9.4 the heights of a random sample of 50 college students showed a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters.

(a) construct a 98% confidence interval for the mean height of all college students.

(b) what can we assert with 98% confidence about the possible size of our error if we estimate the mean height of all college students to be 174.5 centimeters?

$N = 50$, $\bar{x} = 174.5$, $\sigma = 6.9$

IN ORDER TO FIND THE 98% CONFIDENCE INTERVAL FOR THE MEAN

WE NEED TO FIND $98\% = (1 - \alpha) \cdot 100\%$

FROM $98\% = 100(1 - \alpha)\%$

$0.98 = 1 - \alpha$

$\alpha = 1 - 0.98$

$\Rightarrow \alpha = 0.02$

BY USING THE FOLLOWING THEOREM

IF $\bar{x}$ IS THE MEAN OF A RANDOM SAMPLE OF SIZE N FROM A POPULATION WITH KNOWN VARIANCE σ^2,

A $100(1-\alpha)\%$ CONFIDENCE INTERVAL FOR THE MEAN IS GIVEN BY,

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{N}} < \mu < \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{N}}$$

WHERE $Z_{\alpha/2}$ IS THE Z-VALUE LEAVING AN AREA OF $\alpha/2$ TO THE RIGHT

IN THIS CASE $\alpha/2 = 0.02/2 = 0.01$

$\sigma = 6.9$ | $N = 50$ | $\bar{x} = 174.5$ | $\alpha = 0.02$

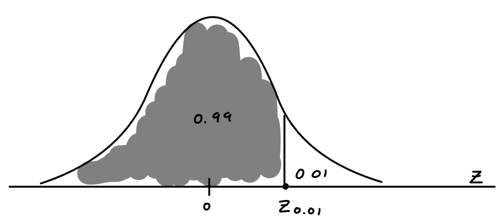
THE $Z_{0.01}$ IS THE Z VALUE LEAVING AN AREA OF 0.01 TO THE RIGHT

AREA $= 1 - 0.01 = 0.99$ TO THE LEFT

$174 - Z_{0.01} \cdot 6.9/\sqrt{50}\ \mu < 174.5 + Z_{0.01} \cdot 6.9/\sqrt{50}$ BY USING THE NORMAL PROBABILITY TABLE WE SEE $Z_{0.01} = 2.33$

$174 - 2.33 \cdot 6.9/\sqrt{50} < \mu < 174.5 + 2.33 \cdot 6.9/\sqrt{50}$

$\boxed{172 < \mu < 176.8}$



IF $\bar{x}$ IS USED TO ESTIMATE μ, WE CAN BE $100(1-\alpha)\%$ CONFIDENT THAT THE ERROR WILL NO EXCEED $Z_{\alpha/2}\ \sigma/\sqrt{N}$

IF $\bar{x} = 174.5$ IS USED AS AN ESTIMATE OF MEAN, WE CAN BE 98% CONFIDENT THAT THE ERROR WILL NOT EXCEED,

$Z_{0.01} \cdot 6.9/\sqrt{50} = 2.33 \cdot 0.976 = 2.274$

THUS, WE CAN ASSERT WITH 98% CONFIDENCE THAT THE POSSIBLE SIZE OF OUR ERROR WILL NOT EXCEED 2.274 cm

9.10 a random sample of 12 graduates of a certain secretarial school typed an average of 79.3 words per minute with a standard deviation of 7.8 words per minute. Assuming a normal distribution for the number of words typed per minute, find a 95% confidence interval for the average number of words typed by all graduates of this school.

12 GRADUATES, 79.3 AVERAGE, $\sigma = 7.8$, 95% CONFIDENCE INTERVAL

$N = 12$ $\quad$ $98\% = 100(1-\alpha)\%$

$\bar{x} = 79.3$ $\quad$ $0.98 = 1 - \alpha$

$s = 7.8$ $\quad$ $\alpha = 0.05$

$\alpha = 0.05$

IF $\bar{x}$ & s ARE THE MEAN & STANDARD DEVIATION OF A RANDOM SAMPLE FROM A NORMAL POPULATION WITH UNKNOWN VARIANCE σ^2, A $100(1-\alpha)\%$ CONFIDENCE INTERVAL FOR THE MEAN IS WHERE $t_{\alpha/2}$ IS THE t-VALUE WITH $v = N-1$ DEGREES OF FREEDOM, LEAVING AN AREA OF $\alpha/2$ TO THE RIGHT

$$\bar{x} - t_{\alpha/2}(s/\sqrt{N}) < \mu < \bar{x} + t_{\alpha/2}(s/\sqrt{N})$$

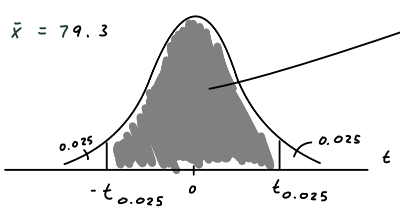
$$79.3 - t_{0.025}(7.8/\sqrt{12}) < \mu < 79.3 + t_{0.025}(7.8/\sqrt{12})$$

$\alpha/2 = 0.025$ $\quad$ THE $t_{0.025}$ IS THE T-VALUE LEAVING AN AREA OF 0.025 TO THE RIGHT

$s = 7.8$

$N = 12$ $\quad$ $1-(0.025+0.025) = 0.95$

$\bar{x} = 79.3$



USING THE TABLE OF CRITICAL VALUES OF THE t-DISTRIBUTION

$t_{0.025} = 2.201$ WITH $v = 12 - 1 = 11$ DEGREES OF FREEDOM

$$\bar{x} - t_{\alpha/2}(s/\sqrt{N}) < \mu < \bar{x} + t_{\alpha/2}(s/\sqrt{N})$$

$$79.3 - t_{0.025}(7.8/\sqrt{12}) < \mu < 79.3 + t_{0.025}(7.8/\sqrt{12})$$

$$79.3 - 2.201(7.8/\sqrt{12}) < \mu < 79.3 + 2.201(7.8/\sqrt{12})$$

THE REQUIRED 95% CONFIDENCE INTERVAL IS $\Longrightarrow$ $74.34 < \mu < 84.26$

9.14 the following measurements were recorded for the drying time, in hours, of a certain brand of latex paint:

3.4	2.5	4.8	2.9	3.6
2.8	3.3	5.6	3.7	2.8
4.4	4.0	5.2	3.0	4.8

assuming that the measurements represent a random sample from a normal population, find a 95% prediction interval for the drying time for the next trial of the paint.

$N = 15$, IN ORDER TO FIND THE SAMPLE MEAN $\bar{x}$, STANDARD DEVIATION s

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i = (3.4 + 2.5 + 4.8 + 3.6 + \ldots + 4.8)/15 = 3.8$$

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2 = (3.4 - 3.8)^2 + (2.5 - 3.8)^2 + (4.8 - 3.8)^2 + \ldots + (4.8 - 3.8)^2 = 0.9427$$

$$s = \sqrt{s^2} = \sqrt{0.94} \approx 0.97$$

THEREFORE WE NEED TO FIND A 95% PREDICTION INTERVAL FOR THE NEXT TRIAL OF THE PAINT.

$$95\% = 100(1-\alpha)\% \Rightarrow 1 - \alpha = 0.95 \Rightarrow \alpha = 0.5$$

FROM A NORMAL DISTRIBUTION OF MEASUREMENTS WITH UNKNOWN MEAN & UNKNOWN VARIANCE σ^2

A $100(1-\alpha)\%$ PREDICTION INTERVAL OF A FUTURE OBSERVATION x_0 IS,

$$\bar{x} - t_{\alpha/2}\, s\sqrt{1 + 1/n} < x_0 < \bar{x} - t_{\alpha/2}\, s\sqrt{1+1/n}$$

WHERE $t_{\alpha/2}$ IS THE T-VALUE WITH $v = n-1$ DEGREES OF FREEDOM TO THE RIGHT

$\alpha/2 = 0.05/2 = 0.025$

$s = 0.97$

$n = 15$

$\bar{x} = 3.8$

$$3.8 - t_{0.025} \cdot 0.97\sqrt{1 + 1/15} < x_0 < t_{0.025} \cdot 0.97\sqrt{1+1/15}$$

$$3.8 - 2.145 \cdot 0.97\sqrt{1+1/15} < x_0 < 2.145 \cdot 0.97\sqrt{1+1/15}$$

$$1.65 < x_0 < 5.95$$

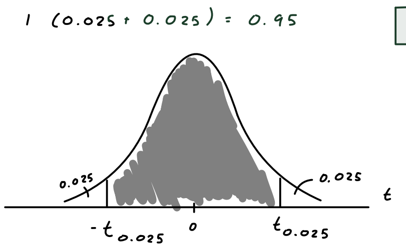
$\therefore$ 95% CONFIDENCE INTERVAL $\boxed{1.65 < x_0 < 5.95}$

$t_{0.025}$ STILL MUST BE FOUND

$t_{\alpha/2} = t_{0.025}$ IS THE T VALUE LEAVING AN AREA 0.025 TO THE RIGHT.

$1\ (0.025 + 0.025) = 0.95$

$\boxed{t_{0.025} = 2.145}$

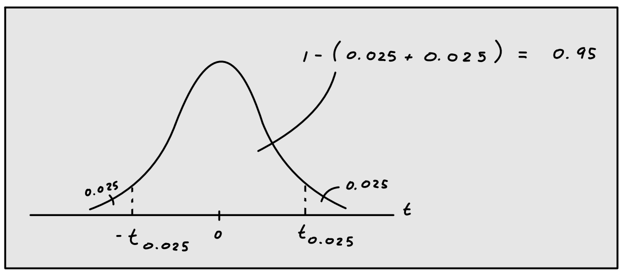


9.16 consider exercise 9.10. compute the 95% prediction interval for the next observed number of words per minute typed by a graduate of the secretarial school.

$N = 20$ SINGLE SERVING ALPHA BITS

$\bar{x} = 11.3$ AVERAGE SUGAR CONTENT

$s = 2.45$ GRAMS



$95\% = 100(1-\alpha)\%$

$1 - \alpha = 0.95$

$\alpha = 0.05$

FOR A NORMAL DISTRIBUTION OF MEASUREMENTS WITH UNKNOWN MEAN AND UNKNOWN VARIANCE σ^2, A $100(1-\alpha)\%$ PREDICTION INTERVAL OF A FUTURE OBSERVATION X_0 IS, WHERE $t_{\alpha/2}$ IS THE T VALUE WITH $N=1$ DEGREES OF FREEDOM, LEAVING AN AREA OF $\alpha/2$ TO THE RIGHT.

$\alpha/2 = 0.05/2 = 0.025$

$s = 2.45$

$N = 20$

$\bar{x} = 11.3$

$$\bar{x} - t_{\alpha/2}\, s\sqrt{1+1/N} < x_0 < \bar{x} + t_{\alpha/2}\, s\sqrt{1+1/N}$$

$$11.3 - t_{0.025} \cdot 2.45\sqrt{1+1/20} < X_0 \; 11.3 + t_{0.025} \cdot 2.45\sqrt{1+1/20}$$

IT REMAINS TO FIND $t_{0.025}$

$t_{\alpha/2} = t_{0.025}$ IS THE T VALUE LEAVING AN AREA 0.025 TO THE RIGHT

TABLE OF CRITICAL VALUES OF THE T-DISTRIBUTION, THE T VALUE WITH $v = 20 - 1 = 19$ DEG OF FREEDOM

9.17 consider exercise 9.9. Compute a 95% prediction interval for the sugar content of the next single serving of Alpha-Bits.

$n = 20$, $\bar{x} = 11.3$ GRAMS, $s = 2.45$ GRAMS

$95\% = 100(1-\alpha)\% \Rightarrow \alpha = 1 - 0.95 \Rightarrow \alpha = 0.05$

FOR A NORMAL DISTRIBUTION OF MEASUREMENTS, A $100(1-\alpha)\%$ PREDICTION INTERVAL OF A FUTURE OBSERVATION x_0 IS

$t_{\alpha/2}$ IS THE T VALUE WITH $v = n - 1$ DEGREES OF FREEDOM, AREA $= \alpha/2$ TO THE RIGHT

$\alpha/2 = 0.05/2 = 0.025$

$s = 2.45$

$n = 20$

$\bar{x} = 11.3$

$$\bar{x} - t_{\alpha/2}\, s\sqrt{1 + 1/n} < x_0 < \bar{x} + t_{\alpha/2}\, s\sqrt{1 + 1/n}$$

$$11.3 - t_{0.025} \cdot 2.45\sqrt{1 + 1/20} < x_0 < 11.3 + t_{0.025} \cdot 2.45\sqrt{1 + 1/20}$$

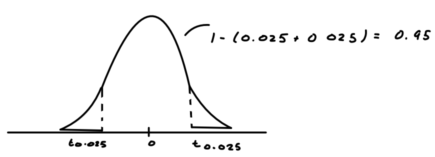
$$11.3 - 2.093 \cdot 2.45\sqrt{1 + 1/20} < x_0 < 11.3 + 2.093 \cdot 2.45$$

$$\boxed{6.05 < x_0 < 16.55}$$

$v = 20 - 1 = 19$ DEGREES OF FREEDOM

AREA OF 0.025 TO THE RIGHT

IS $t_{0.025} = 2.093$



9. 9 regular consumption of presweetened cereals contributes to tooth decay, heart disease, and other degenerative diseases, according to studies conducted by Dr. W. H. Bowen of the National Institute of Health and Dr. J. Yudben, Professor of Nutrition and Dietetics at the University of London. In a random sample consisting of 20 similar single servings of Alpha-Bits, the average sugar content was 11.3 grams with a standard deviation of 2.45 grams. Assuming that the sugar contents are normally distributed, construct a 95% confidence interval for the mean sugar content
for single servings of Alpha-Bits.

WE ARE GIVEN $N = 20$, $s = 2.45$, $\bar{x} = 11.3$

$95\% = 100(1-\alpha)\% \Rightarrow \alpha = 0.05$

$$\bar{x} - t_{\alpha/2}\, s/\sqrt{N} < \mu < \bar{x} + t_{\alpha/2}\, s/\sqrt{N}$$

IN THIS CASE WE HAVE, $\alpha/2 = 0.05/2 = 0.025$

$S = 45$, $N = 20$, $\bar{x} = 11.3$

$$11.3 - t_{0.025}\left(45/\sqrt{20}\right) < \mu < 11.3 + t_{0.025}\left(2.45/\sqrt{20}\right)$$

ACCORDING TO THE TABLE OF CRITICAL VALUES OF THE T-DISTRIBUTION

$t_{0.025} = 2.093$, $\nu = 20 - 1 = 19$

$$11.3 - 2.093\left(45/\sqrt{20}\right) < \mu < 11.3 + 2.093\left(2.45/\sqrt{20}\right)$$

$$10.15 < \mu < 12.45$$

$\therefore$ THE REQUIRED CONFIDENC INTERVAL IS $10.15 < \mu < 12.45$

9.38 two catalysts in a batch chemical process, are being compared for their effect on the output of the process reaction. A sample of 12 batches was prepared using catalyst 1, and a sample of 10 batches was prepared using catalyst 2. The 12 batches for which catalyst 1 was used in the reaction gave an average yield of 85 with a sample standard deviation of 4, and the 10 batches for which catalyst 2 was used gave an average yield of 81 and a sample standard deviation of 5. Find a 90% confidence interval for the difference between the population means, assuming that the populations are approximately normally distributed with equal variances.

FIRST RANDOM SAMPLE

- RANDOM SAMPLE SIZE OF $N_1 = 12$ NORMAL POPULATION
- STANDARD DEVIATION $S_1 = 4$
- MEAN OF $\bar{X}_1 = 85$

SECOND RANDOM SAMPLE

- RANDOM SAMPLE SIZE OF $N_2 = 10$ NORMAL POPULATION
- STANDARD DEVIATION $S_2 = 5$
- MEAN OF $\bar{X}_2 = 81$

WE WANT TO FIND A 90% CONFIDENCE INTERVAL FOR $\mu_1 - \mu_2$

$90\% = 100(1-\alpha)\% \Rightarrow \alpha = 0.01$

THEOREM FOR CONFIDENCE INTERVAL FOR $\mu_1 - \mu_2$, UNKNOWN VARIANCE σ_1 & σ_2

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}\, S_p \sqrt{\frac{1}{N_1} + \frac{1}{N_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}\, S_p \sqrt{\frac{1}{N_1} + \frac{1}{N_2}}$$

WHERE S_p IS THE POOLED ESTIMATE OF THE POPULATION DEVIATION

IN OUR CASE WE HAVE THE FOLLOWING $\alpha/2 = 0.01/2 = 0.05$

$S_1^2 = 4$ $\quad S_2^2 = 5$

$\bar{x}_1 = 85$ $\quad \bar{X}_2 = 81$

$N_1 = 12$ $\quad N_2 = 10$

NOW THE POOLED ESTIMATE OF THE POPULATION STANDARD DEVIATION

$$S_p^2 = \frac{(N_1 - 1) S_p^2 \, (N_2 - 1) S_2^2}{N_1 + N_2 - 2} = \frac{(11 \cdot 6) + (9 \cdot 25)}{20} = 20.05$$

$$S_p = \sqrt{S_p^2} = \sqrt{20.05} = 4.48$$

$$\left((85 - 81) \pm t_{0.05} \cdot 4.48 \sqrt{\frac{1}{12} + \frac{1}{10}} \right)$$

$$4 - t_{0.05} \cdot 4.48 \sqrt{1/12 + 1/10} < \mu_1 - \mu_2 < 4 + t_{0.05} \cdot 4.48 \sqrt{1/12 + 1/10}$$

THE $t_{\alpha/2} = t_{0.05} = 1.725$, WITH $V = N_1 + N_2 - 2 = 20$ DEGREES OF FREEDOM

$$4 - 1.725 \cdot 4.48 \sqrt{1/10 + 1/12} < \mu_1 - \mu_2 < 4 + 1.725 \cdot 4.48 \sqrt{1/10 + 1/12}$$

$$4 - 3.307 < \mu_1 - \mu_2 < 4 + 3.307$$

$$\boxed{0.692 < \mu_1 - \mu_2 < 7.307}$$

9.43 a taxi company is trying to decide whether to purchase brand A or brand B tires for its fleet of taxis. to estimate the difference in the two brands, an experiment is conducted using 12 of each brand. the tires are run until they wear out. the results are

brand a: $\bar{x}_1 = 36{,}300$ kilometer, $s_1 = 5000$ kilometers
brand b: $\bar{x}_2 = 38{,}100$ kilometers, $s_2 = 6100$ kilometers.

compute a 95% confidence interval for $\mu A - \mu B$ assuming the populations to be approximately normally distributed. you may not assume that the variances are equal.

$N_1 = 12$ $\qquad 95\% = 100(1-\alpha)\%$

$S_1 = 5000$ $\qquad \Rightarrow \alpha = 0.05$

$\bar{X}_1 = 36300$ $\qquad \Rightarrow \alpha/2 = 0.025$

$N_2 = 12$

$S_2 = 6100$

$\bar{X}_2 = 38100$

FROM THE THEOREM FOR CONFIDENCE INTERVAL FOR $\mu_1 - \mu_2$ WITH UNKNOWN VARIANCE σ_1 & σ_2

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{S_1^2}{N_1} + \frac{S_2^2}{N_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{S_1^2}{N_1} + \frac{S_2^2}{N_2}}$$

$$v = \frac{(S_1^2/N_1 + S_2^2/N_2)^2}{\left[\frac{\left(\frac{S_1^2}{N_1}\right)^2}{(N_1-1)} + \frac{\left(\frac{S_2^2}{N_2}\right)^2}{(N_2-1)}\right]} \xrightarrow{\text{FIND DEGREES OF FREEDOM}} df = \frac{\left(\frac{S_1^2}{N_1} + \frac{S_2^2}{N_2}\right)^2}{\frac{\left(\frac{S_1^2}{N_1}\right)^2}{N_1-1} + \frac{\left(\frac{S_2^2}{N_2}\right)^2}{N_2-1}} = \frac{\left(\frac{5000^2}{12} + \frac{6100^2}{12}\right)^2}{\frac{\left(\frac{5000^2}{12}\right)^2}{12-1} + \frac{\left(\frac{6100^2}{12}\right)^2}{12-1}} \approx 21$$

$v = 21$ DEGREES OF FREEDOM & $t_{0.025} = 2.08$

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{S_1^2/N_1 + S_2^2/N_2} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{S_1^2/N_1 + S_2^2/N_2}$$

$$(36300 - 38100) - 2.08 \cdot \sqrt{5000^2/12 + 6100^2/12} < \mu_1 - \mu_2 < (36300 - 38100) + 2.08 \cdot \sqrt{5000^2/12 + 6100^2/12}$$

$$-1800 - 4735.9 < \mu_1 - \mu_2 < -1800 + 4735.9$$

$$\boxed{-6535.9 < \mu_1 - \mu_2 < 2935.9}$$

9.48 an automotive company is considering two types of batteries for its automobile. sample information on battery life is collected for 20 batteries of type A and 20 batteries of type B. The summary statistics are x̄A = 32.91, x̄B = 30.47, sA = 1.57, and sB = 1.74. Assume the data on each battery are normally distributed and assume σA = σB

(a) find a 95% confidence interval on µA – µB

(b) draw a conclusion from (a) that provides insight into whether A or B should be adopted.

WE ARE GIVEN, $N_A = 20$, $N_B = 20$, $\bar{X}_A = 32.91$, $\bar{X}_2 = 30.47$, $S_A = 1.57$, $S_B = 1.74$

WE NEED TO FIND A 95% CONFIDENCE INTERVAL ON $\mu_A - \mu_B$

$95\% = 100(1-\alpha)\% \Rightarrow \alpha = 0.05$ & $\alpha/2 = 0.025$

$t_{0.025} = 2.021$ & 38 DEGREES OF FREEDOM

THE DIFFENCENCE IN MEANS IS $\bar{X}_A - \bar{X}_B = 32.91 - 30.47 = 2.44$

THE POOLED ESTIMATE S_P^2 OF THE VARIANCE σ^2 IS

$$S_P^2 = \frac{(N_A - 1)S_A^2 + (N_B - 1)S_B^2}{N_A + N_B - 2} = \frac{19(1.57)^2 + 19(1.74)^2}{38} = 2.746$$

$$S_P = \sqrt{S_P^2} = \sqrt{2.746} = 1.6572$$

95% CONFIDENCE INTERVAL FOR $\mu_A - \mu_B$ IS,

$$(\bar{X}_A - \bar{X}_B) - t_{\alpha/2} \cdot S_P \sqrt{1/N_1 + 1/N_2} < \mu_A - \mu_B < (\bar{X}_A - \bar{X}_B) + t_{\alpha/2} \cdot S_P \sqrt{1/N_1 + 1/N_2}$$

$$2.44 - (2.021)(1.6572)\sqrt{1/20 + 1/20} < \mu_A - \mu_B < 2.44 + (2.021)(1.6572)\sqrt{1/20 + 1/20}$$

$$2.44 - 1.059 < \mu_A - \mu_B < 2.44 + 1.059$$

$$\boxed{1.38 < \mu_A - \mu_B < 3.499}$$

THE CONCLUSION TO DRAW FROM IS THAT TYPE A BATTERY HAS A LONGER LIFE, & WE SHOULD CHOOSE A BECAUSE THE DIFFERENCE BETWEEN A & B IS POSITIVE

9.55 a new rocket-launching system is being considered for deployment of small, short-range rockets. the existing system has p = 0.8 as the probability of a successful launch. a sample of 40 experimental launches is made with the new system, and 34 are successful.

(a) construct a 95% confidence interval for p.
(b) would you conclude that the new system is better?

N = 40 LAUNCHES

X = 34 SUCCESSES

PROPORTION OF SUCCESS IS, $\hat{p} = X/N = 34/40 = 0.85$

$\hat{q} = 1 - \hat{p} = 1 - 0.85 = 0.15$

$95\% = 100(1-\alpha)\% \Rightarrow \alpha = 0.05$ & $\alpha/2 = 0.025$

BASED ON THE LARGE SAMPLE CONFIDENCE INTERVALS FOR P

IF $\hat{P}$ IS THE PROPORTION OF SUCCESS IN A RANDOM SAMPLE OF SIZE N & $\hat{q} = 1 - \hat{p}$

AN APPROXIMATE $100(1-\alpha)\%$ CONFIDENCE INTERVAL, FOR THE BINOMIAL PARAMETER P IS

WHERE $Z_{\alpha/2}$ IS THE Z VALUE LEAVING AN AREA OF $\alpha/2$ TO THE RIGHT

$$\hat{p} - Z_{\alpha/2}\sqrt{\hat{p}\hat{q}/N} < p < \hat{p} + Z_{\alpha/2}\sqrt{\hat{p}\hat{q}/N}$$

$Z_{\alpha/2} = Z_{0.025} = 1.96$ BASED ON THE NORMAL PROBABILITY TABLE

USING THE THEORM, THE 95% CONFIDENCE INTERVAL IS

$$\hat{p} - Z_{\alpha/2} \cdot \sqrt{\hat{p}(1-\hat{p})/N} < p < \hat{p} + Z_{\alpha/2} \cdot \sqrt{\hat{p}(1-\hat{p})/N}$$

$$0.85 - 1.96\sqrt{0.85 \cdot 0.15/40} < p < 0.85 + 1.96\sqrt{0.85 \cdot 0.15/40}$$

$$0.85 - 0.1107 < p < 0.85 + 0.1107$$

$$\boxed{0.7393 < p < 0.9607}$$

THE CURRENT P = 0.8 PROBABILITY OF SUCCESS & 95% CONFIDENCE FOR
THE NEW CONFIDENCE INTERVAL $0.7393 < p < 0.9607$ MEANS THE NEW SYSTEM DOESNT IMPLY THAT A NEW SYSTEM
WOULD BE BETTER.

morgan bergen

apr 18 2023

9.66 ten engineering schools in the United States were surveyed. the sample contained 250 electrical engineers, 80 being women; 175 chemical engineers, 40 being women. compute a 90% confidence interval for the difference between the proportions of women in these two fields of engineering. Is there a significant difference between the two proportions?

$N_1 = 250$
$N_2 = 175$
$X_1 = 80$
$X_2 = 40$

IF $\hat{P}_1$ & $\hat{P}_2$ ARE THE PROPORTIONS OF SUCCESSES IN RANDOM SAMPLES OF SIZES N_1 & N_2 $\hat{q}_1 = 1 - \hat{p}$ & $\hat{q}_2 = 1 - \hat{P}$, AN APPROXIMATE $100(1-\alpha)\%$ CONFIDENCE INTERVAL FOR THE DIFFERENCE OF TWO BINOMIAL PARAMETERS, $P_1 - P_2$ IS GIVEN BY, WHERE $z_{\alpha/2}$ IS THE Z VALUE LEAVING AN AREA OF $\alpha/2$ TO THE RIGHT

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\hat{p}_1 \hat{q}_1 / N_1 + \hat{p}_2 \hat{q}_1 / N_2}$$

$90\% = 100(1-\alpha)\% \Rightarrow \alpha = 0.1$ & $\alpha/2 = 0.05$

$\hat{P}_1 = X_1 / N_1 = 80/250 = 0.32$
$\hat{P}_2 = X_2 / N_2 = 40/175 = 0.2286$
$\hat{q}_1 = 1 - \hat{p}_1 = 1 - 0.32 = 0.68$
$\hat{q}_2 = 1 - \hat{P}_2 = 1 - 0.2286 = 0.7714$

$z_{0.05} = 1.645$

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\hat{p}_1 \hat{q}_1 / N_1 + \hat{p}_2 \hat{q}_1 / N_2}$$

$$(0.32 - 0.2286) \pm 1.645 \cdot \sqrt{0.32 \cdot 0.68 / 250 + 0.2286 \cdot 0.7714 / 175}$$

0.0914 ± 0.07128

$\boxed{(0.201,\ 0.1627)}$