Esercizio 1. Let M an L-structure and let $\psi(x), \varphi(x,y) \in L$. Write a sentence true in M exactly when

a. $\psi(M) \in \{\varphi(a,M) : a \in M\}$;

b. $\{\varphi(a,M) : a \in M\}$ contains at least two sets;

c. $\{\varphi(a,M) : a \in M\}$ contains only sets that are pairwise disjoint.

Esercizio 2. Let M be a structure in the signature of graphs (but not necessarily a graph). Write a sentence ψ such that,

a. $M \models \psi$ if and only if there is an $A \subseteq M$ such that $r^M \subseteq A \times \neg A$.

Esercizio 3. Let $M \preceq N$ and let $\varphi(x,y) \in L$. Suppose there are finitely many sets of the form $\varphi(a,N)$ for some $a \in N^{|x|}$. Prove that all these sets are definable over M.