probability and statistics

morgan bergen

05 some discrete probability distributions

march 20 2023

- introduction and motivation
- binomial and multinomial distributions
- negative binomial and geometric distributions
- poisson distribution and the poisson process

5.4 in a certain city district, the need for money to buy drugs is stated as the reason for 75% of all thefts. Find the probability that among the next 5 theft cases reported in this district,

a. exactly 2 resulted from the need for money to buy drugs

b. at most 3 resulted from the need for money to buy drugs

5.10 a nationwide survey of college seniors by the university of Michigan revealed that almost 70% disapprove of daily pot smoking, according to a report in parade.

if 12 seniors are selected at random and asked their opinion, find the probability that the number who disapprove of smoking pot daily is

a. anywhere from 7 to 9

b. at most 5

c. not less than 8

5.25 suppose that for a very large shipment of integrated-circuit chips, the probability of failure for any one chip is 0.10.

assuming that the assumptions underlying the binomial distributions are met, find the probability that at most 3 chips fail in a random sample of 20.

5.26 assuming that 6 in 10 automobile accidents are due mainly to a speed violation, find the probability that among 8 automobile accidents, 6 will be due mainly to a speed violation

a. by using the formula for the binomial distribution

b. by using table a.1

5.27 if the probability that a fluorescent light has a useful life of at least 800 hours in 0.9, find the probabilities that among 20 such lights

a. exactly 18 will have a useful life of at least 800 hours

b. at least 15 will have a useful life of at least 800 hours

c. at least 2 will not have a useful life of at least 800 hours

5.56 on average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection?

a. exactly 5 accidents will occur?

b. fewer than 3 accidents will occur?

c. at least 2 accidents will occur?

5.62 the probability that a student at a high school fails a screening test for scoliosis is 0.004. of the next 1875 students at the school who screened for scoliosis, find the probability that

a. fewer than 5 fail the test

b. 8, 9, or 10 fail the test

5.66 changes in airport procedures require considerable planning. arrival rates of aircraft are important factors that must be taken into account.

suppose small aircraft arrive at a certain airport, according to a poisson process, at the rate of 6 per hour. Thus the poisson parameter for arrivals over a period of hours is

$\mu = 6t$

a. what is the probability that exactly 4 small aircraft arrive during a 1-hour period?

b. what is the probability that at least 4 arrive during a 1-hour period?

c. if we define a working day as 12 hours, what is the probability that at least 75 small aircraft arrive during a working day?

5.67 the number of customers arriving per hour at a certain automobile service facility is assumed to follow a poison distribution with mean $\lamda = 7$

a. compute the probability that more than 10 customers will arrive in a 2 hour period

b. what is the mean number of arrivals during a 2-hour period?

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- **a.** exactly 2 resulted from the need for money to buy drugs
- **b.** at most 3 resulted from the need for money to buy drugs

LET X DENOTE THE NUMBER OF THEFTS RESULTING FROM THE NEED FOR MONEY FOR SUBSTANCES

TRIALS ARE INDEPENDENT, THEN TRIALS ARE INDEPENDENT THEN $X \sim B(n, p)$

$$X \sim B(n, p) \Rightarrow P(X = x) = \binom{n}{x} p^x (1 - x)^{n-x}$$

$p = 0.75$ & $n = 5$

$$P(X = 2) = \binom{5}{2} (0.75)^2 (1 - 0.75)^{5-2} = \frac{5!}{2!(5-2)!} = (0.75)^2 (0.25)^3$$

- $\boxed{P(X = 2) \simeq 0.08789}$

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

$$= \binom{5}{0} (0.75)^0 (0.25)^5 + \binom{5}{1} (0.75)^1 (0.25)^4 + \binom{5}{2} (0.75)^2 (0.25)^3 + \binom{5}{3} (0.75)^3 (0.25)^2$$

$$= (0.25) + 5(0.75)(0.25)^4 + 10(0.75)^2 (0.25)^3 + 10(0.75)^3 (0.25)^2$$

- $\boxed{P(X \leq 2) \simeq 0.36719}$

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5.10 a nationwide survey of college seniors by the university of Michigan revealed that almost 70% disapprove of daily pot smoking, according to a report in parade.

if 12 seniors are selected at random and asked their opinion, find the probability that the number who disapprove of smoking pot daily is

- **a.** anywhere from 7 to 9
- **b.** at most 5
- **c.** not less than 8

GIVEN THE FOLLOWING,

- 70% PROB THAT STUDENTS DISAPPROVE CANNABIS
- 30% PROB THAT STUDENTS APPROVE
- 12 COLLEGE SENIORS WILL BE SURVEYED

BINOMIAL PROBABILITY SUMS

$$P(X=x) = f(x) = b(x; n, p) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

- BINOMIAL RANDOM VARIABLE THE NUMBER OF X SUCCESSESS IN N BERNOULLI TRIALS
- PROBABILITY DISTRIBUTION OF THIS DISCRETE RANDOM VARIABLE IS THE BINOMIAL DISTRIBUTION
- IT'S VALUES WILL BE DENOTED BY $b(x; n, p)$
- $b(x; n, p)$ DEPENDS ON THE NUMBER OF TRIALS
- $b(x; n, p)$ DEPENDS ON THE NUMBER OF SUCCESSES ON A GIVEN TRIAL

a. OBJECTIVE IS TO FIND THAT THE TOTAL NUMBER OF STUDENTS WHO DISAPPROVE WITH THE GIVEN INFORMATION PROVIDED

SO WE NEED TO FIND

$$P(7 \le X \le 9) = \sum_{i=7}^{9} P(X=i) = \sum_{i=7}^{9} b(i; 12, 0.7) = \underbrace{\sum_{i=0}^{9} b(i; 12, 0.7)}_{N=12,\ R=9,\ P=0.} - \underbrace{\sum_{i=0}^{6} b(i; 12, 0.7)}_{N=12,\ R=6,\ P=0.7} = 0.7472 - 0.1178 = 0.0366$$

728 *Appendix A Statistical Tables and Proofs*

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
12	0	0.2824	0.0687	0.0317	0.0138	0.0022	0.0002	0.0000			
	1	0.6590	0.2749	0.1584	0.0850	0.0196	0.0032	0.0003	0.0000		
	2	0.8891	0.5583	0.3907	0.2528	0.0834	0.0193	0.0028	0.0002	0.0000	
	3	0.9744	0.7946	0.6488	0.4925	0.2253	0.0730	0.0153	0.0017	0.0001	
	4	0.9957	0.9274	0.8424	0.7237	0.4382	0.1938	0.0573	0.0095	0.0006	0.0000
	5	0.9995	0.9806	0.9456	0.8822	0.6652	0.3872	0.1582	0.0386	0.0039	0.0001
	6	0.9999	0.9961	0.9857	0.9614	0.8418	0.6128	0.3348	0.1178	0.0194	0.0005
	7	1.0000	0.9994	0.9972	0.9905	0.9427	0.8062	0.5618	0.2763	0.0726	0.0043
	8		0.9999	0.9996	0.9983	0.9847	0.9270	0.7747	0.5075	0.2054	0.0256
	9		1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.4417	0.1109
	10				1.0000	0.9997	0.9968	0.9804	0.9150	0.7251	0.3410
	11					1.0000	0.9998	0.9978	0.9862	0.9313	0.7176
	12						1.0000	1.0000	1.0000	1.0000	1.0000

$$\boxed{\therefore\ P(7 \le X \le 9) = \sum_{i=7}^{9} P(X=i) = 0.036}$$

728 *Appendix A Statistical Tables and Proofs*

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
12	0	0.2824	0.0687	0.0317	0.0138	0.0022	0.0002	0.0000			
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	4	0.9957	0.9274	0.8424	0.7237	0.4382	0.1938	0.0573	0.0095	0.0006	0.0000
	5	0.9995	0.9806	0.9456	0.8822	0.6652	0.3872	0.1582	0.0386	0.0039	0.0001
	6	0.9999	0.9961	0.9857	0.9614	0.8418	0.6128	0.3348	0.1178	0.0194	0.0005
	7	1.0000	0.9994	0.9972	0.9905	0.9427	0.8062	0.5618	0.2763	0.0726	0.0043
	8		0.9999	0.9996	0.9983	0.9847	0.9270	0.7747	0.5075	0.2054	0.0256
	9		1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.4417	0.1109
	10				1.0000	0.9997	0.9968	0.9804	0.9150	0.7251	0.3410
	11					1.0000	0.9998	0.9978	0.9862	0.9313	0.7176
	12						1.0000	1.0000	1.0000	1.0000	1.0000

b. AT MOST 5 THUS $P(X \le 5)$

$$P(X \le 5) = \sum_{i=0}^{5} P(X=i) = \sum_{i=0}^{5} b(i; 12, 0.7) = 0.0386$$

$$\boxed{\therefore\ P(X \le 5) = \sum_{i=0}^{5} P(X=i) = 0.0386}$$

c NOT LESS THAN 8

$$P(X > 8) = 1 - P(X > 8) = 1 - P(X \le 7) = 1 - \sum_{i=0}^{7} P(X=i)$$

728 *Appendix A Statistical Tables and Proofs*

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
12	0	0.2824	0.0687	0.0317	0.0138	0.0022	0.0002	0.0000			
	1	0.6590	0.2749	0.1584	0.0850	0.0196	0.0032	0.0003	0.0000		
	2	0.8891	0.5583	0.3907	0.2528	0.0834	0.0193	0.0028	0.0002	0.0000	
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	8		0.9999	0.9996	0.9983	0.9847	0.9270	0.7747	0.5075	0.2054	0.0256
	9		1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.4417	0.1109
	10				1.0000	0.9997	0.9968	0.9804	0.9150	0.7251	0.3410
	11					1.0000	0.9998	0.9978	0.9862	0.9313	0.7176
	12						1.0000	1.0000	1.0000	1.0000	1.0000

$$= 1 - \sum_{i=0}^{7} b(i; 12, 0.7) = 1 - 0.2763 = 0.7237$$

$$\boxed{\therefore\ P(X > 8) = 1 - \sum_{i=0}^{7} P(X=i) = 0.7237}$$

5.25 suppose that for a very large shipment of integrated-circuit chips, the probability of failure for any one chip is 0.10.

assuming that the assumptions underlying the binomial distributions are met, find the probability that at most 3 chips fail in a random sample of 20.

LET RANDOM VARIABLE X BE THE NUMBER OF CHIPS FAILED AMONG THE 20 RANDOM CHIPS

- LET FAILURE OF A CHIP BE REPRESENTED AS A SUCCESS
- PROBABILITY OF SUCCESS $p = 0.10$
- EACH TRIAL IS INDEPENDENT
- X HAS A BINOMIAL DISTRIBUTION WITH PARAMETERS $n = 20$ & $p = 0.10$

THE PROBABILITY MASS FUNCTION OF X IS,

$$P(X = x) = b(x; 20, 10) \quad , \qquad \text{WHERE } x = 0, 1, 2, 3, \ldots, 20$$

$$= \binom{20}{x} (10)^x (1 - 0.10)^{20-x}$$

$$= \binom{6}{x} (0.10)^x (0.90)^{20-x}, \qquad \text{WHERE } x = 0, 1, 2, \ldots, 20$$

IN ORDER TO FIND THE PROBABILITY THAT AT MOST 3 CHIPS FAIL IN A RANDOM SAMPLE OF 20

- $$\boxed{P(X \le 3) = \sum_{x=0}^{3} b(x; 20, 0.10) = 0.8670}$$

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

		p									
n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
19	0	0.1351	0.0144	0.0042	0.0011	0.0001					
	1	0.4203	0.0829	0.0310	0.0104	0.0008	0.0000				
	2	0.7054	0.2369	0.1113	0.0462	0.0055	0.0004	0.0000			
	3	0.8850	0.4551	0.2631	0.1332	0.0230	0.0022	0.0001			
	4	0.9648	0.6733	0.4654	0.2822	0.0696	0.0096	0.0006	0.0000		
	5	0.9914	0.8369	0.6678	0.4739	0.1629	0.0318	0.0031	0.0001		
	6	0.9983	0.9324	0.8251	0.6655	0.3081	0.0835	0.0116	0.0006		
	7	0.9997	0.9767	0.9225	0.8180	0.4878	0.1796	0.0352	0.0028	0.0000	
	8	1.0000	0.9933	0.9713	0.9161	0.6675	0.3238	0.0885	0.0105	0.0003	
	9		0.9984	0.9911	0.9674	0.8139	0.5000	0.1861	0.0326	0.0016	
	10		0.9997	0.9977	0.9895	0.9115	0.6762	0.3325	0.0839	0.0067	0.0000
	11		1.0000	0.9995	0.9972	0.9648	0.8204	0.5122	0.1820	0.0233	0.0003
	12			0.9999	0.9994	0.9884	0.9165	0.6919	0.3345	0.0676	0.0017
	13			1.0000	0.9999	0.9969	0.9682	0.8371	0.5261	0.1631	0.0086
	14				1.0000	0.9994	0.9904	0.9304	0.7178	0.3267	0.0352
	15					0.9999	0.9978	0.9770	0.8668	0.5449	0.1150
	16					1.0000	0.9996	0.9945	0.9538	0.7631	0.2946
	17						1.0000	0.9992	0.9896	0.9171	0.5797
	18							0.9999	0.9989	0.9856	0.8649
	19							1.0000	1.0000	1.0000	1.0000
20	0	0.1216	0.0115	0.0032	0.0008	0.0000					
	1	0.3917	0.0692	0.0243	0.0076	0.0005	0.0000				
	2	0.6769	0.2061	0.0913	0.0355	0.0036	0.0002				
	3	$\boxed{0.8670}$	0.4114	0.2252	0.1071	0.0160	0.0013	0.0000			
	4	0.9568	0.6296	0.4148	0.2375	0.0510	0.0059	0.0003			

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5.26 assuming that 6 in 10 automobile accidents are due mainly to a speed violation, find the probability that among 8 automobile accidents, 6 will be due mainly to a speed violation

a. by using the formula for the binomial distribution

b. by using table a.1

GIVEN THE FOLLOWING,

- 60 % OF ACCIDENTS OCCUR FROM SPEED
- FIND P THAT OUT OF 8, 6 ARE DUE TO SPEED
- LET X BE THE NUMBER OF ACCIDENTS DUE TO SPEED OUT OF THE 8 PICKED AT RANDOM
- THEN X FOLLOWS A $b(x;8,0.6) \Rightarrow N=8,\ p=0.6$,

$$P(X=6) = b(x;8,0.60), \quad \text{WHERE } x=0,1,\dots,8$$

p = SUCCESS PROB

$q = 1-p$

$$= \binom{N}{x} p^x q^{N-x} = \binom{8}{x} (0.60)^x (1-0.60)^{8-x}$$

$$P(X=6) = \binom{8}{x} (0.60)^x (0.4)^{8-x}, \quad \text{WHERE } x=0,1,2,\dots,8$$

a. USING OUR DERIVED EQUATION

$$P(X=6) = b(6;8,0.6) = \binom{8}{6}(0.60)^6(0.4)^{8-6} = \binom{8}{6}(0.6)^6(0.4)^2 = \frac{8!}{6!\,2!}(0.6)^6(0.4)^2 = 0.2090189$$

- $\boxed{P(X=6) = 0.2090}$

b. USING THE BINOMIAL PROBABILITY SUMS $\sum_{x=0}^{r} b(x;n,p)$

- $\boxed{P(X=6) = b(6;8,0.60) = 0.8936}$

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^{r} b(x;n,p)$

n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
8	0	0.4305	0.1678	0.1001	0.0576	0.0168	0.0039	0.0007	0.0001	0.0000	
	1	0.8131	0.5033	0.3671	0.2553	0.1064	0.0352	0.0085	0.0013	0.0001	
	2	0.9619	0.7969	0.6785	0.5518	0.3154	0.1445	0.0498	0.0113	0.0012	0.0000
	3	0.9950	0.9437	0.8862	0.8059	0.5941	0.3633	0.1737	0.0580	0.0104	0.0004
	4	0.9996	0.9896	0.9727	0.9420	0.8263	0.6367	0.4059	0.1941	0.0563	0.0050
	5	1.0000	0.9988	0.9958	0.9887	0.9502	0.8555	0.6846	0.4482	0.2031	0.0381
	6		0.9999	0.9996	0.9987	0.9915	0.9648	0.8936	0.7447	0.4967	0.1869
	7		1.0000	1.0000	0.9999	0.9993	0.9961	0.9832	0.9424	0.8322	0.5695
	8				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

5.27 if the probability that a fluorescent light has a useful life of at least 800 hours in 0.9, find the probabilities that among 20 such lights

a. exactly 18 will have a useful life of at least 800 hours

b. at least 15 will have a useful life of at least 800 hours

c. at least 2 will not have a useful life of at least 800 hours

GIVEN THE FOLLOWING,

- LET RANDOM VARIABLE X REPRESENT A USEFUL LIFE
- A SUCCESSFUL TRIAL WILL IN A USEFUL LIFE
- $p = 0.9$ FOR PROBABILITY OF SUCCESS
- ALL TRIALS ARE INDEPENDENT, X THEREFORE HAS A BINOMIAL DISTRIBUTION WITH PARAMETERS $n = 20$ & $p = 0.9$

THE PROBABILITY MASS FUNCTION OF X IS,

$$P(X = x) = b(x;\ 20, 0.9) = \binom{20}{x}(0.9)^x(1-p)^{n-x} = \binom{20}{x}(0.9)^x(0.1)^{20-x}, \quad \text{WHERE } x = 0, 1, \dots, 20$$

a. EXACTLY 18 WILL HAVE A USEFUL LIFE

$$P(X = 18) = b(x = 18;\ 20, 0.9) = \binom{20}{18}(0.9)^{18}(0.1)^2 = \frac{20!}{18!(20-18)!}(0.9)^{18}(0.1)^2 = 0.2852$$

- $\boxed{P(X = 18) = 0.2852}$

b. AT LEAST 15 WILL HAVE A USEFUL LIFE

$$\begin{aligned} P(x \geq 15) &= 1 - P(X \leq 14) \\ &= 1 - \sum_{x=0}^{14} b(x;\ 20, 0.90) \\ &= 1 - 0.0113 \end{aligned}$$

- $\boxed{P(X \geq 15) = 0.9887}$

c. AT LEAST 2 WILL NOT HAVE A USEFUL LIFE

- $\boxed{P(X \leq 18) = \sum_{x=0}^{18} b(x;\ 20, 0.9) = 0.6083}$

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5.56 on average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection?

a. exactly 5 accidents will occur?

b. fewer than 3 accidents will occur?

c. at least 2 accidents will occur?

LET X REPRESENT THE NUMBER OF ACCIDENTS IN A MONTH

- AVERAGE NUMBER OF ACCIDENTS PER MONTH $\lambda = 3$
- SINCE THE GIVEN INTERVAL OF TIME IS 1 MONTH, $t = 1$
- X HAS A POISSON DISTRIBUTION WITH PARAMETER $\lambda t = 3$

THE PMF IS,

$$P(X=x) = p(x;\lambda t), \quad \text{WHERE} \quad x = 0, 1, 2,$$

$$= \frac{e^{-\lambda t}(\lambda t)^{x}}{x!}$$

a. EXACTLY 5 ACCIDENTS OCCUR

$$\boxed{P(X=5) = p(5,3) = \frac{e^{-3}(3)^5}{5!} = 0.1008}$$

b. ANY MONTH FEWER THAN 3 WILL OCCUR

$$P(X<3) = P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= p(0;3) + p(1;3) + p(2,3)$$

$$= \frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!}$$

$$= 0.0498 + 0.1494 + 0.2240$$

$$\boxed{P(X<3) = 0.4232}$$

c. AT LEAST 2 ACCIDENTS WILL

$$P(X \ge 2) = 1 - P(X \le 1)$$

$$= 1 - P(X=0) - P(X=1)$$

$$= 1 - p(0;3) - p(1;3)$$

$$= 1 - 0.0498 - 0.1494$$

$$\boxed{P(X \ge 2) = 0.8008}$$

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5.62 the probability that a student at a high school fails a screening test for scoliosis is 0.004. of the next 1875 students at the school who screened for scoliosis, find the probability that

a. fewer than 5 fail the test

b. 8, 9, or 10 fail the test

LET X REPRESENT THE NUMBER OF STUDENTS WHO FAIL OUT OF THE NEXT 1875 STUDENTS. CONSIDER IT SUCCESS IF A STUDENT FAILS THE SCREENING TEST THUS THE PROBABILITY OF SUCCESS IN EACH TRIAL IS $p = 0.004$

BECAUSE THE TRIALS ARE INDEPENDENT, X HAS BINOMIAL DISTRIBUTION WITH PARAMETERS $n = 1875$ & $p = 0.004$

$\mu = n \cdot p = 1875 \cdot 0.004 = 7.5$

USING THE POISSON APPROXIMATION, X HAS A POISSON DISTRIBUTION WITH PARAMETER $\mu = 7.5$

THE PROBABILITY MASS FUNCTION OF X IS THUS,

$$p(x; 7.5) = \frac{e^{-7.5}(7.5)^x}{x!} \quad ; \quad x = 0, 1, 2, 3, \ldots$$

NOW WE CAN CALCULATE THE PROB THAT FEWER THAN 5 FAIL THE TEST

$$P(X < 5) = P(X \leq 4)$$
$$= P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$
$$= \frac{e^{-7.5}(7.5)^0}{0!} + \frac{e^{-7.5}(7.5)^1}{1!} + \frac{e^{-7.5}(7.5)^2}{2!} + \frac{e^{-7.5}(7.5)^3}{3!} + \frac{e^{-7.5}(7.5)^4}{4!}$$
$$= 0.00055 + 0.00415 + 0.01556 + 0.03889 + 0.07292$$

- $\boxed{P(X \leq 5) = 0.1321}$

NEXT WE CAN CALCULATE THE PROBABILITY THAT 8, 9, OR 10 FAIL THE TEST

$$P(8 \leq X \leq 10) = P(X=8) + P(X=9) + P(X=10)$$
$$= \frac{e^{-7.5}(7.5)^8}{8!} + \frac{e^{-7.5}(7.5)^9}{9!} + \frac{e^{-7.5}(7.5)^{10}}{10!}$$
$$= 0.13733 + 0.11444 + 0.08583$$

- $\boxed{P(8 \leq X \leq 10) = 0.3376}$

probability and statistics

morgan bergen

05 some discrete probability distributions

march 20 2023

- introduction and motivation
- binomial and multinomial distributions
- negative binomial and geometric distributions
- poisson distribution and the poisson process

5.66 changes in airport procedures require considerable planning. arrival rates of aircraft are important factors that must be taken into account.

suppose small aircraft arrive at a certain airport, according to a poisson process, at the rate of 6 per hour. Thus the poisson parameter for arrivals over a period of hours is

$\mu = 6t$

a. what is the probability that exactly 4 small aircraft arrive during a 1-hour period?

b. what is the probability that at least 4 arrive during a 1-hour period?

c. if we define a working day as 12 hours, what is the probability that at least 75 small aircraft arrive during a working day?

LET X REPRESENT THE NUMBER OF AIRCRAFT ARRIVALS DURING A 1 HOUR PERIOD

THE POISSON PARAMETER FOR ARRIVALS FOR A PERIOD OF t HOURS IS $\mu = 6t$

THE PROBABILITY MASS FUNCTION OF X IS

$$P(X = x) = \frac{e^{-6}(6)^x}{x!} \quad ; \quad x = 0, 1, 2, \ldots$$

NOW WE CAN FIND THE PROBABILITY THAT EXACTLY 4 SMALL AIRCRAFT ARRIVE DURING A 1-HOUR PERIOD

$$P(X = 4) = \frac{e^{-6}(6)^4}{4!} = 0.1338561$$

- $\boxed{P(X = 4) \simeq 0.1339}$

NEXT WE CAN CALC THE PROB THAT AT LEAST 4 ARRIVE

$$\begin{aligned} P(X \geq 4) &= 1 - P(X \leq 3) \\ &= 1 - \left(P(X=3) + P(X=2) + P(X=1) + P(X=0)\right) \\ &= 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) \\ &= 1 - p(0;6) - p(1;6) - p(2;6) - p(3;6) \\ &= 1 - \frac{e^{-6}(6)^0}{0!} - \frac{e^{-6}(6)^1}{1!} - \frac{e^{-6}(6)^2}{2!} - \frac{e^{-6}(6)^3}{3!} \\ &= 1 - 0.002479 - 0.014873 - 0.044618 - 0.089235 \\ &= 1 - 0.151204 \\ &= 0.8488 \end{aligned}$$

- $\boxed{P(X \geq 4) = 0.8488}$

LASTLY LETS FIND THAT AT LEAST 75 SMALL AIRCRAFT ARRIVE DURING A 12 HOUR WORKING DAY

$t = 12$ HOUR

$\mu = 6t = 6 \cdot 12 = 72$ HOURS

$$\begin{aligned} P(X \geq 75) &= 1 - P(X \leq 74) \\ &= 1 - \sum_{x=0}^{74} \frac{e^{-72}(12)^x}{x!} = 1 - 0.6227 = 0.3773 \end{aligned}$$

- $\boxed{P(X \geq 75) = 0.3773}$

probability and statistics

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march 20 2023

05 some discrete probability distributions

5.67 the number of customers arriving per hour at a certain automobile service facility is assumed to follow a poison distribution with mean $\lamda = 7$

a. compute the probability that more than 10 customers will arrive in a 2 hour period

b. what is the mean number of arrivals during a 2-hour period?

$$P(X > 10) = 1 - P(X \leq 10)$$
$$= 1 - \sum_{x=0}^{10} p(x; (\mu = 7 \cdot 2))$$
$$= 1 - \sum_{x=0}^{10} p(x; 14)$$
$$= 1 - \sum_{x=0}^{10} \frac{e^{-14} (14)^x}{x!}$$
$$= 1 - 0.175681$$

- $\boxed{P(X > 10) = 0.8243}$

NEXT THE MEAN FOR A 2 HOUR PERIOD IS JUST $E(X) = \mu \cdot 2 = 14$

- $\boxed{E(X) = 14}$