$$\min \sum^{N} (x_k - x_k^d)^T Q_k (x_k - x_k^d) + u_k^T R_k u_k$$

$$x_{k+1} = A_k x_k + B_k u_k . \quad k = 0, \dots, N-1$$

$$C_k^x x_k \le b_k^x . \quad k = 1, \dots, N$$

$$C_k^u u_k \le b_k^u . \quad k = 0, \dots, N-1$$

$$u_k^{lb} \le u_k \le u_k^{ub}$$

两者分开是因为qpOASES的Constraints个数不计u的lower Bound和upper Bound.

$$\Rightarrow$$

$$\min \frac{1}{2} u^T H u + f^T u$$

$$C u \le B .$$

$$u^{lb} \le u \le u^{ub}$$

$$u = [u_0, u_1, u_2, \dots, u_{N-1}].$$

(1) $x_{k+1} = A_k x_k + B_k u_k$. $N = 0, 1, 2, \dots, N-1$

$x_1 = A_0 x_0 + B_0 u_0$

$x_2 = A_1 x_1 + B_1 u_1 = A_1 (A_0 x_0 + B_0 u_0) + B_1 u_1$

$= A_1 A_0 x_0 + A_1 B_0 u_0 + B_1 u_1$

$x_3 = A_2 x_2 + B_2 u_2 = A_2 [(A_1 A_0 x_0 + A_1 B_0 u_0) + B_1 u_1]$

$+ B_2 u_2 = A_2 A_1 A_0 x_0 + A_2 A_1 B_0 u_0 + A_2 B_1 u_1 + B_2 u_2$

$\Rightarrow x_{k+1} = A_k A_{k-1} \dots A_0 x_0 + A_k A_{k-1} \dots A_1 B_0 u_0$

$+ A_k A_{k-1} \dots A_2 B_1 u_1 + A_k A_{k-1} \dots A_3 B_2 u_2$

$$\Rightarrow X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \underbrace{\begin{bmatrix} A_0 \\ A_1 A_0 \\ \vdots \\ A_{N-1} A_{N-2} \cdots A_0 \end{bmatrix}}_{S_x} x_0 + \underbrace{\begin{bmatrix} B_0 & 0 & 0 & \cdots & 0 \\ A_1 B_0 & B_1 & 0 & \cdots & 0 \\ A_2 A_1 B_0 & A_2 B_1 & B_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ A_{N-1} \cdots A_1 B_0 & A_{N-1} \cdots A_2 B_1 & \cdots & \cdots & B_{N-1} \end{bmatrix}}_{S_u} \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ \vdots \\ u_{N-1} \end{bmatrix}$$

$X = S_x x_0 + S_u U$

② $C_k^x x_k \le b_k^x$. $k = 1, 2, \cdots N$.

$$\underbrace{\begin{bmatrix} C_1^x & & & 0 \\ & C_2^x & & \\ & & \ddots & \\ 0 & & & C_N^x \end{bmatrix}}_{C_x} \underbrace{\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix}}_{X} \le \underbrace{\begin{bmatrix} b_1^x \\ b_2^x \\ \vdots \\ b_N^x \end{bmatrix}}_{B_x}$$

$$LBx \le C_x (S_x x_0 + S_u u) \le uB_x$$

$$\Rightarrow LBx - C_x S_x x_0 \le C_x S_u u \le uB_x - C_x S_x x_0$$

③ $C_k^u u_k \le b_k^u$

$$\underbrace{\begin{bmatrix} C_0^u & & & 0 \\ & C_1^u & & \\ & & \ddots & \\ 0 & & & C_{N-1}^u \end{bmatrix}}_{C_u} \underbrace{\begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_{N-1} \end{bmatrix}}_{U} \le \underbrace{\begin{bmatrix} b_0^u \\ b_1^u \\ \vdots \\ b_{N-1}^u \end{bmatrix}}_{B_u}$$

$$LBu \le C_u u \le uB_u$$

$$\Rightarrow C = \begin{bmatrix} C_x S_u \\ C_u \end{bmatrix}, \quad C_{ub} = \begin{bmatrix} uB_x - C_x S_x x_0 \\ uB_u \end{bmatrix} \quad C_{Lb} = \begin{bmatrix} LB_x - C_x S_x x_0 \\ LB_u \end{bmatrix}$$

$$\min \sum_{k=0}^{N} (x_{k+1} - x_{k+1}^{des})^T Q_k (x_{k+1} - x_{k+1}^{des}) + u_k^T R_k u_k$$

$$\min \begin{bmatrix} x_1 - x_1^{des} \\ x_2 - x_2^{des} \\ \vdots \\ x_N - x_N^{des} \end{bmatrix}^T \underbrace{\begin{bmatrix} Q_0 & & & \\ & Q_1 & & \\ & & \ddots & \\ & & & Q_{N-1} \end{bmatrix}}_{Q} \begin{bmatrix} x_1 - x_1^{des} \\ x_2 - x_2^{des} \\ \vdots \\ x_N - x_N^{des} \end{bmatrix}$$

$$+ \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_{N-1} \end{bmatrix}^T \begin{bmatrix} R_0 & & & \\ & R_1 & & \\ & & \ddots & \\ & & & R_{N-1} \end{bmatrix} \begin{bmatrix} u_0 \\ \vdots \\ u_{N-1} \end{bmatrix}$$

$$\min (x - x^{des})^T Q (x - x^{des}) + u^T R u$$

$$\min (x^T - (x^{des})^T) Q (x - x^{des}) + u^T R u$$

$$\min \; x^T Q x - x^T Q x^{des} - (x^{des})^T Q x + (x^{des})^T Q x^{des} + u^T R u$$

$$\min (S_x x_0 + S_u u)^T Q (S_x x_0 + S_u u)$$
$$- (S_x x_0 + S_u u)^T Q x^{des}$$
$$- (x^{des})^T Q (S_x x_0 + S_u u) + (x^{des})^T Q x^{des} + u^T R u$$

$$\min \ \cancel{(S_x x_0)^T Q S_x x_0} + (S_u u)^T Q S_x x_0 + (S_x x_0)^T Q S_u u$$

$$+ (S_u u)^T Q S_u u - \cancel{(S_x x_0)^T Q x^{des}} - (S_u u)^T Q x^{des}$$

$$- \cancel{(x^{des})^T Q S_x x_0} - (x^{des})^T Q S_u u + \cancel{(x^{des})^T Q x^{des}} + u^T R u$$

$$\min \ (Q S_x x_0)^T S_u u + (S_x x_0)^T Q S_u u + u^T S_u^T Q S_u u$$

$$- (Q x^{des})^T S_u u - (x^{des})^T Q S_u u + u^T R u$$

$$\min \ u^T (R + S_u^T Q S_u) u + \Big[(Q S_x x_0)^T S_u + (S_x x_0)^T Q S_u$$

$$- (Q x^{des})^T S_u - (x^{des})^T Q S_u \Big] u$$

$$\min \ \frac{1}{2} u^T H u + f^T u$$

$$\Rightarrow H = 2 (R + S_u^T Q S_u)$$

$$f = \left[(Q S_x x_0)^T S_u + (S_x x_0)^T Q S_u - (Q x^{des})^T S_u - (x^{des})^T Q S_u \right]^T$$

$$f = S_u^T Q S_x x_0 + S_u^T Q^T S_x x_0 - S_u^T Q x^{des} - S_u^T Q^T x^{des}$$

$$\begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \cdots 0 \\ I & 0 & 0 & 0 \cdots 0 \\ 0 & I & 0 & 0 \cdots 0 \\ \vdots & & \ddots & \\ 0 & 0 \cdots 0 & 0 & I \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_0$$

$$X^* = M X + N x_0$$

若 $y_k = C_k x_k + D_k u_k$

$$\underbrace{\min \left(y_{k+1} - y_{k+1}^{ref} \right)^T Q \left(y_{k+1} - y_{k+1}^{ref} \right) + u^T R u}$$

$$X = S_x x_0 + S_u u. \Rightarrow X^* = M(S_x x_0 + S_u u) + N x_0$$

$$= \underbrace{(M S_x + N)}_{:= S_x} x_0 + \underbrace{M S_u}_{:= S_u} u$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} C_1 & & & \\ & C_2 & & \\ & & \ddots & \\ & & & C_N \end{bmatrix} X^* + \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & \ddots & \\ & & & D_N \end{bmatrix} u$$

$$= C_y X^* + D_y u$$

$$= C_y (S_x x_0 + S_u u) + D_y u$$

$$= C_y S_x x_0 + (C_y S_u + D_y) u.$$

$$S_y = C_y S_x. \quad S_{yu} = C_y S_u + D_y$$

$$\Rightarrow H = 2 (R + S_{yu}^T Q S_{yu})$$

$$f = S_{yu}^T Q S_y x_0 + S_{yu}^T Q^T S_y x_0 - S_{yu}^T Q y^{des} - S_{yu}^T Q^T y^{des}$$

Output Constraints:

$$C_{ky}\, y_k \le b_{ky}$$

$$\begin{bmatrix} C_{1y} & & & \\ & C_{2y} & & \\ & & \ddots & \\ & & & C_{Ny} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \le \begin{bmatrix} b_{1y} \\ b_{2y} \\ \vdots \\ b_{Ny} \end{bmatrix}$$

$$C_{sy}\, Y \le B_{sy}$$

$$C_{sy}\left(C_y X + D_y U\right) \le B_{sy}$$

$$C_{sy}\left(S_y x_0 + S_{yu} U + D_y U\right) \le B_{sy}$$

$$C_{sy} S_y x_0 + C_{sy} S_{yu} U + C_{sy} D_y U \le B_{sy}$$

$$\left(C_{sy} S_{yu} + C_{sy} D_y\right) U \le B_{sy} - C_{sy} S_y x_0$$

$$\Rightarrow C = \begin{bmatrix} C_x S_u \\ C_u \\ C_{sy} S_{yu} + \cancel{C_{sy} D_y} \end{bmatrix}, \quad B = \begin{bmatrix} B_x - C_x S_x x_0 \\ B_u \\ B_{sy} - C_{sy} S_y x_0 \end{bmatrix}$$

Correction :

$$C_{sy} Y \leq B_{sy}$$

$$C_{sy} (S_y x_0 + S_{yu} U) \leq B_{sy}$$

$$C_{sy} S_{yu} U \leq B_{sy} - C_{sy} S_y x_0$$

$$\sum (Y_{k+1} - Y_k)^T W (Y_{k+1} - Y_k).$$

$$\{N-1\}\begin{bmatrix} Y_1 - Y_0 \\ Y_2 - Y_1 \\ \vdots \\ Y_{N-1} - Y_{N-2} \end{bmatrix} = \underbrace{\begin{bmatrix} -I & I & & & \\ 0 & -I & I & & \\ & & \ddots & \ddots & \\ & & & -I & I \end{bmatrix}}_{\substack{K \\ (N-1)\times N}} \begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_{N-1} \end{bmatrix}$$

$$(KY)^T W KY$$

$$Y^T (K W K) Y \quad := K^*$$

$$(N-1)\times \dim\{Y_k\}.$$

$$Y = S_y x_0 + S_{yu} U.$$

$$(S_y x_0 + S_{yu} U)^T K^* (S_y x_0 + S_{yu} U).$$

$$= (x_0^T S_y^T + u^T S_{yu}^T) K^* (S_y x_0 + S_{yu} u).$$

$$= (x_0^T S_y^T K^* + u^T S_{yu}^T K^*) (S_y x_0 + S_{yu} u).$$

$$= x_0^T S_y^T K^* S_y x_0 + u^T (S_{yu}^T K^* S_y x_0)$$

$$+ (x_0^T S_y^T K^* S_{yu}) u + u^T \underbrace{S_{yu}^T K^* S_{yu}}_{} u.$$