

Esercizio 1. Let $a = \langle a_i : i \in I \rangle$ be an (infinite) A -indiscernible sequence and let $I \subseteq J$ with $|J| \leq \kappa$. Then there is an A -indiscernible sequence $c = \langle c_i : i \in J \rangle$ such that $c \upharpoonright I = a$.

Esercizio 2. Let $a \equiv_A b$ and let $q(x) = \text{tp}(a/A) = \text{tp}(b/A)$. Prove that if $q(x)$ extends to a global A -invariant type $p(x) \in S(\mathcal{U})$ then there is a sequence $c = \langle c_i : i < \omega \rangle$ such that a, c and b, c are both sequences of A -indiscernibles.

Esercizio 3. Prove that the following are equivalent for every \mathcal{D} that is externally definable by a stable formula

1. \mathcal{D} is invariant over every model M containing A ;
2. $\mathcal{D} \in \text{acl}^{\text{eq}} A$.