Esercizio 1. Assume L is countable and let $M \preceq N$ have arbitrary (large) cardinality. Let $A \subseteq N$ be countable. Adapt the construction used to prove the downward Löwenheim-Skolem Theorem to prove there is a countable model K such that $A \subseteq K \preceq N$ and $K \cap M \preceq N$ (in particular, $K \cap M$ is a model).

Esercizio 2. Give an alternative proof of the exercise above (using the claim of the downward Löwenheim-Skolem Theorem instead of its proof). Hint: the elementary chain lemma may help.

Esercizio 3. Consider $\mathbb{R}$ in the language of strict orders. Prove that $\mathbb{R} \setminus \{0\} \preceq \mathbb{R}$. Are these two structures isomorphic?