

ECE 105 - Physics of Electrical Engineering 1

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Forces and Motion

Force is a vector, and therefore includes direction. For any vector a , $-a$ has the same magnitude but opposite direction.

Coordinate Systems

Given \vec{AB} we can find A or B 's position based on the position of the other one

$$\vec{O}_B = \vec{O}_A + \vec{AB}$$

for any \vec{O}_x is the location of x relative to the origin.

Components

We can break any vector into **components** by finding the angle between it and the plane we want to model it off of.

Example: for $\vec{A} = 6@20^\circ$, we can find it's components with relation to the standard x - y plane with

$$\begin{aligned}\vec{A}_y &= \vec{A} \cos 20^\circ \\ \vec{A}_x &= \vec{A} \sin 20^\circ\end{aligned}$$

Constant Acceleration

$$\begin{aligned}\vec{a} &= \frac{\Delta \vec{v}}{\Delta t} \\ \vec{a} &= \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \\ \vec{a} \Delta t &= \vec{v}_f - \vec{v}_i \\ \vec{v}_f &= \vec{v}_i + \vec{a} \Delta t\end{aligned}$$

For the position vector $\vec{d}, \vec{d}_f = \vec{d}_i + \vec{v}_i \Delta t + \frac{1}{2} \vec{a} (\Delta t)^2$

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}(\Delta \vec{d})$$

Relative Motion

For any three objects a, b , and c

$$v_{ca}^{\vec{}} = v_{cb}^{\vec{}} + v_{ba}^{\vec{}}$$

read "the velocity of c with respect to a is equal to the velocity of c with respect to b *plus* the velocity of b with respect to a .

Circular Motion

$$\begin{aligned}a_c &= \frac{v^2}{r} \\ \theta &\approx \frac{\Delta x}{r} \text{ for very close points.}\end{aligned}$$

For circle with center O and radius $r_0, r_1 \dots$ connected to object on circumference with velocity $\vec{v}_0, \vec{v}_1 \dots$ tangent to circumference, $\vec{a} = \frac{\vec{v}_1 - \vec{v}_0}{t}$. For arc length between object (at different times $t_0, t_1 \dots$) $s, \theta = \frac{s}{r}$. For small $\theta \ll 1, |\vec{v}_1 - \vec{v}_0| = \theta \vec{v}$.

$$|\vec{a}| = \frac{\vec{v}\theta}{t}$$

\vec{a} is perpendicular to \vec{v}

$$\vec{v} = \frac{r_1 - r_0}{t} = \frac{r\theta}{t}$$

$$\therefore \vec{a} = \frac{\vec{v}\theta}{r\frac{\theta}{\vec{v}}} = \frac{\vec{v}^2}{r}$$

$$s = r\theta$$

$$\frac{ds}{dt} = v = r \frac{d\theta}{dt}$$

$$= r\omega$$

$$\omega = \frac{d\theta}{dt}$$

$$a = \frac{v^2}{r}$$

$$= \frac{(r\omega)^2}{r}$$

$$= r\omega^2$$

Types of Forces

A force is a push or pull interaction between two objects, responsible for changing motion.

Springs

When unstretched, no **spring forces** exist. When a string is pushed from equilibrium, its spring force pushes back toward equilibrium.

$$F_s = -k\Delta x$$

Tension

The **tension force** pulls an object toward a rope and a rope toward an object. Ropes can never push.

Normal

The **normal force** "pushes back" against other objects via molecular electromagnetism. It is always perpendicular to the surface for any surface-to-surface contact. Technically, it is a type of spring force.

Friction

Friction is the interaction between an object and a surface. It is a real force which acts opposite the direction of sliding, and is always tangent to surface.

$f \propto N$ is an experimental fact. $f = \mu N$, where μ is the coefficient of friction. μ is dependant on the type of objects and must be determined experimentally.

Kinetic friction is when objects are sliding relative to each other and **static friction** is when objects are not yet sliding

$$f_s \leq \mu_s N$$

Example: A 50kg person is in a 1000kg elevator at rest. When the elevator begins to rise, the person notices her weight is 600N. How far does the elevator move in 3s?

$$\begin{aligned}\Sigma \vec{F} &= m\vec{a} \\ \vec{a} &= \frac{\vec{F}_n - mg}{m} \\ &= \frac{600 - 50g}{50} \\ &= 2.2\text{m/s}^2\end{aligned}$$

$$\begin{aligned}d &= \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \\ &= 0 + \frac{1}{2}(2.2)9 \\ &= 9.9\text{m}\end{aligned}$$

Energy

An object can be said to have a total **energy** equal to the sum of the various forms of energy it may possess.

Kinetic Energy

The **kinetic energy** of an object is determined by its mass and velocity

$$K = \frac{mv^2}{2}$$

For any object with a changing velocity

$$\begin{aligned}v_f^2 &= v_i^2 + 2ad \\ \frac{1}{2}mv_f^2 &= \frac{1}{2}mv_i^2 + mad \\ K_f &= K_i + \Sigma \vec{F}d \\ \Delta K &= \Sigma \vec{F}d\end{aligned}$$

Potential Gravitational Energy

Potential gravitational energy is a measure of stored energy of an object based on its height. It is essentially non-sensical to determine an object's "absolute" potential gravitational energy, thus we often simply solve for the difference in energy.

For a distance h_f above a reference height h_i

$$U_g = mg(h_f - h_i)$$

thus if an object moves from h_i to h_f

$$\begin{aligned}\Delta U_g &= U_{gf} - U_{gi} \\ &= mgh_f - mgh_i \\ &= mg\Delta h\end{aligned}$$

Spring Energy

A **spring's energy** is based on its spring constant k and how far it is compressed from its equilibrium point

$$U_s = \frac{kx^2}{2}$$

Collisions

If a **collision** is isolated, then energy is conserved. Elastic collisions also conserve energy. For all real or inelastic collisions, energy is lost.

Work

Just as energy is a way of keeping track of motion, **work** is a mechanical means for transferring energy equal to the applied force multiplied by the distance it operates along

$$dW = \vec{F}d\vec{s}$$

It can be used to compute the change in energy of a system between two states, as the total work done by non-conservative forces (ie friction) will be equal to the work done by conservative forces (ie gravity, springs, motion)

For a system involving friction, motion, gravity, and a spring, we have

$$\Delta E_{th} = \Delta K + \Delta U_g + \Delta U_s$$

or, if we compute the value of the thermal work done by friction as energy (using $U_f = \mu Nd$, where d is the distance during which the object undergoes friction), we get

$$0 = \Delta K + \Delta U_g + \Delta U_s + \Delta U_f$$

Rotation (of a non-deformable, rigid bodied object)

For any point on an object in **circular rotation**

$$\begin{aligned}\omega &= \frac{d\theta}{dt} \\ s &= r\theta \\ v &= \frac{ds}{dt} = \frac{rd\theta}{dt} = r\omega \\ a_r &= \frac{v^2}{r} = r\omega^2 \\ a_t &= \frac{dv}{dt} = \frac{rd\omega}{dt} = r\alpha\end{aligned}$$

where ω is the angular frequency, s is the arc length of a circle, and α is the angular acceleration

Centre of Mass

For a uniform mass distribution, the **centre of mass** is in the geometric centre. Otherwise

$$x_{centre} = \frac{m_1x_1 + \dots + m_nx_n}{m_1 + \dots + m_n}$$

Gravity acts as if all the mass is located at the centre of mass.

Rotational Energy

$$E_k = \frac{mv^2}{2} = \frac{mr^2\omega^2}{2} = \frac{I\omega^2}{2}$$

where I is the moment of inertia.

Moment of Inertia

$$I = m_1x_1^2 + \dots + m_nx_n^2$$

For a thin rod of length L and uniform mass m ,

$$I = \frac{mL^2}{12}$$

For a filled ring of radius r and uniform mass m (regardless of length, ie. cylinders)

$$I = \frac{mr^2}{2}$$

For a hollow ring of radius r and uniform mass m (regardless of length, ie. hollow cylinders)

$$I = mr^2$$

For a filled sphere of radius r and uniform mass m

$$I = \frac{2mr^2}{5}$$

For a hollow sphere of radius r and uniform mass m

$$I = \frac{2mr^2}{3}$$

To find a "new" moment of inertia, where h is the distance to the new pivot

$$I = I_0 + mh^2$$

Torque

Torque is a measure of how much a given applied force "wants" to rotate an object, where r is the direction from an object to its pivot and θ is the angle between r and the applied force F

$$\tau = rF \sin \theta$$

Static Equilibrium

Any object in **static equilibrium** undergoes no motion at all.

$$\Sigma F = \Sigma \tau = 0$$

Rotational Dynamics

$$\Sigma \tau = I\alpha$$

Oscillations

As **oscillation** is a periodic motion about an equilibrium position.

Simple Harmonic Motion

Any object in **simple harmonic motion** follows a sinusoidal shape in terms of its distance from its equilibrium position. Note:

$$\omega = 2\pi f$$

$$\begin{aligned}x &= A \cos(\omega t + \phi) \\v &= -A\omega \sin(\omega t + \phi)\end{aligned}$$

As you can see

$$v_{\max} = A\omega$$

Oscillation Dynamics

$$\begin{aligned}x(t) &= A \cos(\omega t + \phi) \\v(t) &= -A\omega \sin(\omega t + \phi) \\a(t) &= -A\omega^2 \cos(\omega t + \phi) \\a &= -\omega^2 x \\a_{\max} &= -A\omega^2\end{aligned}$$

Simple Harmonic Motion Dynamics

$$\begin{aligned}x &= A \cos(\omega t + \phi) \\\omega &= 2\pi f = \frac{2\pi}{T} \\v &= -A\omega \sin(\omega t + \phi) \\a &= -A\omega^2 \cos(\omega t + \phi) \\\frac{d^2x}{dt^2} &= -\omega^2 x\end{aligned}$$

Therefore, if

$$a = -Cx$$

we know that a solution of x is

$$x = A \cos(\sqrt{C}t + \phi)$$

For an ideal spring where

$$Fs = -kx$$

by dividng by mass we get

$$\omega = \sqrt{\frac{k}{m}}$$

Simple Pendulum

For a **simple pendulum**

$$a_r = \frac{v^2}{l}$$

and

$$a_t = \alpha l$$

$$mg \sin \theta = m\alpha l$$

$$\alpha = \frac{g}{l} \sin \theta$$

$$\frac{d^2\theta}{dt^2} = \alpha$$

If $\theta \ll 1$ then $\sin \theta = \theta$

Physical Pendula

For a **physical pendulum**, the centre of mass is the location where gravity acts, and thus we have

$$\Sigma \tau = I\alpha$$

$$mgx \sin \theta = I\alpha$$

$$\theta'' = \frac{mgx}{I} \sin \theta$$

$$\approx \frac{mgx}{I} \theta$$

$$\omega = \sqrt{\frac{mgx}{I}}$$

This tends to

$$\omega = \sqrt{n \frac{g}{l}}$$

where n is some real number.

Energy Conservation

For any object in simple harmonic motion, **energy must be conserved**. Thus we have

$$\begin{aligned} E &= \frac{1}{2} I \omega_{max}^2 \\ &= mgh \end{aligned}$$

Waves

Waves are physical areas of increased or decreased energy, which travel in simple harmonic motion. They can be visualised on a horizontal line with regular "humps".

Wave Propagation

The **propagation** of a wave is the direction in which it travels. The form of the wave does not change as it propagates. A wave can be modelled by the equation

$$y = f(x \pm vt)$$

travelling to the left/right (plus/minus) where $y = f(x)$ is the equation of the wave.

We can find the transverse travelling wave by assuming the shape is preserved.

Harmonic Waves

Harmonic waves have a sinusoidal form.

For a stationary harmonic wave, we have

$$\begin{aligned} y = f(x) &= A \sin\left(2\pi \frac{x \pm vt}{\lambda} + \phi\right) \\ &= A \sin\left(\frac{2\pi x}{\lambda} - \omega t + \phi\right) \\ &= A \sin(kx - \omega t + \phi) \end{aligned}$$

where

$$k = \frac{2\pi}{\lambda}$$

Thus the speed of any particle on the wave (in the up/down direction, where x is constant) is

$$v = -A\omega \sin(kx - \omega t + \phi)$$

Waves on a String

The velocity of a **wave on a string** is based solely on the properties of the string. We define the mass density as $\mu = \frac{m}{l}$ so that

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension in the string.

Laws of Superposition

For $y_1 = f_1(x_1t)$ and $y_2 = f_2(x_2t)$

$$y(x, t) = f_1(x_1, t) + f_2(x_2, t)$$

The **superposition** of two waves is **constructive** if it results in a larger amplitude and **destructive** if it results in a small amplitude.

Consider two harmonic waves which are identical except for a phase shift travelling in the same direction on the same string.