

**Esercizio 1.** Let  $\Phi \subseteq L$  be a set of sentences and suppose that  $\vdash \psi \leftrightarrow \bigvee \Phi$  for some sentence  $\psi$ . Prove that there is a finite  $\Phi_0 \subseteq \Phi$  such that  $\vdash \psi \leftrightarrow \bigvee \Phi_0$ .

Nota. Anche se  $\bigvee \Phi$  non è una formula del prim'ordine la semantica è quella naturale:  $M \models \bigvee \Phi$  se qualche formula in  $\Phi$  è vera in  $M$ .

**Esercizio 2.** Let  $a, b, c \in N \models T_{\text{rg}}$ . Prove that  $r(a, N) = r(b, N) \cap r(c, N)$  occurs only in the trivial case  $a = b = c$ .

**Esercizio 3.** Prove that if  $M$  is either the infinite complete graph, the infinite empty graph, or the random graph and  $M_1, M_2 \subseteq M$  are such that  $M_1 \sqcup M_2 = M$ , then  $M_1 \simeq M$  or  $M_2 \simeq M$ .

**Esercizio 4.** Prove that for every  $b \in N \models T_{\text{rg}}$  the set  $r(b, N)$  is a random graph. Assume  $N$  is countable. Is every random graph  $M \subseteq N$  of the form  $r(b, N)$  for some  $b \in N$ ? (Suggerimento: per la seconda domanda si usi l'esercizio precedente.)