

morgan Baccus

Cpts 350

Homework #2

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Traditional Case:

$$\begin{array}{r} \overset{2}{1} \overset{3}{2} 3 4 \\ * 5 6 7 8 \\ \hline 4 8 7 2 \\ \times \times \times 8 0 \\ \times \times \times \times 0 0 \\ \times \times \times \times 0 0 0 \\ \hline \end{array}$$

} n number of rows
here, $n=4$

+ $\times \times \times \times \times \times$ \leftarrow add all the numbers from above

Two four-digit numbers will result in at least a 7-digit number.
The operations to calculate this are a finite amount.

We must multiply and sum each digit to calculate the result. Multiplying the digits is n^2 number of operations and summing all the digits is also n^2 operations.

Thus, the total number of operations in the traditional case is $T(n) = n^2 + n^2$ for n number of digits.

For two four-digits this would be $T(4) = 4^2 + 4^2 = 32$.

New Way:

$$\begin{array}{r}
 \begin{array}{r}
 (12)(34) \\
 * (56)(78) \\
 \hline
 (936)(2652)
 \end{array} \\
 + (672)(1904)(00) \\
 \hline
 \text{xxxxx}
 \end{array}$$

$\left. \begin{array}{l} \text{ } \end{array} \right\} n/2 \text{ number of rows}$
 We shift by 00 in one row since our alphabet is now $\{00, \dots, 99\}$

By grouping the digits in two's, we are reducing the number of operations by $1/2$.

We must multiply and sum each pair of digits to calculate the result. Multiplying the pairs of digits is $n^2/2$ number of operations and summing them is also $n^2/2$ operations.

Thus, the total number of operations in the new way is

$$N(n) = \frac{n^2}{2} + \frac{n^2}{2} \text{ for } n \text{ number of digits.}$$

For two four-digit numbers this would be $N(4) = \left(\frac{4^2}{2}\right) + \left(\frac{4^2}{2}\right) = 8$
 Which is four times faster than the traditional way.

$$\text{The speed up ratio} = \lim_{n \rightarrow \infty} \frac{T(n)}{N(n)} = \lim_{n \rightarrow \infty} \frac{n^2 + n^2}{\left(\frac{n^2}{2}\right) + \left(\frac{n^2}{2}\right)}$$