Esercizio 1. Let $p(x) \subseteq L(B)$ and $p_n(x) \subseteq L(A)$, for $n < \omega$, be consistent types such that

$$p(x) \quad \rightarrow \quad \bigvee_{n<\omega} p_n(x)$$

Prove that there is an $n < \omega$ and a formula $\varphi(x) \in L(A)$ consistent with $p(x)$ such that

$$p(x) \wedge \varphi(x) \quad \rightarrow \quad p_n(x).$$

Esercizio 2. Assume L is countable and that T is complete. Suppose that for every finite tuple x there is a model M that realizes only finitely many types in $S_x(T)$. Prove that T is ω-categorical.

Esercizio 3. Assume L is countable and let T be strongly minimal. Prove that the following are equivalent

1. T is ω-categorical;
2. the algebraic closure of a finite set is finite.