MATH 239 — Combinatorics

Kevin James

Spring 2014

---

1 Combinatorics

Combinatorics is discrete math dealing with 'counting questions' and graph theory. For example:

"How many binary strings of length n are there?": 2^n

"How many binary strings of length n are there which do not contain the (continuous) substring '0101'?"

"How many ways can you make change for a dollar? (in Canada!)"

"How many k-element subsets are there in an n-element set?": n choose $k = \frac{n!}{k!(n-k)!}$

"How many ways are there to order the numbers from 1 to n with no constraints?": $n!$

"Given 123 letters addressed to the 123 students in this class and 123 associated envelopes, how many ways are there to put one letter in each envelope such that nobody gets the right letter?": $\approx \frac{123!}{e}$

"How many n-polyminos (tetris blocks) are there?": $2 \to 1$, $3 \to 2$, $4 \to 5|7$

"How many prime numbers p are there such that $p+2$ is also prime?": infinite?

"How many rooted binary trees are there with n vertices?"

"Can the vertices of a graph be colored by n colors in such a way that every adjacent vertex is given a different color?"

"Can a graph be drawn such that no two edges are crossing?"