probability and statistics

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06 some continuous probability distributions

april 04 2023

- continuous uniform distribution
- normal distribution
- gamma and exponential distribution
- chi-squared distribution

6.1 given a continuous uniform distribution, show that

(a) $\mu = \dfrac{A+B}{2}$

(b) $\sigma^2 = \dfrac{(B+A)}{12}$

LET X BE THE RV WHICH FOLLOWS A CONTINUOUS UNIFORM DISTRIBUTION WITH PROBABILITY DENSITY FUNCTION,

$$f(x) = \frac{1}{B-A} \quad ; \quad A \leq x \leq B$$

THE MEAN OF X IS AS FOLLOWS,

$$\mu = E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx = \int_A^B \frac{x}{B-A}\, dx = \frac{1}{B-A}\int_A^B \frac{x}{2}\, dx$$

$$= \frac{1}{B-A}\left[\frac{x^2}{2}\right]_A^B$$

$$= \frac{1}{B-A} \cdot \frac{B^2 - A^2}{2}$$

$$= \frac{1}{B-A} \cdot \frac{(B-A)(B+A)}{2}$$

$$\therefore \boxed{E(X) = \frac{(A+B)}{2}}$$

THE VARIANCE OF X IS AS FOLLOWS,

$$\text{VAR}(X) = E(X^2) - ((E(X)))^2$$

$$= \int_A^B x^2\left(\frac{1}{B-A}\right) dx - \left(\frac{A+B}{2}\right)^2 = \frac{1}{B-A}\int_A^B x^2\, dx - \left(\frac{A+B}{2}\right)^2$$

$$= \frac{1}{B-A}\left(\frac{x^3}{3}\right)_A^B - \left(\frac{A+B}{2}\right)^2$$

$$= \frac{B^3 - A^3}{3(B-A)} - \left(\frac{A+B}{2}\right)^2$$

$$= \frac{A^2 + AB + B^2}{3} + \frac{A^2 + 2AB + B^2}{4}$$

$$= \frac{4A^2 + 4AB + 4B^2 - 3A^2 - 6AB - 3B^2}{12}$$

$$= \frac{B^2 - 2AB + A^2}{12}$$

$$\therefore \boxed{\text{VAR}(X) = \frac{(B-A)^2}{12}}$$

6.4 a bus arrives every 10 minutes at a bus stop it is assumed that the waiting time for a particular individual is a random variable with a continuous uniform distribution.

(a) what is the probability that the individual waits more than 7 minutes?

(b) what is the probability that the individual waits between 2 and 7 minutes?

- LET RANDOM VARIABLE X REPRESENT WAITING TIME FOR A PARTICULAR INDIVIDUAL
- X FOLLOWS A CONTINUOUS UNIFORM DISTRIBUTION & ITS PROBABILITY DENSITY FUNCTION IS AS FOLLOWS,

$$f(x) = \begin{cases} 1/10 & ; \; 0 \le x \le 10 \\ 0 & ; \; \text{ELSEWHERE} \end{cases}$$

- TO CALCULATE THE PROBABILITY THAT THE INDIVIDUAL WAITS MORE THAN 7 MINUTES YOU MUST TAKE THE INTEGRAL FROM YOU $P(\;)$ CONSTRAINTS, THE FUNCTION

$$P(X \ge 7) = \int_7^{10} \frac{1}{10}\,dx = \frac{x}{10}\Big|_7^{10} = \frac{10}{10} - \frac{7}{10} = \frac{3}{10} = 0.3$$

$$\therefore \boxed{P(X \ge 7) = 0.3}$$

$$P(2 \le x \le 7) = \int_2^7 f(x)\,dx = \int_2^7 \frac{1}{10}\,dx = \frac{x}{10}\Big|_2^7 = \frac{7}{10} - \frac{2}{10} = \frac{5}{10} = \frac{1}{2}$$

$$\therefore \boxed{P(2 \le x \le 7) = 0.5}$$

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6.5 given a standard normal distribution, find the area under the curve that lies

USING THE NORMAL PROBABILITY TABLE

(a) to the left of z = –1.39

$P(Z < -1.39) = 0.0823$

(b) to the right of z = 1.96

$P(Z > 1.96) = 1 - P(Z < 1.96) = 1 - 0.9750 = 0.025$

(c) between z = –2.16 and z = –0.65

$P(-2.16 < Z < -0.65) = P(Z < -0.65) - P(Z < -2.16) = 0.2578 - 0.0154 = 0.2424$

(d) to the left of z = 1.43

$P(Z < 1.43) = 0.9236$

(e) to the right of z = –0.89

$P(Z > -0.896) = 1 - P(Z < -0.89) = 1 - 0.1867 = 0.8133$

(f) between z = –0.48 and z = 1.74

$P(-0.48 < Z < 1.74) = P(Z < 1.74) - P(Z < 0.48) = 0.9591 - 0.3156 = 0.6435$

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6.6 find the value of z if the area under the standard normal curve

(a) to the right of z is 0.3622

$P(Z > z) = 1 - P(Z \leq z) = 1 - 0.3622 = 0.6378$, $P(Z \leq 0.6378) = 0.3526$

(b) to the left of z is 0.1131

$P(Z < 0.1131) = -1.21$

(c) between 0 and z, with z > 0, is 0.4838

$P(0 < Z < z) = P(Z < z) - P(Z < 0)$

$P(Z < z) - P(Z < 0) = 0.4838$

$P(Z < z) - 0.5 = 0.4838$

$P(Z < z) = 0.9838$

$P(Z < 0.9838) = 2.14$

(d) between −z and z, with z > 0, is 0.9500

$P(-z < Z < z) = P(Z < z) - P(Z < z) = 0.95$

$P(Z < z) - (1 - P(Z < z)) = 0.95$

$2P(Z < z) = 1 + 0.95$

$P(Z < z) = 0.975$

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6.7 given a standard normal distribution, find the value of k such that

(a) $P(Z > k) = 0.2946$

(b) $P(Z < k) = 0.0427$

(c) $P(-0.93 < Z < k) = 0.7235$

$P(Z < k) = 0.2934$

$P(Z > k) = 1 - P(Z \le k) = 1 - 0.2946 = 0.7054 \quad k = 0.54$

$P(Z < k) = 0.0427, \quad k = -1.72$

$P(Z < k) = 0.0427, \quad k = -1.72$

$P(-0.93 < Z < k) = 0.7235$

$P(-0.93 < Z < k) = P(-0.93 < Z < k) + P(Z < -0.93)$

$= 0.7235 + 0.1762$

$= 0.8997, \quad k = 1.28$

6.8 **given a normal distribution with μ = 30 and σ = 6, find**

(a) the normal curve area to the right of x = 17

(b) the normal curve area to the left of x = 22

(c) the normal curve area between x = 32 and x = 41

(d) the value of x that has 80% of the normal curve area to the left

(e) the two values of x that contain the middle 75% of the normal curve area

NORMAL DISTRIBUTION WITH $\mu = 30$ & $\sigma = 6$
AREA TO THE RIGHT OF $x = 17$

$$z = \frac{x - \mu}{\sigma} = \frac{17 - 30}{6.0} \approx -2.17$$

$$P(x > 17) = P(z > -2.17) = 1 - P(z \leq -2.17) = 1 - 0.0150 = 0.9850$$

AREA TO THE LEFT OF $x = 22$

$$z = \frac{x - \mu}{\sigma} = \frac{22 - 30}{6.0} \approx -1.33$$

$$P(x < 22) = P(z < -1.33) = 0.0918$$

AREA UNDER THE NORMAL CURVE BETWEEN $x = 32$ & $x = 41$

$$z_1 = x_1 - \mu / \sigma = 32 - 30 / 6.0 \doteq 0.33$$
$$z_2 = x_2 - \mu / \sigma = 41 - 30 / 6.0 \simeq 1.83$$

$$P(32 < X < 41) = P(X < 41) - P(X < 32) = P(z_2 < 1.83) - P(z_1 < 0.33) = 0.9664 - 0.6293$$

$$P(32 < X < 41) = 0.3317$$

VALUE OF X THAT HAS 80% OF THE AREA TO THE LEFT

Z VALUE W/ AREA 0.80 TO LEFT IS 0.842

$$z = x - \mu / \sigma$$
$$x = \sigma z + \mu = 6 \cdot 0.842 + 30$$
$$x = 35.05$$

TWO X VALUES THAT CONTAIN THE MIDDLE 75% OF THE AREA

$$P(-z \leq Z \leq z) = 0.75$$
$$P(Z \leq z) - P(Z \leq -z) = 0.75$$
$$P(Z \leq z) - (1 - P(Z \leq z)) = 0.75$$
$$2P(Z \leq z) - 1 = 0.75$$
$$2P(Z \leq z) = 1.75$$
$$P(Z \leq z) = 0.875$$

$$z_1 = -1.15 \quad \& \quad z_2 = 1.15$$
$$z_1 = x_1 - \mu / \sigma = \sigma z_1 + \mu = 6 \cdot (-1.15) + 30 = 23.1$$
$$z_2 = x_2 - \mu / \sigma = \sigma z_2 + \mu = 6 \cdot (1.15) + 30 = 36.9$$

6.12 the loaves of rye bread distributed to local stores by a certain bakery have an average length of 30 centimeters and a standard deviation of 2 centimeters. assuming that the lengths are normally distributed, what percentage of the loaves are

(a) longer than 31.7 centimeters?

(b) between 29.3 and 33.5 centimeters in length?

(c) shorter than 25.5 centimeters?

X IS LOAVES NORMALLY DISTRIBUTED

$\mu = 30, \ \sigma = 2$

PERCENTAGE OF LOAVES ARE LONGER THAN 31.7 CM

$z = x - \mu / \sigma = 31.7 - 30 / 2 = 0.85$

$P(X > 31.7) = P(Z > 0.85) = 1 - P(Z < 0.85) = 1 - 0.8023 = 0.1977 = 19.77\%$

PERCENTAGE OF LOAVES BETWEEN 29.3 & 33.5 CM

$z_1 = x_1 - \mu / \sigma = 29.3 - 30 / 2 = -0.35$

$z_2 = x_2 - \mu / \sigma = 33.5 - 30 / 2 = 1.75$

$P(29.3 < X < 31.7) = P(-0.35 < Z < 1.75) = P(Z < 1.75) - P(Z < -0.35) = 0.9599 - 0.3632 = 0.5967 = 59.67\%$

PERCENTAGE SHORTER THAN 25.5 CM

$P(X < 25.5) = P(Z < -2.25) = 0.0122 = 1.22\%$

6.18 the heights of 1000 students are normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. assuming that the heights are recorded to the nearest half-centimeter, how many of these students would you expect to have heights

(a) less than 160.0 centimeters?

(b) between 171.5 and 182.0 centimeters inclusive?

(c) equal to 175.0 centimeters?

(d) greater than or equal to 188.0 centimeters?

X WILL BE THE RANDOM VARIABLE THAT REP HEIGHT OF STUDENTS NORMAL DISTRIBUTION $\mu = 174.5$ & $\sigma = 6.9$

$P(X \le 160.0) = P(X - \mu/\sigma \le 160 - 174.5/6.9) = P(Z \le -14.5/6.9) = P(Z \le -2.10) = 0.0179$

$E(X) = NP = 1000 \cdot 0.0179 = 17.9 \simeq 18$

$\boxed{\therefore E(X) \simeq 18}$

STUDENTS HEIGHTS BETWEEN 171.5 & 182.0 CM

$$P(X \le 182) = P\left(\frac{X-\mu}{\sigma} \le \frac{182 - 174.5}{6.9}\right) = P(Z \le 7.5/6.9) = P(Z \le 1.0870) = 0.8621$$

$$P(X \le 171.5) = P\left(\frac{X-\mu}{\sigma} \le \frac{171.5 - 174.5}{6.9}\right) = P(Z \le -3/6.9) = P(Z \le -0.4348) = 0.3336$$

$P(171.5 \le X \le 182) = P(X \le 182) - P(X \le 171.5) = 0.8621 - 0.3336 = 0.5285$

$E(X) = NP = 1000 \cdot 0.5285 = 528.5 \simeq 529$

$\boxed{\therefore E(X) \simeq 529}$

NUMBER OF STUDENTS HEIGHTS EQUAL TO 175.0 CM

$$P(X \le 175.5) = P\left(\frac{X-\mu}{\sigma} \le \frac{175 - 174.5}{6.9}\right) = P(Z \le 0.14) = 0.5557$$

$$P(X \le 174.5) = P\left(\frac{X-\mu}{\sigma} \le \frac{174 - 174.5}{6.9}\right) = P(Z \le 0) = 0.5$$

$P(175 - 0.5 \le X \le 175 + 0.5) = P(174.5 \le X \le 175.5) = P(X \le 175.5) - P(X \le 174.5) = 0.557 - 0.5 = 0.0557$

$E(X) = NP = 1000 \cdot 0.0557 = 55.7 \simeq 56$

$\boxed{\therefore E(X) = 56}$

HEIGHTS GREATER THAN OR EQUAL TO 188.0

$$P(X \ge 188.0) = 1 - P(X \le 188.0) = 1 - P\left(\frac{X-\mu}{\sigma} \le \frac{188 - 174.5}{6.9}\right) = 1 - P(Z \le 1.96) = 1 - P(Z \le 1.96) = 1 - 0.975 = 0.025$$

$E(X) = NP = 1000 \cdot 0.025 = 25$

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6.24 a coin is tossed 400 times. Use the normal curve approximation to find the probability of obtaining

(a) between 185 and 210 heads inclusive

(b) exactly 205 heads

(c) fewer than 176 or more than 227 heads

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$N = 400$ & $P = 5$

MEAN OF BINOMIAL DISTRIBUTION $\mu = NP = 400 \cdot 05 = 200$

STANDARD DEVIATION OF BINOMIAL DISTRIBUTION $\sigma = \sqrt{NP(1-P)} = \sqrt{400 \cdot 0.05 \cdot (1-0.5)} = 10$

185 & 210 HEADS INCLUSIVELY

$z_1 = X_1 - \mu / \sigma = 184.5 - 200/10 = -1.55$

$z_2 = X_2 - \mu / \sigma = 210.5 - 200/10 = 1.05$

$$P(185 \le X \le 210) = \sum_{x=185}^{210} b(x; 400, 0.5) = P(184.5 \le X \le 210.5) = P(-1.55 \le Z \le 1.05) = 0.8531 - 0.0606 = 0.7925$$

PROBABILITY OF 205 HEADS

$z_1 = X_1 - \mu/\sigma = 204.5 - 200/10 = 0.45$

$z_2 = X_2 - \mu/\sigma = 205.5 - 200/10 = 0.55$

$$P(X = 205) = b(205; 400, 0.5) = P(0.45 \le Z \le 0.55) = P(Z \le 0.55) - P(Z \le 0.45) = 0.7088 - 0.6736 = 0.0352$$

FEWER THAN 176 OR MORE THAN 227 HEADS

$z_1 = X_1 - \mu/\sigma = 175.5 - 200/10 = -2.45$

$z_2 = X_2 - \mu/\sigma = 227.5 - 200/10 = 2.75$

$$P(176 \le X \le 227) = \sum_{x=176}^{227} b(x; 400, 0.5) = P(-2.45 \le Z \le 2.75) = P(Z \le 2.75) - P(Z \le -2.45) = 0.9970 - 0.0071 = 0.9899$$

$$P(X < 179) + P(X > 227) = 1 - P(176 \le X \le 227) = 1 - 0.9899 = 0.0101$$

6.26 **a process yields 10% defective items. if 100 items are randomly selected from the process, what is the probability that the number of defectives**

(a) exceeds 13?

(b) is less than 8?

N = 100 , PROBABILITY OF SUCCESS $p = 10\% = 0.1$

PROBABILITY OF FAILURE $q = 1 - p = 0.9$

X IS THE RANDOM VARIABLE THAT REP THE NUMBER OF DEFECTIVE ITEMS OUT OF THE 100 SELECTED

X HAS A BINOMIAL DISTRO W/ $N = 100$, $p = 0.1$, $q = 0.9$

THE MEAN IS $\mu = N \cdot p = 100 \cdot 0.1 = 10$

THE STANDARD DEVIATION IS $\sigma = \sqrt{Npq} = \sqrt{100 \cdot 0.1 \cdot 0.9} = 3$

NEXT TO FIND THAT THE PROBABILITY THAT THE NUMBER OF DEFECTS EXCEEDS 13 AKA TO FIND AN AREA TO THE RIGHT OF $x = 13.5$

$$z = \frac{x - \mu}{\sigma} = \frac{13.5 - 10}{3} = 1.16\overline{7} \approx 1.17$$

$$P(X > 13) = P(z > 1.17) = 1 - P(z < 1.17) = 1 - 0.8790 = 0.1210$$

$$\boxed{P(X > 13) = 0.1210}$$

NEXT, THE PROBABILITY THAT THE NUMBER OF DEFECTS IS LESS THAN 8

MEANING WE MUST FIND THE AREA TO THE LEFT OF $x = 7.5$

$$z = x - \mu / \sigma = 7.5 - 10 / 3 = -0.83$$

$$P(X < 8) = P(z < -0.83) = 0.2033$$

$$\boxed{P(X < 8) = 0.2033}$$

6.29 if 20% of the residents in a U.S. city prefer a white telephone over any other color available, what is the probability that among the next 1000 telephones installed was

(a) between 170 and 185 inclusive will be white?

(b) at least 210 but not more than 225 will be white?

$N = 1000$, $p = 0.20$, $q = 1 - p = 1 - 0.20 = 0.80$

SINCE THIS IS A PASS/FAIL FOR EACH TRIAL X WILL HAVE A BINOMIAL DISTRIBUTION

THE MEAN $\mu = NP = 1000 \cdot 0.2 = 200$

THE STANDARD DEVIATION IS $\sigma = \sqrt{NPQ} = \sqrt{1000 \cdot 0.2 \cdot 0.8} = \sqrt{160} = 12.6491$

$N = 1000$, $p = 0.2$, $q = 0.8$

$\mu = 200$, $\sigma = 12.6491$

TO CALCULATE THAT THE PROBABILITY IS BETWEEN 170 & 185

$(170 < X < 185) \rightarrow (169.5 < X < 185.5) \rightarrow P(-2.41 < Z < -1.15)$

$Z_1 = X_1 - \mu / \sigma = (169.5 - 200)/12.6491 = -2.41$

$Z_2 = X_2 - \mu / \sigma = (185.5 - 200)/12.6491 = -1.15$

$$P(170 \le X \le 185) = P(-2.41 < Z < -1.15)$$
$$= P(Z < -1.15) - P(Z < -2.41)$$
$$= 0.1251 - 0.0080$$

$\therefore \boxed{P(170 \le X \le 185) = 0.1171}$

AT LEAST 210 BUT NOT MORE THAN 225, NEXT

$P(210 \le X \le 225) \rightarrow P(209.5 \le X \le 225.5) \rightarrow P($

$Z_1 = X_1 - \mu / \sigma = (209.5 - 200)/12.6491 = 0.75$

$Z_2 = X_2 - \mu / \sigma = (225.5 - 200)/12.6491 \simeq 2.02$

$$P(210 \le X \le 225) = P(0.75 \le Z \le 2.02)$$
$$= P(2.02 < Z) - P(0.75 < Z)$$
$$= 0.9783 - 0.7734$$

$\therefore \boxed{P(210 \le X \le 225) = 0.2049}$

6.32 a pharmaceutical company knows that approximately 5% of its birth-control pills have an ingredient that is below the minimum strength, thus rendering the pill ineffective. What is the probability that fewer than 10 in a sample of 200 pills will be ineffective?

$n = 200$, $p = 0.05$, $q = 1 - 0.05 = 0.95$

X HAS A BINOMIAL DISTRIBUTION

$\mu = np = 200 \cdot 0.05 = 10$

$\sigma = \sqrt{npq} = \sqrt{200 \cdot 0.95 \cdot 0.05} = \sqrt{9.5} \simeq 3.082$

THUS TO FIND THE PROBABILITY THAT FEWER THAN 10 WILL BE INEFFECTIVE REQUIRES US TO FIND THE AREA UNDER THE CURVE TO THE LEFT OF $x = 9.5$

$z = x - \mu / \sigma = 9.5 - 10 / 3.082 = -0.16$

$\therefore \boxed{P(X < 10) = P(Z < -0.16) = 0.4364}$

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6.34 a pair of dice is rolled 180 times. What is the probability that a total of 7 occurs

(a) at least 25 times?

(b) between 33 and 41 times inclusive?

(c) exactly 30 times?

$N = 180$ TRIALS INDEPENDENT, IF WE CONSIDER IT A SUCCESS TO ROLL A PAIR OF DICE A TOTAL OF 7 OCCURS

$(1,6), (2,5), (3,4), (4,3), (5,2), (6,1) = 6$ OCCURENCES

THUS THE TOTAL CASES IS $36 = 6 \cdot 6$

PROBABILITY OF EACH TRIAL $p = 6/36 = 0.1667$

SUCCEEDING IS $p = 0.1667$

FAILING IS $q = 1 - p = 1 - 0.1667 = 0.8333$

LET X BE THE RV THAT REPRESENTS THE NUMBER OF SUCCESSES FOR 180 TRIALS, IN A BINOMIAL DISTRI

THUS THERE IS $N = 180$ & $P = 0.1667$

$\mu = NP = 180 \cdot 0.1667 = 30$

$\sigma = \sqrt{NPq} = \sqrt{180 \cdot 0.1667 \quad 0.8333} = \sqrt{25} = 5$

NOW IF WE WANT TO FIND THAT THE PROBABILITY THAT A ROLE WILL BE SUCCESSFUL AT LEAST 25 TIMES WE MUST FIND TO THE RIGHT OF $X = 24.5$

$z = x - \mu / \sigma = (24.5 - 30)/5 = -1.1$

$P(x \geq 25) = P(Z \geq -1.1) = 1 - P(Z \leq -1.1) = 1 - 0.1357 = 0.8643$

NEXT TO FIND THE SUCCESSFUL ROLES HAPPEN BETWEEN 33 & 41

$P(33 \leq X \leq 41) \rightarrow P(32.5 \leq X \leq 41.5) \rightarrow P(z_1 \leq Z \leq z_2)$

$z_1 = x_1 - \mu / \sigma = 32.5 - 30/5 = 0.5$

$z_2 = x_2 - \mu / \sigma = 32.5 - 30/5 = 2.3$

$P(33 \leq X \leq 41) = P(0.5 < Z < 2.3)$

$= P(Z < 2.30) - P(Z < 0.5) = 0.9893 - 0.6915 = 0.2978$

$\boxed{\therefore P(33 \leq X \leq 41) = 0.2978}$

EXACTLY 30 MEANING WE MUST FIND THE AREA BETWEEN $x_1 = 29.5$ & $x_2 = 30.5$

$z_1 = x_1 - \mu / \sigma = 29.5 - 30/5 = -0.1$

$z_2 = x_2 - \mu / \sigma = 30.5 - 30/5 = 0.1$

$P(X = 30) = P(-0.10 \leq Z \leq 0.10) = P(Z \leq 0.10) - P(Z \leq -0.10)$

$= 0.5398 - 0.4602$

$= 0.0796$

$\boxed{\therefore P(X = 30) = 0.0796}$

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6.55 computer response time is an important application of the gamma and exponential distributions. Suppose that a study of a certain computer system reveals that the response time, in seconds, has an exponential distribution with a mean of 3 seconds.

(a) what is the probability that response time exceeds 5 seconds?

(b) what is the probability that response time exceeds 10 seconds?

$\beta = 3$ SECONDS , THE PROBABILITY DENSITY FUNCTION OF EXPONENTIAL DISTRIBUTION

$$f(x;\beta) = \tfrac{1}{3} e^{-x/\beta} = \tfrac{1}{3} e^{-x/3} ; \quad x \geq 0$$

$$P(X > 5) = 1 - P(X \leq 5) = 1 - \tfrac{1}{3}\int_0^5 e^{-x/5}\, dx = 1 - (1 - e^{-5/3}) = e^{-5/3} = 0.1889$$

$$\boxed{\therefore P(X > 5) = 0.1889}$$

THE PROBABILITY THAT THAT TIME EXCEEDS 10 SECOND

$$P(X > 10) = 1 - P(X \leq 10) = 1 - \frac{1}{3}\int_0^{10} e^{-x/3}\, dx = 1 - (1 - e^{-10/3}) = 0.0357$$

$$\boxed{\therefore P(X > 10) = 0.0357}$$