

Esercizio 1. For every Δ -type p the following are equivalent

1. p is prime;
2. $p \vdash \bigvee_{i=1}^n \varphi_i \Rightarrow p \vdash \varphi_i$ for some $i \leq n$, for every n and every $\varphi_1, \dots, \varphi_n \in \Delta$.

(Basta una frase per spiegare da cosa segue.)

Esercizio 2. Let $B \subseteq \mathbb{P}$ and let $c \in \mathbb{P}$ be such that $\bigwedge C \not\leq c$ for every finite non-empty $C \subseteq B$.

Prove that the following are equivalent

1. B is a maximal filter relative to c ;
2. $a \notin B \Rightarrow b \wedge a \leq c$ for some $b \in B$.

Esercizio 3. Let $F \subseteq \mathbb{P}$ be a non-principal filter. Is F always contained in a maximal non-principal filter?

Esercizio 4. Suppose we defined $S(\mathbb{P})$ as the set of relatively maximal filters. Which essential property would not hold?