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CptS 317
Homework #8

1) Given the following grammar: $G: S \rightarrow OS|SI|O|I$.

The generated strings are:

$O, 2, OO, OI, II$ | OOO, OOI, OII, IIO

IO is missing

$\left. \begin{array}{l} 010 \\ 100 \\ 101 \\ 110 \end{array} \right\}$ are missing

So we have found that the language is of the form:

$O^+ I^*$ or $O^n I^m$ | $n, m > 0$

Thus, zero or zeros cannot come after I .


2) Given the following grammar $G: S \rightarrow s0|s1|\Lambda$
the generated strings are:

$\{\lambda, 0, 1, 00, 01, 10, 11, 000, 010, 011, 100, \dots\}$

So, the grammar is generated every possible string

with $\Sigma = \langle 0, 1 \rangle$
 ↓
 input alphabet

So, the given grammar generates complete language

i.e.: $(0+1)^*$ or start \rightarrow 

3) Given the following grammar $G: S \rightarrow OS11/\Lambda$

now generating strings are $\langle \Lambda, 011, 001111, 00011111, \dots \rangle$

so clearly, grammar generated strings of the form:

$$L = \{ 0^n 1^{2^n} \mid n \geq 0 \}$$

$$4) S \rightarrow A10$$

$$5) S \rightarrow AB$$

$$A \rightarrow 00011A \mid 00011$$

$$B \rightarrow 01B \mid 01$$

$$6) S \rightarrow 00S1 \mid A$$

$$A \rightarrow 0A \mid 0$$

$$7) S \rightarrow 0S11 \mid A$$

$$A \rightarrow 1A \mid 1$$

$$8) S \rightarrow 00S1 \mid A \mid B \mid 01$$

$$A \rightarrow 0A \mid 0$$

$$B \rightarrow 1B \mid 1$$

$$\begin{aligned}
 9) \quad & S \rightarrow ASB \mid AB \mid ab \\
 & A \rightarrow AS \mid a \mid \lambda \\
 & B \rightarrow SB \mid A \mid b
 \end{aligned}$$

eliminate λ production:

Here we have one Null production.

$A \rightarrow \lambda$ is the null production.

Since $\boxed{S \rightarrow ASB}$
 \searrow
 $S \rightarrow SB$

~~$S \rightarrow ASB$~~
 \searrow
 $S \rightarrow B$

$\boxed{A \rightarrow AS}$
 \searrow
 $A \rightarrow S$

$\boxed{B \rightarrow A}$
 \searrow
 $B \rightarrow \lambda$

◦◦ New productions are:

$$S \rightarrow ASB \mid AB \mid SB \mid B \mid ab$$

$$A \rightarrow AS \mid a$$

$$B \rightarrow SB \mid A \mid b \mid \lambda$$

We still have a λ production.

$\boxed{S \rightarrow ASB}$
 \searrow
 $S \rightarrow AS$

$\boxed{S \rightarrow AB}$
 \searrow
 $S \rightarrow A$

$\boxed{B \rightarrow SB}$
 \searrow
 $B \rightarrow S$

◦◦ Final productions are:

$$S \rightarrow ASB \mid AB \mid SB \mid AS \mid A \mid B \mid ab$$

$$A \rightarrow AS \mid a$$

$$B \rightarrow SB \mid A \mid S \mid b$$

10) Eliminate Unit Productions:

$$S \rightarrow B \mid SB \mid ab$$

$$A \rightarrow a \mid SA$$

$$B \rightarrow A \mid SB \mid b$$

The unit productions we have in the grammar are:

$$S \rightarrow B$$

$$B \rightarrow A$$

(i) $S \rightarrow B$ (eliminate it)

Since $B \rightarrow A \mid SB \mid b$,

We can add the productions

$$S \rightarrow A \mid SB \mid b$$

(ii) $B \rightarrow A$

Since $A \rightarrow a \mid SA$,

We can add productions as:

$$B \rightarrow a \mid SA$$

\therefore The final productions are:

$$S \rightarrow B \mid SB \mid ab \mid A \mid ab$$

$$A \rightarrow a \mid SA$$

$$B \rightarrow A \mid SB \mid SA \mid a \mid b$$

11) Grammar into CNF:

$$\begin{aligned} S &\rightarrow AbBaS \mid ASB \mid AB \mid ab \\ A &\rightarrow bB \mid a \\ B &\rightarrow SBb \mid b \end{aligned}$$

To transform to CNF, we need to eliminate A-production, unit productions, and useless productions.

But in the above grammar we don't have any. But the above grammar still isn't in CNF.

Replace:

- b by x
- a by y
- SB by z

$$S \rightarrow AxByS \mid Az \mid AB \mid Yx$$

$$A \rightarrow xB \mid a$$

$$B \rightarrow zx \mid b$$

$$x \rightarrow b$$

$$y \rightarrow a$$

$$z \rightarrow SB$$

Still, the production

$S \rightarrow AxByS$ is not in CNF.

Replace:

- ~~A~~x by w
- B~~y~~ by c
- wc by d

$$S \rightarrow DS \mid Az \mid AB \mid Yx$$

$$A \rightarrow xB \mid a$$

$$B \rightarrow zx \mid b$$

$$z \rightarrow SB$$

$$w \rightarrow Ax$$

$$c \rightarrow By$$

$$d \rightarrow wc$$

$$x \rightarrow b$$

$$y \rightarrow a$$