

cpts350 hw6

1. Let  $G$  be a DAG (a graph without loops) and  $u, v$  be two designated nodes (there are many other nodes in  $G$ ). Design an efficient algorithm to count the number of paths from  $u$  to  $v$ .
2. Let  $G$  be a DAG (a graph without loops) and  $u, v$  be two designated nodes (there are many other nodes in  $G$ ). In particular, each node in  $G$  is labeled with a color and multiple nodes can share the same color. A good path is one where the number of green nodes is bigger than the number of yellow nodes. Design an efficient algorithm to count the number of good paths from  $u$  to  $v$ .
3. Let  $G$  be a graph (so it may have loops) and  $u, v$  be two designated nodes (there are many other nodes in  $G$ ). In particular, each node in  $G$  is labeled with a color and multiple nodes can share the same color. Suppose that  $\gamma$  is a regular expression on colors (e.g.,  $(\text{green} + \text{yellowyellow})^*\text{yellowblue}$ ). An ugly path is one that the color sequence on the path satisfies the regular expression  $\gamma$ . Design an efficient algorithm to count the number of ugly paths from  $u$  to  $v$  (when the count is infinite, return  $\infty$ ). (Hint: you have two cases to consider, after using a Cartesian product construction: a. the count is infinite – where an SCC algorithm can be used to decide. b. the count is finite, where prob 1 can be used.)
4. Let  $G$  be a graph that may contain loops and hence, the number of paths from a designated start node to a designated end node may be infinite. Unfortunately, you usually can't say that one infinite number is larger than another. Here is the problem: sketch a way to compare the number of paths in two graphs (that both may contain loops). (Hint: google perron, graph, path count).
5. Let  $G$  be a control flow diagram of a C-program (which can be automatically generated). For each node  $u$  in the diagram, one can obtain the total number  $C(u)$  of paths from the root of the diagram to  $u$ . Then, from what you got from 4 above, you may sort all the  $C(u)$ 's for all  $u$  (even though some of  $C(u)$ 's are infinite) and then pick a  $u^*$  that has the maximal  $C(u)$ . Write a mini-paper on how this will address the problem of testing a C-program. (if you want, you can actually publish a paper on this get a Master degree!)
6. (a job interview question for a top tech company) Design an efficient algorithm to compute the number of binary strings with length  $n$  that satisfy

the regular expression  $((0 + 11 + 101)^*(1101))^*$ . (Hint: use Prob 3 above. I will talk about the solutions in class.)