MATH 526 PROBABILITY & STATISTICS

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CHAPTER 1 INTRO TO STATISTICS & DATA ANALYSIS

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1.1 THE FOLLOWING MEASUREMENTS WERE RECORDED FOR DRYING TIME IN HOURS, OF A CERTAIN BRAND OF LATEX PAINT.

3.4	2.5	4.8	2.9	3.6
2.8	3.3	5.6	3.7	2.8
4.4	4.0	5.2	3.0	4.8

ASSUME THE MEASUREMENTS ARE A SIMPLE RANDOM SAMPLE

(A) WHAT IS THE SAMPLE SIZE FOR THE ABOVE SAMPLE?

$$\boxed{\|S\| = 15}$$

(B) CALCULATE THE SAMPLE MEAN FOR THE DATA

$$\bar{x} = \sum_{i=1}^{N} \frac{x_i}{N} = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

$$\bar{x} = \sum_{i=1}^{15} \frac{x_i}{15} = \frac{3.4 + 2.5 + 4.8 + 2.9 + 3.6 + 2.8 + 3.3 + 5.6 + 3.7 + 2.8 + 4.4 + 4.0 + 5.2 + 3.0 + 4.8}{15} = 3.78\bar{6}$$

$$\boxed{\bar{x} \simeq 3.78\bar{6} = \left(\frac{56.8}{15}\right)}$$

(C) CALCULATE THE SAMPLE MEDIAN

$$\tilde{x} = \begin{cases} x_{(N+1)/2}, & \text{IF } N \text{ IS ODD} \\ \frac{1}{2}\left(x_{(N/2)} + x_{(N/2)+1}\right), & \text{IF } N \text{ IS EVEN} \end{cases}$$

$$(\underset{1}{2.5}, \underset{2}{2.8}, \underset{3}{2.8}, \underset{4}{2.9}, \underset{5}{3.0}, \underset{6}{3.3}, \underset{7}{3.4}, \underset{8}{3.6}, 3.7, 4.0, 4.4, 4.8, 5.2, 5.6)$$

$$\tilde{x} = x_{(15+1)/2} = x_8 = 3.6$$

$$\boxed{\tilde{x} = 3.6}$$

CALCULATE THE SAMPLE VARIANCE

$$s^2 = \sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1} = \sum_{i=1}^{15} = \left(2.5 - \left(\frac{56.8}{16}\right)\right)^2 + \left(2.8 - \left(\frac{56.8}{16}\right)\right)^2 + \left(2.9 - \left(\frac{56.8}{16}\right)\right)^2 + \left(3.0 - \left(\frac{56.8}{16}\right)\right)^2 + \left(3.3 - \left(\frac{56.8}{16}\right)\right)^2 + \left(3.4 - \left(\frac{56.8}{16}\right)\right)^2 +$$

SINCE IT'S A TRANSCENDENTAL NUM

$$\left(3.6 - \left(\frac{56.8}{16}\right)\right)^2 + \left(3.7 - \left(\frac{56.8}{16}\right)\right)^2 + \left(4.0 - \left(\frac{56.8}{16}\right)\right)^2 + \left(4.4 - \left(\frac{56.8}{16}\right)\right)^2 + \left(4.8 - \left(\frac{56.8}{16}\right)\right)^2 + \left(5.2 - \left(\frac{56.8}{16}\right)\right)^2 +$$

$$\bar{x} = \left(\frac{56.8}{16}\right) \simeq 3.78\bar{6}$$

$$\left(5.6 - \left(\frac{56.8}{16}\right)\right)^2 \simeq \frac{11.1969\bar{7}}{14} \simeq 0.7997841$$

$$\boxed{s^2 \simeq 0.7997841}$$

CALCULATE THE SAMPLE STANDARD DEVIATION

$$s = \sqrt{s^2} \Rightarrow s = \sqrt{0.7997841} = 0.8943065062$$

$$\boxed{s = 0.8943}$$

(E) COMPUTE THE 20% TRIMMED MEAN FOR THE ABOVE DATASET

TRIM 20% LARGEST, 20% SMALLEST, COMPUTE AVERAGE OF REMAINING VALUES

20% OF 15 IS 3, THUS WE WILL TRIM THE SMALLEST 3 & LARGEST 3

missed 4.8

$$(\cancel{2.5}, \cancel{2.8}, \cancel{2.8}, 2.9, 3.0, 3.3, 3.4, 3.6, 3.7, 4.0, 4.4, \cancel{4.8}, \cancel{5.2}, \cancel{5.6})$$

$$\tilde{x}_{TR(20)} = \frac{2.9 + 3.0 + 3.3 + 3.4 + 3.6 + 3.7 + 4.0 + 4.4}{8} = 3.5375$$

$$\boxed{\tilde{x}_{TR(20)} = 3.5375}$$

1.14 A TIRE MANUFACTURER WANTS TO DETERMINE THE INNER DIAMETER OF A CERTAIN GRADE OF TIRE. IDEALLY, THE DIAMETER WOULD BE 570 mm. THE DATA ARE AS FOLLOWS:

$$\{572, 572, 573, 568, 569, 575, 565, 570\} = S \quad , \quad \|S\| = 8$$

(A) FIND THE SAMPLE MEAN & MEDIAN

$$\bar{x} = \sum_{i=1}^{8} \frac{x_i}{8} = \frac{565 + 568 + 569 + 570 + 572 + 572 + 573 + 575}{8} = 570.5$$

$$\boxed{\bar{x} = 570.5}$$

$$(\underset{1}{565}, \underset{2}{568}, \underset{3}{569}, \underset{4}{570}, \underset{5}{572}, \underset{6}{572}, \underset{7}{573}, \underset{8}{575})$$

$$\tilde{x} = \begin{cases} x_{(n+1)/2}, & \text{IF } N \text{ IS ODD} \\ \frac{1}{2}\left(x_{n/2} + x_{(n/2)+1}\right), & \text{IF } N \text{ IS EVEN} \end{cases}$$

$$\tilde{x} = \frac{1}{2}\left(x_{(8/2)} + x_{(8/2)+1}\right)$$
$$= \frac{1}{2}(x_4 + x_5)$$
$$= \frac{1}{2}(570 + 572)$$

$$\boxed{\tilde{x} = 571}$$

(B) FIND THE SAMPLE VARIANCE, STANDARD DEVIATION, AND RANGE

$$s^2 = \sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1} = \frac{(565-570.5)^2 + (568-570.5)^2 + (569-570.5)^2 + (570-570.5)^2 + (572-570.5)^2 + (572-570.5)^2 + (573-570.5)^2 + (575-570.5)^2}{8-1} = \frac{70}{8-1}$$

$$\boxed{s^2 = 10}$$

$$s = \sqrt{s^2} = \sqrt{10}$$

$$\boxed{s = 3.1627}$$

SAMPLE RANGE $S_{RANGE} = X_{MAX} - X_{MIN}$
$= 575 - 565$

$$\boxed{S_{RANGE} = 10}$$

(C) USING THE CALCULATED STATISTICS IN PARTS (a) AND (b), CAN YOU COMMENT ON THE QUALITY OF THE TIRES?

IDEALLY A QUALITY TIRE IS DEFINED AS 570mm, GIVEN OUR DATA WITH THE MEAN AND MEDIAN AT 570.5 & 571 THE DATA IS QUALITY

bring up std dev.

1.21 THE LENGTHS OF POWER FAILURES, IN MINUTES, ARE RECORDED IN THE FOLLOWING TABLE

{22, 18, 135, 15, 90, 78, 169, 98, 102, 83, 55, 28, 121, 120, ,13, 22, 124, 112, 70, 66, 74, 89, 103, 24, 21, 112, 21, 40,
98, 87, 132, 115, 21, 28, 43, 37, 50, 96, 118, 158, 74, 78, 83, 93, 95}

(A) 1-VAR STATS

$\bar{x} = 74.0\bar{2}$ SAMPLE MEAN

$S_x = 39.257592$ STD. DEV.

(B) 5-NUMBER SUMMARY IS AS FOLLOWS

MIN x = 13.5
$Q_1 = 32.5$
MEDIAN = 78
$Q_3 = 102.5$
MAX X = 158

1.22 THE FOLLOWING DATA ARE MEASUREMENTS OF THE DIAMETER OF 36 RIVET HEADS IN 1/100 OF AN INCH

(A) FIND SAMPLE MEAN, MEDIAN, SAMPLE VAR.

$\bar{x} = 6.726\bar{1}$
MED = 6.725
$S_x = 0.05357$
$S_x^2 = 0.0028$

72 6.77 6.82 6.70 6.78 6.70 6.62 6.75
6.66 6.66 6.64 6.76 6.73 6.80 6.72 6.76
6.76 6.68 6.66 6.62 6.72 6.76 6.70 6.78
6.76 6.67 6.70 6.72 6.74 6.81 6.79 6.78
6.66 6.76 6.76 6.72

b construct a relative frequency histogram of the data

1.24 THE FOLLOWING ARE HISTORICAL DATA ON STAFF SALARIES (DOLLARS PER PUPIL) FOR 30 SCHOOLS SAMPLED IN THE EASTERN PART OF THE U.S. IN THE EARLY 1970s

79 2.99 2.77 2.91 3.10 1.84 2.52 3.22
2.45 2.14 2.67 2.52 2.71 2.75 3.57 3.85
3.36 2.05 2.89 2.83 3.13 2.44 2.10 3.71
3.14 3.54 2.37 2.68 3.51 3.37

(A) FIND SAMPLE MEAN & STD. DEV

$\bar{x} = 2.897\bar{3}$
$S_x = 0.541517$

(B) FIND THE 5 NUMBER SUMMARY

MIN x = 1.84
$Q_1 = 2.52$
MED = 2.86
$Q_3 = 3.36$
MAX X = 3.85