

Morgan Baccus

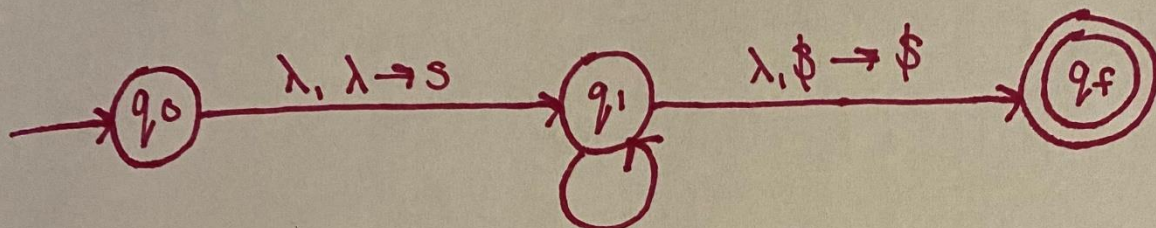
CptS 317

Homework #9

1)  $S \rightarrow aaB \mid bbA \mid AB \mid aabb$

$A \rightarrow BA \mid aBa$

$B \rightarrow AB \mid bAb$



$\lambda, S \rightarrow aaB$

$\lambda, S \rightarrow bbA$

$\lambda, S \rightarrow AB$

$\lambda, S \rightarrow aabb$

$\lambda, A \rightarrow BA$

$\lambda, A \rightarrow aBa$

$\lambda, A \rightarrow a$

$\lambda, B \rightarrow AB$

$\lambda, B \rightarrow bAb$

$\lambda, B \rightarrow b$

$\lambda, a \rightarrow a$

$\lambda, b \rightarrow b$



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2) The PDA of grammar  $G$  can be expressed as  $(V, T, P, s)$ . Let  $V$  contain the initial symbol of  $G$ , being the symbol  $s$ . Also let  $V$  contain the symbols  $X, T$  and  $S$  which correlate to every combination of the stack symbol  $X$ , with the states  $T$  and  $S$ . It then follows that the set  $V$  can be expressed as  $V = \{s, [q_{00}, q_0], [q_0, q_{\text{even}}], [q_0, q_{\text{odd}}], \dots\}$ , onward for all state combination stack symbols for both  $T = \{0, 1\}$  and  $s = \{s\}$ .

The productions of grammar  $G$ , denoted as  $P$ , has 2 forms:

For all  $P$ :  $s \rightarrow q_0 z_0 q_0 \mid q_0 z_0 q_2 \mid \dots$

For all  $P$  swapped for  $s$ :  $s \rightarrow 0z_0 0 \mid 1z_0 1, z_0 \rightarrow 0z_0 0 \mid 1z_0 1 \mid e \mid 0 \mid 1$



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3) By the closure property of regular language, if language  $L$  is regular then the language  $L^r$  (the reverse of strings in the language  $L$ ) is also regular.

Another closure property of regular language is that concatenation of 2 regular languages is regular.

Since language  $L_2$  is regular, by the closure property  $L_2^r$  is also regular.

Then by closure property of concatenation of regular language,  $L_1 L_2^r$  (the concatenation of 2 strings  $x$  and  $y$  in  $L_1$  and  $L_2^r$  respectively) is also regular.

We can see that  $L_3$  is concatenation of  $L_1$  and  $L_2^r$ , hence  $L_3$  is also a regular language.

Since ~~every~~ every regular language is also a context-free language,  $L_3$  is context-free since it is a regular language.



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4) We can prove that  $L$  is not a context-free language with the help of pumping Lemma:

① take a contradiction

② take a string  $w$  divide into 5 parts

$$w = uvxyz$$

③  $w = u v^i x y^i z$  where  $i$  is integer

④ for every  $i$

$w \in L$  then  $L$  is a context-free language  
Otherwise it is not.

Given the language  $L = \{a^{2m} b^{3m} c^{4m} : m \geq 0\}$

①  $L$  is a context-free language.

② string  $w = aa bbb cccc$

$$w = \frac{aa}{u} \frac{bb}{v} \frac{bc}{x} \frac{cc}{y} \frac{c}{z}$$

③  $w = u v^i x y^i z$

$$\text{if } i = 2$$

$$= aa bbbbbb ccccc$$

$$w \neq L$$

So that the given language  $L$  is not a context-free language.