

Morgan Baccus

CptS 350

Homework #12

### Problem 1

Since  $A \leq_m B$ , we have a poly-time computable function  $f$  such that  $\forall x, x \in A \text{ iff } f(x) \in B$ .

Since  $B \leq_m C$ , we have a poly-time computable function  $g$  such that  $\forall y, y \in B \text{ iff } g(y) \in C$ .

To show  $A \leq_m C$ , we need to find a poly-time computable function  $h$  such that  $\forall x, x \in A \text{ iff } h(x) \in C$ . Here, we take  $h = g \circ f$

## Problem 2

We need to guess<sup>is</sup> the question is true and verify.

Guess: There is a sequence of nodes (walk)  $w$  such that the length of the walk  $|w| \leq k$ , where  $k$  is the number of nodes (bound by the size of the walk).

Check 1:  $w$  is indeed a walk on  $G$  in determine poly-time.

Check 2:  $w$  covers every node in  $G$  exactly once in determine poly-time. (can use a hashtable or 2-D array to check since  $k$  is limited).

Check 3: If 1 and 2 are true, return true.  
else, Crash.



### Problem 3

Guess: there is a walk  $w$  that runs the following algorithm in  $T$  steps where  $T \geq k$  (number of nodes in  $G$ ) to bound the size of the walk or run time.

Check 1:  $w$  is a walk on  $G$  in determine poly-time  $T$ .

Check 2:  $w$  covers every node in  $G$  in determine poly-time.  
(Can check using hash table or 2D array since  $k$  is limited).

Check 3: If 1 and 2 are true, return 2. true.  
Else, crash.

### Problem 4

To compare 2 boolean circuits we need to run all possible inputs. Since the input size is  $n$  and there is only 0 and 1 for each possible input, we will spend  $O(2^n)$  to check if  $C_1 = C_2$ .