

# ECE 105 - Physics of Electrical Engineering 1

Kevin Carruthers

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## Forces and Motion

**Force** is a vector, and therefore includes direction. For any vector  $a$ ,  $-a$  has the same magnitude but opposite direction.

## Coordinate Systems

Given  $\vec{AB}$  we can find  $A$  or  $B$ 's position based on the position of the other one

$$\vec{O}_B = \vec{O}_A + \vec{AB}$$

for any  $\vec{O}_x$  is the location of  $x$  relative to the origin.

## Components

We can break any vector into **components** by finding the angle between it and the plane we want to model it off of.

Example: for  $\vec{A} = 6@20^\circ$ , we can find it's components with relation to the standard  $x$ - $y$  plane with

$$\begin{aligned}\vec{A}_y &= \vec{A} \cos 20^\circ \\ \vec{A}_x &= \vec{A} \sin 20^\circ\end{aligned}$$

## Constant Acceleration

$$\begin{aligned}\vec{a} &= \frac{\Delta \vec{v}}{\Delta t} \\ \vec{a} &= \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \\ \vec{a} \Delta t &= \vec{v}_f - \vec{v}_i \\ \vec{v}_f &= \vec{v}_i + \vec{a} \Delta t\end{aligned}$$

For the position vector  $\vec{d}, \vec{d}_f = \vec{d}_i + \vec{v}_i \Delta t + \frac{1}{2} \vec{a} (\Delta t)^2$

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}(\Delta \vec{d})$$

## Relative Motion

For any three objects  $a, b$ , and  $c$

$$v_{ca}^{\vec{}} = v_{cb}^{\vec{}} + v_{ba}^{\vec{}}$$

read "the velocity of  $c$  with respect to  $a$  is equal to the velocity of  $c$  with respect to  $b$  *plus* the velocity of  $b$  with respect to  $a$ .

## Circular Motion

$$a_c = \frac{v^2}{r}$$

$\theta \approx \frac{\Delta x}{r}$  for very close points.

For circle with center  $O$  and radius  $r_0, r_1 \dots$  connected to object on circumference with velocity  $\vec{v}_0, \vec{v}_1 \dots$  tangent to circumference,  $\vec{a} = \frac{\vec{v}_1 - \vec{v}_0}{t}$ . For arc length between object (at different times  $t_0, t_1 \dots$ )  $s, \theta = \frac{s}{r}$ . For small  $\theta \ll 1, |\vec{v}_1 - \vec{v}_0| = \theta \vec{v}$ .

$$|\vec{a}| = \frac{\vec{v} \theta}{t}$$

$\vec{a}$  is perpendicular to  $\vec{v}$

$$\vec{v} = \frac{r_1 - r_0}{t} = \frac{r \theta}{t}$$

$$\therefore \vec{a} = \frac{\vec{v} \theta}{r \frac{\theta}{\vec{v}}} = \frac{\vec{v}^2}{r}$$

$$\begin{aligned}
 s &= r\theta \\
 \frac{ds}{dt} &= v = r \frac{d\theta}{dt} \\
 &= r\omega \\
 \omega &= \frac{d\theta}{dt}
 \end{aligned}$$

$$\begin{aligned}
 a &= \frac{v^2}{r} \\
 &= \frac{(r\omega)^2}{r} \\
 &= r\omega^2
 \end{aligned}$$

## Types of Forces

A force is a push or pull interaction between two objects, responsible for changing motion.

### Springs

When unstretched, no **spring forces** exist. When a string is pushed from equilibrium, its spring force pushes back toward equilibrium.

$$F_s = -k\Delta x$$

### Tension

The **tension force** pulls an object toward a rope and a rope toward an object. Ropes can never push.

### Normal

The **normal force** "pushes back" against other objects via molecular electromagnetism. It is always perpendicular to the surface for any surface-to-surface contact. Technically, it is a type of spring force.

### Friction

**Friction** is the interaction between an object and a surface. It is a real force which acts opposite the direction of sliding, and is always tangent to surface.

$f \propto N$  is an experimental fact.  $f = \mu N$ , where  $\mu$  is the coefficient of friction.  $\mu$  is dependant on the type of objects and must be determined experimentally.

**Kinetic friction** is when objects are sliding relative to each other and **static friction** is when objects are not yet sliding

$$f_s \leq \mu_s N$$

Example: A 50kg person is in a 1000kg elevator at rest. When the elevator begins to rise, the person notices her weight is 600N. How far does the elevator move in 3s?

$$\begin{aligned}\Sigma \vec{F} &= m\vec{a} \\ \vec{a} &= \frac{\vec{F}_n - mg}{m} \\ &= \frac{600 - 50g}{50} \\ &= 2.2\text{m/s}^2\end{aligned}$$

$$\begin{aligned}d &= \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \\ &= 0 + \frac{1}{2}(2.2)9 \\ &= 9.9\text{m}\end{aligned}$$

## Energy

An object can be said to have a total **energy** equal to the sum of the various forms of energy it may possess.

### Kinetic Energy

The **kinetic energy** of an object is determined by its mass and velocity

$$K = \frac{mv^2}{2}$$

For any object with a changing velocity

$$\begin{aligned}v_f^2 &= v_i^2 + 2ad \\ \frac{1}{2}mv_f^2 &= \frac{1}{2}mv_i^2 + mad \\ K_f &= K_i + \Sigma \vec{F}d \\ \Delta K &= \Sigma \vec{F}d\end{aligned}$$

## Potential Gravitational Energy

**Potential gravitational energy** is a measure of stored energy of an object based on its height. It is essentially non-sensical to determine an object's "absolute" potential gravitational energy, thus we often simply solve for the difference in energy.

For a distance  $h_f$  above a reference height  $h_i$

$$U_g = mg(h_f - h_i)$$

thus if an object moves from  $h_i$  to  $h_f$

$$\begin{aligned}\Delta U_g &= U_{gf} - U_{gi} \\ &= mgh_f - mgh_i \\ &= mg\Delta h\end{aligned}$$

## Spring Energy

A **spring's energy** is based on its spring constant  $k$  and how far it is compressed from its equilibrium point

$$U_s = \frac{kx^2}{2}$$

## Collisions

If a **collision** is isolated, then energy is conserved. Elastic collisions also conserve energy. For all real or inelastic collisions, energy is lost.

## Work

Just as energy is a way of keeping track of motion, **work** is a mechanical means for transferring energy equal to the applied force multiplied by the distance it operates along

$$dW = \vec{F}d\vec{s}$$

It can be used to compute the change in energy of a system between two states, as the total work done by non-conservative forces (ie friction) will be equal to the work done by conservative forces (ie gravity, springs, motion)

For a system involving friction, motion, gravity, and a spring, we have

$$\Delta E_{th} = \Delta K + \Delta U_g + \Delta U_s$$

or, if we compute the value of the thermal work done by friction as energy (using  $U_f = \mu Nd$ , where  $d$  is the distance during which the object undergoes friction), we get

$$0 = \Delta K + \Delta U_g + \Delta U_s + \Delta U_f$$

## Rotation (of a non-deformable, rigid bodied object)

For any point on an object in **circular rotation**

$$\begin{aligned}\omega &= \frac{d\theta}{dt} \\ s &= r\theta \\ v &= \frac{ds}{dt} = \frac{rd\theta}{dt} = r\omega \\ a_r &= \frac{v^2}{r} = r\omega^2 \\ a_t &= \frac{dv}{dt} = \frac{rd\omega}{dt} = r\alpha\end{aligned}$$

where  $\omega$  is the angular frequency,  $s$  is the arc length of a circle, and  $\alpha$  is the angular acceleration

## Centre of Mass

For a uniform mass distribution, the **centre of mass** is in the geometric centre. Otherwise

$$x_{centre} = \frac{m_1x_1 + \dots + m_nx_n}{m_1 + \dots + m_n}$$

Gravity acts as if all the mass is located at the centre of mass.

## Rotational Energy

$$E_k = \frac{mv^2}{2} = \frac{mr^2\omega^2}{2} = \frac{I\omega^2}{2}$$

where  $I$  is the moment of inertia.

## Moment of Inertia

$$I = m_1x_1^2 + \dots + m_nx_n^2$$

For a thin rod of length  $L$  and uniform mass  $m$ ,

$$I = \frac{mL^2}{12}$$

For a filled ring of radius  $r$  and uniform mass  $m$  (regardless of length, ie. cylinders)

$$I = \frac{mr^2}{2}$$

For a hollow ring of radius  $r$  and uniform mass  $m$  (regardless of length, ie. hollow cylinders)

$$I = mr^2$$

For a filled sphere of radius  $r$  and uniform mass  $m$

$$I = \frac{2mr^2}{5}$$

For a hollow sphere of radius  $r$  and uniform mass  $m$

$$I = \frac{2mr^2}{3}$$

To find a "new" moment of inertia, where  $h$  is the distance to the new pivot

$$I = I_0 + mh^2$$

## Torque

**Torque** is a measure of how much a given applied force "wants" to rotate an object, where  $r$  is the direction from an object to its pivot and  $\theta$  is the angle between  $r$  and the applied force  $F$

$$\tau = rF \sin \theta$$

## Static Equilibrium

Any object in **static equilibrium** undergoes no motion at all.

$$\Sigma F = \Sigma \tau = 0$$

## Rotational Dynamics

$$\Sigma \tau = I\alpha$$

## Oscillations

As **oscillation** is a periodic motion about an equilibrium position.

## Simple Harmonic Motion

Any object in **simple harmonic motion** follows a sinusoidal shape in terms of its distance from its equilibrium position. Note:

$$\omega = 2\pi f$$

$$\begin{aligned}x &= A \cos(\omega t + \phi) \\v &= -A\omega \sin(\omega t + \phi)\end{aligned}$$

As you can see

$$v_{\max} = A\omega$$

## Oscillation Dynamics

$$\begin{aligned}x(t) &= A \cos(\omega t + \phi) \\v(t) &= -A\omega \sin(\omega t + \phi) \\a(t) &= -A\omega^2 \cos(\omega t + \phi) \\a &= -\omega^2 x \\a_{\max} &= -A\omega^2\end{aligned}$$

## Simple Harmonic Motion Dynamics

$$\begin{aligned}x &= A \cos(\omega t + \phi) \\\omega &= 2\pi f = \frac{2\pi}{T} \\v &= -A\omega \sin(\omega t + \phi) \\a &= -A\omega^2 \cos(\omega t + \phi) \\\frac{d^2x}{dt^2} &= -\omega^2 x\end{aligned}$$

Therefore, if

$$a = -Cx$$

we know that a solution of x is

$$x = A \cos(\sqrt{C}t + \phi)$$

For an ideal spring where

$$Fs = -kx$$



by dividng by mass we get

$$\omega = \sqrt{\frac{k}{m}}$$

## Simple Pendulum

For a **simple pendulum**

$$a_r = \frac{v^2}{l}$$

and

$$a_t = \alpha l$$

$$mg \sin \theta = m\alpha l$$

$$\alpha = \frac{g}{l} \sin \theta$$

$$\frac{d^2\theta}{dt^2} = \alpha$$

If  $\theta \ll 1$  then  $\sin \theta = \theta$

## Physical Pendula

For a **physical pendulum**, the centre of mass is the location where gravity acts, and thus we have

$$\Sigma \tau = I\alpha$$

$$mgx \sin \theta = I\alpha$$

$$\theta'' = \frac{mgx}{I} \sin \theta$$

$$\approx \frac{mgx}{I} \theta$$

$$\omega = \sqrt{\frac{mgx}{I}}$$

This tends to

$$\omega = \sqrt{n \frac{g}{l}}$$

where  $n$  is some real number.

## Energy Conservation

For any object in simple harmonic motion, **energy must be conserved**. Thus we have

$$\begin{aligned} E &= \frac{1}{2} I \omega_{max}^2 \\ &= mgh \end{aligned}$$

## Waves

**Waves** are physical areas of increased or decreased energy, which travel in simple harmonic motion. They can be visualised on a horizontal line with regular "humps".

## Wave Propagation

The **propagation** of a wave is the direction in which it travels. The form of the wave does not change as it propagates. A wave can be modelled by the equation

$$y = f(x \pm vt)$$

travelling to the left/right (plus/minus) where  $y = f(x)$  is the equation of the wave.

We can find the transverse travelling wave by assuming the shape is preserved.

## Harmonic Waves

**Harmonic waves** have a sinusoidal form.

For a stationary harmonic wave, we have

$$\begin{aligned} y = f(x) &= A \sin\left(2\pi \frac{x \pm vt}{\lambda} + \phi\right) \\ &= A \sin\left(\frac{2\pi x}{\lambda} - \omega t + \phi\right) \\ &= A \sin(kx - \omega t + \phi) \end{aligned}$$

where

$$k = \frac{2\pi}{\lambda}$$

Thus the speed of any particle on the wave (in the up/down direction, where  $x$  is constant) is

$$v = -A\omega \sin(kx - \omega t + \phi)$$

## Waves on a String

The velocity of a **wave on a string** is based solely on the properties of the string. We define the mass density as  $\mu = \frac{m}{l}$  so that

$$v = \sqrt{\frac{T}{\mu}}$$

where  $T$  is the tension in the string.

## Laws of Superposition

For  $y_1 = f_1(x_1t)$  and  $y_2 = f_2(x_2t)$

$$y(x, t) = f_1(x_1, t) + f_2(x_2, t)$$

The **superposition** of two waves is **constructive** if it results in a larger amplitude and **destructive** if it results in a small amplitude.

Consider two harmonic waves which are identical except for a phase shift travelling in the same direction on the same string.