Computational dynamics

Computer Laboratory 2:
Introduction to Elastodynamics in FEniCSx.
Time Discretization & Numerical/Rayleigh Damping

Dr. Miguel Ángel Moreno-Mateos

Institute of Applied Mechanics (LTM)

Friedrich-Alexander-Universität Erlangen-Nürnberg

Summer semester 2025

INTRODUCTION

In structure dynamics we often work with systems of form

$$\boldsymbol{M} \cdot \ddot{d}(t) + \boldsymbol{C} \cdot \dot{d}(t) + \boldsymbol{K} \cdot d(t) = \boldsymbol{F}(t)$$

$$d(0) = d_0$$

$$\dot{d}(0) = v_0$$

Viscous damping matrix

- Discrete damping
- Material /visco damping
- Damping due to contact / plasticity
- A) NUMERICAL DAMPING
- …
- B) RAYLEIGH DAMPING: $\mathbf{C} = a\mathbf{M} + b\mathbf{K}$

Parameters

Determined from:

- Experience
- Literature / technical guidelines (DIN, ASME, KTA, …)

[Modal reduction to SDOF: done on blackboard]

Damping constant: $\xi_{(l)} = \frac{1}{2}\left[\frac{a}{\omega_{(l)}} + b\omega_{(l)}\right]$

A) NUMERICAL DAMPING

- Numerical damping refers to the artificial dissipation of energy introduced by the Newmark method, especially at high frequencies.
- This can be either desired (to suppress spurious oscillations) or unwanted (if it distorts the physical response).
- Whether and how numerical damping appears depends on the choice of Newmark parameters.

Newmark update equations:

$$u_{n+1} = u_n + \Delta t \dot{u}_n + \frac{\Delta t^2}{2}\left[[1-2\beta]\ddot{u}_n + 2\beta \ddot{u}_{n+1}\right]$$

$$\dot{u}_{n+1} = \dot{u}_n + \Delta t\left[[1-\gamma]\ddot{u}_n + \gamma \ddot{u}_{n+1}\right]$$

1) **No damping** (second-order accurate, unconditionally stable, energy-conserving):

$$\gamma = \frac{1}{2}; \quad \beta = \frac{1}{4}$$

2) With **damping to suppress high-frequency spurious oscillations** (unconditionally stable):

$$\gamma > \frac{1}{2}; \quad \beta \geq \frac{1}{4}\left[\gamma + \frac{1}{2}\right]^2$$

3) Tuned damping (generalized-α):

$$\gamma = \frac{1}{2} + \alpha; \quad \beta = \frac{1}{4}[1+\alpha]^2, \qquad \alpha \in [0,0.3]$$

B) RAYLEIGH DAMPING

$$\boldsymbol{M} \cdot \ddot{d}(t) + \boldsymbol{C} \cdot \dot{d}(t) + \boldsymbol{K} \cdot d(t) = \boldsymbol{F}(t)$$

$$\boldsymbol{C} = a\boldsymbol{M} + b\boldsymbol{K}$$

$$\begin{aligned} &\boldsymbol{M} \cdot \ddot{d}(t) + \\ &a\boldsymbol{M} \cdot \dot{d}(t) + \\ &b\boldsymbol{K} \cdot \dot{d}(t) + \\ &\boldsymbol{K} \cdot d(t) = \boldsymbol{F}(t) \end{aligned}$$

```
########### NEWMARK METHOD (time discretisation) ######################
# HIGH-FREQUENCY DISSIPATION (NUMERICAL DAMPING) IN NEWMARK METHOD:
gamma = 1
beta = 0.5

########### RAYLEIGH DAMPING ########################################
# Note: Not necessary for HF dissipation if γ and β are properly chosen.
eta_m = 0 #1e-4
eta_k = 0 #1e-4

def avg(x_old,x_new,alpha):
    return alpha*x_old + (1-alpha)*x_new
def update_a(u,u_old,v_old,a_old):
    dt_ = dt
    beta_ = beta
    return (u-u_old-dt_*v_old)/beta_/dt_**2 - (1-2*beta_)/2/beta_*a_old
def update_v(a,u_old,v_old,a_old):
    dt_ = dt
    gamma_ = gamma
    return v_old + dt_*((1-gamma_)*a_old + gamma_*a)
a_new = update_a(u,u_old,v_old,a_old)
v_new = update_v(a_new,u_old,v_old,a_old)
```

```
####### VARIATIONAL PROBLEM - WEAK FORM
Res_u =
  rho * ufl.inner(avg(a_old,a_new,alpha_m),v)) * dx\
+ eta_m * rho * ufl.inner(avg(v_old,v_new,alpha_m),v)\
+ eta_k * (ufl.inner(Piso(Fv, 0), ufl.grad(v)) + ufl.inner(Pvol(Fv, 0), ufl.grad(v)))\
+ (ufl.inner(Piso(F, 0), ufl.grad(v)) + ufl.inner(Pvol(F, 0), ufl.grad(v)) - ufl.inner(traction,v) * dss(1)
```

```
# Reminder
Fv = I + ufl.grad(avg(v_old,v_new,alpha_f))
```

TASKS

Wait until
$>$ 7 cycles.

1. Explore high-frequency spurious oscillations with

   No damping (second-order accurate, unconditionally stable, energy-conserving): $\gamma = \frac{1}{2}; \beta = \frac{1}{4}$.

2. Suppress high-frequency oscillations with numerical damping

   $\gamma > \frac{1}{2}; \beta \geq \frac{1}{4}\left[\gamma + \frac{1}{2}\right]^2$.

   Compare 2 cases.

   - Does it eliminate spurious oscillations?
   - Does it introduce damping in the oscillation?

3. Add Rayleigh damping.

   Choose parameters a and b. Compare 2 cases.

   $\mathbf{C} = a\mathbf{M} + b\mathbf{K}$

Call the instructor for doubts and after successful completion of each task and report it. Enjoy!