Answer key

[redacted]

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Question 1:(10 points) Assuming that 7 in 10 automobile accidents are due mainly to a speed violation, find the probability that among 6 automobiles accidents, 4 will be due mainly to a speed violation using the formula for binomial distribution.

$n=6 \quad p=0.7$

$$p(X=4)=b(4;6,0.7)=\binom{6}{4}0.7^4(1-0.7)^{(6-4)}$$

$$=\binom{6}{4}0.7^4\,0.3^2=0.3241$$

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Question 2:(10 points) Suppose that the average number of accidents occurring weekly on a particular stretch of a highway equals 3. Assuming that the number of accidents occurring weekly is a Poisson random variable. What is the probability that there is at least one accident this week?

X = # of accidents occurring weekly

X follows poisson distribution with $\lambda t = 3$

$$p(X\geq 1)=1-p(X<1)=1-p(X=0)=1-\frac{e^{-3}3^0}{0!}$$

$$=1-e^{-3}$$

$$=0.9502$$

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Question 3:(15 points) A packing plant fills bags with cement. The weight X kg of a bag of cement can be modeled by a normal distribution with mean 50kg and standard deviation 2 kg
(a) Find $P(X > 53)$.
(b) Find the weight that is exceeded by 99% of the bags.

$X \sim N(50, 2)$

8 (a). $P(X>53) = P\left(\frac{X-50}{2} > \frac{53-50}{2}\right) = P(Z > \frac{3}{2})$

$= 1 - P(Z<1.5) = 0.0668$

7 (b). $P(Z > Z^*) = 0.99 \Rightarrow P(Z<Z^*) = 1-0.99 = 0.01$

$\Rightarrow Z^* = -2.326 \Rightarrow \frac{X-50}{2} = -2.326 \Rightarrow X = 50 - 2\times 2.326$

$\Rightarrow X = 45.348$

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Question 4:(15 points) The probability that a patient recovers from a delicate heart operation is 0.9. Of the next 100 patients having this operation, what is the probability that fewer than 86 survive? (Hint: consider the normal approximation to the Binomial).

5 Let X = # of patients who survives

Then X has the Binomial distribution with $n=100$, $p=0.9$

$E(X) = np = 100\times 0.9 = 90 \quad \sigma_X^2 = npq = 100\times 0.9 \times 0.1 = 9$

5 Hence $Z = \frac{X - E(X)}{\sigma_X} = \frac{X-np}{\sqrt{npq}}$ is approximately standard normal.

$Z = \frac{X-90}{3} \sim N(0,1)$

5 → $P(X<86) = P(X \le 85) \approx P\left(Z \le \frac{85+0.5-90}{3}\right) = P(Z \le -1.5)$

$= 0.0668$

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Question 5:(20 points) The length for one individual to be served in a post office is a random variable having an exponential distribution with mean of 5 minutes. What is the probability that at least 2 among the next 4 persons will have a serving time less than 4 minutes?

$X =$ the length for one individual to be served

X is exponential distribution with $\beta = 5$.

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$$P(X<4) = \int_0^4 \frac{1}{5} e^{-\frac{x}{5}} dx = -e^{-\frac{x}{5}} \Big|_0^4 = 0.5507$$

Thus the prob with a serving time less than 4 minutes is 0.5507.

Let $Y =$ # of persons among the next 4 persons with a serving time less than 4 minutes

Y has a binomial distr. with $p = 0.5507$ and $n = 4$

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$$P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - P(Y=0) - P(Y=1)$$

$$= 1 - \binom{4}{0} 0.5507^0 (1-0.5507)^4 - \binom{4}{1} 0.5507^1 (1-0.5507)^3$$

$$= 1 - 0.0408 - 0.1998 = 0.7594$$

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Question 6:(15 points) The breaking strength X of a certain river used in a machine engine has a mean 5000 psi and standard deviation 400 psi. A random sample of 36 rivets is taken. Consider the distribution of $\bar{X}$, the sample mean breaking strength. (a) Write down the **Central Limit Theorem** and when the normal approximation is good. (b) Use Central Limit Theorem to solve the probability that the sample mean falls between 4800 psi and 5200 psi?

$X \sim N(5000, 400)$

$n=36$

(a) Central Limit Theorem: If $\bar{X}$ is the mean of a random sample of size n taken from a population with mean μ and finite variance σ^2, then the limiting form of the distribution of

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$$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$$

is the standard normal distribution $Z \sim N(0,1)$ as $n \to \infty$.

The normal approximation for $\bar{X}$ will generally be good if $n \geq 30$, provided the population distribution is not terribly skewed. If $n<30$, the approximation is good only if the population is not too different from a normal distribution. If the population is known to be normal, the sampling distribution of $\bar{X}$ will follow a normal distribution exactly, no matter how small the size of the samples

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5 (b) By C.L.T. $P(4800 < \bar{X} < 5200) = P\left(\frac{4800-5000}{400/\sqrt{36}} < Z < \frac{5200-5000}{400/\sqrt{36}}\right)$

$= P(-3 < Z < 3) \approx 0.9974.$