Trigonometric and inverse trigonometric functions

Function	Domain	Range	First derivative
$\sin x$	$\mathbb{R}$	$[-1,1]$	$\cos x$
$\cos x$	$\mathbb{R}$	$[-1,1]$	$-\sin x$
$\tan x$	$\mathbb{R} \setminus \{\pi(k+1/2) : k \in \mathbb{Z}\}$	$\mathbb{R}$	$\sec^2 x$
$\sec x$	$\mathbb{R} \setminus \{\pi(k+1/2) : k \in \mathbb{Z}\}$	$(-\infty,-1] \cup [1,\infty)$	$\sec x \tan x$
$\csc x$	$\mathbb{R} \setminus \{\pi k : k \in \mathbb{Z}\}$	$(-\infty,-1] \cup [1,\infty)$	$-\csc x \cot x$
$\cot x$	$\mathbb{R} \setminus \{\pi k : k \in \mathbb{Z}\}$	$\mathbb{R}$	$-\csc^2 x$
$\arcsin x$	$[-1,1]$	$[-\pi/2,\pi/2]$	$\frac{1}{\sqrt{1-x^2}},\ x \in (-1,1)$
$\arccos x$	$[-1,1]$	$[0,\pi]$	$-\frac{1}{\sqrt{1-x^2}},\ x \in (-1,1)$
$\arctan x$	$\mathbb{R}$	$(-\pi/2,\pi/2)$	$\frac{1}{1+x^2},\ x \in \mathbb{R}$
$\operatorname{arcsec} x$	$\lvert x\rvert \geq 1$	$[0,\pi/2) \cup (\pi/2,\pi]$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}},\ \lvert x\rvert > 1$
$\operatorname{arccsc} x$	$\lvert x\rvert \geq 1$	$[-\pi/2,0) \cup (0,\pi/2]$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}},\ \lvert x\rvert > 1$
$\operatorname{arccot} x$	$\mathbb{R}$	$(0,\pi)$	$-\frac{1}{1+x^2},\ x \in \mathbb{R}$

Useful trigonometric identities and integrals

$$\cos^2 x + \sin^2 x = 1$$

$$\sin(2x) = 2\sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x, \qquad \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin x \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)], \qquad \sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C, \qquad \int \csc x \, dx = -\ln|\csc x + \cot x| + C$$

*Antiderivatives of inverse trig functions can be derived via integration by parts—try it!

Inverse trigonometric substitution rules

If the integral involves	Substitution	Identity
$a^2 - x^2$	$x = a\sin\theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$a^2 + x^2$	$x = a\tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$x^2 - a^2$	$x = a\sec\theta, \quad 0 \leq \theta < \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

Hyperbolic and inverse hyperbolic functions

Function	Domain	Range	First derivative
$\sinh x = \frac{e^x - e^{-x}}{2}$	$\mathbb{R}$	$\mathbb{R}$	$\cosh x$
$\cosh x = \frac{e^x + e^{-x}}{2}$	$\mathbb{R}$	$[1, \infty)$	$\sinh x$
$\tanh x$	$\mathbb{R}$	$(-1, 1)$	$\operatorname{sech}^2 x$
$\operatorname{sech} x$	$\mathbb{R}$	$(0, 1]$	$-\operatorname{sech} x \tanh x$
$\operatorname{csch} x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$	$-\operatorname{csch} x \coth x$
$\coth x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, -1) \cup (1, \infty)$	$-\operatorname{csch}^2 x$
$\operatorname{arcsinh} x = \ln(x + \sqrt{x^2 + 1})$	$\mathbb{R}$	$\mathbb{R}$	$\frac{1}{\sqrt{1+x^2}},\ x \in \mathbb{R}$
$\operatorname{arccosh} x = \ln(x + \sqrt{x^2 - 1})$	$[1, \infty)$	$[0, \infty)$	$\frac{1}{\sqrt{x^2-1}},\ x \in (1, \infty)$
$\operatorname{arctanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$	$(-1, 1)$	$\mathbb{R}$	$\frac{1}{1-x^2},\ x \in (-1, 1)$
$\operatorname{arcsech} x = \ln \left(\frac{1}{x} + \frac{\sqrt{1-x^2}}{x} \right)$	$(0, 1]$	$[0, \infty)$	$-\frac{1}{x\sqrt{1-x^2}},\ x \in (0, 1)$
$\operatorname{arccsch} x = \ln \left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{\lvert x \rvert} \right)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$	$-\frac{1}{\lvert x \rvert \sqrt{x^2+1}},\ x \neq 0$
$\operatorname{arccoth} x = \frac{1}{2} \ln \left(\frac{1-x}{1+x} \right)$	$(-\infty, -1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$	$\frac{1}{1-x^2},\ \lvert x \rvert > 1$

Useful hyperbolic identities

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh(2x) = 2 \sinh x \cosh x$$

$$\cosh(2x) = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y, \qquad \cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\sinh x \cosh y = \frac{1}{2}[\sinh(x - y) + \sinh(x + y)], \qquad \sinh x \sinh y = \frac{1}{2}[\cosh(x + y) - \cosh(x - y)]$$

$$\cosh x \cosh y = \frac{1}{2}[\cosh(x - y) + \cosh(x + y)]$$

Osborn's Rule

In general, to obtain an identity for hyperbolic functions from an analogous trigonometric identity, replace each trigonometric function by the corresponding hyperbolic function and change the sign of every product or implied product of two sines.

Inverse hyperbolic substitution rules

If the integral involves	Substitution	Identity
$a^2 - x^2$	$x = a \tanh u$	$1 - \tanh^2 u = \operatorname{sech}^2 u$
$a^2 + x^2$	$x = a \sinh u$	$1 + \sinh^2 u = \cosh^2 u$
$x^2 - a^2$	$x = a \cosh u$	$\cosh^2 u - 1 = \sinh^2 u$