk aversion. Our baseline specification employs a CRRA parameter of 1, the central estimate from Chetty (2006), corresponding to logarithmic utility over continuation wealth.

Decision weights. Our demand model implies the following relationship between the decision

[20] This rules out the possibility that lottery demand is driven by convexity in the value function, as proposed by Friedman and Savage (1948). The theory that utility might be convex at high amounts of money is largely inconsistent with the bulk of the evidence. The robust stylized fact, first postulated as part of the "fourfold pattern of risk attitudes" by Tversky and Kahneman (1992), is that individuals are risk-seeking over low-probability gains, but risk-averse over moderate to high probability gains. This pattern is consistent with typical probability weighting functions, but not with convex utility functions. To illustrate, we conducted a supplementary survey with 200 subjects, summarized in Appendix E.3. In this survey, subjects could choose between a certain prize and an option with a 50 percent chance of a higher prize that yielded slightly lower expected value than the certain option; thus, choosing the risky option implies risk-seeking preferences. We find that for gambles on the order of hundreds of dollars, 12 percent of respondents chose the risky option (13 percent among those who purchased a lotto ticket in the past 12 months), while for gambles on the order of hundreds of millions of dollars, 2 percent of respondents chose the risky option (6 percent among those who purchased a lotto ticket in the past 12 months). These results are consistent with the fourfold pattern of risk attitudes, and inconsistent with the convexity conjecture of Friedman and Savage (1948).

weights Φ_k and the measured semi-elasticities:[21]

$$\frac{\zeta_k}{|\zeta_p|} = \frac{\Phi_k}{\pi_k} m'(w_k). \tag{17}$$

Thus, once $m(w_k)$ is specified, our semi-elasticity estimates from Section 3 provide direct estimates of the decision weights on the jackpot and the second-highest prize. To estimate the decision weights on smaller prizes, we adopt a two-parameter specification for the probability weighting function. Specifically, we utilize the "neo-additive" weighting function, axiomatized in Chateauneuf, Eichberger, and Grant (2007). This weighting function is discontinuous at the endpoints $\pi = 0$ and $\pi = 1$, and linear on the interval $(0, 1)$, with intercept b_0 and slope b_1. As we show in Appendix F.1, other standard parameterizations of probability weighting functions (Tversky and Kahneman 1992; Goldstein and Einhorn 1987; Prelec 1998) cannot simultaneously fit the estimates of Φ_1 and Φ_2 implied by equation (17). These other parameterizations imply that the ratio of second-prize to jackpot expected value semi-elasticities must be significantly larger than what we estimate, and in fact larger than 1.

Distribution of taste shocks. The distribution of ε is derived from our estimates of the semi-elasticities at various levels of lottery demand. Figure 2 documents a remarkably linear relationship between ln(sales) and the jackpot expected value first-stage fitted values, which implies a constant semi-elasticity of demand over the range of jackpot values in our data. We therefore assume a constant semi-elasticity across jackpot values over \$40 million. Below this level, we use an interpolation procedure to ensure that demand falls continuously to zero at the point where price exceeds the certainty equivalent. Appendix F.2 provides further details on our procedure for calibrating the distribution of ε.[22]

5.2 Model Calibration

We summarize the intuition behind the calibration here; Appendix F.2 presents details.

We calibrate the model assuming a single, representative lottery game with price, prizes, and probabilities set to match the current values for Powerball from Table 1. We assume that all prizes are reduced by 30 percent to account for income taxes.

The positive model of demand is fully characterized by the weighting function parameters b_0 and b_1. We first calibrate a representative consumer model using our estimates of aggregate demand semi-elasticities from Section 3.

To estimate the bias share parameter χ, we augment our observational data with an estimate

[21] To see this, note that changing the ticket price by $-dp < 0$ induces an increase in lottery purchases equal to $ds = |\zeta_p|\, s \cdot dp$. The increase in the expected value of prize level k that would generate the same rise in demand, denoted dx_k, satisfies $dp = \frac{\Phi_k}{\pi_k} m'(w_k) dx_k$. The change in demand generated by this jackpot increase is $ds = \zeta_k s \cdot dx_k$. Therefore, $|\zeta_p|\, s \frac{\Phi_k}{\pi_k} m'(aw_k) \cdot dx_k = \zeta_k s \cdot dx_k$, which gives the equation in the text.

[22] For additional intuition on why this assumption fully identifies the model, note that in the utility specification from equation (15) above, any two lotteries with the same net-of-price certainty equivalent $\sum_k \Phi_k m(w_k) - p$ have identical demand. Thus, our constant semi-elasticity assumption amounts to a mapping between the net certainty equivalent and quantity demanded, which allows us to compute demand for lotteries with any jackpot and price.

of the average counterfactual level of lottery spending if consumers were unbiased, based on the survey results in Section 4.3. Acknowledging that this is a strong assumption, we interpret the share of consumption explained by bias from Section 4.3 as the *causal* effect of bias, allowing us to compute the counterfactual normative level of consumption for each individual in our survey. Using equation (16), we then compute the bias share χ that would generate this normative level of consumption.

We allow heterogeneity by exploiting the survey microdata on household income and lottery spending. We partition income into the three bins displayed in Figure 4: less than \$50,000, \$50,000 to \$100,000, and more than \$100,000. Within each income bin, we specify three types of individuals: those who consume zero lotteries, and among positive consumers, those with below- vs. above-median consumption. We assume non-consumers never purchase a lottery ticket in any counterfactual scenario. Among the remaining types, we draw the average level of lottery spending and counterfactual unbiased spending from our survey, and we assume our semi-elasticity estimates are homogeneous across types. This allows us to compute type-specific parameters.

5.3 Parameter Estimates

Table 7 presents the estimated model parameters b_0, b_1, and χ in both the representative agent and heterogeneous specifications. The probability weighting function intercept term b_0, though small in absolute terms, is large relative to the jackpot probability π_1, implying substantial overweighting of small probabilities. Approximately 40 percent of the wedge between decision weights and objective probabilities is explained by bias, with lower-income, above-median lottery consumers being the most biased.

6 Implications for Optimal Lottery Design

We focus our analysis on the optimal choice of two attributes to which consumer responses are best identified in our data: ticket price and jackpot size. We present two calculations. First, we implement the sufficient statistics formulas from Section 2.2 to separately compute the optimal ticket price and jackpot (while holding the other fixed). As is standard in applications of sufficient statistics approaches, we employ the simplifying approximation that the elasticities and other key statistics are unaffected by the policy changes we consider; Kleven (Forthcoming) would call this a "structural" approach. Second, we jointly compute the optimal price and jackpot using the structural model from the previous section. The structural model allows the key statistics to adjust in response to policy changes and also allows us to estimate welfare. When separately computing the optimal ticket price or jackpot (while holding the other fixed), the structural model delivers very similar answers to our sufficient statistics formulas, which suggests that the estimates are not sensitive to specific parametric assumptions.

We assume that social marginal welfare weights $g(\theta)$ reflect the marginal utility of consumption and are declining with net-of-tax income $c(\theta)$, which we impute based on our survey measure of pre-tax income using the pre- to post-tax income mapping from Piketty, Saez, and Zucman (2018). We follow Saez (2002b) in setting these weights proportional to $c(\theta)^{-\nu}$, with $\nu = 1$ in our baseline specifications.

6.1 Optimal Policy from Sufficient Statistics Formulas

From Proposition 1, the sufficient statistics formula for the lottery ticket's markup over the marginal cost is $\bar{\gamma}(1+\sigma) - Cov\left[g(\theta), s(\theta)\right]/\bar{\zeta}_p \bar{s}$. We can rearrange this to give an expression for the optimal lottery ticket price, which we explain further below:

$$p = \bar{\gamma}(1+\sigma) - \frac{Cov\left[g(\theta), s(\theta)\right]}{\bar{\zeta}_p \bar{s}} + \sum_k \pi_k w_k + o \tag{18}$$

$$= 1.40(1+0.10) - \frac{0.45}{|-0.4958| \times 7.61} + 0.63 + 0.2 \tag{19}$$

$$\approx 2.32. \tag{20}$$

Equation (19) substitutes in our estimate of each statistic. The first term is the bias correction term. We approximate money-metric bias $\gamma(\theta)$ using our survey data from Section 4.3: we divide each individual's amount of consumption explained by bias by our estimated price semi-elasticity of demand, $\gamma_i = \hat{\boldsymbol{\tau}}\tilde{\boldsymbol{b}}_i/\bar{\zeta}_p$. For example, if bias increases consumption by 60 percent and a \$1 price increase reduces consumption by 30 percent, then bias would be $\gamma_i = 60\%/(30\%/\$1) = \$2$. Allcott, Lockwood, and Taubinsky (2019) formalize this approach. This implies an average marginal bias of $\bar{\gamma} \approx \$1.40$ per ticket.[23] To estimate the progressivity of bias correction σ, we combine the bias estimates with each individual's welfare weight g. This gives $\sigma = 0.10$, reflecting the higher bias among lower-income individuals.

The second term, $-\frac{Cov[g(\theta),s(\theta)]}{\bar{\zeta}_p \bar{s}}$, quantifies the optimal price reduction due to redistributive concerns. Since lottery spending doesn't decline much with income, this covariance is small. We can use this term to quantify the importance of causal income effects, as discussed in Section 2.2.4. Our two estimates of causal income elasticities from Appendix Table A12 suggest that either 82 percent or 275 percent of the downward slope in lottery spending across incomes is attributable to income-correlated preference heterogeneity, rather than causal income effects.[24] Assuming that

[23] Specifically, we assume a homogeneous price semi-elasticity, so that

$$\bar{\gamma} = \frac{\mathbb{E}[\gamma_i \bar{\zeta}_p s_i]}{\bar{\zeta}_p \bar{s}}$$

Our survey measures expenditures, which must be converted into a measure of quantity $s(\theta)$ to compute the covariances in equation (18); we use $p = \$2$. The covariance terms in which $s(\theta)$ appears are divided by the mean of $s(\theta)$, so the results are insensitive to the price used for this conversion.

[24] Causal income effects plus between-income preference heterogeneity must sum to the observed cross-sectional profile of spending in Figure 4. Thus our causal income elasticity of -0.02 accounts for 18 percent of the cross-sectional income elasticity (-0.111) reported in Appendix Table A12, implying that the remaining 82 percent is attributable to preference heterogeneity. Our alternative causal income elasticity of 0.194 suggests that causal income effects have the opposite sign of the cross-sectional profile—i.e., lotteries are a normal good—implying that the preference for lotteries is declining steeply enough with income as to more than offset the positive causal income effects.

these percentages are constant across the income distribution, Proposition C1 in Appendix C shows that to account for causal income effects, we should rescale $Cov\left[g(\theta), s(\theta)\right]$ by either 0.82 or 2.75 in the optimal price formula. But because $Cov\left[g(\theta), s(\theta)\right]$ is small, neither adjustment changes the results very much.

The final two terms in equation (18) reflect the lottery ticket's marginal cost. $\sum_k \pi_k w_k$ is the net-of-income-tax expected payout of prizes, which we set to reflect the current Powerball format in Table 1, reduced by the 30 percent income tax rate. o is the overhead cost, which we assume is 10 percent of the current \$2 price, following the discussion in Section 1.

This calculation gives an optimal price of \$2.32, which is close to the current Powerball ticket price of \$2. The estimate of money-metric bias matters a lot for the optimal price: the corrective term $\bar{\gamma}(1+\sigma)$ is about \$1.50, and so the optimal ticket price would be significantly lower in the absence of bias.

Similarly, we can use Proposition 1 to estimate the optimal jackpot amount. We rearrange equation (7) to isolate the jackpot expected value from the summation on the left-hand side, and we use the fact that the average cost of a ticket rises one-for-one with a change in the jackpot expected value, implying $\frac{\partial C}{\partial a} = \bar{s}$, where $a = \pi_1 w_1$ is the jackpot expected value. This gives the following result:

$$\pi_1 w_1 = -\bar{\gamma}(1+\sigma_1) + \frac{\bar{\kappa} - \bar{\rho} + Cov\left[g(\theta), \kappa(\theta) - \rho(\theta)\right] - \bar{s}}{\bar{\zeta}_1 \bar{s}} + p - \sum_{k=2}^{K} \pi_k w_k - o \tag{21}$$

$$= -1.40(1+0.10) + \frac{14.65 - 5.59 + 0.07 - 7.61}{0.9543 \times 7.61} + 2 - 0.22 - 0.2 \tag{22}$$

$$\approx 0.24. \tag{23}$$

The key new terms for this calculation are $\bar{\kappa}$, the average monthly willingness-to-pay to increase the jackpot expected value by \$1 (through w_1 rather than π_1), and $\bar{\rho}$, the average portion of that WTP that is driven by bias. From Section 2, $\bar{\kappa} = -\bar{\zeta}_1/\bar{\zeta}_p \cdot \bar{s} = \$14.65$ per month. To estimate $\bar{\rho}$, we assume that the normative share of WTP for a higher jackpot is the same as the normative share of WTP for lottery tickets. We approximate the latter by (WTP $-\gamma$)/WTP, based on the WTP for their median ticket purchased, computed using a demand curve with constant semi-elasticity $\bar{\zeta}_p$.[25] This average normative share is 0.86, and $\bar{\rho} \approx \$5.59$ per month. $\bar{\gamma}$ and σ are computed as before, but weighting individuals by demand responses to jackpot expected value changes, rather than price changes.[26]

The optimal net-of-tax jackpot expected value is \$0.24. The current average Powerball net-of-tax jackpot expected value is \$0.41.

[25]Letting p_0 and $s_0(\theta)$ denote type θ's status quo price and consumption, a constant semi-elasticity demand function locally satisfies $\ln(s(p;\theta) - s_0(\theta)) = \zeta_p(p - p_0)$. The WTP for the median unit is thus the p that satisfies this equation when $s(p;\theta) = s_0(\theta)/2$ and $p_0 = 2$. We impose a floor of zero on $\kappa - \rho$, which binds for a small number of observations, since it is unlikely that the normative value declines when the jackpot rises.

[26]Specifically, we assume a homogeneous prize semi-elasticity, so that $\bar{\gamma} = \frac{\mathbb{E}[\gamma_i \bar{\zeta}_1 s_i]}{\bar{\zeta}_1 \bar{s}}$.

6.2 Optimal Policy from Structural Model

We now study optimal policy using the heterogeneous structural model calibrated in Section 5.3. Figure 10 plots the estimated surplus (per \$100 of aggregate spending) that each consumer type derives from the status quo representative lottery. Total perceived surplus is decomposed into lottery revenues, which we assume are distributed evenly across individuals, and perceived consumer surplus, reflecting consumers' WTP for lotteries. Behavioral biases reduce actual (normative) consumer surplus below perceived surplus, to the point that actual surplus may be negative. We plot actual surplus both for our baseline bias estimates and under the alternative assumption that bias share $\chi(\theta)$ is 2 times as large. We find substantial heterogeneity in surplus conditional on income, with heavier consumers deriving far more perceived utility from lotteries, while also incurring larger bias costs, especially in the "2x bias" specification. Low-income individuals with above-median consumption receive approximately zero actual surplus under our baseline bias estimates; the point estimate is just slightly negative.

Figure 11 presents our baseline optimal policy estimates. Both panels display plots of social welfare (relative to a counterfactual with no lottery) across a range of prices (Panel a) and jackpots (Panel b). These figures report results for three different assumptions about bias. The dashed green lines report results under the assumption that all observed demand is fully normatively justified ($\chi(\theta) \equiv 0$). Unsurprisingly, lotteries generate substantial surplus in this case. The optimal "effective tax rate"—by which we mean the implicit tax rate (share of ticket price that is a markup over marginal cost) after reducing marginal cost to account for revenues recovered by a 30 percent income tax—is lower than in the status quo because there is no corrective benefit from reducing consumption. In fact, redistributive considerations favor a small subsidy, because poorer consumers buy more lottery tickets. Relative to the current Powerball design, decreasing the price (Panel a) or increasing the jackpot (Panel b) would increase welfare in this case.

The solid blue lines in Figure 11 show the welfare gain from the lottery given our empirical estimates of bias. The lottery generates lower welfare gains, although welfare remains positive under the status quo price and jackpot. Holding prizes fixed, the optimal price is somewhat higher than the status quo of \$2 per ticket, although the welfare benefit of increasing the price beyond \$2 is small.[27] Holding price fixed, the optimal jackpot is somewhat lower than the status quo, although the welfare difference between the optimum and the status quo is again small. Both of these estimates are reassuringly close to the estimates from our sufficient statistics formulas, suggesting that the estimates are not sensitive to specific parametric assumptions about how our key statistics vary out-of-sample with policy parameters.

Finally, the dot-dashed red lines plot the welfare gains assuming that the bias share $\chi(\theta)$ is 2 times our empirical estimates. In this case, the status quo lottery reduces welfare.

We also jointly solve for the optimal *combination* of ticket price and jackpot size. Table 8 presents results. The third column presents the effective tax rate. Row 1 presents estimates under

[27] This flatness of welfare across higher prices comes from the fact that the demand response $ds(\theta)/dp$ becomes small as price rises, so there are negligible corrective benefits from further price adjustments.

our baseline assumptions. The optimal price is \$4.73, the optimal jackpot expected value is \$0.92, and the optimal effective tax rate is 72 percent. The optimal effective tax rate is moderately higher than the current Mega Millions and Powerball effective rates of about 60 percent; the effective taxes on instant games are even lower. Our model's optimal price and jackpot are much different from the status quo, however. This is because our estimates imply substantial normative utility from higher jackpots, so a revenue-neutral perturbation that jointly raises jackpots and prices above status quo levels would increase consumer surplus.

The remaining rows present optimal design under alternative assumptions. Rows 2 and 3 report alternative bias assumptions. When consumers are completely unbiased (row 2), the optimal effective tax rate is close to zero, and in fact is slightly negative due to regressivity concerns. When consumers are 50 percent more biased than baseline (row 3), the optimal effective tax rate increases. At even higher levels of bias, such as the "2x bias" specification shown in Figures 10 and 11, welfare is maximized by eliminating the lottery altogether.

Rows 4 and 5 consider alternative assumptions about the curvature of individual utility. Our baseline assumption was a CRRA parameter of 1 (Chetty 2006), and we consider alternative values of 0.8 and 1.5. Values outside this range generate extreme implications for consumers' willingness to pay for lottery tickets suggestive of model misspecification.[28]

Rows 6 and 7 consider weaker and stronger redistributive preferences, with welfare weights proportional to $c(\theta)^{-0.25}$ and $c(\theta)^{-4}$, rather than $c(\theta)^{-1}$ as in our baseline, where $c(\theta)$ denotes type θ's net-of-tax income. Row 8 considers the possibility that our estimated decision weight on lower prizes reflects inattention to *variation* in the California second prize, rather than a low weight on those prizes' expected value. This specification assumes a "structural" second-prize semi-elasticity of $\bar{\zeta}_2 = \bar{\zeta}_1$ as a conservative upper bound. Row 9 assumes $b_1 = 1$ for all consumers, implying that decision weights depart from expected utility maximization only for the jackpot and, correspondingly, only the jackpot decision weight is subject to bias. Row 10 examines the case where the jackpot varies over time, cycling over 10 values corresponding to the average value of each decile of Powerball jackpots in our data. In this case, jackpot expected value is adjusted by rescaling all values of the jackpot proportionally. Row 11 reports results when we estimate bias with the ORIV measurement error correction. The results in rows 6–11 are all broadly similar to row 1.

[28]In our model, the CRRA parameter affects WTP for lottery tickets. With more curvature, the value function $m(w)$ is less sensitive to variation in the jackpot, requiring a higher decision weight Φ_1 to rationalize our empirical estimates of $\hat{\bar{\zeta}}_1$. This in turn implies more utility from the jackpot and a higher WTP. CRRA values below 0.8 imply low WTP for the representative lottery ticket, to the point that if the jackpot declines to the Powerball jackpot reset value, demand falls to zero. In our baseline specification with CRRA=1, individuals' average WTP for the representative lottery ticket conditional on purchase—which must be higher than the \$2 ticket price—is \$4.76. On the other hand, a CRRA value of 1.5 implies a much higher WTP of \$14.38 for each purchased ticket. In the short 200-subject supplementary survey described in Appendix E.3, we find that for a Mega Millions lottery with a \$250 million jackpot, participants have a mean and median WTP of \$6.11 (with 95 percent confidence interval $(\$5.25, \$6.97)$) and \$4.00, respectively. The mean and median WTP among those who have purchased a lotto ticket in the last 12 months are similar: \$6.22 (with 95 percent confidence interval $(\$4.52, \$7.92)$) and \$4.00, respectively. These estimates are in the ballpark of our model's prediction at a CRRA parameter of 1, and reinforce our claim that alternative assumptions would deliver implausible predictions about WTP for lottery tickets.

The remaining rows explore the role of differences in bias and consumption across the income distribution. Rows 12 and 13 report results when the bias share $\chi(\theta)$ is assumed to be the same at all incomes, while still varying across the three consumption groups (row 12), or homogeneous across incomes and consumption groups (row 13). Row 14 reports results when we assume a steeper decline in expenditures across income groups, assuming consumption in the bottom (top) income bin is the highest (lowest) level in the 95 percent confidence intervals of consumption in each bin.

7 Conclusion

People have long debated whether state-run lotteries are a regressive "tax on people who are bad at math" or a win-win that generates both enjoyment and government revenues. In this paper, we formalize a model of socially optimal lottery design and provide a novel set of empirical results necessary to implement the model. We find that aggregate demand responds more to a \$1 change in jackpot expected value than it does to a \$1 price change or to a \$1 change in second prize expected value, a result consistent with a particular form of probability weighting. In our new nationally representative survey, lottery spending is correlated with proxies of behavioral bias such as innumeracy and poor statistical reasoning; our bias proxies explain 43 percent of lottery spending. Using these empirical moments, we calibrate a structural model of lottery demand and implement sufficient statistics formulas to study optimal policy.

In our model, lotteries increase overall welfare, although the gains are unequally distributed: below-median spenders benefit from redistributed revenues, while above-median spenders receive the least—and not necessarily positive—surplus. This suggests that quantity restrictions such as monthly spending limits could increase welfare. The model's socially optimal implicit tax rate is slightly higher than the current Mega Millions and Powerball designs, which suggests that recent trends toward higher implicit taxes in those games may have increased welfare.

Our optimal policy and welfare results hinge on estimates of bias, which highlights the importance of our survey work but also motivates additional research in that area. There are many other caveats to our results, and we think of this paper as just a first step toward studying state-run lotteries through the lens of behavioral optimal taxation. Our theoretical and empirical techniques may also be more broadly useful for studying regulation of non-price attributes in the presence of behavioral bias.

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Table 1: **Mega Millions and Powerball Prices and Prize Structures**

	Mega Millions			Powerball		
Start date	June 22, 2005	October 19, 2013	October 28, 2017	January 7, 2009	January 15, 2012	October 7, 2015
Ticket price	$1	$1	$2	$1	$2	$2
Format	5/56 + 1/46	5/75 + 1/15	5/70 + 1/25	5/59 + 1/39	5/59 + 1/35	5/69 + 1/26
Jackpot (average)	$58 million	$97 million	$173 million	$66 million	$112 million	$170 million
Reset value	$12 million	$15 million	$40 million	$20 million	$40 million	$40 million
Probability	1/175,711,536	1/258,890,850	1/302,575,350	1/195,249,054	1/175,223,510	1/292,201,338
Expected value	$0.33	$0.38	$0.57	$0.34	$0.64	$0.58
Second prize	$250,000	$1 million	$1 million	$200,000	$1 million	$1 million
Probability	1/3,904,701	1/18,492,204	1/12,607,306	1/5,138,133	1/5,153,633	1/11,688,054
Expected value	$0.064	$0.054	$0.079	$0.039	$0.19	$0.086
Third prize	$10,000	$5,000	$10,000	$10,000	$10,000	$50,000
Probability	1/689,065	1/739,688	1/931,001	1/723,145	1/648,976	1/913,129
Expected value	$0.015	$0.0068	$0.011	$0.014	$0.015	$0.055
Fourth prize	$150	$500	$500	$100	$100	$100
Probability	1/15,313	1/52,835	1/38,792	1/19,030	1/19,088	1/36,525
Expected value	$0.0098	$0.0095	$0.013	$0.0053	$0.0052	$0.0027
Fifth prize	$150	$50	$200	$100	$100	$100
Probability	1/13,781	1/10,720	1/14,547	1/13,644	1/12,245	1/14,494
Expected value	$0.011	$0.0047	$0.014	$0.0073	$0.0082	$0.0069
Sixth prize	$7	$5	$10	$7	$7	$7
Probability	1/306	1/766	1/606	1/359	1/360	1/580
Expected value	$0.023	$0.0065	$0.016	$0.019	$0.019	$0.012
Seventh prize	$10	$5	$10	$7	$7	$7
Probability	1/844	1/473	1/693	1/787	1/706	1/701
Expected value	$0.012	$0.011	$0.014	$0.0089	$0.0099	$0.01
Eighth prize	$3	$2	$4	$4	$4	$4
Probability	1/141	1/56	1/89	1/123	1/111	1/92
Expected value	$0.021	$0.035	$0.045	$0.032	$0.036	$0.043
Ninth prize	$2	$1	$2	$3	$4	$4
Probability	1/75	1/21	1/37	1/62	1/55	1/38
Expected value	$0.027	$0.047	$0.055	$0.049	$0.072	$0.10
Probability, any prize	1/40	1/15	1/24	1/35	1/32	1/25
Lower prize expected value	$0.18	$0.17	$0.25	$0.17	$0.36	$0.32
Total expected value	$0.51	$0.55	$0.82	$0.51	$1.00	$0.90

Notes: This table reports the prizes, win probabilities, and expected values corresponding to each prize level and the overall ticket for all Mega Millions and Powerball formats used since 2010. The non-jackpot prize amounts are the fixed prizes offered in states other than California. The expected value is computed simply as the prize (or average prize) multiplied by the win probability.

Table 2: **Descriptive Statistics: Mega Millions and Powerball Sales and Prize Data**

	Obs.	Mean	Std. dev.	Min	Max
Jackpot ($millions)	2,035	116.3	115.1	12.0	1,600.0
California 2nd prize pool ($000s)	1,705	893.4	982.7	39.3	7,660.3
Nationwide ticket sales (millions)	2,035	23.0	30.6	8.8	651.9
California ticket sales (millions)	1,705	3.3	5.1	0.9	120.2

Notes: This table presents descriptive statistics for draw-level Mega Millions and Powerball sales and prize data from June 2010 through February 2020. Jackpot amounts and sales data are from LottoReport.com; California second prize amounts are from https://www.calottery.com/draw-games/. Jackpots are advertised jackpot amounts, and ticket sales exclude the Just the Jackpot, Power Play, and Megaplier add-ons.

Table 3: **Prize Semi-Elasticity Estimates**

(a) **Jackpot Semi-Elasticity: National Sales**

	(1) OLS	(2) IV	(3) IV
Jackpot expected value ($)	0.9984***	0.9437***	0.9543***
	(0.0309)	(0.0383)	(0.0374)
Lags in $\boldsymbol{H}$	0	1	4
Quadratic terms in $\boldsymbol{H}$	No	No	Yes
Observations	2,035	2,035	2,035

(b) **Jackpot and Second Prize Semi-Elasticities: California Sales Only**

	(1) OLS	(2) IV	(3) IV	(4) IV
Jackpot expected value ($)	1.0764***	0.9972***	1.0191***	0.9917***
	(0.0418)	(0.0470)	(0.0494)	(0.0538)
2nd prize expected value ($)	0.0441	0.0578	0.0457	-0.0301
	(0.0390)	(0.0744)	(0.0633)	(0.0723)
2nd prize expected value ×				0.1238**
below-median jackpot expected value				(0.0522)
Below-median jackpot expected value				-0.0583***
				(0.0145)
Lags in $\boldsymbol{H}$	0	1	4	4
Quadratic terms in $\boldsymbol{H}$	No	No	Yes	Yes
Observations	1,705	1,701	1,695	1,695

Notes: This table presents estimates of equation (10), a regression of the natural log of sales on prize expected values, controlling for a vector of lagged logged sales and prize expected values as well as game-format, game-regional coverage, game-quarter of sample, and game-weekend fixed effects. The IV regressions instrument for prize expected values with a forecast based on the previous period prize amount and an indicator for whether the prize was won in the previous period. Panel (a) uses nationwide sales, while Panel (b) uses California sales only. The samples include all Mega Millions and Powerball drawings from June 2010 to February 2020; the sample in Panel (b) is smaller because California did not join Powerball until April 2013. Newey-West standard errors allowing up to ten lags are in parentheses. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

Table 4: **Price Semi-Elasticity**

	(1) Pooled	(2) Powerball	(3) Mega Millions	(4) Pooled
Price × 12-month window	-0.4958***	-0.5588***	-0.4305***	-0.4978***
	(0.0435)	(0.0573)	(0.0568)	(0.0473)
12-month window	0.7412***	0.8565***	0.6223***	0.7417***
	(0.0745)	(0.0971)	(0.0954)	(0.0768)
Jackpot expected value ($)	0.8770***	0.8412***	0.9019***	
	(0.0459)	(0.0406)	(0.0777)	
Jackpot ($millions)				0.0034***
				(0.0004)
Observations	625	312	313	625

Notes: This table presents estimates of equation (11), a regression of the natural log of sales on ticket price interacted with an indicator for the 12-month window—six months before and six months after—around a price change event and the 12-month window indicator, controlling for either the jackpot expected value or the jackpot amount. Columns 1 and 4 pool the data from both the Powerball and Mega Millions price changes and also include a price-change event fixed effect, while columns 2 and 3 consider each price change in isolation. We use Newey-West standard errors with up to ten lags. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

Table 5: **Descriptive Statistics: 2020 Survey Data**

(a) **Demographics**

	Obs.	Mean	Std. dev.	Min	Max
Household income ($000s)	2,879	72.12	53.08	5	250
Years of education	2,879	14.32	2.26	4	20
Age	2,879	48.82	16.79	18	91
1(Male)	2,879	0.50	0.50	0	1
1(White)	2,879	0.66	0.47	0	1
1(Black)	2,879	0.11	0.31	0	1
1(Hispanic)	2,879	0.16	0.36	0	1
Household size	2,879	3.04	1.62	1	6
1(Married)	2,879	0.53	0.50	0	1
1(Employed)	2,879	0.63	0.48	0	1
1(Urban)	2,879	0.83	0.37	0	1
1(Attend church)	2,879	0.36	0.48	0	1
Political ideology	2,878	3.83	1.59	1	7

(b) **Spending and Income Effects**

	Obs.	Mean	Std. dev.	Min	Max
Monthly lottery spending ($)	2,877	15.16	38.04	0	1,000
Income change (%)	2,871	0.17	18.52	-50	50
Spending change (%)	2,870	-5.46	19.17	-50	50
Self-reported income effect (%)	2,855	-1.41	16.28	-50	50

(c) **Proxies for Preferences and Biases**

	Obs.	Mean	Std. dev.	Min	Max
Unwillingness to take risks	2,879	-3.93	1.38	-7	-1
Financial risk aversion	2,879	3.05	0.82	1	4
Lottery seems fun	2,875	0.16	1.83	-3	3
Enjoy thinking about winning	2,871	0.81	1.93	-3	3
Self-control problems	2,875	-0.34	1.10	-3	3
Financial literacy	2,879	0.77	0.25	0	1
Financial numeracy	2,879	0.63	0.32	0	1
Gambler's Fallacy	2,879	0.29	0.39	0	1
Non-belief in Law of Large Numbers	2,879	0.42	0.18	0.00	0.93
Expected value miscalculation	2,879	0.69	0.37	0	1
Overconfidence	2,865	-0.01	0.51	-4.95	4.95
Expected returns	2,871	0.28	0.20	0.05	0.95
Predicted life satisfaction	2,850	2.34	4.76	-10	10

Notes: This table presents descriptive statistics for our 2020 AmeriSpeak survey. Panel (a) presents demographics, Panel (b) presents spending and income effects, and Panel (c) presents proxies for preferences and biases. Section 4.1 summarizes the coding of these variables.

Table 6: **Regressions of Monthly Lottery Spending on Bias Proxies**

	(1) OLS	(2) OLS	(3) OLS	(4) OLS	(5) OLS	(6) ORIV
Self-control problems	0.281***	0.414***	0.293***	0.280***	0.279***	0.708***
	(0.029)	(0.033)	(0.031)	(0.030)	(0.035)	(0.143)
Financial illiteracy	0.123***	0.190***	0.142***	0.127***	0.137***	0.182*
	(0.032)	(0.034)	(0.030)	(0.032)	(0.040)	(0.099)
Statistical mistakes	0.101***	0.157***	0.133***	0.097***	0.096***	0.019
	(0.027)	(0.031)	(0.027)	(0.027)	(0.033)	(0.097)
Overconfidence	0.029	0.034	0.032	0.027		
	(0.023)	(0.029)	(0.024)	(0.024)		
Expected returns	0.068***	0.199***	0.098***	0.068***	0.051*	-0.025
	(0.024)	(0.028)	(0.025)	(0.024)	(0.029)	(0.079)
Predicted life satisfaction	0.006	0.153***	0.024	0.005	-0.009	-0.082
	(0.025)	(0.027)	(0.025)	(0.024)	(0.029)	(0.097)
Risk aversion	-0.012		-0.013	-0.009	-0.012	0.028
	(0.026)		(0.025)	(0.027)	(0.031)	(0.046)
Lottery seems fun	0.614***		0.604***	0.608***	0.606***	1.082***
	(0.029)		(0.029)	(0.029)	(0.033)	(0.085)
Enjoy thinking about winning	0.167***		0.171***	0.176***	0.175***	0.064
	(0.029)		(0.029)	(0.029)	(0.033)	(0.084)
ln(household income)	0.104***			0.104***	0.099**	0.096**
	(0.036)			(0.036)	(0.043)	(0.046)
ln(years of education)	-0.652***			-0.635***	-0.552***	-0.160
	(0.170)			(0.173)	(0.192)	(0.211)
Other demographics	Yes	No	No	Yes	Yes	Yes
State fixed effects	Yes	No	No	No	Yes	Yes
R^2	0.41	0.16	0.36	0.39	0.40	0.57
Observations	2,810	2,810	2,810	2,810	2,072	4,144
Clusters	2,810	2,810	2,810	2,810	2,072	2,072

Notes: This table presents estimates of equation (12), a regression of ln(1+*monthly lottery spending*) on bias proxies, preference proxies, demographic controls, and state fixed effects using data from our AmeriSpeak surveys. "Other demographics" includes age, household size, political ideology, and indicators for male, black, white, Hispanic, married, employed, urban area, and attends religious services at least once a month. Columns 1–5 present OLS estimates. Column 6 presents Obviously Related Instrumental Variables estimates: we estimate equation (12) in a stacked dataset with the 2021 bias and preference proxies below the 2020 bias and preference proxies, instrumenting for the 2020 variables with their 2021 values and vice versa, while clustering standard errors by respondent. Robust standard errors are in parentheses. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

Table 7: **Estimates of Parameters in Structural Model**

Representative agent model						
	b_0	b_1	χ			
	1.12×10^{-6}	0.27	0.41			
Model with heterogeneity						
	Below median s			Above median s		
	b_0	b_1	χ	b_0	b_1	χ
Low incomes	2.16×10^{-6}	0.36	0.31	2.16×10^{-6}	0.36	0.42
Middle incomes	1.08×10^{-6}	0.27	0.21	1.08×10^{-6}	0.27	0.4
High incomes	5.8×10^{-7}	0.23	0.2	5.8×10^{-7}	0.23	0.43

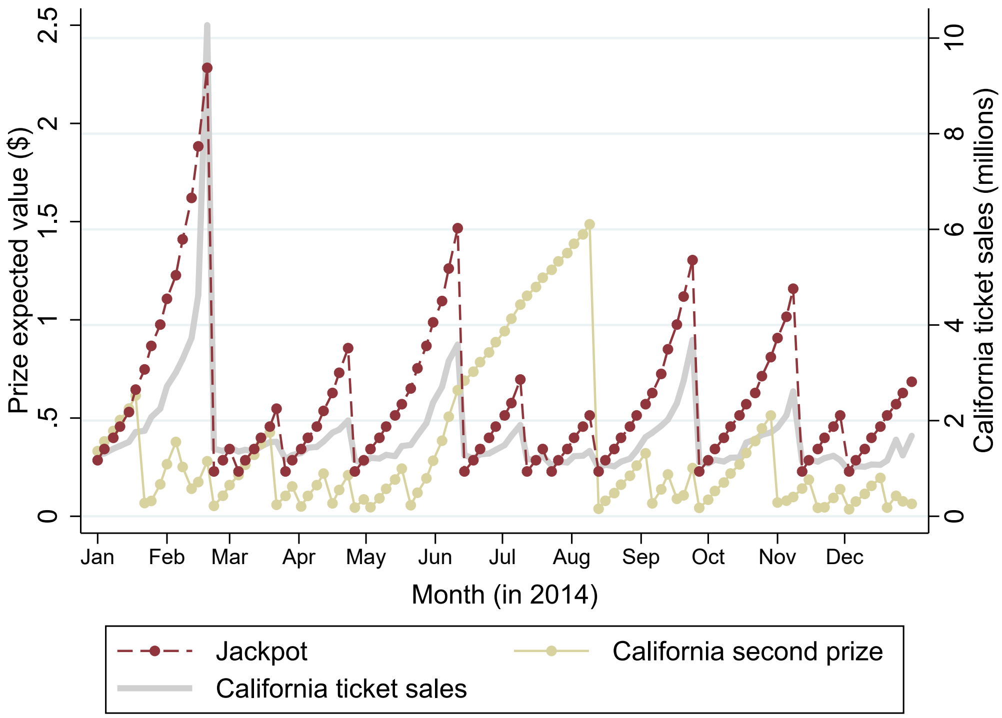
Notes: This table reports estimates of parameters for the structural models, both in the representative agent case, and in the heterogeneous agent case with three income levels and two levels of consumption. Parameters b_0 and b_1 are the intercept and slope parameters of the neo-additive probability weighting function. The parameter χ represents the share of the departure from expected utility weighting that is attributed to bias, as opposed to normative preferences. (Lottery non-consumers are omitted from the lower panel.) In the heterogeneous agent specification, semi-elasticities are assumed to be constant, resulting in homogeneous values of b_0 and b_1 conditional on income. See Section 5.1 for details.

Table 8: **Optimal Lottery Tax and Attributes Under Alternative Assumptions**

	Ticket price ($)	Average jackpot expected value ($)	Effective tax rate
1. Baseline	4.73	0.92	0.72
2. Completely unbiased	1.97	1.58	-0.01
3. 50 percent more biased	5.16	0.56	0.81
4. CRRA = 0.8	3.89	0.92	0.65
5. CRRA = 1.5	9.59	0.87	0.87
6. Weaker redistribution	4.75	0.94	0.71
7. Stronger redistribution	4.70	0.88	0.72
8. Higher value of $\bar{\zeta}_2=\bar{\zeta}_1$	5.05	0.96	0.73
9. All bias is on jackpot	4.76	0.92	0.72
10. Variable jackpot	4.47	0.73	0.72
11. Measurement error correction	4.86	0.86	0.74
12. Same bias share across incomes	4.70	0.92	0.71
13. Same bias share for everyone	3.64	1.22	0.55
14. Steeper decline across incomes	4.62	0.88	0.72

Notes: This table reports key features of the optimal representative lottery according to our structural model. The first two columns report the jointly optimal price and jackpot expected value of a lottery ticket resembling a current Powerball ticket. The third column reports the optimal effective tax rate, calculated as the share of price that is a mark-up over marginal cost (net-of-tax total ticket expected value plus overhead). See Section 6.2 for details about the specifications considered in each row.

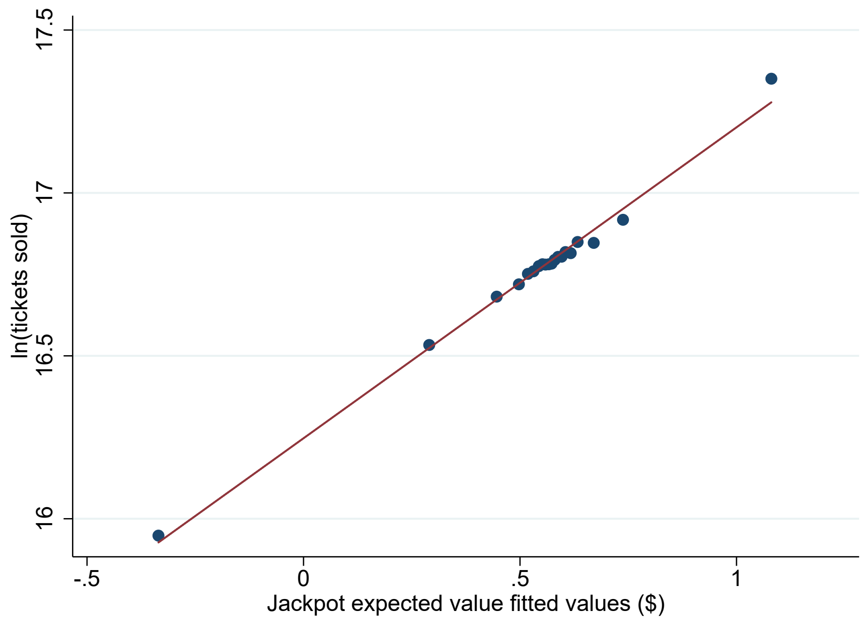
Figure 1: **Powerball Prizes and Ticket Sales in 2014**



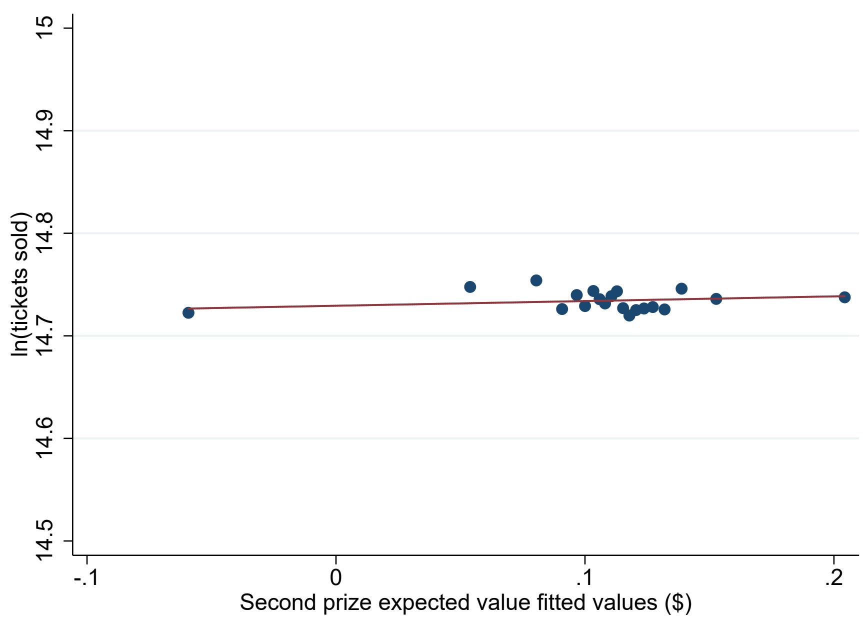
Notes: This figure presents the expected values of the jackpot and California second prize as well as California ticket sales for each Powerball drawing in 2014.

Figure 2: **Responsiveness of Ticket Sales to Jackpot and California Second Prize**

(a) **National Sales versus Jackpot Expected Value First-Stage Fitted Values**



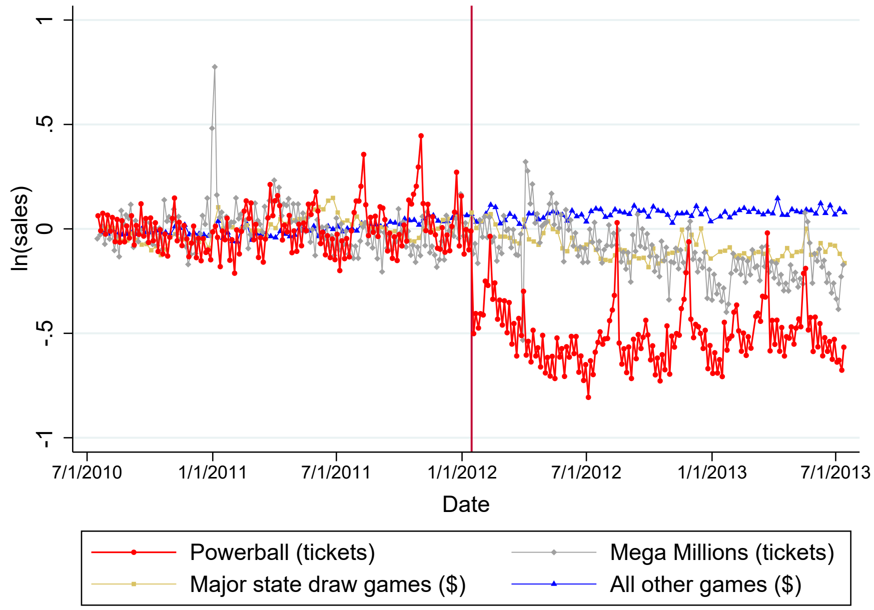
(b) **California Sales versus Second Prize Expected Value First-Stage Fitted Values**



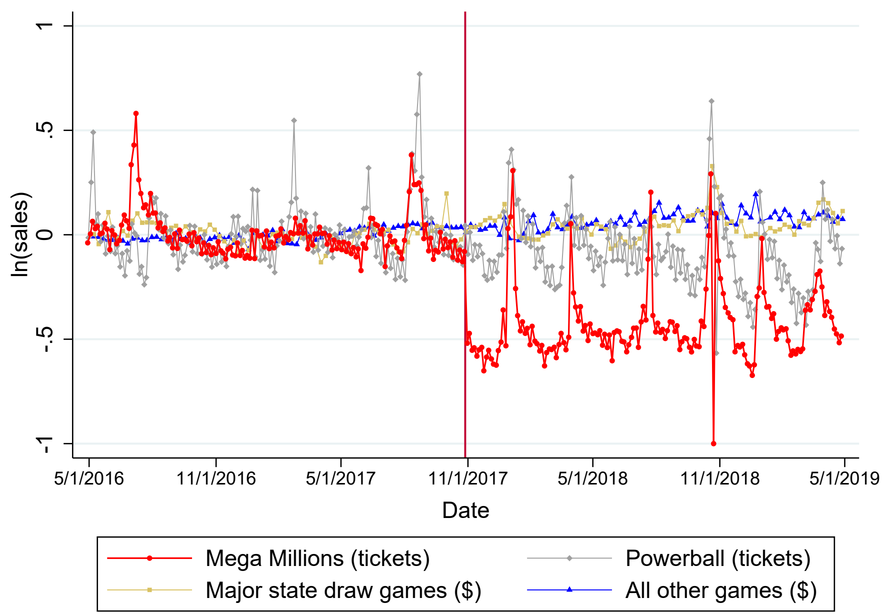
Notes: Panel (a) presents a binned scatter plot of the natural log of national sales against the jackpot expected value fitted values from the first stage of equation (10), residual of the controls in that equation. Panel (b) presents a binned scatter plot of the natural log of California sales against the California second prize expected value fitted values from the first stage of equation (10), residual of the jackpot expected value first-stage fitted values and the other controls in that equation. The sample includes all Mega Millions and Powerball drawings from June 2010 through February 2020.

Figure 3: **Price Change Event Studies**

(a) **Powerball**

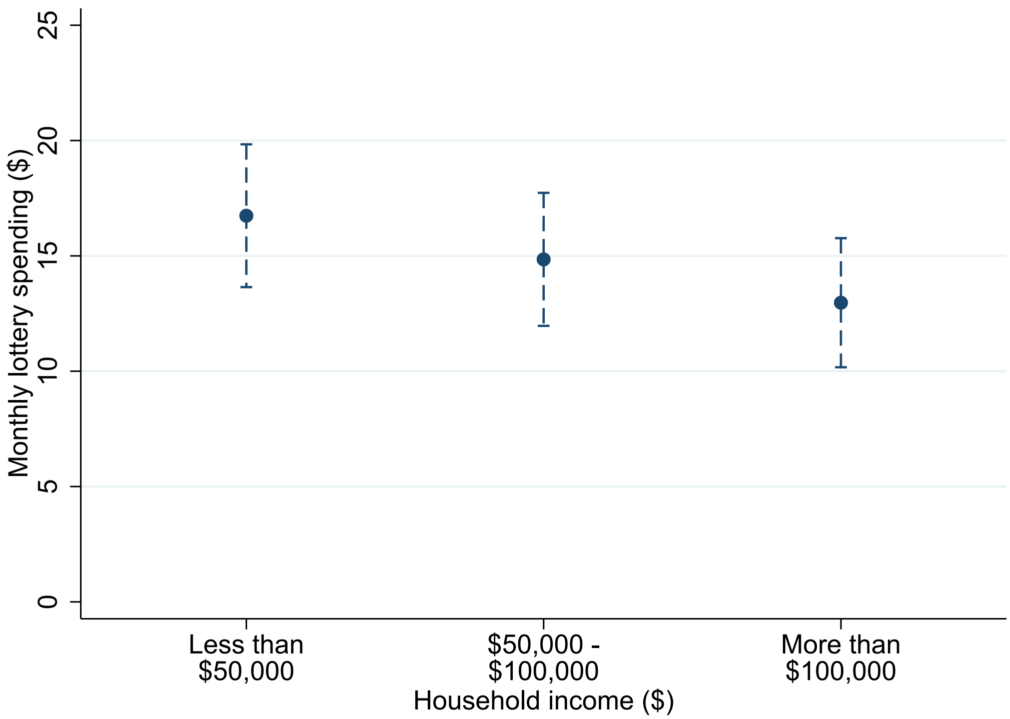


(b) **Mega Millions**



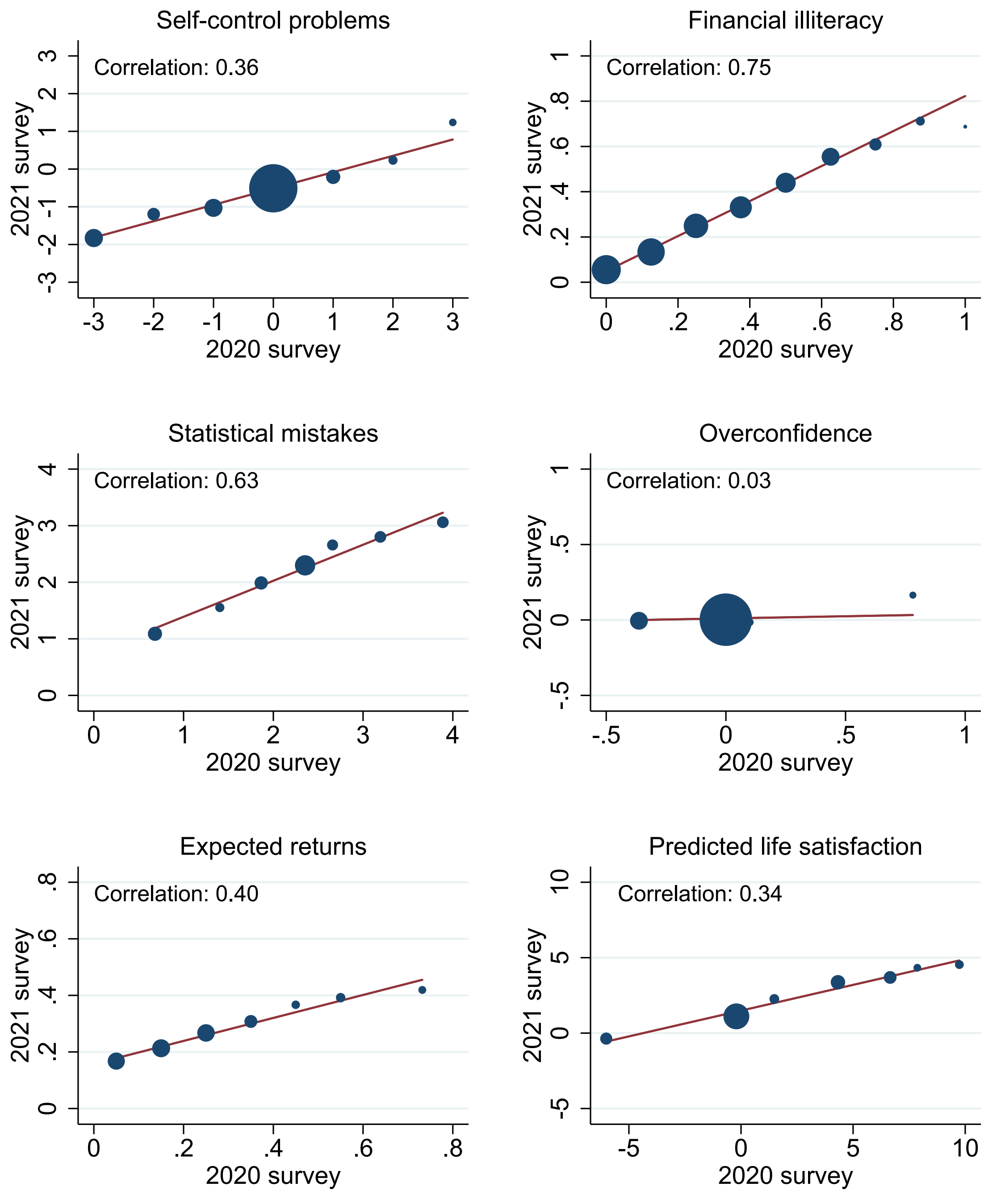
Notes: These figures present the natural log of Mega Millions and Powerball ticket sales (residual of jackpot expected values) and the natural log of other games' sales in dollars (residual of week fixed effects) before and after price increases, which are indicated by the vertical red lines. The levels of Mega Millions and Powerball sales are adjusted so that the average natural log of sales is zero before the price change when jackpots are within \$10 million of the reset value, while the levels of other games' sales are adjusted so that the average natural log of sales is zero before the price change. In Panel (a), the Powerball ticket price increased from \$1 to \$2 on January 15, 2012. In Panel (b), the Mega Millions ticket price increased from \$1 to \$2 on October 28, 2017. One value in Panel (b) from a Mega Millions drawing with a record jackpot is winsorized at -1.

Figure 4: **Lottery Spending by Income**



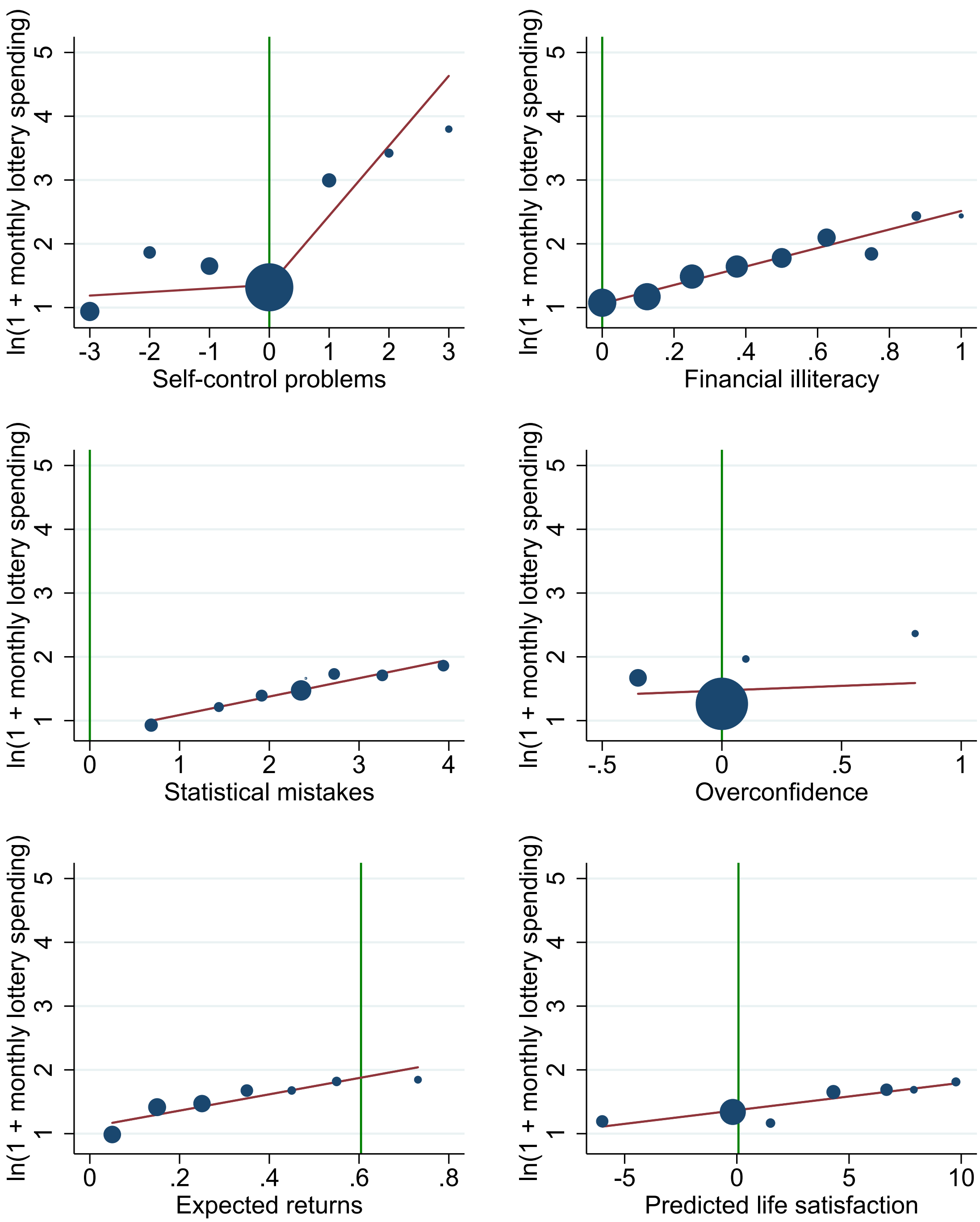
Notes: This figure presents average monthly lottery spending within household income groups, with 95 percent confidence intervals, using data from our AmeriSpeak survey. Observations are weighted for national representativeness.

Figure 5: **Test-Retest Reliability of Bias Proxies**



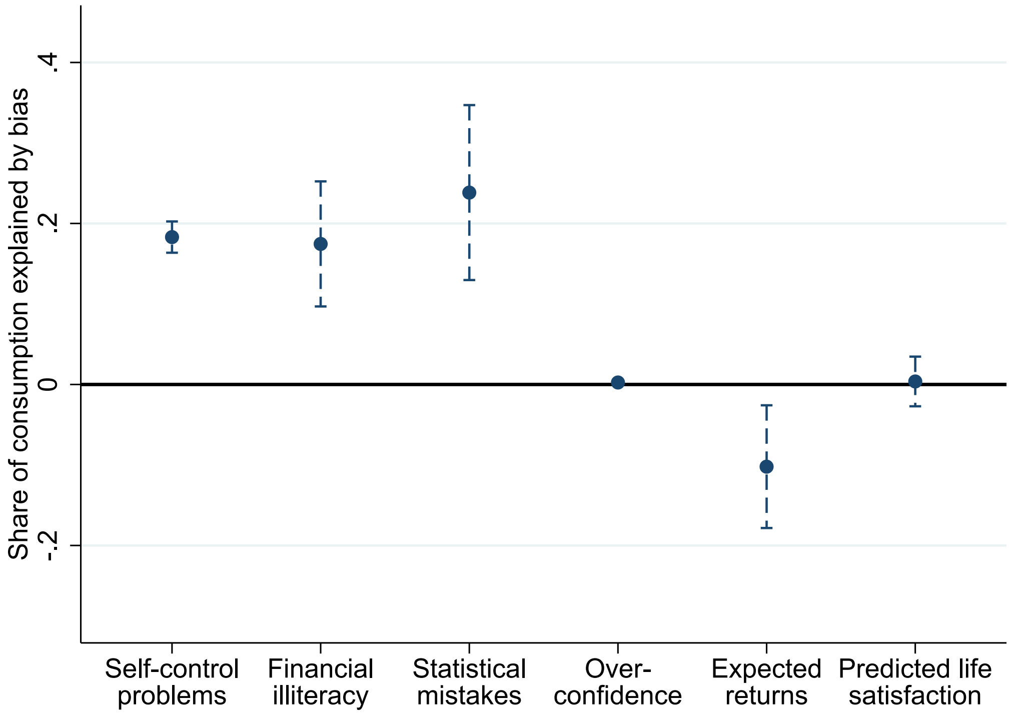
Notes: This figure presents binned scatter plots of the 2020 vs. 2021 elicitations of our six bias proxies, using data from our AmeriSpeak surveys.

Figure 6: **Relationship Between Lottery Spending and Bias Proxies**



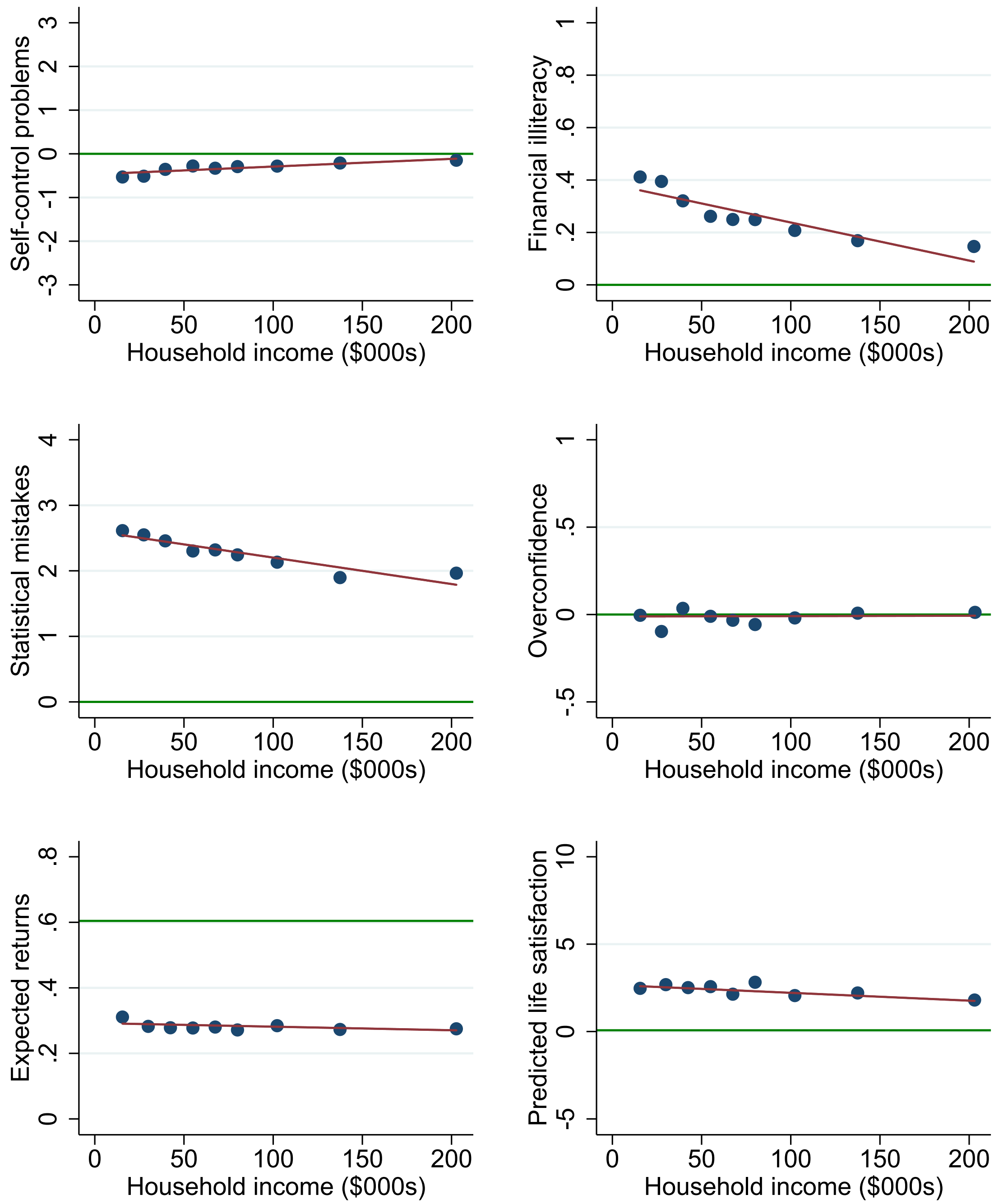
Notes: This figure presents binned scatter plots of ln(1+monthly lottery spending) versus our six bias proxies, using data from our 2020 AmeriSpeak survey. The vertical line on each panel corresponds to the correct or "unbiased" value of the bias proxy.

Figure 7: **Share of Lottery Spending Explained by Biases**



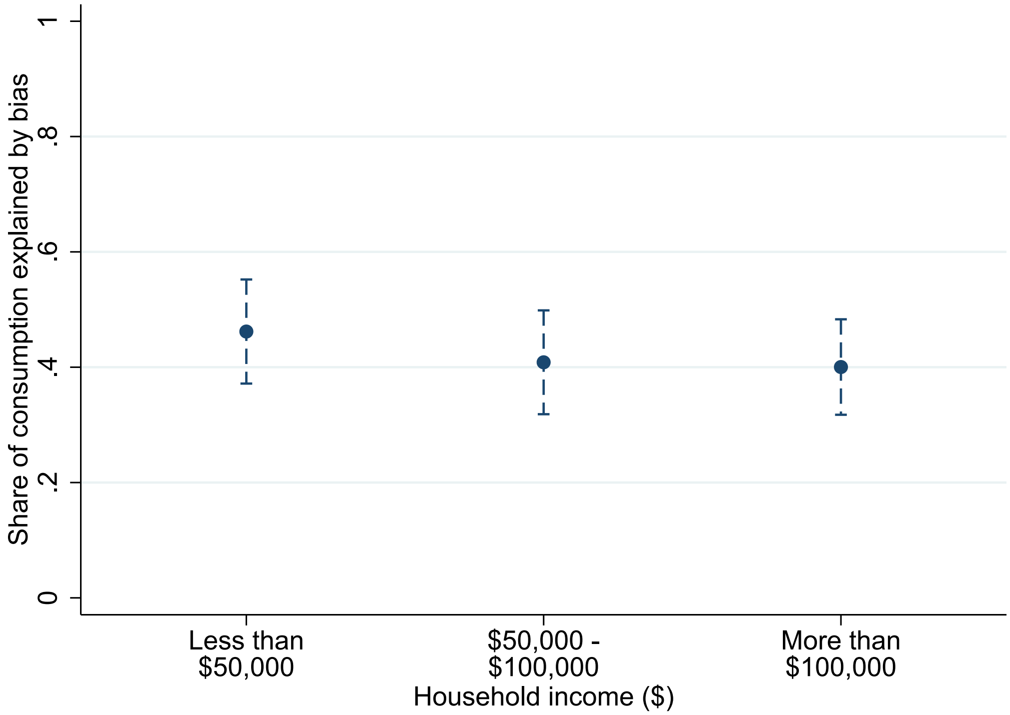
Notes: This figure plots the share of lottery spending explained by each of our six bias proxies, with 95 percent confidence intervals. Predicted unbiased consumption is $\hat{s}_{ik}^{V} = \frac{s_i+1}{\exp(\hat{\tau}_k \tilde{b}_{ik})} - 1$, where s_i is *monthly lottery spending*, $\hat{\tau}_k$ is the OLS estimate from column 1 of Table 6, and $\tilde{b}_{ik} = \frac{b_{ik}-b_k^V}{SD(b_{ik})}$ is the difference between person i's proxy b_{ik} and the unbiased value b_k^V in standard deviation units. We winsorize at $\hat{s}_i^V \geq 0$, and we fix $\hat{s}_{ik}^V = 0$ if $s_i = 0$. The share of consumption explained by each bias proxy is $\frac{\sum_i \left(s_i - \hat{s}_{ik}^V\right)}{\sum_i s_i}$.

Figure 8: **Relationship Between Income and Bias Proxies**



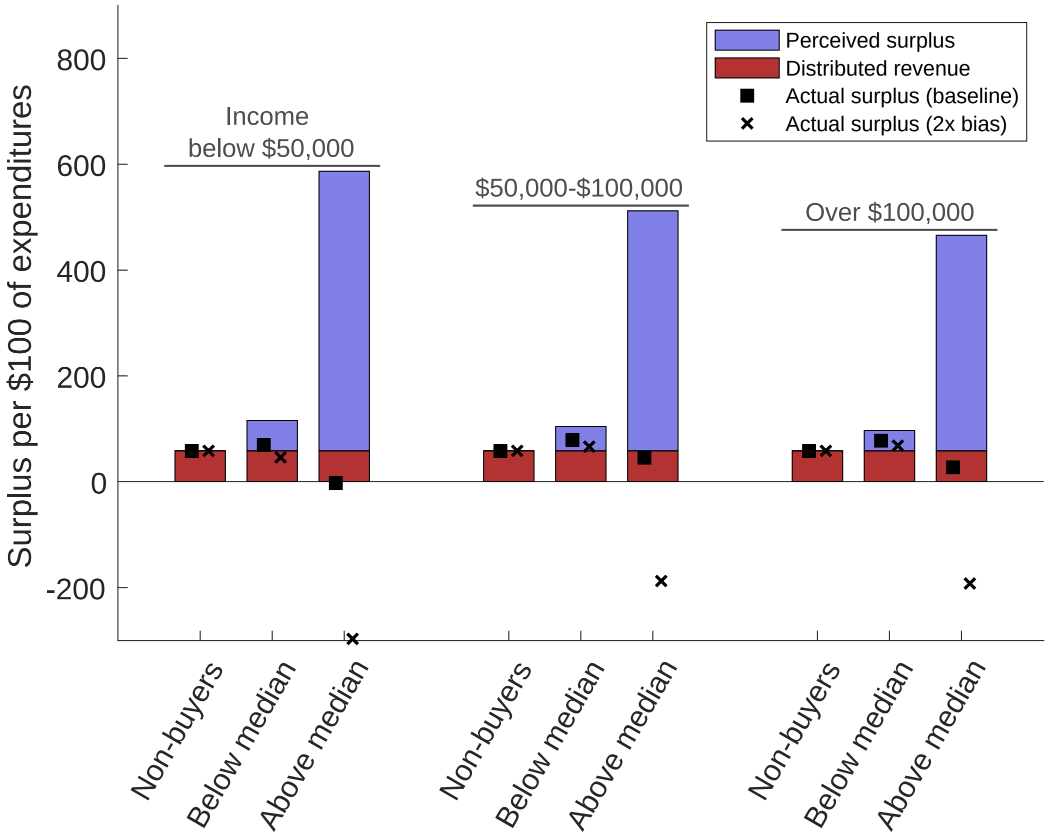
Notes: This figure presents binned scatter plots of our six bias proxies by household income, using data from our 2020 AmeriSpeak survey. The horizontal line on each panel corresponds to the correct or "unbiased" value of the bias proxy.

Figure 9: **Share of Lottery Spending Explained by Bias within Income Groups**



Notes: This figure plots the share of lottery spending explained by bias within household income groups, with 95 percent confidence intervals. Predicted unbiased consumption is $\hat{s}_i^V = \frac{s_i+1}{\exp(\hat{\boldsymbol{\tau}}\tilde{\boldsymbol{b}}_i)} - 1$, where s_i is *monthly lottery spending*, $\hat{\boldsymbol{\tau}}$ is the OLS estimate from column 1 of Table 6, and $\tilde{b}_{ik} = \frac{b_{ik} - b_k^V}{SD(b_{ik})}$ is the difference between person i's proxy b_{ik} and the unbiased value b_k^V in standard deviation units. We winsorize at $\hat{s}_i^V \geq 0$, and we fix $\hat{s}_{ik}^V = 0$ if $s_i = 0$. The share of consumption explained by bias is $\frac{\sum_i \left(s_i - \hat{s}_i^V\right)}{\sum_i s_i}$.

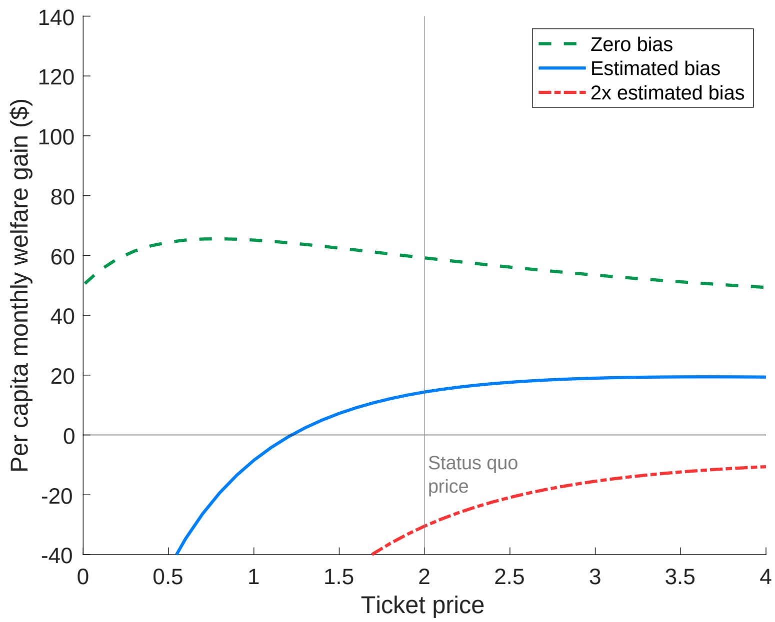
Figure 10: **Estimated Surplus from Lotteries in the Status Quo**



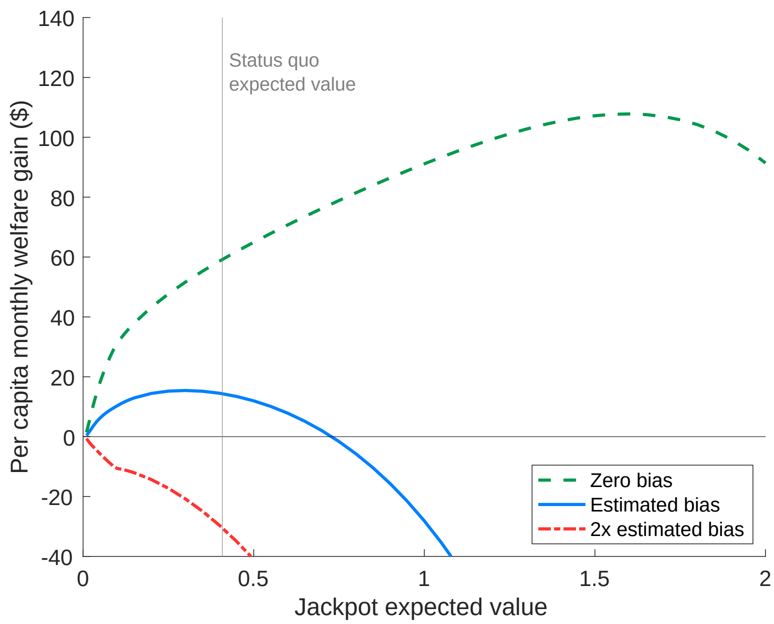
Notes: This figure plots the estimated surplus from the status quo representative lottery in our structural model, relative to a setting with no lottery. Income bins are partitioned into those who purchase no lottery tickets, those who purchase less than the median amount (conditional on purchasing), and those who purchase more than the median.

Figure 11: **Effect of Lottery Attributes on Social Welfare**

(a) **Variation in Ticket Price**



(b) **Variation in Jackpot Size**



Notes: These figures plot the simulated social welfare gain from a representative lottery relative to no lottery, when varying ticket price (Panel a) or jackpot size (Panel b). The baseline representative lottery is based on a standard $2 Powerball ticket with a jackpot pool of $170 million. Prizes are reduced by 30 percent to account for income taxes.

Online Appendix

The Optimal Design of State-Run Lotteries

Benjamin B. Lockwood ⓡ Hunt Allcott ⓡ Dmitry Taubinsky ⓡ Afras Sial

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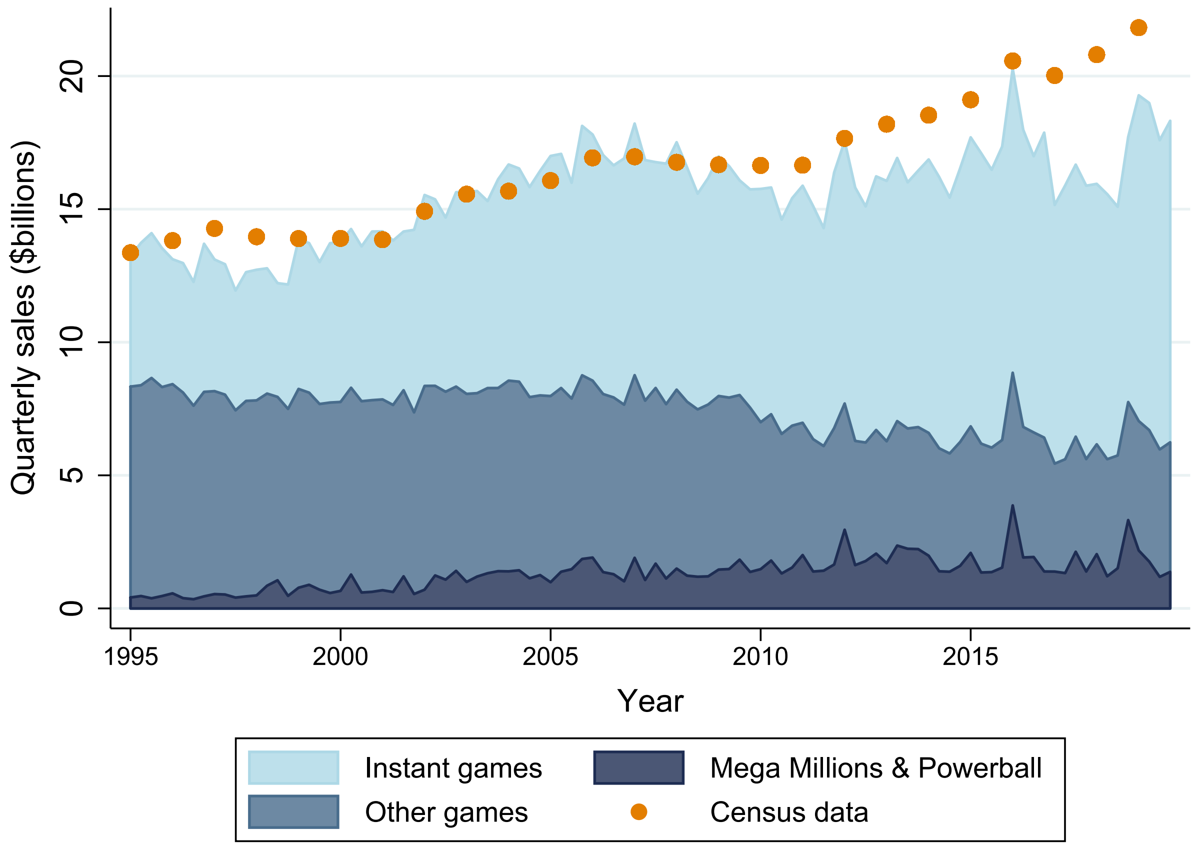
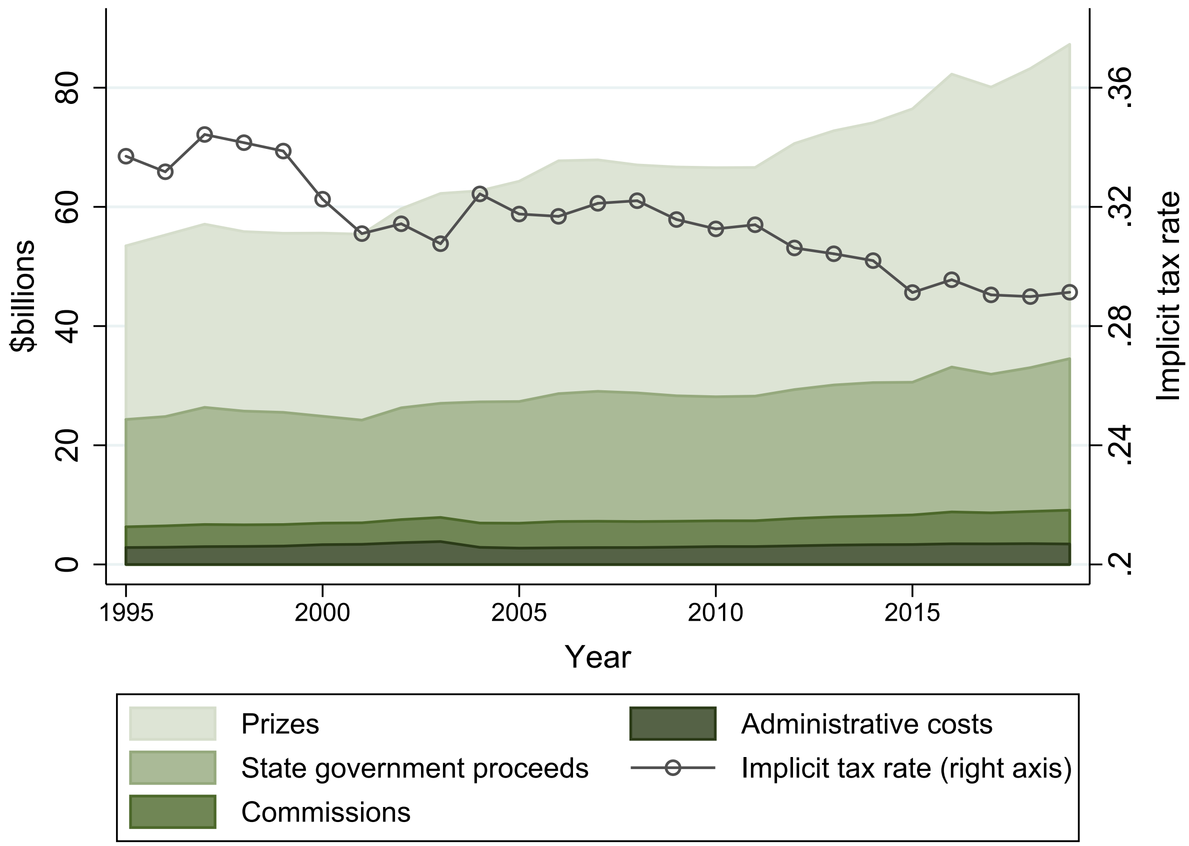
A Background Appendix

Figure A1: **Lottery Sales by Game Type over Time**



Notes: This figure presents total U.S. lottery sales by type of game, using data from La Fleur's. Census data are from the Census of Governments, inflated to account for the assumption that retailers receive 6.5 percent of sales as commissions, the midpoint of the typical range (North American Association of State and Provincial Lotteries 2021a). Monetary amounts are in real 2019 dollars.

Figure A2: **Lottery Sales Allocation and Implicit Tax Rate over Time**



Notes: This figure presents the allocation of the proceeds of U.S. lottery sales and the implicit lottery tax rate using data from the Census of Governments, assuming that commissions equal 6.5 percent of sales, the midpoint of the typical range (North American Association of State and Provincial Lotteries 2021a). The implicit tax rate equals state government proceeds divided by total sales. Monetary amounts are in real 2019 dollars.

B Special Cases of Proposition 1

To provide intuition for the implications of our formulas, we consider a number of special cases.

No bias, homogeneous preferences. When $\gamma(\theta) \equiv 0$ and $s(z)$, $\kappa(z)$ and $\rho(z)$ are constant across the population, Proposition 1 implies $p = C'_s(\boldsymbol{a}, \bar{s})$ and $\bar{\kappa}(a) = C'_a(a, \bar{s})$. In other words, the price is equal to the marginal cost of an additional lottery ticket, while a is set such that lottery buyers' surplus from an increase in a is equal to the marginal cost of increasing a.

In the simple example where there is a single prize of size $w = a$ and expected payout πa, the conditions reduce to $p = \pi a$ and $\bar{\kappa}(a) = \pi \bar{s}$. With utility function $u(a) = (1+\phi)\pi m(a)$, this implies that p and a are determined by the conditions $p = \pi a + o$ and $(1+\phi)\pi m'(a) = \pi$, where o is the additional administrative cost from each lottery ticket. The second condition uniquely determines

a, which then determines the price p. For example, if $m(x) = \ln(1+x)$, then the optimal lottery structure is $a = \max\{\phi, 0\}$ and $p = \pi \max\{\phi, 0\} + o$. This implies that $a > 0$ at the optimum if and only if $\phi > 0$; i.e., if additional entertainment utility leads individuals to value lotteries above their monetary value.

No bias, heterogeneous preferences. When $\gamma(\theta) \equiv 0$, Proposition 1 implies the conditions $p - C_s'(a, \bar{s}) = -\frac{Cov[s(z),g(z)]}{|\bar{\zeta}_p|\bar{s}}$ and $\bar{\kappa} = C_a'(a, \bar{s}) + Cov[s(z), g(z)]\frac{\bar{\zeta}_a}{|\bar{\zeta}_p|} - Cov[\kappa(z), g(z)]$. The first condition is analogous to Diamond's (1975) "many-person Ramsey tax rule," which states that the tax is proportional to its degree of progressivity and is inversely proportional to the elasticity. The condition for $\bar{\kappa}$, which results from substituting (6) into (7), is new, and states that lottery buyers' surplus from a marginal increase in a must equal the marginal cost of increasing a plus the degree to which increasing a is more progressive than decreasing p.

Homogeneous bias and preferences. When $s(z)$, $\gamma(z)$, $\kappa(z)$ and $\rho(z)$ are homogeneous across the income distribution, Proposition 1 implies $p - C_s'(a, \bar{s}) = \bar{\gamma}$ and $\bar{\kappa}(a) = C_a'(a, \bar{s}) + \bar{\rho}(a)$. In this case, the price is set above a lottery ticket's expected value when $\bar{\gamma} > 0$, so as to discourage lottery consumption. Moreover, the optimal level of attribute a is set such that $\bar{\kappa}(a) > C_a'$ when individuals overvalue not just the absolute utility of the lottery ticket but also changes in a (i.e., $\bar{\rho}(a) > 0$). This implies that the optimal choice of a is lower than what it would be when individuals correctly evaluate lottery tickets (under the reasonable assumption that $\bar{\kappa}(a)$ is decreasing in a due to the concavity of m). Thus, individuals are effectively taxed in two ways relative to the no-bias benchmark. First, the price is set to be higher than the expected monetary value of a lottery ticket. Second, utility-increasing attributes of the lottery ticket (e.g., its prize levels) are set to be lower than what would be optimal in the absence of bias.

Consider the simple example where there is a single prize of size $w = a$, and $u = (1+\phi)\pi m(a)$ and $v = (1+\phi^V)\pi m(a)$. We have $\bar{\kappa}(a) = (1+\phi)\pi m'(a)\bar{s}$, $\bar{\rho}(a) = (\phi - \phi^V)\pi m'(a)\bar{s}$, and $\gamma = (\phi - \phi^V)\pi m(a)$. At an interior optimum, we thus have the first-order conditions $p - \pi a = (\phi - \phi^V)\pi m(a)$ and $(1+\phi)m'(a) = 1 + (\phi - \phi^V)m'(a)$. Note that if ϕ^V is sufficiently low, it is optimal to choose $a = 0$.

Revenue-maximizing lottery structure. The revenue-maximizing lottery structure can be obtained from our calculations by ignoring the effects on consumer surplus. The revenue effects of changing a and p are $p\frac{d\bar{s}}{da} - \frac{d}{da}C(a, \bar{s}(a))$ and $p\frac{d\bar{s}}{dp} - \frac{d}{dp}C(a, \bar{s}(a))$, respectively. This leads to the conditions $p - C_s'(a, \bar{s}) = \frac{1}{|\bar{\zeta}_p|}$ and $\frac{C_a'(a,\bar{s})}{\bar{s}} = \frac{\bar{\zeta}_a}{|\bar{\zeta}_p|}$. The first condition is just the standard inverse elasticity rule for product pricing. The second condition states that the per-ticket marginal cost of increasing a has to equal the ratio of the semi-elasticities. The intuition for the second condition is that increasing a by da and increasing p by $dp = da\frac{C_a'(a,\bar{s})}{\bar{s}}$ has a zero direct effect on government revenue; thus, this perturbation cannot affect consumer demand if a and p are set optimally. For example, when a corresponds to expected payout of a lottery ticket, so that $C_s' = a$ and $C_a'/\bar{s} = 1$, the optimal choice of p and a must satisfy $p - a = 1/\left|\bar{\zeta}_p\right|$ and $\left|\bar{\zeta}_p\right| = \bar{\zeta}_a$.

C Theory Appendix: A More General Model and Generalizations of Proposition 1

C.1 A More General Model

We consider a more general model in which individuals first choose income z and then choose whether or not to buy lottery tickets on various occasions. Specifically, we assume that individuals choose their income in period $t = 0$, and then choose whether or not to buy a lottery ticket on choice occasions $t = 1, \ldots, t^*$. Individuals realize a taste shock ε_t at the beginning of each period that determines their utility from the lottery ticket. Individuals' utility given a vector of shocks $\boldsymbol{\varepsilon}$ and a vector $\boldsymbol{x} = (x_1, \ldots, x_{t^*}) \in \{0,1\}^{t^*}$ of lottery ticket purchase decisions is given by $U(x, c, z; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon}) = G(n(c;\theta) + \sum_t u(x_t; \boldsymbol{a}, \theta, \varepsilon_t) - \psi(z;\theta))$, where $\boldsymbol{a}$ is a vector of attributes, with jth component a_j. Without loss of generality, we can consider individual's optimization problem as a static problem where the vector of shocks $\boldsymbol{\varepsilon}$ is realized in period 1 and individuals choose a consumption plan $\boldsymbol{x}$ for all t^* periods.

Lottery demand is a random variable $\boldsymbol{s}_\theta(\boldsymbol{\varepsilon})$ that maps shocks $\boldsymbol{\varepsilon}$ to a total number of lottery tickets purchased. We let $\bar{s}_\theta$ denote expected lottery purchases. We assume that $\boldsymbol{\varepsilon}$ is smoothly distributed, so that $\bar{s}_\theta$ is smooth in p and a_j. For shorthand, we will sometimes write $\boldsymbol{u}(s; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon}) = \text{argmax}_{\boldsymbol{x}}\{\sum_t u(x_t; \boldsymbol{a}, \theta, \varepsilon_t) | \sum x_t = s\}$; that is, $\boldsymbol{u}$ is utility obtained after a vector of shocks $\boldsymbol{\varepsilon}$ is realized and the person executes an optimal consumption plan given the constraint that s lottery tickets are purchased.

In contrast to the body of the paper, this functional form allows for exogenous shocks to income to change lottery consumption, but we still maintain the assumption of weak separability. The weak separability assumption could also be relaxed by following the approach of ALT, and replacing the (causal) income elasticity of lottery demand with the elasticity of lottery demand with respect to changes in earnings z, where appropriate.

We let $\kappa_j(p, \boldsymbol{a}, z; \theta)$ denote the valuation of a marginal increase in the jth attribute of $\boldsymbol{a}$. We let $V(x, c, z; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon}) = G(n(c;\theta) + \sum_t v(x_t; \boldsymbol{a}, \theta, \varepsilon_t) - \psi(z;\theta))$ denote normative utility. We define $\boldsymbol{v}(s; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon})$ analogous to $\boldsymbol{u}(s; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon})$.

We let $C(\boldsymbol{a}, \bar{s})$ be the cost of supplying lottery tickets, with C'_j denoting the derivative with respect to the jth component a_j of $\boldsymbol{a}$, and C'_s denoting the derivative with respect to $\bar{s}$.

Note that we assume that the utility from gambling does not depend on the history of prior gambling decisions (i.e., no habit formation) for expositional simplicity. Our results, which apply the Envelope Theorem to the expected perceived utility function, do not require this form of stationarity.

C.2 Assumptions

We make the following assumptions:

Assumption 1. *Utility from numeraire consumption, $n(c)$, has bounded relative risk aversion: there is $r > 0$ such that $|cn''(c)/n'(c)| < r$ is for all c.*

Assumption 2. *Lottery expenditures are a small share of the total budget, so that terms of order $\frac{\bar{s}_\theta}{z(\theta)-T(z(\theta))-p\bar{s}_\theta}$ and $\frac{pVar[s_\theta(\varepsilon)]/\bar{s}_\theta}{z(\theta)-T(z(\theta))-p\bar{s}_\theta}$ are negligible.*

Assumption 3. *Constant social marginal welfare weights conditional on income: $g(\theta,\boldsymbol{\varepsilon}) = g(\theta',\boldsymbol{\varepsilon})$ if $z(\theta) = z(\theta')$.*

Assumption 4. *U and V are smooth functions that are strictly concave in c, s, and z, and μ is differentiable with full support.*

Assumption 5. *The optimal income tax function $T(\cdot)$ is twice differentiable, and each consumer's choice of income z admits a unique global optimum, with the second-order condition holding strictly at the optimum.*

Assumption 6. *$\bar{s}_\theta$ and $\kappa_j(\theta)$ are orthogonal to the income elasticity ζ_z conditional on income.*

Assumption 7. *Income effects on labor supply are negligible.*

Assumptions 1 and 2 ensure unpredicted variation in an individual's lottery expenditures does not have consequential effects on her marginal utility from numeraire consumption, as is clarified in Lemma C1 and its proof. Assumption 2 also implies that the difference between compensated and uncompensated demand for lottery tickets is negligible (see ALT). The term $\frac{\bar{s}_\theta}{z(\theta)-T(z(\theta))-p\bar{s}_\theta}$ is negligible simply when lotteries are a small share of total epxenditures. By the Central Limit Theorem, the term $\frac{pVar[s_\theta(\varepsilon)]/\bar{s}_\theta}{z(\theta)-T(z(\theta))-p\bar{s}_\theta}$ approaches zero when the number of choice occasions grows large while $\bar{s}_\theta$ stays constant. Thus

Assumption 3 is analogous to Assumption 1 in Saez (2002a). It holds immediately if types are homogeneous conditional on income. More generally, Saez (2002a) argues this is a reasonable normative requirement even under heterogeneity "if we want to model a government that does not want to discriminate between different consumption patterns...." Therefore we sometimes write $g(z)$ to denote the welfare weight directly as a function of earnings.

Assumptions 4 and 5 ensure that the income distribution does not exhibit any atoms and consumers' labor supply and consumption decisions respond smoothly to perturbations of the tax system (Jacquet and Lehmann Forthcoming).

C.3 Elasticity Concepts and Sufficient Statistics

All statistics are understood to be endogenous to the tax regime (t, T), though we suppress those arguments for notational simplicity. We begin by defining the elasticities related to sin good consumption.

Price and attribute elasticities

- $\zeta_p(\theta)$: the price semi-elasticity of demand for s from type θ, formally equal to $\left(\frac{d\bar{s}_\theta}{dp}\right)\frac{1}{\bar{s}_\theta}$
- $\zeta_{a_j}(\theta)$: the price semi-elasticity of demand for s from type θ, formally equal to $\left(\frac{d\bar{s}_\theta}{da_j}\right)\frac{1}{\bar{s}_\theta}$
- $\xi(\theta)$: the causal income elasticity of demand for s, equal to $\frac{d}{dz}\bar{s}_\theta\left(p, \boldsymbol{a}, z; \theta\right) \cdot \frac{z}{s}$.

Income Elasticities

We define labor supply responses to include any "circularities" due to the curvature of the income tax function, which is assumed to be differentiable. Thus, following Jacquet and Lehmann (Forthcoming), we define a tax function $\hat{T}$ which has been locally perturbed around the income level z_0 by raising the marginal tax rate by τ and reducing the tax level by ν:

$$\hat{T}(z; z_0, \tau, \nu) := T(z) + \tau(z - z_0) - \nu. \tag{24}$$

Let $z^*(\theta)$ denote a type θ's choice of earnings under the status quo income tax T, and let $\hat{z}(\theta; \tau, \nu)$ denote θ's choice of earnings under the perturbed income tax $\hat{T}(z; z^*(\theta), \tau, \nu)$. Then the compensated elasticity of taxable income is defined in terms of the response of $\hat{z}$ to τ, evaluated at $\tau = \nu = 0$:

$$\zeta_z^c(\theta) := \left(-\left.\frac{\partial \hat{z}(\theta; \tau, 0)}{\partial \tau}\right|_{\tau=0}\right)\frac{1 - T'(z^*(\theta))}{z^*(\theta)}. \tag{25}$$

The income tax is similarly defined in terms of the response of $\hat{z}$ to a tax credit ν (this statistic will be nonpositive if leisure is a non-inferior good):

$$\eta_z(\theta) := \left(\left.\frac{\partial \hat{z}(\theta; 0, \nu)}{\partial \nu}\right|_{\nu=0}\right)\left(1 - T'(z^*(\theta))\right). \tag{26}$$

These definitions are comparable to those in Saez (2001), except that they include circularities and thus permit a representation of the optimal income tax in terms of the actual earnings density, rather than the "virtual density" employed in that paper.

Bias

We continue defining bias γ analogous to the definition in the body of the paper: it is the value that $\gamma(p, a, y; \theta, \varepsilon)$ that satisfies

$$u(1; \boldsymbol{a}, \theta, \varepsilon) - v(1; \boldsymbol{a}, \theta, \varepsilon) = n(y - p + \gamma; \theta) - n(y - p; \theta)$$

where y is disposable income. In other words, γ is the degree, in units of dollars, by which the individual overestimates the value of the lottery ticket.

We define

$$\begin{aligned}\gamma(z;\boldsymbol{a},\theta) = \mathbb{E}[&\gamma(p,\boldsymbol{a},z-T(z)-s_\theta(\boldsymbol{\varepsilon});\theta,\varepsilon)\\ &|\boldsymbol{u}(s_\theta(\boldsymbol{\varepsilon})+1;\boldsymbol{a},\theta,\boldsymbol{\varepsilon})-\boldsymbol{u}(s_\theta(\boldsymbol{\varepsilon});\boldsymbol{a},\theta,\boldsymbol{\varepsilon}) = n(z-T(z)-p(s_\theta(\boldsymbol{\varepsilon})+1);\theta)\\ &-n(z-T(z)-s_\theta(\boldsymbol{\varepsilon});\theta), z(\theta)=z]\end{aligned}$$

In other words, $\gamma(z)$ is the average bias of z-earners who are on the margin of purchasing an additional lottery ticket. The statistics $\bar{\gamma}$, σ_p and σ_{a_j} are constructed as in the body of the paper. By definition, the social welfare impact of inducing a marginal z-earner to purchase one fewer lottery ticket is $\gamma(z)g(z)$.

We define ρ_j more generally as the difference $\rho_j = \kappa_j - \kappa_j^V$, where κ_j^V is the willingness to pay for a marginal change in a_j that would result if consumers chose according to normative preferences V. We define σ_p as in the body of the paper, and we define σ_{a_j} analogous to σ_a in the body of the paper.

Aggregation

With some abuse of notation, we write $s(z)$, $\gamma(z)$, $\kappa_j(z)$ and so forth to denote the averages among z-earners. We denote population averages of these statistics using "bar" notation. For example, average consumption of s is denoted $\bar{s}$. The cumulative density function of the income distribution is denoted $H(z)$, which we assume possesses a density function $h(z)$.

Causal Income Effects and Preference Heterogeneity

Following ALT, we distinguish between two sources of cross-sectional variation in $s(z)$: income effects and (decision) preference heterogeneity. Let $\bar{s}'(z)$ denote the cross-sectional change in s with respect to income z at a particular point in the income distribution. This total derivative can be decomposed into two partial derivatives: the (causal) income effect, $s'_{inc}(z)$, and between-income preference heterogeneity $s'_{pref}(z)$. The causal income effect depends on the empirically estimable income elasticity of s: $s'_{inc}(z) = \mathbb{E}\left[\xi(\theta)/z \mid z(\theta)=z\right]$. Between-income preference heterogeneity is the residual: $s'_{pref}(z) = \bar{s}'(z) - s'_{inc}(z)$. The key sufficient statistic for preference heterogeneity, "cumulative between-income preference heterogeneity" is defined as:

$$\begin{aligned} s_{pref}(z) &:= \int_{x=z_{min}}^{z} s'_{pref}(x)dx \\ s_{inc}(z) &:= \int_{x=z_{min}}^{z} s'_{inc}(x)dx \end{aligned}$$

The $s_{pref}(z)$ term quantifies the amount of lottery consumption at income z, relative to the lowest income level z_{min}, that can be attributed to preference heterogeneity rather than income effects.

Analogously, we define $\kappa'_j(z)$ to be the cross-sectional heterogeneity in the valuation of a marginal increase in a_j. We define $\kappa'_{j,inc}(z) := \mathbb{E}\left[\frac{\partial}{\partial z}\kappa_j(p,\boldsymbol{a},z;\theta)|z(\theta)=z\right]$, and let $\kappa'_{pref}(z) = \kappa'_j(z) - \kappa'_{j,inc}(z)$ be the residual. We define

$$\kappa_{j,pref}(z) := \int_{x=z_{min}}^{z} \kappa'_{j,pref}(x)dx$$
$$\kappa_{j,inc}(z) := \int_{x=z_{min}}^{z} \kappa'_{j,inc}(x)dx.$$

C.4 Results and Derivations

Preliminary Lemmas

Although in contrast to standard optimal tax models, ours features discrete choice of a commodity, we show that we can still establish an approximate Roy Identity in our model under assumptions 1 and 2, and thus derive a simple expression for how changes in lottery attributes affect labor supply. This is the content of Lemma C1 below.

Lemma C1. *The change in earnings of type θ induced by a small change da_j is equal to the change in earnings that would be induced by imposing a type-specific $dT^\theta(z) = -da_j \cdot \kappa_j(p,\boldsymbol{a},z;\theta)$. Under assumptions 1 and 2, the change in earnings of type θ induced by a small change dp in the price is equal to $dT^\theta(z) = dp \cdot \bar{s}(p,\boldsymbol{a},z;\theta)$ up to negligible terms.*

Proof. The first statement follows from Lemma 1 of Saez (2002a). To prove the second statement, assume, without loss, that G is linear. The Envelope Theorem implies that a change dp in the price of the lottery has an expected utility impact of $\mathbb{E}_\varepsilon\left[n'(z-T(z)-ps_\theta(\varepsilon))s_\theta(\varepsilon)\right]dp$. Similarly, a change $dy = dp\bar{s}_\theta$ in after-tax income has an expected utility impact of $\mathbb{E}_\varepsilon\left[n'(z-T(z)-ps_\theta(\varepsilon))\right]\bar{s}_\theta dp$. Up to second order, the difference between these two terms is

$$\begin{aligned}|\mathbb{E}\left[n''(z-T(z)-\bar{s}_\theta)((s_\theta(\varepsilon))^2 - s_\theta(\varepsilon)\bar{s}_\theta)\right]dp| &= |n''(z-T(z)-\bar{s}_\theta)dp|Var[s_\theta] \\ &\leq rn'(z-T(z)-\bar{s}_\theta)\frac{pVar[s_\theta(\varepsilon)]/\bar{s}_\theta}{z(\theta)-T(z(\theta))-p\bar{s}_\theta}|dp|\end{aligned}$$

Thus, for $dT^\theta(z) = dp \cdot \bar{s}(p,\boldsymbol{a},z;\theta)$,

$$\frac{dz(\theta)}{dp}/\frac{dz(\theta)}{dT^\theta} = 1 + O\left(\frac{pVar[s_\theta(\varepsilon)]/\bar{s}_\theta}{z(\theta)-T(z(\theta))-p\bar{s}_\theta}\right)$$

□

The next lemma will also prove useful in the derivations below.

Lemma C2. *Let $q(x)$ be any continuously differentiable function with $q(z_{min}) = 0$. Then*

$$\int_{z=z_{min}}^{\infty}\int_{x=z}^{\infty}(1-g(x))h(x)dxq'(z)dz = \int_{z=z_{min}}^{\infty} q(z)(1-g(z)h(z)dz.$$

Proof. This follows from integration by parts and the fact that $\int_{z=z_{min}}^{\infty}(1-g(z))h(z)=0$ at the optimum. $\square$

The Main Result

Proposition C1. *If p,$\boldsymbol{a}$, and T are set optimally, then*

$$p-\frac{\partial C}{\partial \bar{s}}=\bar{\gamma}_p(1+\sigma_p)-\frac{Cov\left[s_{pref}(z),g(z)\right]}{|\bar{\zeta}_p|\bar{s}}$$
$$p-\frac{\partial C}{\partial \bar{s}}=\bar{\gamma}_{a_j}(1+\sigma_{a_j})-\frac{\bar{\kappa}_j-\bar{\rho}_j-\frac{\partial C}{\partial a_j}+Cov[(\kappa_{j,pref}(z)-\rho_j(z)),g(z)]}{\bar{\zeta}_{a_j}\bar{s}}$$

for all a_j, with equality when $a_j>0$.

If the income tax T is not necessarily optimal, but p and $\boldsymbol{a}$ are set optimally, then

$$p-C_s'=\bar{\gamma}(1+\sigma_p)-\frac{\mathbb{E}\left[s(z)(g(z)-1)\right]}{|\bar{\zeta}|\bar{s}}-\frac{1}{|\bar{\zeta}_p|\bar{s}}\mathbb{E}\left[\frac{T'(z)}{1-T'(z)}\zeta_z(z)zs_{inc}'(z)\right]$$
$$p-\frac{\partial C}{\partial a_j}=\bar{\gamma}(1+\sigma_{a_j})-\frac{\mathbb{E}\left[(\kappa_j(z)-\rho_j(z))g(z)\right]-\frac{\partial C}{\partial a_j}}{\bar{\zeta}_{a_j}\bar{s}}+\frac{1}{\bar{\zeta}_{a_j}\bar{s}}\mathbb{E}\left[\frac{T'(z)}{1-T'(z)}\zeta_z(z)z\kappa_{j,inc}'(z)\right]$$

The intuition behind the "regressivity costs" and "consumer welfare" terms comes from considering a joint reform where a change in p or a_j is accompanied by a corresponding change in the income tax T that leaves labor supply preserved. When there are no causal income effects, a change in p or a_j has no effect on labor supply, and thus no accompanying change in the income tax T is necessary; in this case, $s_{pref}(z)=s(z)$ and $\kappa_{j,pref}(z)=\kappa_j(z)$. When lotteries are a normal good, an increase in the price, for example, generates a higher tax burden on those choosing to earn more, and thus creates disincentives for higher labor supply equivalent to the change produced by an increase in the marginal income tax rate—this is formalized in Lemma 1. Thus, an increase in p must be accompanied by a decrease in the income tax, which leads the net tax burden of the reform to be proportional to $s_{pref}(z)$ rather than $s(z)$.

Proposition C1 generalizes the classic Atkinson and Stiglitz (1976) result in three ways. First, note that in the case of no correlated preference heterogeneity, $s_{pref}\equiv 0$, and thus the optimal price equals the marginal cost. This is analogous to the classic Atkinson and Stiglitz (1976) result that when consumption preferences are homogeneous, commodity taxes are not useful for redistribution in the presence of nonlinear income taxation. Second, our results allow us to establish an Atkinson-Stiglitz type result for optimal attribute regulation. In the absence of biases, and when $\kappa_{j,pref}\equiv 0$, meaning that preferences for the attribute are uncorrelated with earnings ability, the optimal attribute choice must satisfy $\bar{\kappa}_j=C_j'+(p-C_s')\left|\bar{\zeta}_{a_j}\right|\bar{s}$. In other words, consumers' average marginal valuation of each attribute component must equal the marginal cost of increasing that attribute component. This again parallels the classic Atkinson and Stiglitz (1976) results, but extends to the case of attribute regulation. Third, while the case of $s_{pref}=0$, and $\kappa_{j,pref}\equiv 0$ is a special case corresponding to the assumptions of Atkinson and Stiglitz (1976), our general result in Proposition

C1 provides a characterization of optimal regulation under a much broader set of assumptions.

Proof. Consider first increasing the marginal tax rate between z^* and $z^* + dz$ by a small amount $d\tau$. Assumption 2 implies that the effects of small changes in z induced by this perturbation have negligible effects on the consumption s (this is a simple extension of the derivations in ALT of the proof of Proposition 1). Following the derivations of Saez (2001) or ALT, and utilizing assumption 7, the optimal income tax T must thus satisfy

$$\frac{T'(z^*)}{1-T'(z^*)} = \frac{\int_{x=z^*}^{\infty}(1-g(x))dH(x)}{\bar{\zeta}_z(z^*)z^*h(z^*)} \tag{27}$$

Consider now the effect of increasing the price p by dp. The total welfare effect, written in terms of the marginal value of public funds, can be decomposed into the following components:

- *Mechanical revenue effect*: the reform mechanically raises revenue from each consumer by $dp \cdot s(\theta)$, for a total of $dp\bar{s}$.

- *Mechanical welfare effect*: As in the proof of Lemma 1, the reform mechanically reduces each consumer's net income by $dp \cdot s_\theta(\varepsilon)$. To isolate the mechanical effect, we compute the loss in welfare as if this reduction all comes from composite consumption c. Under assumption 2, and using the derivations in the proof of Lemma 1, we can write this $dp \cdot \bar{s}_\theta g(\theta)$ up to negligible higher-order terms. Thus the total mechanical welfare effect is $-dp\mathbb{E}\left[s(z)g(z)\right]$

- *Revenue effects of substitution.* The reform changes costs by $C_s' \frac{d\bar{s}}{dp}dp$, and changes earnings by $p\frac{d\bar{s}}{dp}$. The net effect is thus $\left(p - C_s'\right)\bar{\zeta}\bar{s}dp$

- *Bias-correcting effects of substitution:* the reform causes each consumer to decrease their s consumption by $dp \cdot \zeta(\theta)/p$. This generates a behavioral welfare effect equal to

$$-dp\mathbb{E}\left[g(z)\gamma(z)\zeta_p(z)s(z)\right] = -dp\bar{\gamma}_p(1+\sigma_p)\bar{\zeta}\bar{s}$$

  .

- *Effect on earnings:* The reform causes a change in income tax revenue collected from type θ equal to $\frac{dz(\theta)}{dp}T'(z(\theta))$. Lemma 1 implies that $\frac{dz(\theta)}{dp} = -\zeta_z(\theta)\left(\frac{z(\theta)}{1-T'(z(\theta))}\right)\frac{\partial s(p,\boldsymbol{a},z;\theta)}{\partial z}$. By assumption 6, this generates a total fiscal externality through the income tax equal to $-dp \cdot \mathbb{E}\left[\frac{T'(z)}{1-T'(z)}\zeta_z(z)zs'_{inc}(z)\right]$.

- *Indirect effects on sin good consumption:* The change in earnings affects consumption indirectly. However, relative to the other effects above, these effects are of order $\frac{\bar{s}_\theta}{z(\theta)-T(z(\theta))-p\bar{s}_\theta}$ and therefore negligible by Assumption 2 (formally, this is easily proven by extending the calculations of ALT in their proof of Proposition 1).

Combining these components, and taking into account that the income tax T is set optimally, the total welfare effect of the price change is equal to

$$\frac{dW}{dp} = \mathbb{E}\left[s(z)(1-g(z))\right] + \left(p - C_s'\right)\bar{\zeta}_p\bar{s} - \bar{\gamma}(1+\sigma_p)\bar{\zeta}_p\bar{s} \tag{28}$$

$$- \mathbb{E}\left[\frac{T'(z)}{1-T'(z)}\zeta_z(z)zs_{inc}'(z)\right] \tag{29}$$

$$= \mathbb{E}\left[s(z)(1-g(z))\right] + \left(p - C_s'\right)\bar{\zeta}_p\bar{s} - \bar{\gamma}_p(1+\sigma_p)\bar{\zeta}_p\bar{s}$$

$$- \int_{z=z_{min}}^{\infty}\int_{x=z}^{\infty}(1-g(x)h(x)dxs_{inc}'(z)dz \tag{30}$$

$$= \mathbb{E}\left[s(z)(1-g(z))\right] + \left(p - C_s'\right)\bar{\zeta}_p\bar{s} - \bar{\gamma}_p(1+\sigma_p)\bar{\zeta}_p\bar{s}$$

$$- \int_{z=z_{min}}^{\infty}(1-g(z))s_{inc}(z)h(z)dz \tag{31}$$

$$= -Cov\left[s_{pref}(z), g(z)\right] + \left(p - C_s'\right)\bar{\zeta}_p\bar{s} - \bar{\gamma}_p(1+\sigma_p)\bar{\zeta}_p\bar{s}$$

In the computations above, expression (30) follows from (27), while expression (31) follows from Lemma C2. At the optimum, $\frac{dW}{dp} = 0$, which implies the first order condition

$$p - C_s' = \bar{\gamma}_p(1+\sigma_p) - \frac{Cov\left[s_{pref}(z), g(z)\right]}{|\bar{\zeta}_p|\bar{s}} \tag{32}$$

Next consider the effects of increasing a_j. The total welfare effect, written in terms of the marginal value of public funds, can be decomposed into the following components:

- *Mechanical welfare effect*: The reform mechanically reduces changes consumers' perceived utility by $\kappa_j(\theta)$ dollars, and consumers' normative utility by $\kappa_j(\theta) - \rho_j(\theta)$ dollars. Thus the total mechanical welfare effect is $\mathbb{E}\left[(\bar{\kappa}_j(z) - \bar{\rho}_j(z))g(z)\right]da_j$
- *Revenue effects.* The reform changes costs by $C_j'da_j + C_p'\frac{d\bar{s}}{da_j}da_j$, and changes earnings by $p\frac{d\bar{s}}{da_j}$. The net effect is thus $\left(p - C_s'\right)\bar{\zeta}_{a_j}\bar{s}da_j - C_j'da_j$
- *Bias-correcting effects of substitution:* The reform causes each consumer to decrease their s consumption by $dp \cdot \zeta_{a_j}(\theta)/p$. This generates a behavioral welfare effect equal to

$$-da_j\mathbb{E}\left[g(z)\bar{\gamma}(z)\zeta_{a_j}(z)s(z)\right] = -da_j\bar{\gamma}_{a_j}(1+\sigma_{a_j})\bar{\zeta}_{a_j}\bar{s}$$

- *Effect on earnings:* The reform causes a change in income tax revenue collected from type θ equal to $\frac{dz(\theta)}{da_j}T'(z(\theta))$. Lemma 1 implies that $\frac{dz(\theta)}{da_j} = \zeta_z(\theta)\left(\frac{z(\theta)}{1-T'(z(\theta))}\right)\frac{\partial k_j(p,\boldsymbol{a},z;\theta)}{\partial z}$. By assumption 6, this generates a total fiscal externality through the income tax equal to $da_j \cdot \mathbb{E}\left[\frac{T'(z)}{1-T'(z)}\zeta_z(z)z\kappa_{j,inc}'(z)\right]$.
- *Indirect effects on sin good consumption:* The change in earnings affects consumption indirectly. However, relative to the other effects above, these effects are of order $\frac{\bar{s}_\theta}{z(\theta)-T(z(\theta))-p\bar{s}_\theta}$

and therefore negligible by Assumption 2 (formally, this is easily proven by extending the calculations of ALT in their proof of Proposition 1).

Combining these components, and taking into account that the income tax T is set optimally, the total welfare effect of the price change is equal to

$$\frac{dW}{da_j} = \mathbb{E}\left[(\kappa_j(z) - \rho_j(z))g(z)\right] + \left(p - C_s'\right)\bar{\zeta}_{a_j}\bar{s} - C_j' - \bar{\gamma}_{a_j}(1+\sigma_{a_j})\bar{\zeta}_{a_j}\bar{s} \tag{33}$$

$$+ \mathbb{E}\left[\frac{T'(z)}{1-T'(z)}\zeta_z(z)z\kappa_{j,inc}'(z)\right] \tag{34}$$

$$= \mathbb{E}\left[(\kappa_j(z) - \rho_j(z))g(z)\right] + \left(p - C_s'\right)\bar{\zeta}_{a_j}\bar{s} - C_j' - \bar{\gamma}_{a_j}(1+\sigma_{a_j})\bar{\zeta}_{a_j}\bar{s}$$
$$+ \int_{z=z_{min}}^{\infty}\int_{x=z}^{\infty}(1-g(x)h(x)dx\kappa_{j,inc}'(z)dz$$
$$= \mathbb{E}\left[(\kappa_j(z) - \rho_j(z))g(z)\right] + \left(p - C_s'\right)\bar{\zeta}_{a_j}\bar{s} - C_j' - \bar{\gamma}_{a_j}(1+\sigma_{a_j})\bar{\zeta}_{a_j}\bar{s}$$
$$+ \int_{z=z_{min}}^{\infty}(1-g(z))\kappa_{j,inc}(z)h(z)dz$$
$$= \bar{\kappa}_j - \bar{\rho}_j + Cov[(\kappa_{j,pref}(z) - \rho_j(z)), g(z)] + \left(p - C_s'\right)\bar{\zeta}_{a_j}\bar{s} - C_j' - \bar{\gamma}_{a_j}(1+\sigma_{a_j})\bar{\zeta}_{a_j}\bar{s}$$

At the optimum $\frac{dW}{da_j} = 0$ if $a_j > 0$ and $\frac{dW}{da_j} < 0$ if $a_j = 0$, which implies the first-order condition

$$\bar{\kappa}_j - \bar{\rho}_j + Cov[(\kappa_{j,pref}(z) - \rho_j(z)), g(z)] \leq \bar{\gamma}_{a_j}(1+\sigma_p)\left|\bar{\zeta}_{a_j}\right|\bar{s} - \left(p - C_s'\right)\left|\bar{\zeta}_{a_j}\right| + C_j'.$$

with equality when $a_j > 0$. Rearranging gives the second condition in the Proposition.

Finally, note that the first-order conditions implied by (28)—(29) and (33)—(34) also allow us to characterize the optimal p and $\boldsymbol{a}$ even when the income tax is not optimal. □

D Aggregate Lottery Demand Appendix

Table A1: **First Stages for Prize Semi-Elasticity Estimates**

(a) **Jackpot Semi-Elasticity: Nationwide Data**

	(1) Jackpot EV ($)	(2) Jackpot EV ($)
Jackpot expected value forecast ($)	1.0299***	1.1022***
	(0.0203)	(0.0343)
Lags in ***H***	1	4
Quadratic terms in ***H***	No	Yes
F-statistic		1,035
R^2	0.94	0.95
Observations	2,035	2,035

(b) **Jackpot and Second Prize Semi-Elasticities: California Data**

	(1) Jackpot EV ($)	(2) Jackpot EV ($)	(3) 2nd prize EV ($)	(4) 2nd prize EV ($)
Jackpot expected value forecast ($)	1.0217***	1.0985***	0.0249***	0.0265***
	(0.0250)	(0.0380)	(0.0035)	(0.0045)
2nd prize expected value forecast ($)	-0.0908	-0.0945**	1.0056***	1.0045***
	(0.0589)	(0.0475)	(0.0169)	(0.0159)
Lags in ***H***	1	4	1	4
Quadratic terms in ***H***	No	Yes	No	Yes
F-statistic	1,627	841	3,567	4,025
R^2	0.94	0.94	0.85	0.85
Observations	1,701	1,695	1,701	1,695

Notes: This table presents first stage estimates of equation (10). The first stages regress prize expected values on a forecast based on the previous period prize amount and an indicator for whether the prize was won in the previous period, controlling for a vector of lagged logged sales and prize expected values as well as game-format, game-regional coverage, game-quarter of sample, and game-weekend fixed effects. Panel (a) uses nationwide data, while Panel (b) uses California data only. The samples include all Mega Millions and Powerball drawings from June 2010 to February 2020; the sample in Panel (b) is smaller because California did not join Powerball until April 2013. Newey-West standard errors allowing up to ten lags are in parentheses. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

Table A2: **Prize Semi-Elasticity Estimates with Alternative Standard Errors**

(a) **Jackpot Semi-Elasticity: National Sales**

	(1) OLS	(2) IV	(3) IV
Jackpot expected value ($)	0.9984	0.9437	0.9543
	(0.0309)	(0.0380)	(0.0374)
	(0.0309)	(0.0383)	(0.0374)
	(0.0315)	(0.0391)	(0.0378)
	(0.0292)	(0.0381)	(0.0366)
	(0.0307)	(0.0392)	(0.0380)
	(0.0301)	(0.0364)	(0.0339)
Lags in $\boldsymbol{H}$	0	1	4
Quadratic terms in $\boldsymbol{H}$	No	No	Yes
Observations	2,035	2,035	2,035

(b) **Jackpot and Second Prize Semi-Elasticities: California Sales Only**

	(1) OLS	(2) IV	(3) IV
Jackpot expected value ($)	1.0764	0.9972	1.0191
	(0.0412)	(0.0461)	(0.0492)
	(0.0418)	(0.0470)	(0.0494)
	(0.0434)	(0.0482)	(0.0504)
	(0.0402)	(0.0468)	(0.0485)
	(0.0425)	(0.0482)	(0.0504)
	(0.0422)	(0.0450)	(0.0461)
2nd prize expected value ($)	0.0441	0.0578	0.0457
	(0.0403)	(0.0743)	(0.0639)
	(0.0390)	(0.0744)	(0.0633)
	(0.0389)	(0.0710)	(0.0602)
	(0.0379)	(0.0764)	(0.0653)
	(0.0379)	(0.0733)	(0.0636)
	(0.0347)	(0.0706)	(0.0585)
Lags in $\boldsymbol{H}$	0	1	4
Quadratic terms in $\boldsymbol{H}$	No	No	Yes
Observations	1,705	1,701	1,695

Notes: This table presents estimates of equation (10), a regression of the natural log of sales on prize expected values, controlling for a vector of lagged logged sales and prize expected values as well as game-format, game-regional coverage, game-quarter of sample, and game-weekend fixed effects. The IV regressions instrument for prize expected values with a forecast based on the previous period prize amount and an indicator for whether the prize was won in the previous period. Panel (a) uses nationwide sales, while Panel (b) uses California sales only. The samples include all Mega Millions and Powerball drawings from June 2010 to February 2020; the sample in Panel (b) is smaller because California did not join Powerball until April 2013. The standard errors, in order from top to bottom, are Newey-West with five lags, Newey-West with ten lags, Newey-West with twenty-five lags, robust standard errors clustered by game and month, robust standard errors clustered by game and quarter, and robust standard errors clustered by game and half year.

Table A3: **Regressions of Jackpot Expected Value on Forecast Instruments**

	(1) EV, t (\$)	(2) EV, $t-1$ (\$)	(3) EV, $t-2$ (\$)	(4) EV, $t-3$ (\$)	(5) EV, $t-4$ (\$)
Jackpot expected value forecast, t (\$)	0.8127***	-0.1590***	-0.0522***	-0.0130***	-0.0007
	(0.0266)	(0.0393)	(0.0137)	(0.0038)	(0.0021)
Jackpot expected value forecast, $t-1$ (\$)	-0.0081**	0.9951***	-0.1116***	-0.0318***	-0.0036
	(0.0041)	(0.0562)	(0.0320)	(0.0107)	(0.0036)
Jackpot expected value forecast, $t-2$ (\$)	-0.0076	-0.0043	1.0540***	-0.1034***	-0.0165***
	(0.0049)	(0.0043)	(0.0582)	(0.0294)	(0.0058)
Jackpot expected value forecast, $t-3$ (\$)	-0.0093	-0.0154*	-0.0076	1.1180***	-0.0624***
	(0.0060)	(0.0081)	(0.0054)	(0.0619)	(0.0211)
Jackpot expected value forecast, $t-4$ (\$)	-0.0099	-0.0081	-0.0083	0.0072	1.1436***
	(0.0138)	(0.0103)	(0.0134)	(0.0126)	(0.0419)
R^2	0.84	0.87	0.88	0.91	0.95
Observations	2,035	2,035	2,035	2,035	2,035

Notes: This table presents estimates of a regression of jackpot expected values on leads and lags of the jackpot expected value forecast, controlling for a vector of lagged logged sales and jackpot expected values as well as game-format, game-regional coverage, game-quarter of sample, and game-weekend fixed effects. We instrument for prize expected values with a forecast based on the previous period prize amount and an indicator for whether the prize was won in the previous period. The samples include all Mega Millions and Powerball drawings from June 2010 to February 2020. Newey-West standard errors allowing up to ten lags are in parentheses. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

D.1 Substitution Across Games

D.1.1 Substitution in Response to Jackpot Variation

Our modeling considers only one good subject to behavioral bias. If changes in prices or attributes cause substitution to other goods subject to behavioral bias, optimal policy would have to account for this (Allcott, Lockwood, and Taubinsky 2019). To test for substitution between Mega Millions and Powerball, we re-estimate equation (10) with two changes. First, the dependent variable is in levels instead of logs, which allows us to easily construct a diversion ratio by dividing regression coefficients. Second, we add the jackpot for game $-j$. The regression is

$$\bar{s}_{jt} = \bar{\zeta}_j \pi_{1jt} w_{1,j,t} + \bar{\zeta}_c \pi_{1,-j,t} w_{1,-j,t} + \boldsymbol{\beta}_H \boldsymbol{H}_{j,t-1} + \boldsymbol{\beta}_{-j,H} \boldsymbol{H}_{-j,t-1} + \xi_{jt} + \epsilon_{jt}. \tag{35}$$

As before, we instrument for $\pi_{1jt} w_{1jt}$ and $\pi_{1,-j,t} w_{1,-j,t}$ with $\pi_{1jt} Z_{1jt}$ and $\pi_{1,-j,t} Z_{1,-j,t}$, and we control for lags of sales and jackpots in order to isolate random variation in jackpot amounts. Mega Millions draws are on Tuesday and Friday, while Powerball draws are on Wednesday and Saturday. We define t by matching the Tuesday-Wednesday draws and Friday-Saturday draws for a given week.

Panel (a) of Appendix Table A4 presents OLS and IV estimates. Column 2 shows that when a game's jackpot expected value increases by \$1, that game's sales increase by 50.59 million tickets. However, the other game's ticket sales are statistically unaffected, and the 95 percent confidence intervals exclude effects larger than about 2.4 million tickets.

We can also estimate substitution to lottery games other than the multi-state games. To do that, we collapse the balanced panel of games in the La Fleur's data to the nationwide weekly level. Now let $\bar{s}_{jt}$ be sales in units of dollars, let $\pi_{1t}w_{1t}$ denote the average jackpot expected value across the four draws of Mega Millions and Powerball in week t, and let $\boldsymbol{H}_{t-1}$ be the vector of sales and jackpots for Mega Millions and Powerball for the four previous weeks. The regression is

$$\bar{s}_{jt} = \bar{\zeta}_c \pi_{1t} w_{1t} + \boldsymbol{\beta}_H \boldsymbol{H}_{t-1} + \xi_{jt} + \epsilon_{jt}. \tag{36}$$

We instrument for $\pi_{1t}w_{1t}$ with a weekly version of the rollover instrument.[29] In these weekly data, ξ_{jt} represents quarter-of-sample fixed effects and 52 week-of-year fixed effects, which we have found to improve precision by soaking up seasonality.

Panel (b) of Appendix Table A4 presents the OLS estimates. The IV estimates are very similar; see Appendix Table A5. Each column considers sales of different games. Column 1 shows that when the average Mega Millions and Powerball jackpot expected values increase by $1, their combined weekly ticket sales increase by $267.90 million. Column 2 shows that sales of 13 major state-level draw games increase by $0.96 million, suggesting statistically significant but economically small complementarity.[30] Columns 3 and 4 show no statistically significant effects on instant games and on the combination of all games other than Mega Millions and Powerball. The 95 percent confidence interval in column 4 rules out that a $1 increase in the Mega Millions and Powerball jackpots increases sales of other games by more than $5.1 million or decreases sales by more than $6.4 million, implying economically very limited substitution. Appendix Figure A3 presents visual examples of these null effects for the 2014 data, paralleling Figure 1.

[29] The weekly version of the instrument is the product of the jackpot win probability and the draw-level jackpot forecast, constructed as described in equation (9), averaged over weeks in the same way as the jackpot expected values.

[30] We selected these 13 games because they were the most likely substitutes for Mega Millions and Powerball. We first selected the largest draw game in each state plus additional games where jackpot data were available, then limited to a balanced panel in states that were always Mega Millions and Powerball members. The 13 games are Lotto from Colorado, Lotto! from Connecticut, Lotto from Illinois, Megabucks Doubler from Massachusetts, Multi-Match from Maryland, Tri-State Megabucks Plus from Maine and New Hampshire, Gopher 5 from Minnesota, Lotto from New York, Classic Lotto and Rolling Cash 5 from Ohio, Megabucks from Oregon, Lotto Texas from Texas, and Lotto from Washington.

Table A4: **Cross-Game Substitution**

(a) **Mega Millions and Powerball**

	(1) OLS	(2) IV
Own game jackpot	58.00***	50.59***
expected value ($)	(7.83)	(6.40)
Other game jackpot	0.03	-0.03
expected value ($)	(1.31)	(1.25)
Lags in $\boldsymbol{H}$	0	4
Quadratic terms in $\boldsymbol{H}$	No	Yes
Observations	2,035	2,035
Dependent variable mean	23.0	23.0

(b) **Different Game Types**

	(1) Mega Millions & Powerball	(2) Major state draw games	(3) Instant games	(4) Other state-level games
Jackpot expected value ($)	267.90***	0.96***	0.00	-0.62
	(39.63)	(0.17)	(2.67)	(2.93)
Observations	508	508	508	508
Dependent variable mean	108.2	12.4	509.5	669.2

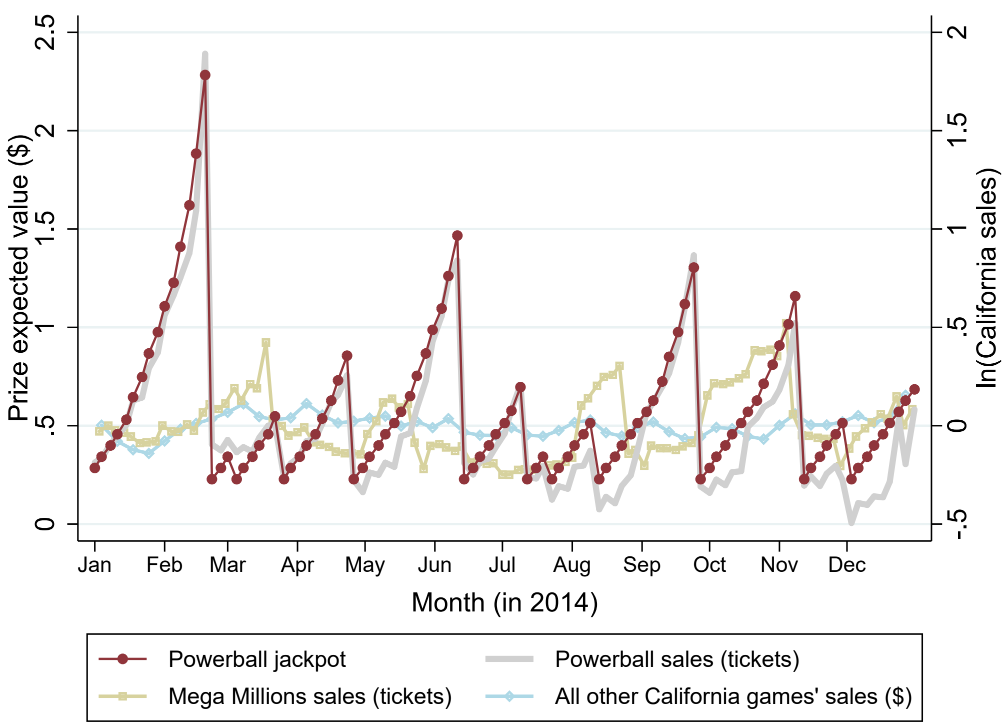
Notes: Panel (a) of this table presents estimates of equation (35), a regression of the level of sales of a multi-state game on the prize expected values of both multi-state games, controlling for a vector of lagged sales and prize expected values of each multi-state game as well as game-format, game-regional coverage, game-week, weekend, and quarter-of-sample fixed effects, using nationwide game-by-draw data. The IV regression instruments for prize expected values with a forecast based on the previous period prize amount and an indicator for whether the prize was won in the previous period. Panel (b) presents estimates of equation (36), a regression of the aggregate level of sales of the lottery games indicated in each column on the week-average prize expected values of Mega Millions and Powerball, controlling for week and quarter-of-sample fixed effects, using a balanced panel of nationwide game-by-week data. Newey-West standard errors allowing up to ten lags are in parentheses. Sales are in millions of tickets in Panel (a) and millions of dollars in Panel (b). *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

Table A5: **Cross-Game Substitution: IV**

	(1) Mega Millions & Powerball	(2) Major state draw games	(3) Instant games	(4) Other state-level games
Jackpot expected value ($)	271.89***	1.14***	-6.63	-8.87*
	(32.73)	(0.18)	(4.22)	(4.62)
Lags in $\boldsymbol{H}$	4	4	4	4
Quadratic terms in $\boldsymbol{H}$	Yes	Yes	Yes	Yes
Observations	508	508	508	508
Dependent variable mean	108.2	12.4	509.5	669.2

Notes: This table presents estimates of equation (36), a regression of the aggregate level of sales in millions of dollars of the lottery games indicated in each column on the week-average prize expected values of Mega Millions and Powerball, controlling for a vector of week-average lagged Mega Millions and Powerball sales and prize expected values as well as game-week and quarter-of-sample fixed effects, using a balanced panel of nationwide game-by-week data. Week-average prize expected values are instrumented with week averages of a forecast based on the previous period prize amount and an indicator for whether the prize was won in the previous period. Newey-West standard errors allowing up to ten lags are in parentheses. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

Figure A3: **Powerball Prizes and Other Games Ticket Sales in 2014**



Notes: This figure presents the expected values of the Powerball jackpot, the natural log of Powerball and Mega Millions California ticket sales for each drawing in 2014, and the natural log of aggregate sales from a balanced panel of all other California game-by-week data. The levels of sales are adjusted so that the average natural log of sales is zero.

D.1.2 Substitution in Response to Price Variation

We can also test for substitution by estimating the effect of game j's price change on other games. To do this, we estimate an analogue to equation (11), except with the substitute game's sales level on the left-hand side, a control for the substitute game's jackpot expected value, and a vector of week-of-year and event fixed effects collectively denoted ξ_t:

$$\bar{s}_{-jt} = \bar{\zeta}_p p_{jt} W_{jt} + \beta_1 W_{jt} \left(+\beta_2 \pi_{1-jt} w_{1-jt}\right) + \xi_t + \epsilon_{jt}. \tag{37}$$

The substitute game expected value control $\pi_{1-jt} w_{1-jt}$ is used only to study substitution to the other major multi-state game, not when we estimate substitution to other games in the La Fleur's data. Table A6 presents results.

Table A6: **Cross-Price Demand Responses**

	(1) Own-price response	(2) Other multi-state game	(3) Major state draw games	(4) All other games
Price × 12-month window	-19.90***	5.84	-0.03	7.85
	(4.47)	(6.70)	(0.37)	(11.52)
12-month window	27.76***	-4.75	0.28	-7.91
	(7.46)	(9.63)	(0.61)	(18.42)
Jackpot expected value ($)	63.94***	89.63***		
	(7.64)	(21.72)		
Observations	312	312	312	312
Dependent variable mean	37.8	38.5	12.6	650.5

Notes: This table presents estimates of equation (37), a regression of the aggregate level of sales of the lottery games indicated in each column on the ticket price of a multi-state game interacted with an indicator for the 12-month window—six months before and six months after—around a price change event for that multi-state game and the 12-month window indicator, controlling for week fixed effects and the week-average jackpot expected value of the game indicated in the column in columns 1 and 2. Each column pools data from a 36-month window around both the Powerball and Mega Millions price changes, using a balanced panel of nationwide game-by-week data, and also includes a price-change event fixed effect. Newey-West standard errors allowing up to ten lags are in parentheses. Sales are in millions of tickets in columns 1–2 and millions of dollars in columns 3–4. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

D.2 Long-Run vs. Short-Run Elasticity

As in many other studies, we have a well-identified short-run elasticity, but our policy analysis requires a long-run elasticity. Consider two models in which these elasticities might differ. First, consumption might be substitutable or complementary over time, e.g. if previous purchases cause people to tire or get excited in the future. Second, consumers might have a desired average spending (e.g. $X per month) that they allocate across draws to maximize expected value. In the limiting case, demand might be fully inelastic to the average jackpot level but highly elastic to variation across draws.

To address these issues, we test for effects of lagged jackpot amounts and also aggregate over time. Define a "jackpot spell" as a group of draws beginning after a jackpot is won and continuing until the next win. The sawtooth pattern in Figures 1 and A3 illustrates that while the length of each jackpot spell varies, these spells are well-defined units of analysis that capture the variation we want to use. We collapse the data to the average $\ln \bar{s}_{jt}$ and average $\pi_{kjt}w_{kjt}$ over each of the 196 complete jackpot spells in our sample, which are 4.7 weeks long on average. Now using t to index jackpot spells, we estimate an analogue of equation (10) including lags indexed by l:

$$\ln \bar{s}_{jt} = \sum_{l=0}^{L} \bar{\zeta}_{1l}\pi_{1j,t-l}w_{1j,t-l} + \boldsymbol{\beta}_H \boldsymbol{H}_{j,t-1} + \xi_{jt} + \epsilon_{jt}, \tag{38}$$

where the fixed effects ξ_{jt} are now game-format, game-regional coverage, and game-year of sample fixed effects based on the first draw in the spell. We instrument for $\pi_{1jt}w_{1jt}$ with an prediction of the spell midpoint jackpot expected value using the reset value $\underline{w}_{1jf(t)}$, the average percent increase $\iota_{1jf(t)}$, and the number of rollovers R_{jt} in the spell: $\left(1+\iota_{1jf(t)}\right)^{R_{jt}/2} \cdot \pi_{1jt}\underline{w}_{1jf(t)}$. Because the sample is smaller, we include only the single previous spell's average sales and jackpot expected value in $\boldsymbol{H}_{j,t-1}$. The first stages are strong, and the jackpot expected value forecast instrument for lag $t-l$ strongly predicts the actual jackpot expected value for lag $t-l$ but not for other lags; see Appendix Table A8.

Table A7 presents results. Columns 1 and 2 present the OLS and IV estimates with no lags, while columns 3 and 4 add three lags. In case aggregating to jackpot spells is still not enough to identify a long-run elasticity, columns 5 and 6 present estimates after aggregating to groups of three jackpot spells. There are 64 complete three-spell groups in our sample, which are 15.2 weeks long on average.

The contemporaneous effects in columns 1, 2, 5, and 6 are comparable to the draw-level estimates from Table 3. The lag coefficients in columns 3 and 4 imply intertemporal complementarity: higher jackpots in recent prior spells cause higher demand now, with an effect that decays toward zero by the third lag. The OLS and IV estimates are very similar, suggesting limited simultaneity bias.

Table A7: **Intertemporal Substitution**

	(1) OLS	(2) IV	(3) OLS	(4) IV	(5) OLS	(6) IV
Jackpot expected value ($)	0.8941***	0.8819***	0.9197***	0.8621***	0.9599***	1.0031***
	(0.0427)	(0.0390)	(0.0505)	(0.0393)	(0.1386)	(0.1328)
Jackpot expected value, $t-1$ ($)			0.1516***	0.1111***		
			(0.0373)	(0.0354)		
Jackpot expected value, $t-2$ ($)			0.0870**	0.0635**		
			(0.0356)	(0.0310)		
Jackpot expected value, $t-3$ ($)			0.0477**	0.0523***		
			(0.0196)	(0.0193)		
Lags in $\boldsymbol{H}$	0	1	0	1	0	1
R^2	0.87	0.90	0.88	0.88	0.74	0.80
Observations	193	191	187	184	61	57

Notes: This table presents estimates of equation (38), a regression of the average natural log of sales on contemporaneous and lagged average jackpot expected values, controlling for a vector of lagged average logged sales and prize expected values as well as game-format, game-regional coverage, and game-year of sample fixed effects based on the first draw in the period of interest. In columns 1–4, we use nationwide game-by-jackpot spell data, where a jackpot spell is defined as a group of draws beginning after a jackpot is won and continuing until the next win. In columns 5–6, we use nationwide game-by-three-spell data, averaging across series of three jackpot spells. The sample includes all complete Mega Millions and Powerball jackpot spells from June 2010 to February 2020. Observation counts exclude singletons. Newey-West standard errors allowing up to ten lags are in parentheses. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

Table A8: **First Stages for Intertemporal Substitution Estimates**

	(1) Spell jackpot EV, t	(2) Spell jackpot EV, t	(3) Spell jackpot EV, $t-1$	(4) Spell jackpot EV, $t-2$	(5) Spell jackpot EV, $t-3$	(6) 3-spell jackpot EV, t
Jackpot expected value	0.8576***	0.8618***	-0.0351	-0.0068	-0.0097	0.9574***
midpoint forecast, t ($)	(0.0775)	(0.0810)	(0.0222)	(0.0233)	(0.0455)	(0.1131)
Jackpot expected value		0.0601*	0.8370***	-0.0072	-0.0107	
midpoint forecast, $t-1$ ($)		(0.0319)	(0.0890)	(0.0243)	(0.0248)	
Jackpot expected value		0.0398	0.0261	0.8694***	-0.0222	
midpoint forecast, $t-2$ ($)		(0.0266)	(0.0343)	(0.0936)	(0.0196)	
Jackpot expected value		0.0344	0.0086	0.0203	0.8541***	
midpoint forecast, $t-3$ ($)		(0.0291)	(0.0285)	(0.0373)	(0.0867)	
Lags in $\boldsymbol{H}$	1	1	1	1	1	1
F-statistic	123	121	93	146	273	72
R^2	0.77	0.78	0.79	0.78	0.77	0.68
Observations	191	184	184	184	184	57

Notes: This table presents first stage estimates of equation (38). The first stages regress average jackpot expected values on forecasts of the midpoint of the jackpot expected value within a jackpot spell or three-spell period based on the number of rollovers during the period as well as the expected value of the jackpot when it resets and the average percent jackpot increase after a rollover corresponding to the game-format of the first draw in the period of interest. In columns 1–5, we use nationwide game-by-jackpot spell data, where a jackpot spell is defined as a group of draws beginning after a jackpot is won and continuing until the next win. In column 6, we use nationwide game-by-three-spell data, averaging across series of three jackpot spells. The sample includes all complete Mega Millions and Powerball jackpot spells from June 2010 to February 2020. Observation counts exclude singletons. Newey-West standard errors allowing up to ten lags are in parentheses. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

D.3 Format Change Appendix

Table A9: **Placebo Tests: Non-Price Format Change Event Studies**

	(1) Mega Millions	(2) Mega Millions	(3) Mega Millions	(4) Powerball	(5) Powerball	(6) Powerball
Post format change × 12-month window	0.3269*	0.1554***	-0.0081	0.3348	0.2941***	0.0549
	(0.1812)	(0.0501)	(0.0500)	(0.2469)	(0.0664)	(0.0768)
12-month window	0.0141	-0.1133***	-0.0095	-0.1912*	-0.1545***	-0.0474
	(0.1025)	(0.0340)	(0.0376)	(0.1056)	(0.0268)	(0.0381)
Jackpot expected value ($)		1.2206***			0.9306***	
		(0.0560)			(0.0428)	
Jackpot ($millions)			0.0050***			0.0036***
			(0.0002)			(0.0003)
Observations	313	313	313	313	313	313

Notes: This table presents estimates of a regression of the natural log of sales on an indicator for a post-format change period interacted with an indicator for the 12-month window—six months before and six months after—around a game-format change event and the 12-month window indicator, controlling for jackpot expected value in columns 2 and 5 and jackpot amounts in columns 3 and 6. In columns 1–3, the sample includes all Mega Millions draws in a 36-month window around the October 19, 2013 format change event. In columns 4–6, the sample includes all Powerball draws in a 36-month window around the October 7, 2015 format change event. We use Newey-West standard errors with up to ten lags. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

E Survey Appendix

E.1 AmeriSpeak Survey Question Text

Variable	Question text
	Spending and income effects
Monthly lottery spending	How many dollars did you spend in total on lottery tickets in an average month in 2019?
Income change	How much income did you earn in 2019 compared to 2018? In 2019, I earned ... [Less than half as much, 25% to 50% less, 10% to 25% less, 5% to 10% less, 1% to 5% less, The exact same amount, 1% to 5% more, 5% to 10% more, 10% to 25% more, 25% to 50% more, Over 50% more]
Spending change	How much money did you spend in total on lottery tickets in 2019 compared to 2018? In 2019, I spent ... [Less than half as much, 25% to 50% less, 10% to 25% less, 5% to 10% less, 1% to 5% less, The exact same amount, 1% to 5% more, 5% to 10% more, 10% to 25% more, 25% to 50% more, Over 50% more]

Self-reported income effect	Imagine you got a raise and your income doubled. How do you think your lottery spending would change? I would spend . . . [Less than half as much, 25% to 50% less, 10% to 25% less, 5% to 10% less, 1% to 5% less, The exact same amount, 1% to 5% more, 5% to 10% more, 10% to 25% more, 25% to 50% more, Over 50% more]
	Preferences
Unwillingness to take risks	In general, how willing or unwilling are you to take risks? [1 Very unwilling, 2, 3, 4, 5, 6, 7 Very willing]
Financial risk aversion	Which of the following statements comes closest to the amount of financial risk that you are willing to take when you save or make investments? [Substantial financial risks expecting to earn substantial returns, Above-average financial risks expecting to earn above-average returns, Average financial risks expecting to earn average returns, No financial risks]
Lottery seems fun	To what extent do you agree or disagree with the following statement: *For me, playing the lottery seems fun.* [-3 Strongly disagree, -2, -1, 0 Neutral, 1, 2, 3 Strongly agree]
Enjoy thinking about winning	To what extent do you agree or disagree with the following statement: *I enjoy thinking about how life would be if I won the lottery.* [-3 Strongly disagree, -2, -1, 0 Neutral, 1, 2, 3 Strongly agree]
	Bias proxies
Self-control problems	It can be hard to exercise self-control, and some people feel that there are things they do too much or too little – for example, exercise, save money, or eat junk food. Do you feel like you play the lottery too little, too much, or the right amount? [-3 Far too little, -2, -1, 0 The right amount, 1, 2, 3 Far too much]
Financial illiteracy	Normally, which asset displays the highest fluctuations over time? [Savings accounts, Bonds, Stocks]
	When an investor spreads her money among different assets, does the risk of losing money: [Increase, Decrease, Stay the same]
	A second hand car dealer is selling a car for $6,000. This is two-thirds of what it cost new. How much did the car cost new? [$_]

If 5 people all have the winning numbers in the lottery and the prize is $2 million, how much will each of them get? [$_]

Let's say you have $200 in a savings account. The account earns 10% interest per year. How much will you have in the account at the end of two years? [$_]

Suppose you had $100 in a savings account and the interest rate was 2% per year. After 5 years, how much do you think you would have in the account if you left the money to grow? [More than $102, Exactly $102, Less than $102]

Imagine that the interest rate on your savings account was 1% per year and inflation was 2% per year. After 1 year, how much would you be able to buy with the money in this account? [More than today, Exactly the same as today, Less than today]

Do you think that the following statement is true or false? "Buying a single company stock usually provides a safer return than a stock mutual fund." [True, False]

Statistical ability

For the next few questions, imagine flipping a coin that has a 50% chance of landing heads and a 50% chance of landing tails. Imagine that after eight flips, you observe the patterns described in the table below. What is the probability, in percent from 0-100, that the next flip is tails? [tails-tails-tails-heads-tails-heads-heads-heads _%, heads-heads-heads-heads-heads-heads-heads-heads _%, heads-tails-heads-tails-tails-tails-tails-tails _%]

Now imagine starting over and flipping a coin 1000 times. What are the chances, in percent from 0-100, that the total number of heads will lie within the following ranges? [Between 481 and 519 heads _%, Between 450 and 550 heads _%, Between 400 and 600 heads _%]

Now we are going to ask you how much people might win from different lotteries. For each lottery described in the table below, please give us your best estimate of what percent (from 0-100) of the lottery revenues are returned to the winners. [Tickets cost $1, and 1 out of every 10 tickets wins $10. _%, Tickets cost $1, and 1 out of every 1,000 tickets wins $500. _%, Tickets cost $1, 1 out of every 400,000,000 tickets wins $200,000,000, **and** 1 out of every 1,000 tickets wins $100. _%, Tickets cost $1, and 1 out of every 300,000,000 tickets wins $200,000,000. _%]

Overconfidence

Imagine **you** could keep buying whatever lottery tickets you want, over and over for a very long time. For every $1000 you spend, how much do you think **you** would win back in prizes, on average? [$0 to $99, $100 to $199, $200 to $299, $300 to $399, $400 to $499, $500 to $599, $600 to $699, $700 to $799, $800 to $899, $900 to $999, $1000 to $1499, $1500 to $1999, $2000 to $5000, More than $5000]

Imagine that the **average** lottery player in the country could keep buying whatever lottery tickets they want, over and over for a very long time. For every $1000 they spend, how much do you think they would win back in prizes, on average? [$0 to $99, $100 to $199, $200 to $299, $300 to $399, $400 to $499, $500 to $599, $600 to $699, $700 to $799, $800 to $899, $900 to $999, $1000 to $1499, $1500 to $1999, $2000 to $5000, More than $5000]

Expected returns

Think about the total amount of money spent on lottery tickets nationwide. What percent do you think is given out in prizes? [0 - 9%, 10 - 19%, 20 - 29%, 30 - 39%, 40 - 49%, 50 - 59%, 60 - 69%, 70 - 79%, 80 - 89%, 90 - 100%]

Predicted life satisfaction

A recent study surveyed Swedish lottery winners. A typical person in the study had won between $100,000 and $800,000 in the lottery about 12 years before the survey. The study compared people who had won more vs. less money to determine the effect of additional lottery winnings.

The survey asked the following question about life satisfaction: "Taking all things together in your life, how satisfied would you say that you are with your life these days?" People responded on a scale from 0 ("Extremely dissatisfied") to 10 ("Extremely satisfied"). The average response was 7.21 out of 10.

Do you think lottery winnings increased life satisfaction, decreased life satisfaction, or had exactly zero effect? [Increased life satisfaction, Decreased life satisfaction, Had exactly zero effect]

By **how much** do you think an additional $100,000 in lottery winnings [increased/decreased] average life satisfaction on the 0-10 scale?

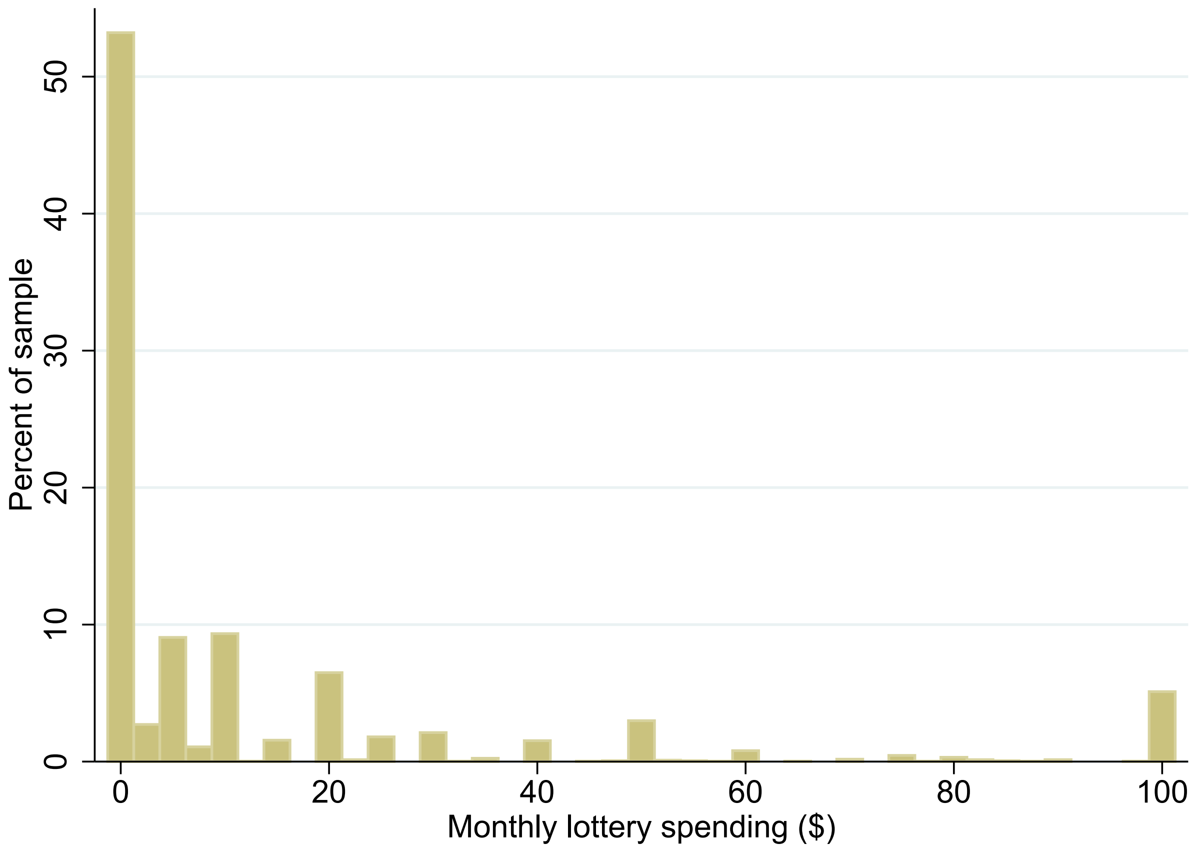
E.2 Additional Tables and Figures from AmeriSpeak Survey

Table A11: **Descriptive Statistics: 2021 Survey Data**

	Obs.	Mean	Std. dev.	Min	Max
Unwillingness to take risks	2,124	-3.96	1.37	-7	-1
Financial risk aversion	2,124	3.04	0.80	1	4
Lottery seems fun	2,124	-0.04	1.76	-3	3
Enjoy thinking about winning	2,122	0.71	1.86	-3	3
Self-control problems	2,124	-0.66	1.27	-3	3
Financial literacy	2,124	0.80	0.25	0	1
Financial numeracy	2,124	0.66	0.32	0	1
Gambler's Fallacy	2,124	0.27	0.39	0	1
Non-belief in Law of Large Numbers	2,124	0.39	0.17	0.00	0.93
Expected value miscalculation	2,124	0.66	0.38	0	1
Overconfidence	2,123	0.01	0.46	-4.95	4.65
Expected returns	2,123	0.27	0.20	0.05	0.95
Predicted life satisfaction	2,112	2.28	4.82	-10	10

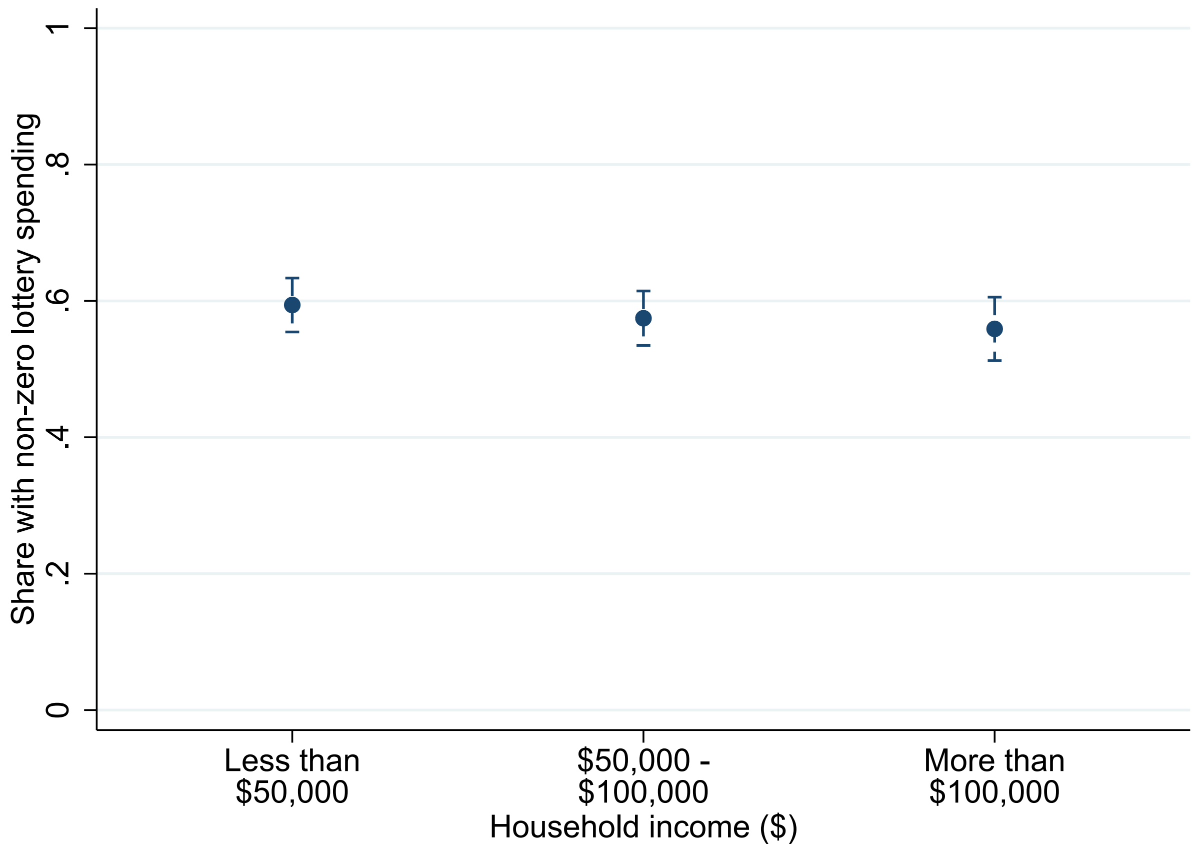
Notes: This table presents descriptive statistics for our 2021 AmeriSpeak survey, which resampled proxies for preferences and biases. Section 4.1 summarizes the coding of these variables.

Figure A4: **Distribution of Monthly Lottery Spending**



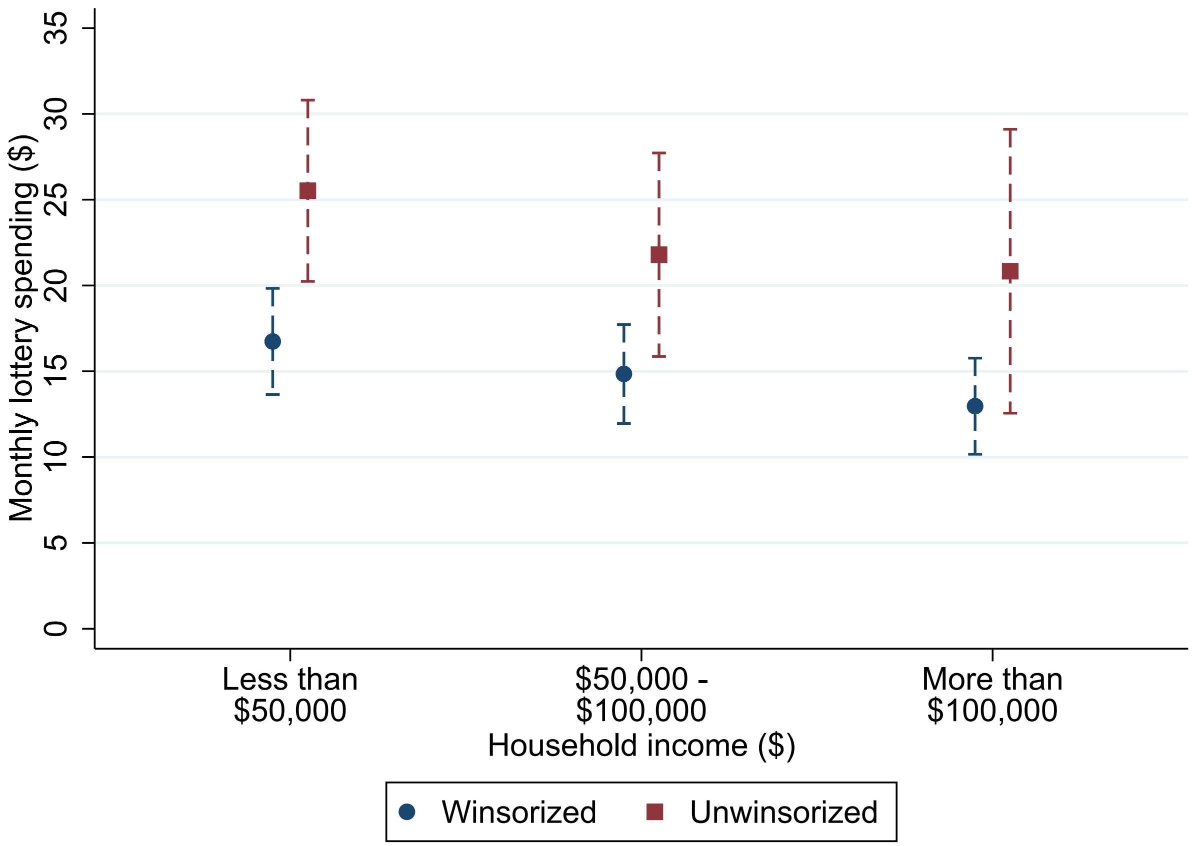
Notes: This figure presents a histogram of monthly lottery spending, using data from our AmeriSpeak survey. For this figure, spending is winsorized at $100 per month.

Figure A5: **Non-Zero Lottery Spending by Income**



Notes: This figure reports the share of people with non-zero monthly lottery spending in 2019 within household income groups, with 95 percent confidence intervals, using data from our AmeriSpeak survey. Observations are weighted for national representativeness.

Figure A6: **Lottery Spending by Income with and without Winsorization**



Notes: This figure presents average monthly lottery spending within household income groups, with 95 percent confidence intervals, using data from our 2020 AmeriSpeak survey. The winsorized values are the same as in Figure 4. The unwinsorized values are the original survey responses. Observations are weighted for national representativeness.

Table A12: **Causal and Cross-Sectional Income Effects**

	(1) Spending change	(2) Self-reported income effect	(3) ln(1 + monthly lottery spending)
Income change	0.194***		
	(0.025)		
ln(household income)			-0.111***
			(0.035)
Constant	-5.483***	-0.014***	1.911***
	(0.353)	(0.003)	(0.144)
Observations	2,862	2,855	2,877

Notes: Columns 1 and 2 report estimates of the causal elasticity of lottery spending with respect to income, using data from our AmeriSpeak survey. *Income change* and *spending change* refer to the self-reported percent change in household income and lottery spending in 2019 compared to 2018, respectively. *Self-reported income effect* is the answer to the question "Imagine you got a raise and your income doubled. How do you think your lottery spending would change?" in percent. Column 3 reports the cross-sectional elasticity of lottery spending with respect to income.

Table A13: **First-Stage Estimates for Preference and Bias Proxies**

	(1) Self-control problems	(2) Financial illiteracy	(3) Statistical mistakes	(4) Expected returns	(5) Predicted life satisfaction	(6) Risk aversion	(7) Lottery seems fun	(8) Enjoy thinking about winning
Self-control problems	0.353***	0.025**	0.008	0.022	-0.023	-0.002	0.008	0.003
	(0.058)	(0.010)	(0.013)	(0.015)	(0.017)	(0.010)	(0.013)	(0.012)
Financial illiteracy	0.064***	0.551***	0.248***	0.050***	-0.003	0.017	0.004	0.007
	(0.020)	(0.018)	(0.015)	(0.017)	(0.019)	(0.011)	(0.015)	(0.015)
Statistical mistakes	0.021	0.155***	0.439***	-0.017	0.026	0.008	0.036***	0.038***
	(0.018)	(0.011)	(0.019)	(0.016)	(0.017)	(0.010)	(0.013)	(0.013)
Expected returns	0.023	0.020**	-0.018	0.356***	0.011	0.012	0.036***	0.009
	(0.015)	(0.009)	(0.011)	(0.023)	(0.014)	(0.010)	(0.012)	(0.012)
Predicted life satisfaction	-0.016	0.005	0.019*	0.010	0.286***	-0.005	0.028**	0.054***
	(0.015)	(0.009)	(0.011)	(0.014)	(0.023)	(0.010)	(0.012)	(0.012)
Risk aversion	-0.027*	0.028***	0.011	0.002	-0.015	0.720***	-0.050***	0.009
	(0.014)	(0.008)	(0.011)	(0.015)	(0.015)	(0.014)	(0.011)	(0.011)
Lottery seems fun	0.028	0.004	0.021	0.071***	0.028	-0.034***	0.472***	0.115***
	(0.019)	(0.011)	(0.014)	(0.018)	(0.018)	(0.012)	(0.021)	(0.017)
Enjoy thinking about winning	0.017	0.005	0.025*	0.005	0.103***	0.022*	0.122***	0.491***
	(0.016)	(0.011)	(0.014)	(0.018)	(0.018)	(0.011)	(0.016)	(0.023)
ln(household income)	0.029	-0.059***	-0.031**	0.024	-0.011	-0.055***	-0.016	0.003
	(0.019)	(0.011)	(0.014)	(0.018)	(0.019)	(0.009)	(0.013)	(0.013)
ln(years of education)	-0.174**	-0.469***	-0.281***	-0.152*	-0.269***	-0.105**	-0.178***	-0.196***
	(0.083)	(0.047)	(0.070)	(0.087)	(0.092)	(0.041)	(0.062)	(0.061)
Other demographics	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
State fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-statistic	50.4	506.9	299.0	225.0	132.7	2,570.9	363.9	312.2
R^2	0.22	0.82	0.91	0.77	0.30	0.60	0.38	0.40
Observations	4,144	4,144	4,144	4,144	4,144	4,144	4,144	4,144
Clusters	2,072	2,072	2,072	2,072	2,072	2,072	2,072	2,072

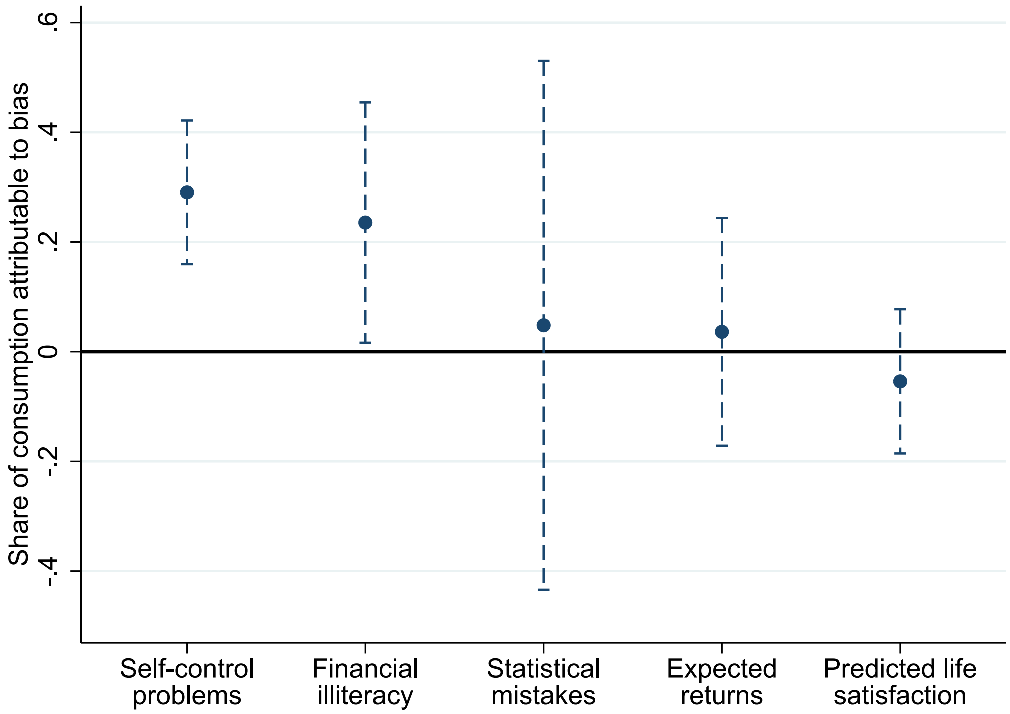
Notes: This table presents first stage estimates of the Obviously Related Instrumental Variables version of equation (12): regressions of 2020 and 2021 bias and preference proxies on 2021 and 2020 bias and preference proxies plus demographic controls and state fixed effects, using data from our AmeriSpeak surveys. "Other demographics" includes age, household size, political ideology, and indicators for male, black, white, Hispanic, married, employed, urban area, and attends religious services at least once a month. Robust standard errors, clustered by respondent, are in parentheses. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

Table A14: **Association of Income and Bias Proxies with Differences in Reported Lottery Spending**

	(1) Household income ($000s)	(2) Self-control problems	(3) Financial illiteracy	(4) Statistical mistakes	(5) Over-confidence	(6) Expected returns	(7) Predicted life satisfaction
Δ monthly lottery spending	-0.000080	-0.000060	-0.000046	0.000001	0.000225	0.000430	0.000061
	(0.013641)	(0.000865)	(0.000303)	(0.000328)	(0.000290)	(0.000393)	(0.000268)
R^2	0.00	0.00	0.00	0.00	0.00	0.03	0.00
Observations	79	79	79	79	79	79	79

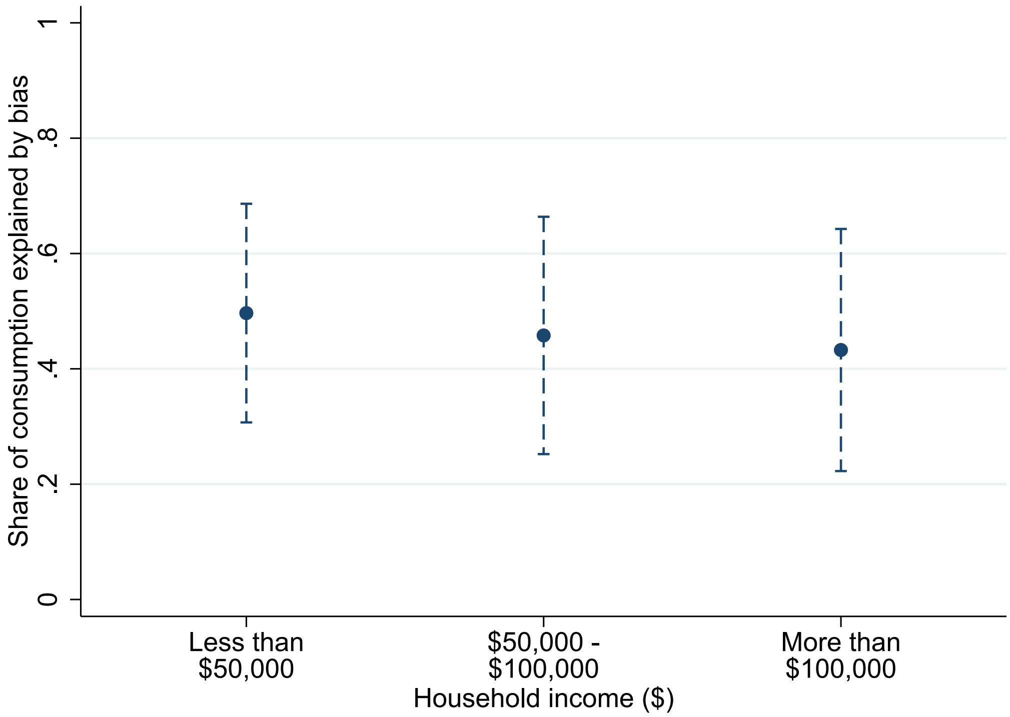
Notes: This table reports the associations of household income and bias proxies with the difference in 2019 *monthly lottery spending* reported on the 2020 vs. 2021 surveys. The sample includes all respondents from whom we re-elicited 2019 *monthly lottery spending* on the 2021 survey, which included only people who had reported spending more than $150 per month or more than 10 percent of their income on lottery tickets in our 2020 survey. Robust standard errors are in parentheses. *, **, ***: statistically significant with 90, 95, and 99 percent confidence, respectively.

Figure A7: **Share of Lottery Spending Explained by Biases (Instrumental Variables Estimates)**



Notes: This figure plots the share of lottery spending explained by each of our six bias proxies, with 95 percent confidence intervals. Predicted unbiased consumption is $\hat{s}^V_{ik} = \frac{s_i+1}{\exp(\hat{\tau}_k \tilde{b}_{ik})} - 1$, where s_i is *monthly lottery spending*, $\hat{\tau}_k$ is the IV estimate from column 6 of Table 6, and $\tilde{b}_{ik} = \frac{b_{ik}-b^V_k}{SD(b_{ik})}$ is the difference between person i's proxy b_{ik} and the unbiased value b^V_k in standard deviation units. We winsorize at $\hat{s}^V_i \geq 0$, and we fix $\hat{s}^V_{ik} = 0$ if $s_i = 0$. The share of consumption explained by each bias proxy is $\frac{\sum_i (s_i - \hat{s}^V_{ik})}{\sum_i s_i}$.

Figure A8: **Share of Lottery Spending Explained by Bias within Income Groups (Instrumental Variables Estimates)**



Notes: This figure plots the share of lottery spending explained by bias within household income groups, with 95 percent confidence intervals. Predicted unbiased consumption is $\hat{s}_i^V = \frac{s_i+1}{\exp(\hat{\boldsymbol{\tau}}\tilde{\boldsymbol{b}}_i)} - 1$, where s_i is *monthly lottery spending*, $\hat{\boldsymbol{\tau}}$ is the IV estimate from column 6 of Table 6, and $\tilde{b}_{ik} = \frac{b_{ik}-b_k^V}{SD(b_{ik})}$ is the difference between person i's proxy b_{ik} and the unbiased value b_k^V in standard deviation units. We winsorize at $\hat{s}_i^V \geq 0$, and we fix $\hat{s}_{ik}^V = 0$ if $s_i = 0$. The share of consumption explained by bias is $\frac{\sum_i \left(s_i - \hat{s}_i^V\right)}{\sum_i s_i}$.

E.3 Supplemental Survey

We administered a brief survey on the online platform Prolific in June 2021 with a final sample of 200 respondents. We restricted recruitment to respondents with (a) residency in the U.S., (b) at least 15 prior submissions on Prolific, and (c) a prior-submission approval rate of at least 95%. We did not allow respondents to take the survey on a mobile device to maintain the legibility of graphics in the survey. The average respondent took 2.8 minutes to complete the survey and was paid $1.50 for their participation.

The survey proceeded in the following steps. First, respondents consented to participate in the survey and entered their unique identifier used for anonymous compensation and communication on the platform. They then answered a set of four questions in which they expressed whether they would hypothetically prefer to receive a smaller certain dollar amount or a chance of receiving a larger dollar amount. Specifically, we asked whether they would prefer (i) $110 for sure or a 50% chance of $200, (ii) $210 for sure or a 50% chance of $400, (iii) $110 million for sure or a 50%

chance of \$200 million, and (iv) \$210 million or a 50% chance of \$400 million. The order in which the four questions were presented was randomized at the respondent level.

Next, we provided respondents with information about a Mega Millions drawing. We informed them that the cost of a Mega Millions ticket is \$2 and displayed a graphic stating that the next estimated jackpot was valued at \$252 million, with a cash option of \$153.9 million. We then displayed another graphic explaining how Mega Millions is played, the fixed lower prize amounts, and the odds of winning at each prize level as well as overall. We elicited respondents' hypothetical WTP for a ticket in the Mega Millions drawing we described.

Finally, we collected standard demographic information from respondents and asked whether they had purchased any lottery games with prize drawings (i.e., "any lottery game in which you pick numbers and win if you match the numbers from a drawing") in the past 12 months. The survey concluded with a request for feedback before redirecting respondents back to the Prolific platform. We excluded 20 respondents because of implausible WTP values of $\geq$ \$50 that seem more consistent with inattention. These high-WTP respondents were just as risk-averse in all of the binary gamble decisions as the subjects in our main sample, which is internally inconsistent behavior that is highly suggestive of these high-WTP responses being "noise."

F Details of Structural Estimation

F.1 Calibration of Decision Weights

This appendix explores the ability of common probability weighting function parameterizations to fit the decision weights implied by our empirical estimates. Thus, for the purposes of this subsection, we suppose decision weights Φ_k are an explicit function of probabilities: $\Phi_k = \Phi(\pi_k)$. Since our elasticity estimates are measured at the population level, this calibration considers representative agent calibrations, and we therefore omit the dependence on type, θ. We consider the four common probability weighting function parameterizations discussed in Wakker's (2010) textbook treatment (see Section 7.2: "Parametric Forms of Weighting Functions"), and Fehr-Duda and Epper (2012). These are applied cumulatively as proposed in Tversky and Kahneman (1992), so that the decision weight on the jackpot is $\mathcal{W}(\pi_1)$, the weight on the second prize is $\mathcal{W}(\pi_1+\pi_2)-\mathcal{W}_{TK}(\pi_1)$, and more generally,

$$\Phi(\pi_k) = \mathcal{W}\left(\sum_{j=1}^{k}\pi_j\right) - \mathcal{W}\left(\sum_{j=1}^{k-1}\pi_j\right). \tag{39}$$

The four candidate parameterizations are:

- Tversky and Kahneman (1992):

$$\mathcal{W}(\pi) = \frac{\pi^b}{\left(\pi^b + (1-\pi)^b\right)^{1/b}} \tag{40}$$

- Goldstein and Einhorn (1987):

$$\mathcal{W}(\pi) = \frac{a\pi^b}{a\pi^b + (1-\pi)^b} \tag{41}$$

- Prelec (1998):

$$\mathcal{W}(\pi) = \left(\exp\left(-\left(-\ln(\pi)\right)^a\right)\right)^b \tag{42}$$

- The "neo-additive" form studied in Chateauneuf, Eichberger, and Grant (2007):

$$\mathcal{W}(\pi) = \begin{cases} 0 & \text{if } \pi = 0 \\ b_0 + b_1\pi & \text{if } 0 < \pi < 1 \end{cases} \tag{43}$$

  Note that in our application, the behavior at $\pi = 1$ is irrelevant, since the residual worst-case outcome is a payoff of zero, with $m(0) = 0$. When applied cumulatively as in equation (39), this neo-additive specification takes a particularly simple form with

$$\Phi(\pi_k) = \begin{cases} b_0 + b_1\pi_1 & \text{if } k = 1 \\ b_1\pi_k & \text{if } k > 1 \end{cases} \tag{44}$$

Each of the first three specifications features a continuous "inverse S-shape" on the interval $[0, 1]$. Figure A9, Panel (a), illustrates this shape, plotting the Tversky and Kahneman (1992) and Prelec (1998) specifications for parameter values previously estimated in the literature, in solid blue and red.[31] (The other displayed parameterizations will be described below.) Panel (b) is constructed by zooming in on the bottom left corner of Panel (a), in order to display the behavior of these parameterizations across the very low probabilities relevant for top lottery prizes; the solid red and blue lines rise so steeply from zero that they are indistinguishable from the vertical axis.

To compare the ability of these functional forms to fit our empirical estimates, our starting point is equation (17) from the text:

$$\frac{\zeta_k}{|\zeta_p|} = \frac{\Phi(\pi_k)}{\pi_k} m'(w_k). \tag{45}$$

We can use our semi-elasticity estimates and our specification of $m(\cdot)$ to compute the decision weights $\Phi(\pi_1)$ and $\Phi(\pi_2)$ non-parametrically; these correspond to points on the probability weighting function. For this calculation, we use values for π_k and w_k that correspond to a current Powerball ticket (net of income taxes), and we assume $m(\cdot)$ comes from a CRRA utility function, all as described in Section 5.1. Using our semi-elasticity estimates of $\zeta_1 = 0.9543$, $\zeta_2 = 0.0457$, and

[31] The Tversky and Kahneman (1992, p. 312) specification (solid blue) plots the parameterization with $b = 0.61$, the preferred estimate in that paper. The Prelec (1998) specification (solid red) plots the parameterization at preferred values reported in Wakker (2010): "Ongoing empirical research suggests that the parameters $a = 0.65$ and $b = 1.0467$, giving intersection with the diagonal at 0.32, are good parameter choices for gains."

$\zeta_p = -0.4958$, from Section 3, this calculation implies[32]

$$\frac{\Phi(\pi_1)}{\pi_1} = 231. \tag{46}$$

and

$$\frac{\Phi(\pi_2)}{\pi_2} = 0.16. \tag{47}$$

Figure A9, Panel (b), plots the points on the probability weighting function implied by these weights, $(\pi_1, \Phi(\pi_1))$ and $(\pi_2, \Phi(\pi_2))$.

As discussed in Section 3.2, our estimate of ζ_2 may be affected by the low salience of variation in the second prize in California, with the implication that if the second prize were as heavily advertised as the jackpot, the resulting ζ_2 would be higher. This concern rests on the observation that *variation* in the second prize (in California) lacks salience; there is no reason to suppose that the average *level* of the prize—which is stable across years—is similarly non-salient. Yet the low level of the second prize (relative to the jackpot) provides information about a natural upper bound for the elasticity ζ_2. Put simply: if the "full salience" semi-elasticity ζ_2 were in fact higher than the (observed) full salience jackpot semi-elasticity ζ_1, then lottery administrators could raise demand at zero cost by reallocating the prize pool away from the jackpot and toward the second prize, while holding the total ticket expected value constant. Yet despite frequent format revisions, we see no cases of Powerball or Mega Millions formats adjusting in this direction; on the contrary, in each of the format reforms in our data, there is a relative reallocation toward a *higher* jackpot expected value relative to the second prize. We interpret this trend as suggestive evidence that the second prize semi-elasticity is weakly lower than the jackpot semi-elasticity, and thus we plot an alternative value for $\Phi(\pi_2)$ in Panel (b) assuming, as a conservative upper bound, that $\zeta_2 = \zeta_1$

We are interested in the features of probability weighting function parameterizations which can match the plotted points in Panel (b). The most straightforward case is the Tversky and Kahneman (1992) weighting function in equation (40): since this specification has only a single parameter b, there is a unique specification which passes through our estimated $\Phi(\pi_1)$; it is plotted as the dotted blue line. An immediate observation is that due to the steep slope of the dotted blue line across low probabilities, it predicts decision weights at π_2 that are far higher than even our upper bound estimate for $\Phi(\pi_2)$.

The red dot-dashed line shows how the failure of the Prelec (1998) class of functions is somewhat different. This two-parameter specification actually *can* be adjusted to pass through both our estimates of $\Phi(\pi_1)$ and $\Phi(\pi_2)$. (We use the upper bound in Panel (b); the specification passing through the lower point estimate is even more extreme.) Although this specification can technically fit both points, it results in an implausible weighting function beyond these two points. The extreme nature of this specification can be seen by returning to Panel (a), which also plots this

[32]This calculation is simplified relative to our final calibration of the structural model, described in the next Appendix section, which accounts for non-local price change that generates our estimate of ζ_p and the fact that our ζ_1 and ζ_2 estimates these are estimated using variation in pre-income-tax prizes.

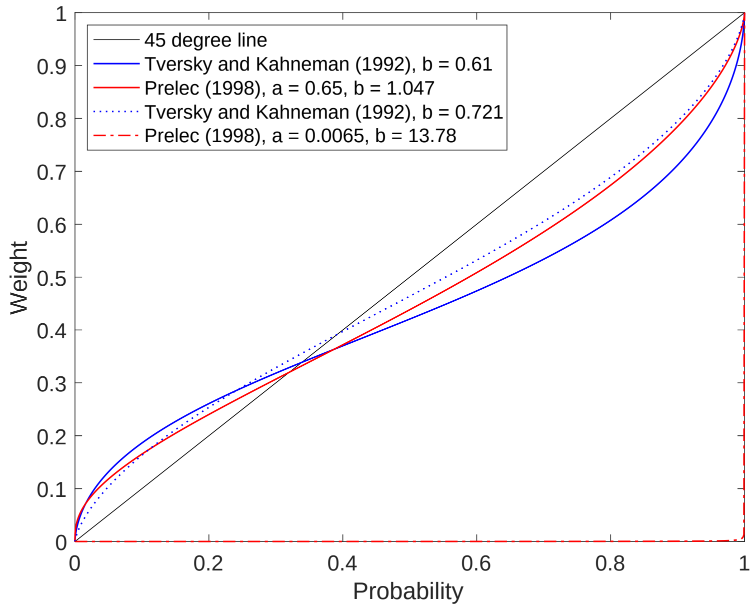
specification, demonstrating that it places exceedingly low weights on almost the entire interval [0,1], before rising sharply at high probabilities to reach $\mathcal{W}(1) = 1$. Such a probability weighting function would imply that an agent would be unwilling to pay more than \$0.10 for a lottery ticket that pays out \$100 with a probability of 95%. This leads us to conclude that the Prelec (1998) specification cannot match our results under conventional parameterizations. The two-parameter Goldstein and Einhorn (1987) parameterization produces results very similar to the Prelec (1998) specification, and we thus omit it to minimize redundancy.

Other papers have found slightly different estimates of the three inverse S-shape functions than the ones reported plotted in Figure A9. See, e.g., also, Camerer and Ho (1994) Wu and Gonzalez (1996), Abdellaoui (2000), Filiz-Ozbay et al. (2015). However, our arguments above show that none of these slightly different calibrations of the standard inverse S-shape functions can match our semi-elasticity estimates.

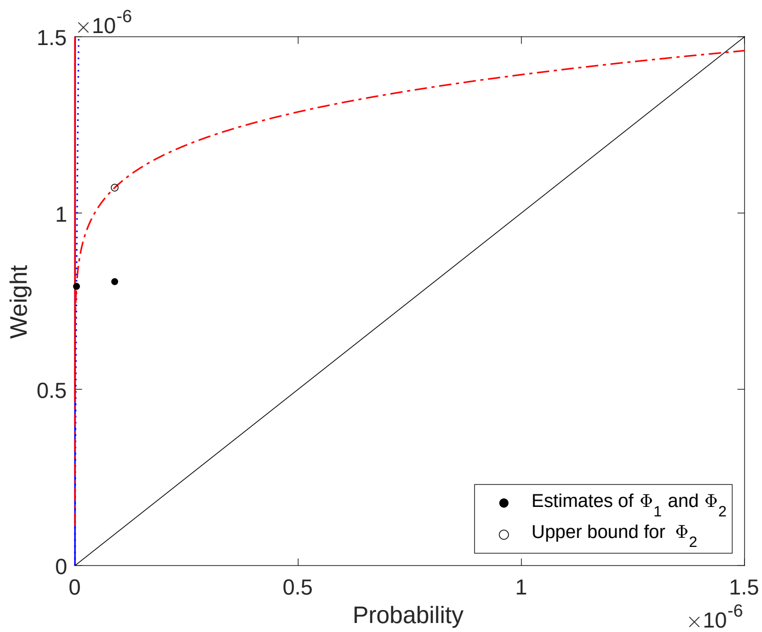
Finally, we consider the neo-additive parameterization. This weighting function simply corresponds to a straight line with a positive vertical intercept (at which the probability weighting function is discontinuous) passing through the estimated points for $\Phi(\pi_1)$ and $\Phi(\pi_2)$ in Panel (b). As such, it is readily apparent that this specification can easily match either set of points. Yet it is also apparent that although our estimate of $\Phi(\pi_1)$ provides a tight estimate of the vertical intercept parameter b_0, the uncertainty in our estimate of ζ_2 admits a wide range of potential values for the slope parameter b_1. As we show in our structural simulations, the key implications for optimal lottery design are insensitive to this slope parameter.

Figure A9: **Probability Weighting Function Parameterizations**

(a) **Plotted on the Interval [0,1]**



(b) **Plotted Over Small Probabilities**



Notes: These figures plot two common parameterizations of common probably weighting functions. The solid lines correspond to parameter values favored in the prospect theory literature. Panel (a) plots these functions across the full range of probabilities from 0 to 1, while Panel (b) "zooms in" on the bottom left corner of Panel (a), while displaying points motivated by the elasticity estimates from Section 3 (see text for details).

F.2 Implementation of Structural Model

This appendix describes implementation details of the structural model described in Section 5. Equations are written as functions of individual types θ, in order to span the heterogeneous agent model.

Equation (15) in the text illustrates that once we have specified decision weights $\Phi_k(\theta)$ and the value function $m(w;\theta)$, the net-of-price certainty equivalent (in brackets) is a sufficient statistic for type θ's lottery demand. That is, any two lotteries with the same net-of-price certainty equivalent will generate the same demand. We will use this feature repeatedly, so it is useful to formally define this "net certainty equivalent":

$$NCE(L;\theta) = \sum_k \Phi_k(\theta) m\left(w_k;\theta\right) - p \tag{48}$$

In what follows, we calibrate the structural model above for a single representative lottery, which we then manipulate to find characteristics of the optimal representative lottery. To describe the model calibration, we proceed in two steps. In Step 1, we demonstrate how this model is exactly identified by the functional form assumptions in Section 5.1 and by estimates for the following type-specific parameters:

- $s(\theta)$: consumption of a representative lottery,
- $s^V(\theta)$: debiased consumption of that representative lottery,
- $\zeta_1(\theta)$, $\zeta_2(\theta)$: semi-elasticities of demand with respect to the expected value of the jackpot and the second prize, generated by (local) variation in the prizes w_1 and w_2 of the representative lottery,
- $\zeta_p(\theta)$: semi-elasticity of demand with calculated based on a (non-local) discrete price change in the representative lottery,
- $y(\theta)$: individual expected continuation wealth,
- $g(\theta)$: welfare weights.

In Step 2, we describe how we translate our empirical estimates into the necessary estimates of $\left\{s(\theta), s^V(\theta), \zeta_p(\theta), \zeta_1(\theta), \zeta_2(\theta), y(\theta), g(\theta)\right\}$ above, for each representative agent θ in a discretized grid of types Θ.

Step 1: Model Identification

Value function. We assume agents have a utility function over continuation wealth W with a constant coefficient of relative risk aversion (CRRA) η:

$$\mathcal{U}(W) = \begin{cases} \ln(W) & \text{if } \eta = 1 \\ \frac{W^{1-\eta}-1}{1-\eta} & \text{if } \eta \neq 1 \end{cases} \tag{49}$$

Our value function $m(w;\theta)$ corresponds to the utility *gain* from winning a prize w, normalized by one's local marginal utility of wealth, so that the decision-weighted gain can be expressed in dollars. For an agent whose non-prize continuation wealth is $y(\theta)$, this value function can be written

$$m(w;\theta) = \frac{\mathcal{U}(y(\theta)+w) - \mathcal{U}(y(\theta))}{\mathcal{U}'(y(\theta))} \tag{50}$$

$$= \begin{cases} \frac{\ln(y(\theta)+x) - \ln y(\theta)}{y(\theta)^{-1}} & \text{if } \eta = 1 \\ \frac{\frac{1}{1-\eta}\left((y(\theta)+x)^{1-\eta} - y(\theta)^{1-\eta}\right)}{y(\theta)^{-\eta}} & \text{if } \eta \neq 1 \end{cases} \tag{51}$$

$$= \begin{cases} y(\theta)\ln\left(\frac{y(\theta)+x}{y(\theta)}\right) & \text{if } \eta = 1 \\ \frac{y(\theta)}{1-\eta}\left(\left(\frac{y(\theta)+x}{y(\theta)}\right)^{1-\eta} - 1\right) & \text{if } \eta \neq 1 \end{cases} \tag{52}$$

We can also compute the derivative, which proves useful for the calibrations below:

$$m'(w;\theta) = \frac{\mathcal{U}'(y(\theta)+w)}{\mathcal{U}'(y(\theta))} = \left(\frac{y(\theta)}{y(\theta)+w}\right)^{\eta}. \tag{53}$$

Decision weights. To identify decision weights, we can exploit the insight formalized in equation (17) from the text: decision weights imply a relationship between the relative responsiveness of demand to prices and prizes, quantified empirically by our estimates of ζ_p and ζ_k. Intuitively, if demand reacts more strongly to variation in the size of the jackpot expected value than to than to a change in ticket price (as we find) that is evidence of a high decision weight Φ_1 on the jackpot. Formally, substituting equation (44), with $k = 1$ and $k = 2$, into equation (17) gives two equations which identify the parameters of the neo-additive decision weighting function $b_0(\theta)$ and $b_1(\theta)$ from type-specific estimates of $\zeta_1(\theta)$, $\zeta_2(\theta)$, and $\zeta_p(\theta)$:

$$\frac{\zeta_1(\theta)}{|\zeta_p(\theta)|} = \frac{b_0(\theta) + b_1(\theta)\pi_1}{\pi_1} m'(w_1;\theta) \tag{54}$$

$$\frac{\zeta_2(\theta)}{|\zeta_p(\theta)|} = \frac{b_1(\theta)\pi_2}{\pi_2} m'(w_2;\theta) \tag{55}$$

These equations are exactly correct if $\zeta_1(\theta)$, $\zeta_2(\theta)$, and $\zeta_p(\theta)$ are estimates based on local variation, or are constant. In practice, our observational estimates of ζ_1 and ζ_2 are based on local variation, and ζ_1 appears to be constant across large variations in the jackpot (see Figure 2a). However our estimate of ζ_p is based on a discrete change in price from \$1 to \$2. To account for this, we compute the jackpot change that would generate the same demand response as this discrete price change, assuming that ζ_1 is constant. (This assumption is in keeping with the model of demand presented below; thus this approach effectively ensures that our structural model predicts a demand change in response to a discrete \$1 price change that matches our empirical estimates.) These equations suppress the type-dependent index θ for clarity, but they can be applied on a type-specific basis.

This jackpot change is given by the following equation:

$$\Delta w_1 = \frac{|\zeta_p|}{\zeta_1 \pi_1}. \tag{56}$$

By construction, this jackpot change has the same effect on demand as the discrete price reduction from \$2 to \$1, with $\Delta p = -1$, and thus they have the same effect on the net certainty equivalent in equation (48), so employing equation (44) we have

$$\Phi_1 \left(m(w_1 + \Delta w_1) - m(w_1)\right) = -\Delta p \tag{57}$$

$$\Rightarrow \; b_0 + b_1 \pi_1 = \frac{1}{m(w_1 + \Delta w_1) - m(w_1)}. \tag{58}$$

Finally, dividing equation (17) for $k = 1$ by $k = 2$, we find

$$\frac{\zeta_1}{\zeta_2} = \frac{\frac{\Phi_1}{\pi_1} m'(w_1)}{\frac{\Phi_2}{\pi_2} m'(w_2)} \tag{59}$$

$$= \frac{b_0 + b_1 \pi_1}{b_1 \pi_1} \cdot \frac{m'(w_1)}{m'(w_2)}. \tag{60}$$

Substituting the value for $b_0 + b_1\pi_1$ from equation (58) into the numerator of equation (60), we can solve for b_1, and thus for b_0. Cumulatively applying these decision weights as in Chateauneuf, Eichberger, and Grant (2007), these parameters fully identify the certainty equivalent,

$$\sum_k \Phi_k(\theta) m\left(w_k; \theta\right) = b_0(\theta) m\left(w_1; \theta\right) + b_1(\theta) \sum_k \pi_k m\left(w_k; \theta\right), \tag{61}$$

and they thus fully specify the net certainty equivalent function in equation (48).

Preference shocks. Equations (14) and (15) imply that the preference shock distribution $F_{\varepsilon|\theta}$ is equivalent to specifying a $\mathbb{R} \to \mathbb{R}^+$ function mapping $NCE(L;\theta)$ to demand s, which we denote

$$S(NCE;\theta) = F_{\varepsilon|\theta}\left[NCE\right].$$

Rather than characterizing F directly, we directly characterize the function $S(NCE;\theta)$, which implicitly defines $F_{\varepsilon|\theta}$. We calibrate $S(NCE;\theta)$ using three key assumptions. First, we assume that no agents are willing to pay a positive price for a lottery with prizes that are all zero, i.e., $S(x;\theta) = 0$ for all $x \leq 0$. This implies that the preference shock ε_t has positive support: $F_{\varepsilon|\theta}(0) = 0$. We therefore interpret ε_t as the hassle cost of purchasing a lottery ticket on occasion t; it represents the amount an agent would need to be paid to purchase a degenerate lottery ticket with $p = 0$ and $w_k = 0$ (for all k) on a given occasion. This cost might be larger on days when one is not already entering an establishment that sells lottery tickets, for example.

Second, we assume that demand responds smoothly to changes in net certainty equivalent, that is, $S(x;\theta)$ is continuous and differentiable for all x. Third, motivated by the strikingly linear relationship between log sales and jackpot expected value in Figure 2a, we assume that $S(NCE;\theta)$ has constant semi-elasticity with respect to w_1 over the range of jackpot variation in our data. Note that assuming constant semi-elasticity across *all* values of w_1 would imply positive demand even at arbitrarily negative values of NCE, conflicting with our first assumption that $S(0;\theta) = 0$. We therefore proceed by first characterizing a function $\tilde{S}(NCE;\theta)$ with globally constant $\zeta_1(\theta)$, and we then linearly interpolate $S(NCE;\theta)$ between $S(0;\theta) = 0$ and $S(NCE(L_{min};\theta);\theta) = \tilde{S}(NCE(L_{min};\theta);\theta)$, where L_{min} is the representative lottery with a jackpot of \$40 million.

To characterize the constant jackpot semi-elasticity demand function $\tilde{S}(NCE;\theta)$, we exploit the fact that the level of demand under the baseline (status quo) representative lottery, denoted L^0, is observed in our data. This identifies a point on the demand curve:

$$\tilde{S}(NCE(L^0;\theta);\theta) = s(\theta).$$

(In the equations to follow, we again suppress the type index, although all functions are understood to be computed using type-specific parameters.) To compute demand for an arbitrary lottery L, we first compute the difference in net certainty equivalent from the status quo lottery,

$$\Delta NCE = NCE(L) - NCE(L^0).$$

We then compute the change in jackpot for the baseline lottery which would result in this same change in net certainty equivalent. That is the value Δw_1 which satisfies the following equation:

$$\Phi_1 \left(m(w_1 + \Delta w_1) - m\left(w_1\right)\right) = \Delta NCE,$$

which can be rearranged to solve explicitly:

$$\Delta w_1 = m^{-1}\left(\frac{\Delta NCE}{\Phi_1} + m\left(w_1\right)\right) - w_1.$$

This expression depends on the inverse of the value function from equation (52), which can be written as follows

$$m^{-1}(X) = \begin{cases} y(\theta)\left(\exp\left(\frac{X}{y(\theta)}\right) - 1\right) & \text{if } \eta = 1 \\ y(\theta)\left[\left(X \cdot \frac{1-\eta}{y(\theta)} + 1\right)^{\frac{1}{1-\eta}} - 1\right] & \text{if } \eta \neq 1 \end{cases} \tag{62}$$

Finally, because the semi-elasticity of demand is constant in response to variation in w_1, we can compute the resulting change in demand as $\Delta \ln s = \zeta_1 \cdot \pi_1 \cdot \Delta w_1$. This fully characterizes the constant semi-elasticity demand function:

$$\tilde{S}(NCE(L;\theta);\theta) = \\ s(\theta)\cdot\exp\left[\zeta_1(\theta)\pi_1\cdot\left(m^{-1}\left(\frac{\Delta NCE(L;\theta)-NCE(L^0;\theta)}{\Phi_1(\theta)}+m\left(w_1;\theta\right);\theta\right)-w_1\right)\right]. \quad (63)$$

Having fully specified the function $\tilde{S}(NCE(L;\theta);\theta)$, we can define $S(NCE(L;\theta);\theta)$ by interpolating between $NCE=0$ and $NCD(L_{min};\theta)$ as described above. This fully specifies our positive model of demand.

To calibrate bias, and thus *normative* (debiased) demand, we assume that a constant share χ of the difference $\Phi_k-\pi_k$, i.e., the difference between decision weights and expected utility maximization, is driven by behavioral biases. Therefore we compute the χ for each agent that would rationalize a given debiased level of demand $\ln s^V(\theta)$. To do this, note that we can write the "debiased certainty equivalent" for lottery L as a function of the bias share χ:

$$NCE^V(L,\chi):=\sum_k\left(\pi_k+\left(\Phi_k-\pi_k\right)(1-\chi)\right)m\left(w_k\right)$$

Using the estimate of debiased demand for the baseline lottery, $s^V(\theta)$, we thus compute the bias share $\chi(\theta)$ for each type that satisfies

$$s^V(\theta)=\tilde{S}(NCE^V(L^0,\chi)).$$

Having thus identified $\chi(\theta)$, we can then compute normative demand for an arbitrary lottery L as $S(NCE^V(L,\chi(\theta);\theta);\theta)$.

Welfare. To compute consumer surplus, and thus welfare, it suffices to show how welfare is computed for a given type; it is straightforward to aggregate across types by weighting these changes by the welfare weights $g(\theta)$. Here we can make use of the fact that surplus can be measured as the integral under the Hicksian demand curve, from the point where the quantity demanded is equal to zero. In this model, there are no income effects, and therefore the Hicksian demand curve is found simply by varying p in the demand curve we have already derived: $S(\sum_k\Phi_k(\theta)m\left(w_k;\theta\right)-p;\theta)$. Since we have already imposed that quantity demanded falls to zero when $NCE=0$, the integral under the Hicksian demand curve reflecting welfare from a given lottery L is identical to the integral $\int_{x=0}^{NCE(L;\theta)}S(x;\theta)dx$. This provides the perceived utility surplus, based on consumers' willingness to pay for lottery tickets. To compute normative utility, we subtract the bias costs $\chi(\theta)\sum_k(\Phi_k-\pi_k)m(w_k)$ from perceived utility.

Step 2: Estimation of Input Parameters

We now describe how we use our empirical results to arrive at estimates of the parameters $s(\theta)$, $s^V(\theta)$, $\zeta_p(\theta)$, $\zeta_1(\theta)$, $\zeta_2(\theta)$, $y(\theta)$, and $g(\theta)$ which identify the model.

We assume a baseline CRRA parameter of $\eta = 1$, corresponding to log utility of continuation wealth.

To translate our empirical estimates into the context of this model with a single representative lottery game, we abstract from the diversity of games in the data and assume that our empirical estimates (of demand, elasticities, and the other statistics described below) align with those that would arise from a single lottery game with average attributes. Specifically, we specify a "representative lottery" with the features of a current Powerball lottery ticket described in Table 1, with a price of \$2 per ticket and a jackpot pool equal to the empirical average of \$170 million. To account for administration and overhead costs, which are typically between 5% and 15% of total lottery revenues in the U.S., we assume each lottery ticket has an additional cost of \$0.20 (see Appendix Figure A2).

We then specify a discretized type space. Our heterogeneous agent specification employs nine types, corresponding to the three income bins displayed in Figure 4, with three partitions of agents within each income bin. A share of agents purchase no lottery tickets (we assume these agents are expected utility maximizers, for whom $\Phi_k(\theta) = \pi_k$, implying that their demand is zero for any lottery with a price greater than expected value), while the remaining lottery purchasers partitioned into two groups—above- and below-median consumers—at each income level. Population shares and type-specific averages are computed within each partition of the type space.

We calibrate type-specific estimates of $s(\theta)$, $s^V(\theta)$, $y(\theta)$, and $g(\theta)$ as follows. We draw our estimates of lottery consumption in the status quo, $s(\theta)$, directly from the reported expenditures in our survey, reported in Figure 4. For our baseline estimates, we rescale these amounts by a common factor so that total lottery expenditures match those reported by the U.S. Census for 2019, in Figure A1. To calibrate debiased consumption $s^V(\theta)$, we interpret the share of consumption explained by bias (as estimated in Section 4.3) as the *causal* effect of bias. Thus for each individual, we reduce their reported level of consumption by their estimated quantity effect of bias to arrive at a debiased consumption, which we average within each partition of the type space to reach $s^V(\theta)$. To compute net continuation wealth $y(\theta)$, we compute the average reported income (according to our survey) within each partition of the type space. We then convert this into a measure of net (of tax) income $c(\theta)$ using the mapping between gross and net income from Piketty, Saez, and Zucman (2018). To arrive at continuation wealth, we multiple each type's net income by 20, to coarsely represent expected wealth accounting for future earnings for a representative worker. Following Saez (2002b), we set welfare weights proportional to $c(\theta)^{-\nu}$, with $\nu = 1$ in our baseline specifications, and with robustness checks for $\nu = 0.25$ and $\nu = 4$.

Our estimates of semi-elasticities with respect to price and the jackpot and second prizes, presented in Sections 3.2 and 3.3, rely on population data, and are not estimated separately across individuals. We assume these elasticities are homogeneous, with $\zeta_p(\theta) = \bar{\zeta}_p$ for each θ, etc. (It is straightforward to extend this model to data with heterogeneous elasticity estimates, as they would simply affect the values of $b_0(\theta)$ and $b_1(\theta)$ identified by equations (56)–(60) above.) We divide prize elasticities by one minus the assumed income tax rate of 30%, to convert these into elasticities with respect to the net-of-tax prize in the model.

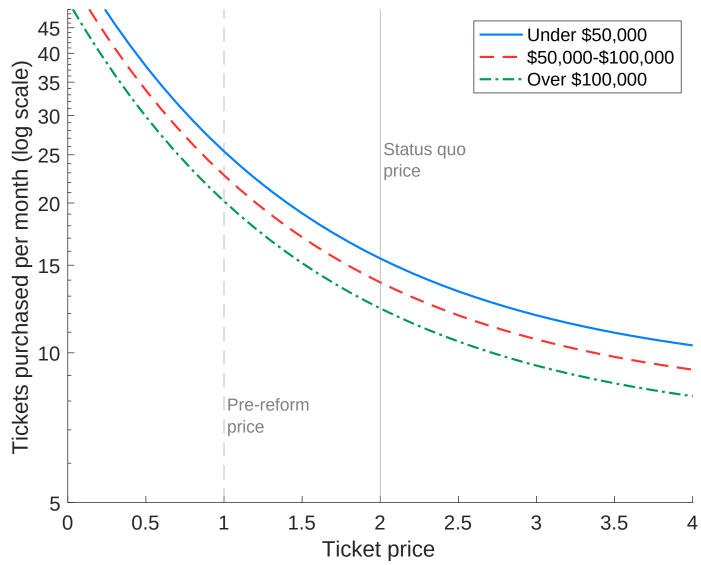
This completes the description of the structural model calibration. Figure A10 illustrates the behavior of the structural demand model, by plotting average simulated monthly demand for lottery tickets in each income bin, in response to variation in the price and the jackpot of the representative lottery. Several identifying features of the model are apparent from these plots. The status quo price and jackpot expected value for the baseline representative lottery ticket are displayed as vertical lines, and the plotted demand curves intersect these status quo values at the quantities corresponding to the expenditure levels in each income bin from Figure 4, scaled up to correspond to total lottery spending in 2019 according to the U.S. Census. (Since the representative ticket price is \$2, these quantities are equal to half the level of dollar expenditures.)

The vertical axis of both plots is displayed with a log scale, so that a constant semi-elasticity of demand with respect to jackpot expected value appears as a constant slope in Panel (b). Our estimate of ζ_1 therefore controls the slope of the linear portions of the demand curves in Panel (b), which is assumed to be constant within groups. The nonlinearity apparent to the left of the status quo jackpot expected value comes from our interpolation between $S(0;\theta)$ and $\tilde{S}(NCE(L_{min};\theta);\theta)$. This interpolation is also evident in Panel (a), where demand becomes concave and falls toward zero when price becomes high.

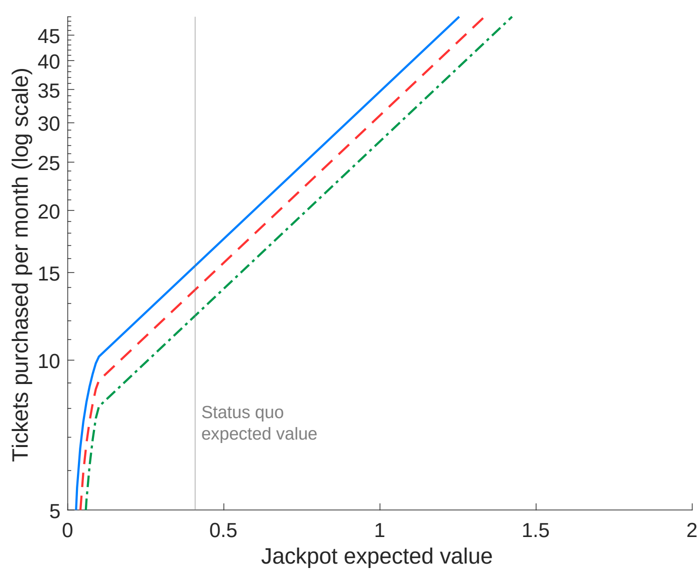
In Panel (a), a ticket price of \$1 is shown with a dashed line—this depicts the discretely different price at which demand is measured prior to the Mega Millions and Powerball price changes described in Table 1. The model is calibrated so that the difference in demand at the price of \$1 vs. the status quo price of \$2 corresponds to our empirical estimate of ζ_p. Perceived surplus in the status quo can be understood as the integral under the demand curves in Panel (a) from the status quo price rightward, to the point where demand falls to zero.

Figure A10: **Simulated Purchases and Expenditures by Income Bin**

(a) **Tickets Purchased varying Price**



(b) **Tickets Purchased varying Jackpot Size**



Notes: This figure plots the simulated average number of tickets purchased in each of three income bins from our structural model, across different values of the representative lottery ticket price and jackpot. Ticket purchases are plotted on a log scale, so that Panel (b) illustrates the structural assumption of a constant semi-elasticity of demand with respect to jackpot expected value across the range of jackpot values in our data. (See the discussion in Section 5.1 and Appendix F.2 for additional details.)