Esercizio 1. Prove that for every Δ-type p the following are equivalent

1. p is prime;
2. $p \vdash \bigvee_{i=1}^{n} \varphi_i \quad \Rightarrow \quad p \vdash \varphi_i$ for some $i \leq n$, for every n and every $\varphi_1, \ldots, \varphi_n \in \Delta$.

(È suffciente una frase che spieghi da cosa segue l'equivalenza.)

Esercizio 2. Let $\mathbb{P}$ be an upper semilattice. Let $B \subseteq \mathbb{P}$ and let $c \in \mathbb{P}$ be such that $\wedge C \nleq c$ for every finite non-empty $C \subseteq B$. Prove that the following are equivalent

1. B is a maximal filter relative to c;
2. $a \notin B \;\Rightarrow\; b \wedge a \leq c$ for some $b \in B$.

N.B. in 2 non si assume che B sia un filtro.

Esercizio 3. Let $\mathbb{P}$ be an lower semilattice. Let $F \subseteq \mathbb{P}$ be a principal filter. Is F always contained in a maximal principal filter?

(Non serve dimostrazione dettagliata, basta un controesempio.)

Esercizio 4. Let $\mathbb{P}$ be a distributive lattice. Suppose we defined $S(\mathbb{P})$ as the set of relatively maximal filters. Which essential (for Stone duality) property would not hold ?

(Sufficiente dire quale e perché senza produrre un controesempio.)