Davide Radaelli <daviderady@gmail.com>

Your thoughts
7 messages

Davide Radaelli <daviderady@gmail.com>   Fri, Feb 11, 2022 at 5:06 AM
To: "Laurens D. Gunnarsen" <l.gunnarsen@comcast.net>

Dear Laurens,

I am writing to you to get your thoughts and advice. I want to love doing math. However all my attempts in the past year to enjoy math have failed. It's deeply frustrating. Writing to you has been extremely helpful to me in the past, and prompted thoughtful reflections. I'm hoping you can help me again.

My goal is to build a love for math in a vacuum. I want to try to do this without a mentor.

I think this is something I mostly need to figure out myself, and I plan to keep on trying. Let me explain my current approach.

Right now, I believe explaining is the central part of the joy of mathematics. Or more directly, understanding, which can be gained by explaining. I see explaining as the mathematical equivalent to playing music. Explaining is play.

This justifies the regime I've imposed on myself since the start of the year. I write an explanation every day. When I write it, I notice inconsistencies in my explanation. I write these inconsistencies on a piece of paper, and they serve as a basis for my exploration the next day. I repeat this cycle every day.

I have a repository of what I wrote at here if it interests you. But be warned that I don't tailor the writing for any audience, and it's honestly rather chaotic most of the time.

So far I've been consistent on doing this task every day. And I'm hoping at some point I start to enjoy it more, and don't have to force myself to do it.

If you feel you have the time for this, and if you find it interesting, would you be open to speaking on a call? Please let me know, and I can adjust accordingly.

Cordially,
Davide Radaelli

Laurens D. Gunnarsen <l.gunnarsen@comcast.net>   Wed, Feb 16, 2022 at 1:01 AM
To: Davide Radaelli <daviderady@gmail.com>

Dear Davide,

First of all, I must beg you to forgive me for failing to reply immediately (or, alas, even consistently) to your messages; the simple truth is that they require of me a significant amount of thought, and these days I find I am often obsessed by a large-scale writing project (a novel, in fact) that demands a lot of my cognitive resources. I know this is an embarrassingly selfish (and therefore mostly specious) justification for my shameful irresponsibility as a correspondent. But I fear it's the best I can do.

Second: I too think you should love doing math. But I fear that your notion of what this means may be a little too narrow to enable you to succeed. I would call your attention to this observation of the late, great Michael Atiyah, who knew whereof he spoke:

"People think mathematics begins when you write down a theorem followed by a proof. That's not the beginning, that's the end. For me the creative place in mathematics comes before you start to put things down on paper, before you try to write a formula. You picture various things, you turn them over in your mind. You're trying to create, just as

a musician is trying to create music, or a poet. There are no rules laid down. You have to do it your own way. But at the end, just as a composer has to put it down on paper, you have to write things down. But the most important stage is understanding. A proof by itself doesn't give you understanding. You can have a long proof and no idea at the end of why it works. But to understand why it works, you have to have a kind of gut reaction to the thing. You've got to feel it."

The key thing to pursue is this gut feeling of which Atiyah speaks, this strange, wordless intuition; for it is more primitive than, and therefore more reliable than, fully explicit and articulate understanding. Indeed, let me also quote to you Andre Weil:

"Nothing is more fruitful – all mathematicians know it – than those obscure analogies, those disturbing reflections of one theory in another; those furtive caresses, those inexplicable discords; nothing also gives more pleasure to the researcher. The day comes when the illusion dissolves; the yoked theories reveal their common source before disappearing. As the *Gita* teaches, one achieves knowledge and indifference at the same time."

I beg you to note Weil's emphasis on what gives most pleasure to the researcher: it is not conclusive understanding, but rather the prolonged contemplation of a tantalizing mystery whose solution he somehow feels might ultimately prove to be just barely within reach. Note, too, the poetic, almost enraptured way in which Weil expresses his passion for such delights, and consider that Weil was a waspish and forbidding personality whose susceptibility to ecstatic pleasures was far from conspicuous. I would propose to you that if you wish to enjoy mathematics, you should embrace its fruitful mysteries much more passionately than its rigorous explanations. Weil is perhaps going too far to say that one achieves knowledge and indifference at the same time, for there is certainly some joy in achieving a final answer; but to yearn exclusively for a final answer is clearly quite profoundly wrong-headed. It's a little like making love exclusively to produce offspring. It's not (only) the product, but (also) the process, that counts.

If you want to learn to love doing mathematics, and to do so without benefit of any personal guidance or encouragement, then you are going to need to focus your attention on exactly the right books. Most books will waste most (or practically all) of your time; only a tiny few will do you enough good to repay the attention they'll demand of you. Here are some good books for your purpose:

https://www.amazon.com/You-Are-Mathematician-Penguin-Science/dp/014017480X

http://www.gang.umass.edu/~franz/Paul_Zeitz_The_Art_and_Craft_of_Problem_SolvingBookosorg.pdf

https://archive.org/details/Induction_And_Analogy_In_Mathematics_1_

https://archive.org/details/AnthonyGardinerDiscoveringMathematics

https://www.amazon.com/Euler-Master-Dolciani-Mathematical-Expositions/dp/0883853280

https://www.amazon.com/Excursions-Calculus-Continuous-Mathematical-Expositions/dp/0883853175

https://www.amazon.com/Integer-Partitions-George-Andrews/dp/0521600901

https://www.amazon.com/Friendly-Introduction-Number-Theory/dp/0132637995

https://www.amazon.com/Pathway-Into-Number-Theory/dp/0521575400

https://www.amazon.com/Integers-Polynomials-Rings-Undergraduate-Mathematics/dp/0387201726

https://www.amazon.com/Knots-Surfaces-Discovering-Mathematics-Mathematical/dp/0821804510

https://www.amazon.com/Combinatorial-Introduction-Topology-Dover-Mathematics/dp/0486679667

https://www.amazon.com/Knot-Book-Colin-Adams/dp/0821836781

https://www.amazon.com/Groups-Symmetry-Discovering-Mathematics-Mathematical/dp/0821804502

https://www.amazon.com/Symmetries-Things-John-H-Conway/dp/1568812205

http://www.aerialpress.com/DYN/

https://www.amazon.com/Visual-Complex-Analysis-Tristan-Needham/dp/0198534469

https://www.amazon.com/Visual-Differential-Geometry-Forms-Mathematical/dp/0691203695

https://www.amazon.com/Indras-Pearls-Vision-Felix-Klein/dp/1107564743

I can augment this list in various ways, but this will likely be enough for the moment; note that the above titles appear with the more elementary closer to the top, and the less elementary near the bottom (indeed, the last few on the list are pretty ambitious, and might keep you busy for quite a while.) The main thing to keep in mind here is this crucial remark of Poincare:

"A mathematical demonstration is not a simple juxtaposition of syllogisms, it is syllogisms *placed in a certain order*, and the order in which these elements are placed is much more important than the elements themselves. If I have the feeling, the intuition, so to speak, of this order, so as to perceive at a glance the reasoning as a whole, I need no longer fear lest I forget one of the elements, for each of them will take its allotted place in the array, and that without any effort of memory on my part.

It seems to me then, in repeating a reasoning learned, that I could have invented it. This is often only an illusion; but even then, even if I am not so gifted as to create it by myself, I myself reinvent it in so far as I repeat it."

Many of the books above will explicitly demand you to solve problems, which will keep you from focusing too narrowly on acquiring knowledge for its own sake, and instead cause you to act and to improvise, which is what you must do if you are to enjoy the practice of mathematics. But even those books that offer you no exercises (such as the last of them) you should always understand as challenging you to imagine how to recreate for yourself the ideas and methods they present; your goal is to develop what Poincare calls "the feeling, the intuition, so to speak, of this order, so as to perceive at a glance the reasoning as a whole."

Again: we are speaking here of a *feeling.* It is an affective state, not a cognitive achievement, that we aim at. The goal is not understanding or enlightenment, but rather an exhilarated sense of exaltation engendered by a perceived coherence or unity of things. This is an almost mystical-ecstatic state, of the sort described by contemplative adepts, or by those who have been under the influence of psychedelic drugs. The goal is to feel godlike -- to feel that even a deity could not be any more empowered than you are yourself by the comprehensive insight you have attained.

For this purpose, it is helpful to remember what it feels like to be dwarfed and astounded and ravished by the grandiose and eternal tremendousness of natural phenomena, like the appearance of the Milky Way in an otherwise profoundly dark night sky, or of the aurora borealis, of of the Grand Canyon. This is how the kind of insight you seek will affect you.

If I had to offer any broad, general advice, I think it would be this: since you seek to love doing mathematics, and since love is an emotion, embrace and cultivate your emotional responses to all kinds of mathematical phenomena, quite without concern for how they may be explained or understood. The phenomena themselves are marvelous, so seek to become better at marveling at them, as things in themselves, rather than at our explanations of them. Accept as not only inevitable but stimulating the spell of mystery they cast upon us, and learn to delight in your enchantment. Don't rush to break free of it, don't rush to understand: instead, tarry in your bafflement and wonder, and revel in it, too. Remember, with knowledge comes indifference -- or, at least, detachment. Stay engaged. Stay on the edge of your seat. Enjoy the ride without thinking exclusively of the destination.

OK. Enough from me for now. Hope this helps. And have fun!

L.
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Davide Radaelli <daviderady@gmail.com>   Sun, Feb 20, 2022 at 12:08 AM
To: maxiwizz@gmail.com

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