2.95 in a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. if the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0.06, what is the probability that an adult over 40 years of age is diagnosed as having cancer?

C DENOTES THE EVENT THAT A PERSON HAS CANCER
C' DENOTES THE EVENT THAT A PERSON DOES NOT HAVE CANCER
D DENOTE THE EVENT THAT A PERSON HAS BEEN DIAGNOSED WITH CANCER

$P(C) = 0.05$ HAS CANCER
$P(C') = 1 - P(C) = 0.95$ NO CANCER
$P(D \mid C) = 0.78$ HAS CANCER DIAGNOSED CORRECTLY
$P(D \mid C') = 0.06$ NO CANCER DIAGNOSED INCORRECTLY
$P(D) = ?$ DIAGNOSED WITH CANCER

ACCORDING TO THEOREM OF TOTAL PROBABILITY

$P(D) = P(D \mid C) \cdot P(C) + P(D \mid C') \cdot P(C')$
$= 0.78 \cdot 0.05 + 0.06 \cdot 0.95$
$= 0.039 + 0.057$

$\boxed{P(D) = 0.096}$

2.115 a certain federal agency employs three consulting firms (A, B, and C) with probabilities 0.40, 0.35, and 0.25, respectively. from past experience it is known that the probability of cost overruns for the firms are 0.05, 0.03, and 0.15, respectively. suppose a cost overrun is experienced by the agency.

a. what is the probability that the consulting firm involved is company C?

b. what is the probability that it is company A?

$P(A) = 0.40$ COMPANY GETS CHOSEN
$P(B) = 0.35$ COMPANY GETS CHOSEN
$P(C) = 0.25$ COMPANY GETS CHOSEN
$P(L \mid A) = 0.05$ COMPANY A EXPERIENCES COST OVERRUNS
$P(L \mid B) = 0.03$ COMPANY B EXPERIENCES COST OVERRUNS
$P(L \mid C) = 0.15$ COMPANY C EXPERIENCES COST OVERRUNS

$P(C \mid L) = ?$
$P(A \mid L) = ?$

USING BAYES THEOREM

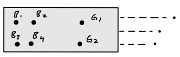
$$P(C \mid L) = \frac{P(C)\,P(L \mid C)}{P(C)\,P(L \mid C) + P(B)\,P(L \mid B) + P(A)\,P(L \mid A)} = \frac{(0.40)(0.05)}{(0.40)(0.05) + (0.35)(0.03) + (0.25)(0.15)} = 0.2941$$

$P(C \mid L) = 2.941$

$$P(A \mid L) = \frac{P(A)\,P(L \mid A)}{P(A)\,P(L \mid A) + P(B)\,P(L \mid B) + P(C)\,P(L \mid C)} = \frac{(0.40)(0.05)}{(0.40)(0.05) + (0.35)(0.03) + (0.25)(0.15)} = 0.294$$

$P(A \mid L) = 0.294$

3.26 from a box containing 4 black balls and 2 green balls, 3 balls are drawn in succession, each ball being replaced in the box before the next draw is made. Find the probability distribution for the number of green balls.



IF X REPRESENTS A RANDOM VARIABLE THAT REPRESENTS THE NUMBER OF GREEN BALLS DRAWN IN 3 ATTEMPTS

SAMPLING WITH REPLACEMENT

$N = 6$ X $G_1\ G_2\ B_1$
GREEN DRAW $= 2/6$ $G_1\ G_2\ B_2$
BLUE DRAW $= 4/6$ $G_1\ G_2\ B_3$
$G_N = 2$
$B_N = 4$

IN ORDER TO OBTAIN THE PROBABKITY DISTRIBUTION FOR RV X

$$P(X = x) = \left(\frac{2}{6}\right)^x \left(\frac{4}{6}\right)^{3-x} = \frac{2^{3-x}}{27}$$

$$\boxed{P(X = x) = \frac{2^{3-x}}{27}}$$

5.16 suppose that airplane engines operate independently and fail with probability equal to 0.4. assuming that a plane makes a safe flight if at least one-half of its engines run, determine whether a 4-engine plane or a 2- engine plane has the higher prob for a successful flight.

PROBABILITY 0.4 OF FAILED ENGINES FOR BOTH 4 & 2
PLANE SUCCEEDS IF 1/2 OF ENGINES RUN
OBJECTIVE IS TO FIND WHETHER A 4 ENGINE OR 2 ENGINE PLANE SUCCEEDS
FAIL $p' = 0.4$
SUCCESS $p = 1 - p' = 0.6$
NUMBER OF ENGINES IS N
X RANDOM VARIABLE REPRESENTS THAT ENGINE RUNS PROPERLY
X BINOMIAL RV WITH $N = 4$, $p = 0.6$
IF MORE THAN HALF OF THE ENGINES SUCCEED
$P(X \geq N/2) = P(X \geq 2)$
4 ENGINE PLANE $N = 4$, $p = 0.6$, $q = 0.4$

$$P(X = x) = b(x; N, p) = \binom{N}{x} p^x q^{N-x} = \binom{N}{x} p^x (1-p)^{N-x}$$

$$P(X = x) = b(x; N, p) = \binom{N}{x} p^x (q)^{N-x} = \binom{4}{x} 0.6^x (0.4)^{4-x}, \quad x = 0, 1, 2, 3, 4$$

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1)$$
$$= 1 - \sum_{i=0}^{1} P(X = i) = 1 - \sum_{i=0}^{1} b(i; 4, 0.6)$$

$$P(X \geq 2) = 1 - \sum_{i=0}^{1} b(i; 4, 0.6) = 1 - 0.1792 = 0.8208$$

$P(X \geq 2) = 0.8208$ FOR 4 ENGINE PLANE

FOR A 2 PLANE ENGINE, $N = 2$ $p = 0.6$, $q = 0.4$

$$P(X = x) = b(x; 2, 0.6) = \binom{N}{x} p^x q^{N-x} = \binom{2}{x} 0.6^x\, 0.4^{2-x}$$

$$P(X \geq 1) = 1 - P(X > 1) = 1 - P(X = 0)$$
$$= 1 - b(x; 2, 0.6) = 1 - \binom{2}{0} 0.6^0\, 0.4^{2-0} = 1 - 0.16 = 0.84$$

$P(X \geq 1) = 0.84$ FOR 2 ENGINE PLANE

$\boxed{P(X \geq 1) = 0.84 > P(X \geq 2) = 0.8208}$
THUS 2 ENGINE PLANE HAS HIGHER PROB

5.87 imperfections in computer circuit boards and computer chips lend themselves to statistical treatment. for a particular type of board, the probability of a diode failure is 0.03 and the board contains 200 diodes.

a. what is the mean number of failures among the diodes?

b. what is the variance?

c. the board will work if there are no defective diodes. what is the probability that a board will work?

P OF FAILURE 0.03 DIODE IN BOARD
BOARD CONTAINS $N = 200$ DIODES
$p = 0.03$, $q = 1 - 0.03 = 0.97$, $N = 200$
TRIALS ARE INDEPENDENT

MEAN NUMBER OF FAILURES
$\boxed{\mu = E(x) = Np = 200 \cdot 0.03 = 6}$

VARIANCE
$\boxed{\sigma^2 = Np(1-p) = 200(0.03)(q) = 200 \cdot 0.03(0.97) = 5.82}$

PROBABILITY THERE ARE $x = 0$ DEFECTS

$$P(X = 0) = p(x; \mu) = e^{-\mu} \frac{\mu^x}{x!}, \quad x = 0, 1, 2, 3, \ldots$$

$$\boxed{P(X = 0) = \frac{e^{-6}(6)^0}{0!} = 0.00248}$$

3.56 the joint density function of the random variables X and Y is

f(x, y) = { 6x | 0 < x < 1, 0 < y < 1 - x; 0 | elsewhere }

a. Show that X and Y are not independent

b. Find P(X > 0.3 | Y = 0.5)

PDF JOINT FOR RV X & Y

$$f(x, y) = \begin{cases} 6x, & 0 < x < 1,\ 0 < y < 1-x \\ 0, & \text{ELSEWHERE} \end{cases}$$

THE RANDOM VARIABLES X & Y WILL BE INDEPENDENT IFF THE EQUALITY HOLDS TRUE FOR ALL VALUES x & y WHICH ARE THE VARIABLES WE ASSUME

$f(x, y) = g(x)\, h(y)$

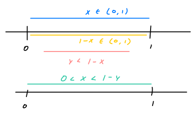
MARGINAL DENSITY FUNCTION $g(x)$ IS,

$$g(x) = \int_y f(x, y)\, dy = \int_0^{1-x} 6x\, dy = 6xy \Big|_0^{1-x} = 6x(1-x) - 6x(0) = 6x - 6x^2 \quad \text{FOR } 0 < x < 1$$

MARGINAL DENSITY FUNCTION $h(x)$ IS,

$$h(y) = \int_x f(x, y) = \int_0^{1-y} 6x\, dx = \frac{6x^2}{2} \Big|_0^{1-y} = 3x^2 \Big|_0^{1-y} = 3(1 - 2y + y^2) = 3 - 6y + 3y^2$$

$6x$, $0 < x < 1 \Rightarrow \int_x = \int_0^{1-y}$ $\quad y \in \langle 0, 1 \rangle$ BECAUSE $y < 1-x$
$0 < y < 1-x \Rightarrow \int_y = \int_0^{1-x}$ $\quad x < 1-y \Rightarrow 0 < x < 1-y$



$f(x, y) = g(x)\, h(y) = (6x - 6x^2)(3 - 6y + 3y^2)$
$6x \neq 12x - 36xy + 18xy - 18x^2 + 36x^2y - 18y^2x^2$
$6x \neq -18y^2x^2 - 18xy + 36x^2y + 12x$

$\boxed{\text{THEREFORE } X \text{ \& } Y \text{ ARE NOT INDEPENDENT}}$

FIND $P(X > 0.3 \mid y = 0.5)$

$P(x > 0.3 \mid y = 0.5) = P(0.3 < x < 1-y \mid y = 0.5)$
$= P(0.3 < x < 1 - 0.5)$
$= P(0.3 < x < 0.5 \mid y = 0.5)$

$$P(0.3 < x < 0.5 \mid y = 0.5) = \int_x f(x \mid 0.5)\, dx$$
$$= \int_{0.3}^{0.5} \frac{f(x, 0.5)}{h(0.5)}\, dx$$
$$= \int_{0.3}^{0.5} \frac{6x}{3(1 - 0.5)^2}\, dx$$
$$= \int_{0.3}^{0.5} 8x\, dx = \frac{8x^2}{2} \Big|_{0.3}^{0.5}$$
$$= 4(0.5)^2 - 4(0.3)^2$$
$$= 0.64$$

$\boxed{\therefore P(x > 0.3 \mid y = 0.5) = 0.64}$

5.71 for a certain type of copper wire, it is known that, on the average, 1.5 flaws occur per millimeter. assuming that the number of flaws is a Poisson random variable, what is the probability that no flaws occur in a certain portion of wire of length 5 millimeters?
what is the mean number of flaws in a portion of length 5 millimeters?

X REPRESENTS THE # OF FLAWS IN 5 mm WIRE
$\lambda = 1.5$ REPRESENTS AVERAGE FLAW PER 1 mm
$t = 5$ REPRESENTS INTERVAL OF LENGTH IS 5 mm
X HAS A POISSON DISTRO
$\lambda t = 7.5$ REPRESENTS POISSON PARAMETER
PROBABILITY MASS FUNCTION
$P(X = x) = p(x; \lambda t)$ WHERE $x = 0, 1, 2, \ldots$
$$P(X = x) = p(x; \lambda t) = e^{-\lambda t} \frac{(\lambda t)^x}{x!}$$

NO FLAWS OCCUR $P(X = 0)$
$$\boxed{P(X = 0) = p(0; \lambda t) = p(0; 7.5) = e^{-7.5} \frac{(7.5)^0}{0!} = e^{-7.5} = 0.000553}$$

MEAN NUMBER OF FLAWS $t = 5$
$\boxed{\mu = E(x) = \lambda t = 5 \cdot 1.5 = 7.5}$

6.66 a certain type of device has an advertised failure rate of 0.01 per hour. the failure rate is constant and the exponential distribution applies. what is the mean time to failure? what is the probability that 200 hours will pass before a failure is observed?

FAILURE RATE $\lambda = 0.01$
LET RANDOM VARIABLE X REPRESENT THE FAILURE RATE OF A CERTAIN DEVICE
X IS AN EXPONENTIAL DISTRIBUTION WITH PARAMETER $\lambda = 0.01$

MEAN TIME TO FAILURE
$\mu = 1/\lambda = 1/0.01 = 100$

PROBABILITY OF FAILURE AFTER 200 HRS
$P(X \geq 200) = 1 - P(X \leq 200) = 1 - (1 - e^{-0.01 \cdot 200}) = 0.1353$
$P(X \geq 200) = 0.1353$