Final exam review problem answer key:

Ch2.

2.95. Consider the events:

C: an adult selected has cancer

D: the adult is diagnosed as having cancer

$P(C) = 0.05$, $P(D|C) = 0.78$, $P(C') = 1 - 0.05 = 0.95$, $P(D|C') = 0.06$

so $P(D) = P(C \cap D) + P(C' \cap D) = (0.05)(0.78) + (0.95)(0.06) = 0.096$

2.115. Consider the events

O: overrun

A: consulting firm A.

B: consulting firm B.

C: consulting firm C.

(a) $$P(C|O) = \frac{P(O|C) \cdot P(C)}{P(O|A) \cdot P(A) + P(O|B) \cdot P(B) + P(O|C) \cdot P(C)}$$

$$= \frac{(0.15)(0.25)}{(0.05)(0.40) + (0.03)(0.35) + (0.15)(0.25)} = \frac{0.0375}{0.0680} = 0.5515$$

2.120 ~~2.110~~. Consider the events:

~~[illegible]~~ D: a person has the rare disease. $P(D) = \frac{1}{500}$

P: the test shows a positive result. $P(P|D) = 0.95$. $P(P|D') = 0.01$

$$P(D|P) = \frac{P(P|D) \cdot P(D)}{P(P|D) \cdot P(D) + P(P|D') \cdot P(D')} = \frac{0.95 \times (1/500)}{(0.95)(1/500) + (0.01)(1 - 1/500)} = 0.1599.$$

Ch3.

3.26. Denote by X the number of green balls in the three draws. Let G and B stand for the colors of green and black, respectively.

Simple Events	x	$P(X=x)$	
BBB	0	$(\frac{2}{3})^3 = 8/27$	
GBB	1	$(1/3)(2/3)^2 = 4/27$	$12/27 = \frac{4}{9}$
BGB	1	$(1/3)(2/3)^2 = 4/27$	
BBG	1	$(1/3)(2/3)^2 = 4/27$	
BGG	2	$(1/3)^2(2/3) = 2/27$	$6/27 = \frac{2}{9}$
GBG	2	$(1/3)^2(2/3) = 2/27$	
GGB	2	$(1/3)^2(2/3) = 2/27$	
GGG	3	$(1/3)^3 = 1/27$.	

The probability mass function for X is then

x	0	1	2	3
$p(X=x)$	$8/27$	$4/9$	9	27

3.56. (a). $h(y) = 6\int_0^{1-y} x\,dx = 3(1-y^2)$. for $0<y<1$.

Since $f(x|y) = \frac{f(x,y)}{h(y)} = \frac{2x}{(1-y)^2}$ for $0<x<1-y$. It involves the variable y, so X and Y are NOT independent.

(b). $P(X>0.3 | Y=0.5) = 8\int_{0.3}^{0.5} x\,dx = 0.64$.

3.68. (a). $g(x) = \int_1^2 (\frac{3x-y}{9})\,dy = \frac{3xy - y^2/2}{9}\Big|_1^2 = \frac{x}{3} - \frac{1}{6}$, for $1<x<3$.

$h(y) = \int_1^3 (\frac{3x-y}{9})\,dx = \frac{4}{3} - \frac{2}{9}y$, for $1<y<2$.

(b). No. Since $g(x) \cdot h(y) \neq f(x,y)$.

(c) $P(X>2) = \int_2^3 (\frac{x}{3} - \frac{1}{6})\,dx = (\frac{x^2}{6} - \frac{x}{6})\Big|_2^3 = \frac{2}{3}$.

Ch4.

4.47. $g(x) = \frac{2}{3}\int_0^1 (x+2y)\,dy = \frac{2}{3}(x+1)$, $0<x<1$.

So $\mu_X = \frac{2}{3}\int_0^1 x(x+1)\,dx = \frac{5}{9}$.

$h(y) = \frac{2}{3}\int_0^1 (x+2y)\,dx = \frac{2}{3}(\frac{1}{2}+2y)$.

So $\mu_Y = \frac{2}{3}\int_0^1 y(\frac{1}{2}+2y)\,dy = \frac{11}{18}$.

$E(XY) = \frac{2}{3}\int_0^1\int_0^1 xy(x+2y)\,dy\,dx = \frac{1}{3}$.

So $\sigma_{XY} = E(XY) - \mu_X\mu_Y = \frac{1}{3} - (\frac{5}{9})(\frac{11}{18}) = -0.0062$.

4.70. (a). $g(x) = \frac{3}{2}\int_0^1 (x^2+y^2)\,dy = \frac{1}{2}(3x^2+1)$ for $0<x<1$ and

$h(y) = \frac{1}{2}(3y^2+1)$ for $0<y<1$.

Since $f(x,y) \neq g(x)h(y)$, X and Y are NOT independent.

(b). $E(X+Y) = E(X)+E(Y) = 2E(X) = \int_0^1 x(3x^2+1)\,dx = \frac{3}{4}+\frac{1}{2} = 5/4$.

$E(XY) = \frac{3}{2}\int_0^1\int_0^1 xy(x^2+y^2)\,dx\,dy = \frac{3}{2}\int_0^1 y(\frac{1}{4}+\frac{y^2}{2})\,dy$

$= \frac{3}{2}[(\frac{1}{4})(\frac{1}{2}) + (\frac{1}{2})(\frac{1}{4})] = \frac{3}{8}$.

(c). $V(X) = E(X^2) - [E(X)]^2 = \frac{1}{2}\int_0^1 x^2(3x^2+1)\,dx - (\frac{5}{8})^2 = \frac{7}{15} - \frac{25}{64} = \frac{73}{960}$.

$V(Y) = \frac{73}{960}$.

$Cov(X,Y) = E(XY) - E(X)E(Y) = \frac{3}{8} - (\frac{5}{8})^2 = -\frac{1}{64}$.

(d) $V(X+Y) = V(X) + V(Y) + 2Cov(X,Y) = 2\cdot\frac{73}{960} - 2\cdot\frac{1}{64} = \frac{29}{240}$.

Ch 5.

5.16. $n=4$. $p=0.6$.

$$p(x \geq 2) = 1 - p(x \leq 1) = 1 - p(x=0) - p(x=1)$$

$$= 1 - \binom{4}{0} 0.6^0 (1-0.6)^4 - \binom{4}{1} 0.6^1 (1-0.6)^3$$

$$= 1 - \cancel{0.2208}\,0.1792 = 0.8208.$$

$n=2$. $p=0.6$

$$p(x \geq 1) = 1 - p(x=0) \cancel{= 1 - 0.1600 = 0.8400}.$$

$$= 1 - \binom{2}{0} 0.6^0 (1-0.6)^2 = 1 - 0.16 = 0.8400.$$

So the 2-engine plane has a slightly higher probability for a successful flight when $p = 0.6$.

5.71. $\mu = \lambda t = (1.5)(5) = 7.5$

$$p(x=0 \mid \lambda t = 7.5) = e^{-7.5} = 5.53 \times 10^{-4}.$$

5.87. (a) $\mu = np = (200)(0.03) = 6$.

(b). $\sigma^2 = npq = (200)(0.03)(0.97) = 5.82$.

(c). $p(x=0) = \dfrac{e^{-6}(6)^0}{0!} = 0.0025$ (Using the poisson approximation).

$p(x=0) = (0.97)^{200} = 0.0023$ (Using the binomial distribution).

Ch 6.

6.8. (a). $z = (17-30)/6 = -2.17$. Area $= 1-0.0150 = 0.9850$

(b). $z = (22-30)/6 = -1.33$ Area $= 0.0918$

(c). $z_1 = (32-30)/6 = 0.33$. $z_2 = (41-30)/6 = 1.83$.

Area $= 0.9664 - 0.6293 = 0.3371$

(d) $z = 0.84$. So $x = 30 + (6)(0.84) = 35.04$.

(e) $z_1 = -1.15$. $z_2 = 1.15$. So $x_1 = 30 + (6)(-1.15) = 23.1$

$x_2 = 30 + (6)(1.15) = 36.9$.

6.31 $\mu = (180)(1/6) = 30$ $\sigma = \sqrt{npq} = \sqrt{(180)(1/6)(5/6)} = 5$.

$z = \dfrac{35.5-30}{5} = 1.1$ $P(X > 35.5) = P(Z > 1.1) = 1 - 0.8643 = 0.1357$.

6.32. $\mu = (200)(0.05) = 10$ $\cancel{\sigma = \sqrt{(400)(1/10)(9/10)} = 6}$.

$\sigma = \sqrt{(200)(0.05)(0.95)} = 3.082$.

$z = (9.5-10)/3.082 = -0.16$

$P(X<10) = P(X \le 9) = P(X < \dfrac{9.5-10}{3.082}) = P(X < -0.16) = 0.4364$.

6.66 (a) $\mu = \beta = 100$ hours.

(b). $P(X \geq 200) = 0.01 \int_{200}^{\infty} e^{-0.01x}\,dx = e^{-2} = 0.1353$.

Ch8.

8.12. (a) $\bar{x} = 11.69$ milligrams.

(b) $s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1} = \frac{n\sum_{i=1}^{n}x_i^2 - (\sum_{i=1}^{n}x_i)^2}{n(n-1)} = \frac{(8)(1168.21) - 93.5^2}{(8)(7)} = 10.776$.

OR using the calculator to find these.

8.23. (a) $\mu = \sum x f(x) = (4)(0.2) + (5)(0.4) + (6)(0.3) + (7)(0.1) = 5.3$.

$\sigma^2 = \sum (x-\mu)^2 f(x) = (4-5.3)^2(0.2) + (5-5.3)^2(0.4) + (6-5.3)^2(0.3) + (7-5.3)^2(0.1)$
$= 0.81$.

(b). $n=36$ $\mu_{\bar{x}} = \mu = 5.3$. $\sigma_{\bar{x}}^2 = \sigma^2/n = 0.81/36 = 0.0225$

(c). $n=36$. $\mu_{\bar{x}} = 5.3$. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.9}{6} = 0.15$.

$z = (5.5-5.3)/0.15 = 1.33$.

So $p(\bar{x} < 5.5) = p(z < 1.33) = 0.9082$.

8.27. $n=50$. $\bar{x}=0.23$ and $\sigma = 0.1$.

$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{0.23-0.2}{0.1/\sqrt{50}} = 2.12$.

So $p(\bar{x} \geq 0.23) = p(z \geq 2.12) = 0.0170$.

Hence the probability of having such observations, given the mean $\mu = 0.20$ is small. Therefore, the mean amount to be 0.20 is not likely to be true.

8.29. $\mu_{\bar{x}_1-\bar{x}_2} = 72-28 = 44$. $\sigma_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{100/64 + 25/100} = 1.346$.

$z = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} = \frac{(44.2-44)}{1.346} = 0.15$ So $p(\bar{x}_1-\bar{x}_2 < 44.2) = p(z < 0.15)$
$= 0.5596$.

Ch8.

8.48. Using a calculator we find $t_{0.025} = 2.131$ $\nu = 15$ d.f.

$$t = \frac{27.5 - 30}{5/4} = -2.00$$ falls between -2.131 and 2.131

So the claim is valid.

8.49. $$t = \frac{24-20}{4.1/3} = 2.927$$ $t_{0.01} = 2.896$. $\nu = 8$.

$2.927 > 2.896$.

So the anwer is No. and $\mu > 20$.