

Esercizio 1. Prove that if T has exactly 2 maximal consistent extension T_1 and T_2 then there is a sentence φ such that $T, \varphi \vdash T_1$ and $T, \neg\varphi \vdash T_2$.

Esercizio 2. Assume L is countable and let $M \subseteq N$ and $A \subseteq N$ be both countable. Prove that there is a countable model K such that $A \subseteq K \leq N$ and $K \cap M \leq N$ (in particular, $K \cap M$ is a model).

Hint 1: adapt the construction used to prove the downward Löwenheim-Skolem.

Hint 2 (alternative construction): construct two countable chains of countable models such that $K_i \cap M \subseteq M_i \leq M$ and $A \cup M_i \subseteq K_{i+1} \leq M$. The required model is $K = \bigcup_{i \in \omega} K_i$ as $K \cap M = \bigcup_{i \in \omega} M_i$.

Esercizio 3. Let L be the language of strict orders augmented with countably many constants $\{c_i : i \in \omega\}$. Let T be the theory that extends T_{dlo} with the axioms $c_i < c_{i+1}$ for all i . Exhibit three non isomorphic models of this theory. Say if an extension lemma similar to 4.1 holds with N one of these models and any model of T as M .