Esercizio 1. Let $\Phi \subseteq L$ be a set of sentences and suppose that $\vdash \psi \leftrightarrow \bigvee \Phi$ for some sentence ψ. Prove that there is a finite $\Phi_0 \subseteq \Phi$ such that $\vdash \psi \leftrightarrow \bigvee \Phi_0$.

Nota. Anche se $\bigvee \Phi$ non è una formula del prim'ordine la semantica è quella naturale: $M \models \bigvee \Phi$ se qualche formula in Φ è vera in M.

Esercizio 2. Let $a, b, c \in N \models T_{\mathrm{rg}}$. Prove that $r(a, N) = r(b, N) \cap r(c, N)$ occurs only in the trivial case $a = b = c$.

Esercizio 3. Prove that if M is either the infinite complete graph, the infinite empty graph, or the countable random graph and $M_1, M_2 \subseteq M$ are such that $M_1 \sqcup M_2 = M$, then $M_1 \simeq M$ or $M_2 \simeq M$.

Esercizio 4. Prove that for every $b \in N \models T_{\mathrm{rg}}$ the set $r(b, N)$ is a random graph. You may assume for simplicity that N is countable. Is every random graph $M \subseteq N$ of the form $r(b, N)$ for some $b \in N$? Suggerimento: per la seconda domanda si usi l'esercizio precedente. Alternativamente si può usare l'esercizio .