(8.12) THE TAR CONTENTS OF 8 BRANDS OF CIGARETTES SELECTED AT RANDOM FROM THE LATEST LIST RELEASED BY THE FEDERAL TRADE COMMISSION ARE AS FOLLOWS:

7.3, 8.6, 10.4, 16.1, 12.2, 15.1, 14.5 AND 9.3 MILLIGRAMS

CALCULATE,

(A) THE MEAN

GIVEN THAT THE DATA SET IS $N=8$
THE MEAN OF THE OBSERVED SAMPLE IS,

$$\bar{x} = \frac{7.3 + 8.6 + 10.4 + 16.1 + 12.2 + 15.1 + 14.5 + 9.3}{N}$$

$$\bar{x} = \frac{93.5}{8}$$

$$\boxed{\bar{x} = 11.6875}$$

(B) THE VARIANCE

SINCE WE KNOW NOW THE MEAN OF THE SAMPLE, WE ARE ABLE TO FIND THE VARIANCE USING THE FOLLOWING FORMULA:

$$s^2 = \frac{1}{N-1} \sum_{i=1} N(x_i - \bar{x})^2$$

$$= \frac{1}{7}\Big((7.3 - 11.6875)^2 + (8.6 - 11.6875)^2 + (10.4 - 11.6875)^2 + (16.1 - 11.6875)^2 + (12.2 - 11.6875)^2 + (15.1 - 11.6875)^2 + (14.5 - 11.6875)^2 + (9.3 - 11.6875)^2 \Big)$$

$$= (1/7)(75.42875)$$

$$\boxed{s^2 \simeq 10.7756}$$

(8.23) THE RANDOM VARIABLE X, REPRESENTING THE NUMBER OF CHERRIES IN A CHERRY PUFF, HAS THE FOLLOWING PDF

x	4	5	6	7
$P(X=x)$	0.2	0.4	0.3	0.1

(A) FIND THE MEAN μ & VARIANCE OF σ^2 OF X

$$\mu_X = 4 \cdot P(X=4) + 5 \cdot P(X=5) + 6 \cdot P(X=6) + 7 \cdot P(X=7)$$

$$= 4 \cdot 0.2 + 5 \cdot 0.4 + 6 \cdot 0.3 + 7 \cdot 0.1$$

$$\boxed{\mu_X = 5.3}$$

$$\sigma_X^2 = (4-\mu_X)^2 \cdot P(X=4) + (5-\mu_X)^2 \cdot P(X=5) + (6-\mu_X)^2 \cdot P(X=6) + (7-\mu_X)^2 \cdot P(X=7)$$

$$= (4-5.3)^2 \cdot 0.2 + (5-5.3)^2 \cdot 0.4 + (6-5.3)^2 \cdot 0.3 + (7-5.3)^2 \cdot 0.1$$

$$\boxed{\sigma_X^2 = 0.81}$$

(B) FIND THE MEAN $\mu_{\bar{X}}$ & VARIANCE $\sigma_{\bar{X}}^2$ OF THE MEAN $\bar{X}$ FOR RANDOM VARIABLE / FOR RANDOM SAMPLE OF 36 CHERRY PUFFS.

$$\mu_{\bar{X}} = \mu_X = 5.3$$

$$\sigma_{\bar{X}}^2 = \frac{\sigma_X^2}{N} = \frac{0.81}{36} = 0.0225$$

$$\boxed{\sigma_{\bar{X}}^2 = 0.0225}$$

$$\boxed{\mu_{\bar{X}} = 5.3}$$

(c) FIND THE PROBABILITY THAT THE AVERAGE NUMBER OF CHERRIES IN 36 PUFFS WILL BE LESS THAN 5.5

$$z = \frac{\bar{x} - \mu_x}{\frac{\sigma_x}{\sqrt{n}}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

HAS A NORMAL DISTRIBUTION WITH THE MEAN 0 & THE STD. DEV. 1. THUS, WE GET

$$P(\bar{x} < 5.5) = P\left(\frac{\bar{x} - \mu_x}{\sigma_{\bar{x}}} < \frac{5.5 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$$

$$= P\left(z < \frac{5.5 - 5.3}{\sqrt{0.0225}}\right)$$

$$= P\left(z < \frac{5.5 - 5.3}{0.15}\right)$$

$$= P(z < 1.33)$$

$$\boxed{P(\bar{x} < 5.5) = 0.9082}$$

(8.26) THE AMOUNT OF TIME THAT A DRIVE-THROUGH BANK TELLER SPENDS ON A CUSTOMER IS A RANDOM VARIABLE WITH A MEAN $\mu = 3.2$ MINUTES & A STANDARD DEVIATION $\sigma = 1.6$ MINUTES

IF A RANDOM SAMPLE OF 64 CUSTOMERS IS OBSERVED, FIND THE PROBABILITY THAT THEIR MEAN TIME AT THE TELLER WINDOWS IS

(A) AT MOST 2.7 MINUTES

$N = 64$, $\mu_x = 3.2$, $\sigma_x = 1.6$

$\mu_{\bar{x}} = \mu_x = 3.2$

$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} = \frac{1.6}{\sqrt{64}} = 0.2$

$z = \frac{\bar{x} - \mu_x}{\left(\frac{\sigma_x}{\sqrt{N}}\right)} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$ WITH A NORMAL DISTRIBUTION WITH A MEAN OF 0 & STD OF 1

$P(\bar{x} \leq 2.7) = P\left(\frac{\bar{x} - \mu_x}{\sigma_{\bar{x}}} \leq \frac{2.7 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$

$= P\left(z \leq \frac{2.7 - 3.2}{0.2}\right)$

$= P(z \leq -1.5)$

$\boxed{P(\bar{x} \leq 2.7) = 0.0062}$

(B) MORE THAN 3.5 MINUTES

$$P(\bar{x} > 3.5) = P\left(\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} > \frac{3.5 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$$

$$= P\left(Z > \frac{3.5 - 3.2}{0.2}\right)$$

$$= P(Z > 1.5)$$

$$= 1 - 0.9332$$

$$\boxed{P(\bar{x} > 3.5) = 0.0668}$$

(C) AT MINUTES OF AT LEAST 3.2 MINUTES BUT LESS THAN

$$P(3.2 \leq \bar{x} < 3.4) = P\left(\frac{3.2 - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \leq \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} < \frac{3.4 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$$

$$= P\left(\frac{3.2 - 3.2}{0.2} \leq Z < \frac{3.4 - 3.2}{0.2}\right)$$

$$\cancel{P(Z \leq}$$

$$= P(0 \leq Z < 1)$$

$$= P(Z < 1) - P(Z < 0)$$

$$= 0.8413 - 0.5$$

$$\boxed{P(3.2 \leq \bar{x} < 3.4) = 0.3413}$$

(8.27) IMPURITY IS 0.2 GRAM PER GRAM OF OUTPUT

STANDARD DEVIATION IS 0.1 GRAM PER GRAM

$\mu = 0.2$, 50 TIMES SAMPLE AVG $\bar{x} = 0.23$

MAKE USE OF CENTORAL LIMIT THM

$\sigma_x = 0.1$ $\qquad \mu_{\bar{x}} = \mu_x = 0.2$

$\mu_x = 0.20$ $\qquad \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} = \frac{0.1}{\sqrt{50}} = 0.01414$

$\bar{x} = 0.23$

MEAN $= 0.2$ $\qquad z = \frac{\bar{x} - \mu_x}{\frac{\sigma_x}{\sqrt{N}}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$

$N = 50$

$$P(0.23 \leq \bar{x}) = P\left(\frac{0.23 - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \leq \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$$

$$= P\left(\frac{0.23 - 0.20}{0.01414} \leq z\right)$$

$$= P(2.122 \leq z)$$

$$= 1 - 0.9831$$

$$\boxed{P(0.23 \leq \bar{x}) = 0.0169}$$

SO IF THE POPULATION MEAN $\mu_x = 0.20$ THERE IS ONLY A 1.69% CHANCE THAT THE SAMPLE MEAN OF A SAMPLE LENGTH 50 WILL BE 0.23 OR LARGER

THUS THE MEAN AMOUNT OF IMPURITY ISNT 0.20

(8.29) X IS THE RANDOM VARIABLE WITH MEAN $\mu_X = 72$ & STANDARD DEV $\sigma_X = 10$ WHICH DESCRIBES THE POPULATION (OF TERRIERS) WHERE RANDOM SAMPLE OF SIZE $N_X = 64$

Y IS THE RANDOM VARIABLE WITH MEAN $\mu_Y = 28$ STANDARD DEV $\sigma_Y = 5$ WHICH DESCRIBES POODLES FROM SAMPLE OF $N_Y = 100$

$$\mu_{\bar{X}-\bar{Y}} = \mu_X - \mu_Y = 72 - 28 = 44$$

$$\sigma_{\bar{X}-\bar{Y}} = \sqrt{\frac{\sigma_X^2}{N_X} + \frac{\sigma_Y^2}{N_Y}} = \sqrt{\frac{100}{64} + \frac{25}{100}} = \sqrt{1.8125} = 1.346$$

$$Z = \frac{\bar{X} - \bar{Y} - \mu_{\bar{X}-\bar{Y}}}{\sigma_{\bar{X}-\bar{Y}}}$$

MEAN OF 0
STD OF 1

$$P(\bar{X} - \bar{Y} \leq 44.2) = P\left(\frac{\bar{X} - \bar{Y} - \mu_{\bar{X}-\bar{Y}}}{\sigma_{\bar{X}-\bar{Y}}} \leq \frac{44.2 - \mu_{\bar{X}-\bar{Y}}}{\sigma_{\bar{X}-\bar{Y}}}\right)$$

$$= P(Z \leq (44.2 - 44)/1.346)$$

$$= P(Z \leq 0.1486)$$

$$\boxed{P(\bar{X} - \bar{Y} \leq 44.2) = 0.5591}$$

(8.30) THE MEAN SCORE FOR FRESHMAN ON AN APTITUDE TEST AT A CERTAIN COLLEGE IS 540, WITH A STANDARD DEVIATION OF 50. ASSUME THE MEANS TO BE MEASURED TO ANY DEGREE OF ACCURACY. WHAT IS THE PROBABILITY THAT TWO GROUPS SELECTED AT RANDOM, CONSISTING OF 32 & 50 STUDENTS, RESPECTIVELY, WILL DIFFER IN THEIR MEAN SCORE BY,

LET X BE THE RANDOM VARIABLE WITH MEAN $\mu_X = 540$ STD $\sigma_X = 50$ WHICH DESCRIBES THE FIRST GROUP OF $N_X = 32$ STUDENTS FROM THE OBSERVED POPULATION

LET Y BE THE RANDOM VARIABLE WITH MEAN $\mu_Y = 540$ STD $\sigma_Y = 50$ WHICH DESCRIBES THE GROUP OF $N_Y = 50$ STUDENTS FROM THE OBSERVED POPULATION.

WITH A SAMPLING DISTRIBUTION OF $\bar{X} - \bar{Y}$ WILL BE APPROXIMATELY NORMAL & WILL HAVE THE FOLLOWING MEAN & STANDARD DEVIATION:

$$\mu_{\bar{X}-\bar{Y}} = \mu_X - \mu_Y = 540 - 540 = 0$$

$$\sigma_{\bar{X}-\bar{Y}} = \sqrt{\frac{\sigma_X^2}{N_X} + \frac{\sigma_Y^2}{N_Y}} = \sqrt{\frac{2500}{32} + \frac{2500}{50}} = \sqrt{128.125} = 11.319$$

$$Z = \left(\frac{\bar{X} - \bar{Y} - \mu_{\bar{X}-\bar{Y}}}{\sigma_{\bar{X}-\bar{Y}}} \right)$$

$$P(20 < |\bar{X} - \bar{Y}|) = P(\bar{X} - \bar{Y} < -20) + P(20 < \bar{X} - \bar{Y})$$

$$= P\left(\frac{\bar{X} - \bar{Y} - \mu_{\bar{X}-\bar{Y}}}{\sigma_{\bar{X}-\bar{Y}}} < \frac{-20 - \mu_{\bar{X}-\bar{Y}}}{\sigma_{\bar{X}-\bar{Y}}} \right)$$

$$+ P\left(\frac{20 - \mu_{\bar{X}-\bar{Y}}}{\sigma_{\bar{X}-\bar{Y}}} < \frac{\bar{X} - \bar{Y} - \mu_{\bar{X}-\bar{Y}}}{\sigma_{\bar{X}-\bar{Y}}} \right)$$

$$= P\left(Z < \frac{-20-0}{11.319} \right) + P\left(\frac{20-0}{11.319} < Z \right)$$

$$= P(Z < -1.767) + P(1.767 < Z)$$

$$= P(Z < -1.767) + P(1.767 < Z)$$

$$= 0.0386 + 1 - 0.9614 = 0.0772$$

$$\boxed{P(20 < |\bar{X} - \bar{Y}|) = 0.07721}$$

(B) AMOUNT BETWEEN 5 & 10 POINTS?

$$P(5 < |\bar{x} - \bar{y}| < 10) = 2 \cdot P(5 < \bar{x} - \bar{y} < 10)$$

$$= 2P\left(\frac{5 - \mu_{\bar{x}-\bar{y}}}{\sigma_{\bar{x}-\bar{y}}} < \frac{\bar{x} - \bar{y} - \mu_{\bar{x}-\bar{y}}}{\sigma_{\bar{x}-\bar{y}}} < \frac{10 - \mu_{\bar{x}-\bar{y}}}{\sigma_{\bar{x}-\bar{y}}}\right)$$

$$= 2P\left(\frac{5-0}{11.314} < Z < \frac{10-0}{11.319}\right)$$

$$= 2P\left(\frac{5}{11.319} < Z < \frac{10}{11.319}\right)$$

$$= 2(P(Z < 0.8835) - P(Z \leq 0.4417))$$

$$= 2(0.8115 - 0.6706)$$

$$= 2 \cdot 0.1409$$

$$\boxed{P(5 < |\bar{x} - \bar{y}| < 10) = 0.2818}$$

(8.48) 30 HOURS, 16 BATTERIES ARE TESTED EACH MONTH

t-VALUES FALLS BETWEEN $-t_{0.025}$ & $t_{0.025}$

WHAT IS THE CONCLUSION SHOULD THE FIRM DRAW FROM A SAMPLE THAT HAS A MEAN OF $\bar{X} = 27.5$ HRS & STANDARD DEVIATION OF $S = 5$ HOURS?

$\mu = 30$, $N = 16$, FOR NORMAL DISTRIBUTIONS, THEN THE RV

$$T = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{N}}}$$ HAS t-DISTRIBUTION WITH $v = N - 1 = 15$ °OF FREEDOM

$\bar{X} = 27.5$ & $S = 5$

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{N}}} = \frac{27.5 - 30}{\frac{5}{\sqrt{16}}} = \frac{-2.5 \cdot 4}{5} = -2$$

$t_{0.025} = 2.131 \Rightarrow -t_{0.025} = -2.131$

THEREFORE, THE COMPUTED t-VALUE INDEED FINDS ITSELF BETWEEN $t_{0.025}$ & $-t_{0.025}$ SO IT'S GOOD FOR A AVG BATTERY LIFE 030 HOURS.

(8.49) A NORMAL POP W/ UNKNONN VARIANCE HAS A MEAN OF 20. IS ONE LIKELY TO OBTAIN A RANDOM ~~VARIABLE~~ SAMPLE OF SIZE 9 FROM THIS POPULATION WITH A MEAN OF 24 & A STANDARD DEV OF 4.1 ?

$N = 9$, $\bar{x} = 24$, $S = 4.1$ FROM NORMAL DISTRIBUTION WITH A MEAN $\mu = 20$

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{N}}} = \frac{24 - 20}{\left(\frac{4.1}{\sqrt{9}}\right)} = \frac{4.3}{4.1} = 2.927$$

$N = 9 \Rightarrow T = \dfrac{\bar{x} - \mu}{\frac{S}{\sqrt{N}}}$ HAS A t-DISTRI WITH $\nu = N - 1 = 8°$ OF FREEDOM

$t_{0.01} = 2.896 < 2.927 \Rightarrow P(T > 2.927) < 0.01$

A SAMPLE UNDER THE ASSUMPTION $\mu = 20$ IS UNLIKELY & REASONABLE CONCLUSION BASED ON THE SAMPLE IS $\mu > 20$

(8.72) GIVEN A NORMAL RANDOM VARIABLE X WITH MEAN 20 & VARIANCE 9, & A RANDOM ~~VARIABLE~~ SAMPLE OF SIZE N TAKEN FROM THE DISTRIBUTION, WHAT SAMPLE SIZE N ~~TAKEN FROM~~ IS NECESSARY IN ORDER THAT

$$P(19.9 \leq \bar{x} \leq 20.1) = 0.95$$

$$\mu_{\bar{x}} = \mu_x = 2 \qquad Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} = \frac{3}{\sqrt{N}}$$

$$0.95 = P(19.9 \leq \bar{x} \leq 20.1)$$

$$= P\left(\frac{19.9 - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \leq \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \leq \frac{20.1 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$$

$$= P\left(\frac{19.9 - 20}{3/\sqrt{N}} \leq Z \leq \frac{20.1 - 20}{(3/\sqrt{N})}\right)$$

$$= P\left(-\frac{\sqrt{N}}{30} \leq Z \leq \frac{\sqrt{N}}{30}\right)$$

$$= P\left(Z \leq \frac{\sqrt{N}}{30}\right) - P\left(Z \leq -\frac{\sqrt{N}}{30}\right)$$

$$= P\left(Z \leq \frac{\sqrt{N}}{30}\right) - \left(1 - P\left(Z \leq \frac{\sqrt{N}}{30}\right)\right)$$

$$= 2P\left(Z \leq \frac{\sqrt{N}}{30}\right) - 1$$

$$2P\left(Z \leq \frac{\sqrt{N}}{30}\right) = 1.95 \Rightarrow P\left(Z \leq \frac{\sqrt{N}}{30}\right) = 0.975 \Rightarrow \frac{\sqrt{N}}{30} = 1.96$$

$$\therefore \boxed{N = 3457}$$