MORGAN BERGEN

SUN FEB 12 2022

H 5, 7, 13, 14, 17, 33, 36, 38, 41, 43, 45, 47, 56

3.5 DETERMINE THE VALUE c SO THAT EACH OF THE FOLLOWING FUNCTIONS CAN SERVE AS A PROBABILITY DISTRIBUTION OF THE DISCRETE RANDOM VARIABLE X.

(A) $f(x) = c(x^2+4)$, FOR $x = 0, 1, 2, 3$

$$= c(0^2+4) + c(1^2+4) + c(2^2+4) + c(3^2+4)$$

$$= c(4) + c(5) + c(8) + c(13)$$

$$1 = c(30)$$

$$\boxed{c = 1/30}$$

(B) $f(x) = c\binom{2}{x}\binom{3}{3-x}$, FOR $x = 0, 1, 2$

$$\sum_x f(x) = 1 \Rightarrow \sum_{x=0}^{2} f(x) = f(0) + f(1) + f(2) = c\binom{2}{0}\binom{3}{3-0} + c\binom{2}{1}\binom{3}{3-1} + c\binom{2}{2}\binom{3}{3-2}$$

$$= c(1)(1) + c(2)(3) + c(1)(3)$$

$$1 = 5c$$

$$\boxed{c = 1/5}$$

(3.7) THE TOTAL NUMBER OF HOURS, MEASURED IN UNITS OF 100 HOURS, THAT A FAMILY RUNS A VACUUM CLEANER OVER A PERIOD OF ONE YEAR IS A CONTINUOUS RANDOM VARIABLE X THAT HAS THE DENSITY FUNCTION,

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 \le x < 2 \\ 0, & \text{ELSEWHERE} \end{cases}$$

FIND THE PROBABILITY THAT OVER A PERIOD OF ONE YEAR, A FAMILY RUNS THEIR VACUUM CLEANER

(A) LESS THAN 120 HOURS

(B) BETWEEN 50 & 100 HOURS

(A) $x < 120/100 \Rightarrow x < 1.2$

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 \le x < 2 \\ 0, & \text{ELSEWHERE} \end{cases}$$

$$P(1 \le x < 2) = \int_1^2 2 - x\,dx = 2x - \frac{x^2}{2}\Big|_1^2 = \left(2(2) - \frac{2^2}{2}\right) - \left(2(1) - \frac{1^2}{2}\right) = (4-2) - (2 - 1/2) = 1/2$$

$$\boxed{P(1 \le x < 2) = 1/2}$$

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(A) $x < 120/100 \Rightarrow x < 1.2$

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$$\boxed{P(1 \leq x < 2) = 1/2}$$

(B) $50 < x < 100 \Longrightarrow 0.5 < x < 1$

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \\ 0, & \text{ELSEWHERE} \end{cases}$$

$$P(0 < x < 1) = \int_0^1 x \, dx = \frac{x^{1+1}}{1+1} \Big|_0^1 = \frac{x^2}{2} \Big|_0^1 = \frac{1}{2}$$

$$\boxed{P(0 < x < 1) = 1/2}$$

(3.13) THE PROBABILITY DISTRIBUTION OF X, THE NUMBER OF IMPERFECTIONS PER 10 METERS OF A SYNTHETIC FABRIC IN CONTINUOUS ROLLS OF UNIFORM WIDTH IS GIVEN BY

x	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

PROBABILITY DISTRIBUTION

CONSTRUCT THE CUMULATIVE DISTRIBUTION FUNCTION OF X

THE CUMULATIVE DISTRIBUTION FUNCTION $F(x)$ OF A DISCRETE RANDOM VARIABLE X WITH PROBABILITY DISTRIBUTION $f(x)$ IS

$$F(x) = P(X \le x) = \sum_{t \le x} f(t), \quad \text{FOR} \quad -\infty < x < \infty$$

$f(0) = 0.41$ — $F(0) = P(X \le 0) = f(0) = 0.41$

$f(1) = 0.37$ — $F(1) = P(X \le 1) = f(0) + f(1) = 0.41 + 0.37 = 0.78$

$f(2) = 0.16$ — $F(2) = P(X \le 2) = f(0) + f(1) + f(2) = 0.41 + 0.37 + 0.16 = 0.94$

$f(3) = 0.05$ — $F(3) = P(X \le 3) = f(0) + f(1) + f(2) + f(3) = 0.41 + 0.37 + 0.16 + 0.05 = 0.99$

$f(4) = 0.01$ — $F(4) = P(X \le 4) = f(0) + f(1) + f(2) + f(3) + f(4) = 0.41 + 0.37 + 0.16 + 0.05 + 0.01 = 1$

$$F(x) = \begin{cases} 0.41, & \text{FOR } x \le 0 \\ 0.78, & \text{FOR } 0 \le x \le 1 \\ 0.94, & \text{FOR } 1 \le x \le 2 \\ 0.99, & \text{FOR } 2 \le x \le 3 \\ 1, & \text{FOR } 3 \le x \le 4 \end{cases}$$

(3.14) THE WAITING TIME, IN HOURS, BETWEEN SUCCESSIVE SPEEDERS SPOTTED BY A RADAR UNIT IS A CONTINUOUS RANDOM VARIABLE WITH CUMULATIVE DISTRIBUTION FUNCTION

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-8x}, & x \geq 0 \end{cases}$$

FIND THE PROBABILITY OF WAITING LESS THAN 12 MINUTES BETWEEN SUCCESSIVE SPEEDERS

(A) USING THE CUMULATIVE DISTRIBUTION FUNCTION OF X

SINCE THE CDF IS IN HOURS WE MUST DO $12/60 = 0.2$ FOR IT'S ASSIGNMENT TO x

$$F(x=0.2) = 1 - e^{-8(x=0.2)} = 1 - e^{(-1.6)} = 0.7981$$

$$\boxed{F(x=0.2) = 0.7981}$$

(B) USING THE PROBABILITY DENSITY FUNCTION OF X

$$f(x) = \frac{d\,F(x)}{dx} = \begin{cases} \dfrac{d\,F(x)}{dx}, & x \leq 0 \\ \dfrac{d\,F(x)}{dx}, & x > 0 \end{cases} \Rightarrow \begin{cases} \dfrac{d\,0}{dx}, & x \leq 0 \\ \dfrac{d\left(1-e^{-8x}\right)}{dx}, & x > 0 \end{cases} \Rightarrow \begin{cases} 0 & x \leq 0 \\ 8e^{-8x} & x > 0 \end{cases}$$

$$f(x) = \begin{cases} 0, & x \leq 0 \\ 8e^{-8x}, & x > 0 \end{cases} \qquad \int_{-\infty}^{0.2} f(x)\,dx = \int_{-\infty}^{0.2} f(x)\,dx = 1 - e^{-8(0.2)} = 0.7981$$

$$\boxed{f(x=0.2) = 0.7981}$$

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(3 17) A CONTINUOUS RANDOM VARIABLE X THAT CAN ASSUME VALUES BETWEEN $x=1$ & $x=3$ HAS A DENSITY FUNCTION GIVEN BY $f(x) = 1/2$

(A) SHOW THAT THE AREA UNDER THE CURVE IS EQUAL TO 1

PROBABILITY DENSITY FUNCTION VERIFICATION

$$\boxed{P(1 < X < 3) = \int_1^3 \frac{1}{2}\,dx = \frac{x}{2}\Big|_1^3 = \frac{3}{2} - \frac{1}{2} = 1 \checkmark}$$

(B) FIND $P(2 < X < 2.5)$

$$P(2 < x < 2.5) = \int_2^{2.5} \frac{1}{2}\,dx = \frac{x}{2}\Big|_2^{2.5} = \frac{2.5}{2} - \frac{2}{2} = \frac{1/2}{2} = \frac{1}{4}$$

$$\boxed{P(2 < x < 2.5 = 1/4}$$

(C) FIND $P(X \leq 1.6)$

$$P(1 < x \leq 1.6) = \int_1^{1.6} \frac{1}{2}\,dx = \frac{x}{2}\Big|_1^{1.6} = \frac{1.6}{2} - \frac{1}{2} = \frac{0.6}{2} = 0.3$$

$$\boxed{P(1 < x \leq 1.6) = 0.3}$$

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HW ~~5, 7, 13, 14, 17~~, 33, 36, 38, 41, 43, 45, 47, 56

(3.33) SUPPOSE A CERTAIN TYPE OF SMALL DATA PROCESSING FIRM IS SO SPECIALIZED THAT SOME HAVE DIFFICULTY MAKING A PROFIT IN THEIR FIRST YEAR OF OPERATION. THE PROBABILITY DENSITY FUNCTION THAT CHARACTERIZES THE PROPORTION Y THAT MAKE A PROFIT IS GIVEN BY

$$f(y) = \begin{cases} ky^4(1-y)^3, & 0 \le y \le 1 \\ 0, & \text{ELSEWHERE} \end{cases}$$

(A) WHAT IS THE VALUE OF K THAT RENDERS THE ABOVE A VALID DENSITY FUNCTION?

(B) FIND THE PROBABILITY THAT AT MOST 50% OF THE FIRMS MAKE A PROFIT IN THE FIRST YEAR

(C) FIND THE PROBABILITY THAT AT LEAST 80% OF THE FIRMS MAKE A PROFIT IN THE FIRST YEAR

$$f(y) = \begin{cases} ky^4(1-y)^3, & 0 \le y \le 1 \\ 0, & \text{ELSEWHERE} \end{cases}$$

$$1 = \int_0^1 ky^4(1-y)^3\,dy = k\int_0^1 y^4(1-y)^3\,dy$$

$$= k\int_0^1 y^4(1-y)(1-y)(1-y)\,dy$$

$$= k\int_0^1 y^4(1-y-y+y^2)(1-y)\,dy$$

$$= k\int_0^1 y^4(1-2y+y^2)(1-y)\,dy$$

$$= k\int_0^1 y^4(1-y-2y+2y^2+y^2-y^3)\,dy$$

$$= k\int_0^1 y^4(1-3y+3y^2-y^3)\,dy$$

$$= k\int_0^1 y^4-3y^5+3y^6-y^7\,dy$$

$$= k\int_0^1 -y^7+3y^6-3y^5+y^4\,dy$$

$$= -k\int_0^1 y^7\,dy + 3k\int_0^1 y^6 - 3k\int_0^1 y^5 + k\int_0^1 y^4\,dy$$

$$= -\frac{K}{8} + \frac{3K}{7} - \frac{K}{2} + \frac{K}{5}$$

$$= \frac{-35K}{280} + \frac{120K}{280} - \frac{140K}{280} + \frac{56K}{280}$$

$$= \frac{1K}{280}$$

$$1 = \frac{K}{280}$$

$\boxed{K = 280}$ THIS IS THE VALUE OF K THAT RENDERS A VALID PDF

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(A) WHAT IS THE VALUE OF k THAT RENDERS THE ABOVE A VALID DENSITY FUNCTION?

(B) FIND THE PROBABILITY THAT AT MOST 50% OF THE FIRMS MAKE A PROFIT IN THE FIRST YEAR

(C) FIND THE PROBABILITY THAT AT LEAST 80% OF THE FIRMS MAKE A PROFIT IN THE FIRST YEAR

50% = 0.5 => $P(Y \le 0.5)$

FOR $0 \le y < 1$, $F(y) = 56y^5(1-y)^3 + 28y^6(1-y)^2 + 8y^7(1-y) + y^8$

$\boxed{P(Y \le 0.5) = 0.3633}$

$\boxed{P(Y > 0.80) = 0.0563}$

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(3.36) ON A LABORATORY ASSIGNMENT, IF THE EQUIPMENT IS WORKING, THE DENSITY FUNCTION OF THE OBSERVED OUTCOME X, IS

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & \text{OTHERWISE} \end{cases}$$

(A) CALCULATE $P(X \le 1/3)$

(B) WHAT IS THE PROBABILITY THAT X WILL EXCEED 0.5?

(C) GIVEN THAT $X \ge 0.5$, WHAT IS THE PROBABILITY THAT X WILL BE LESS THAN 0.75?

WE CAN FIND $P(X \le 1/3)$ BY INTEGRATING THE DENSITY FUNCTION FROM 0 TO 1/3

$$P(a < X < b) = \int_a^b f(x)\,dx$$

$$P(0 < X < 1/3) = \int_0^{1/3} 2(1-x)\,dx = \int_0^{1/3} 2 - 2x\,dx = \left[2x - \frac{2x^2}{2}\right]\Bigg|_0^{1/3} = 2x - x^2\Big|_0^{1/3} = \frac{2}{3} - \frac{1}{9} = \frac{2}{9}$$

$$\boxed{P(X < 1/3) = 2/9}$$

(B) WHAT IS THE PROBABILITY THAT X WILL EXCEED 0.5?

$$P(X > 1/2) = P(1/2 < x < 1) = \int_{1/2}^{1} (2-2x)\,dx = \left[2x - x^2\right]\Big|_{1/2}^{1} = 2-1 + 2(1/2) - (1/2)^2 = 1 - (1 - 1/4)$$

$$= 4/4 - (4/4 - 1/4)$$

$$= 4/4 - 3/4$$

$$= 1/4$$

$$\boxed{P(X > 1/2) = 1/4}$$

(C) GIVEN THAT $X \ge 0.5$, WHAT IS THE PROBABILITY THAT X WILL BE LESS THAN 0.75?

$$P(0.5 \le X < 0.75) = \int_{0.5}^{0.75} 2 - 2x\,dx = \left[2x - x^2\right]\Big|_{0.5}^{0.75} = \left(2(0.75) - (0.75)^2\right) - \left(2(0.5) - (0.5)^2\right) = 0.1875$$

$$\boxed{P(0.5 \le x < 0.75) = 0.1875}$$

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(3.38) IF THE JOINT PROBABILITY DISTRIBUTION OF X & Y IS GIVEN BY

$$f(x,y) = \frac{x+y}{30}, \quad \text{FOR } x = 0, 1, 2, 3; \quad y = 0, 1, 2$$

FIND

(A) $P(X \le 2,\ Y = 1)$;
(B) $P(X > 2,\ Y \le 1)$;
(C) $P(X > Y)$;
(D) $P(X + Y = 4)$;

(A) $P(X \le 2,\ Y = 1) = 0.2$

$$P(X \le 2, Y = 1) = P(X=0, Y=1) + P(X=1, Y=1) + P(X=2, Y=1)$$
$$= f(0,1) + f(1,1) + f(2,1) = \frac{0+1}{30} + \frac{1+1}{30} + \frac{2+1}{30}$$
$$= 1/30 + 2/30 + 3/30 = 6/30$$
$$= 0.2$$

(B) $P(X > 2,\ Y \le 1) = 0.4$

$$P(X > 2, Y \le 1) = P(X=2, Y=0) + P(X=2, Y=1) + P(X=3, Y=0) + P(X=3, Y=1)$$
$$= f(2,0) + f(2,1) + f(3,0) + f(3,1)$$
$$= \frac{2+0}{30} + \frac{2+1}{30} + \frac{3+0}{30} + \frac{3+1}{30}$$
$$= \frac{2}{30} + \frac{3}{30} + \frac{3}{30} + \frac{4}{30} = \frac{12}{30}$$
$$= 0.4$$

(C) $P(X > Y) = 0.6$

$$P(X > Y) = P(X=3, Y=0) + P(X=3, Y=1) + P(X=3, Y=2) + P(X=2, Y=0) + P(X=2, Y=1) + P(X=1, Y=0)$$
$$= f(3,0) + f(3,1) + f(3,2) + f(2,0) + f(2,1) + f(1,0)$$
$$= \frac{3+0}{30} + \frac{3+1}{30} + \frac{3+2}{30} + \frac{2+0}{30} + \frac{2+1}{30} + \frac{1+0}{30}$$
$$= 3/30 + 4/30 + 5/30 + 2/30 + 3/30 + 1/30 = 18/30$$
$$= 0.6$$

(D) $$P(X + Y = 4) = P(2 + 2 = 4) + P(1 + 3 = 4)$$
$$= f(2,2) + f(1,3) = \frac{2+2}{30} + \frac{1+3}{30} = \frac{4}{30} + \frac{4}{30} = \frac{8}{30}$$

HW ~~5~~, ~~7~~, ~~13~~, ~~14~~, ~~17~~, ~~33~~, ~~36~~, ~~38~~, 41, 43, 45, 47, 56

(3.41) A CANDY COMPANY DISTRIBUTES BOXES OF CHOCOLATES WITH A MIXTURE OF CREAMS, TOFFEES, & CORDIALS. SUPPOSE THAT THE WEIGHT OF EACH BOX IS 1 KILOGRAM, BUT THE INDIVIDUAL WEIGHTS OF THE ICE CREAMS, TOFFEES, & CORDIALS VARY FROM BOX TO BOX. FOR A RANDOMLY SELECTED BOX, LET X AND Y RERESENT THE WEIGHTS OF THE CREAMS & THE TOFFEES, RESPECTIVELY, & SUPPOSE THAT THE JOINT DENSITY FUNCTION OF THESE VARIABLES IS

$$f(x,y) = \begin{cases} 24xy, & 0 \le x \le 1,\ 0 \le y \le 1,\ x + y \le 1 \\ 0, & \text{ELSEWHERE} \end{cases}$$

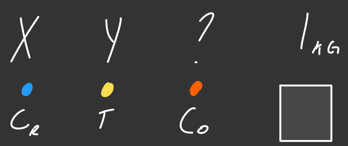
(A) FIND THE PROBABILITY THAT IN A GIVEN BOX THE CORDIALS ACCOUNT FOR MORE THAN 1/2 OF THE WEIGHT.

(B) FIND THE MARGINAL DENSITY FOR THE WEIGHT OF THE CREAMS

(C) FIND THE PROBABILITY THAT THE WEIGHT OF THE TOFFEES IN A BOX IS LESS THAN 1/8 OF A KILOGRAM IF IT IS KNOWN THAT CREAMS CONSTITUTE 3/4 OF THE WEIGHT.

(A) FIND THE PROBABILITY THAT IN A GIVEN BOX THE CORDIALS ACCOUNT FOR MORE THAN 1/2 OF THE WEIGHT.

SINCE X IS THE WEIGHT OF THE CREAMS,
Y IS THE WEIGHT OF THE TOFFEES
TOTAL WEIGHT OF THE BOX IS 1 KG



THE WEIGHT OF CORDIALS IS $(1 - X - Y)$ & TO DETERMINE THAT IT'S GREATER THAN 1/2

$$P(1 - x - y > 1/2) = P(1 - x - y > 1/2) = P((-1)(-x-y) > (-1/2)(-1))$$

$$P(x + y < 1/2) = \int_0^{1/2}\int_0^{1/2 - x} f(x,y)\,dy\,dx = \int_0^{1/2}\int_0^{1/2-x} 24xy\,dy\,dx = \int_0^{1/2} 24x \int_0^{1/2-x} y\,dy\,dx$$

$$= \int_0^{1/2} 24x\left[\frac{y^2}{2}\right]\Bigg|_0^{1/2-x} dx$$

$$= \int_0^{1/2} x\left[12y^2\right]\Bigg|_0^{1/2-x} dx$$

$$= \int_0^{1/2} x\left[12\left(\frac{1}{2} - x\right)^2\right] dx$$

$$= \int_0^{1/2} 3x - 12x^2 + 12x^3\,dx$$

$$= \left[\frac{3x^2}{2} - \frac{12x^3}{3} + \frac{12x^4}{4}\right]\Bigg|_0^{1/2}$$

$$= \frac{3(1/2)^2}{2} - \frac{12(1/2)^3}{3} - \frac{12(1/2)^4}{4}$$

$$= \frac{3}{8} - \frac{1}{2} + \frac{3}{16}$$

$$= \frac{1}{16}$$

$$\boxed{P(x + y < 1/2) = 1/16}$$

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(3.41) A CANDY COMPANY DISTRIBUTES BOXES OF CHOCOLATES WITH A MIXTURE OF CREAMS, TOFFEES, & CORDIALS. SUPPOSE THAT THE WEIGHT OF EACH BOX IS 1 KILOGRAM, BUT THE INDIVIDUAL WEIGHTS OF THE ICE CREAMS, TOFFEES, & CORDIALS VARY FROM BOX TO BOX. FOR A RANDOMLY SELECTED BOX, LET X AND Y RERESENT THE WEIGHTS OF THE CREAMS & THE TOFFEES, RESPECTIVELY, & SUPPOSE THAT THE JOINT DENSITY FUNCTION OF THESE VARIABLES IS

$$f(x,y) = \begin{cases} 24xy & , \quad 0 \le x \le 1,\ 0 \le y \le 1,\ x + y \le 1 \\ 0 & , \quad \text{ELSEWHERE} \end{cases}$$

(B) FIND THE MARGINAL DENSITY FOR THE WEIGHT OF THE CREAMS

$$g(x) = \int_{-\infty}^{\infty} 2/3\,(x + 2y)\,dy = \int_0^{1-x} 24xy\,dy = 24x\left(\frac{y^2}{2}\right)\Bigg|_0^{1-x} = 24x\left(\frac{(1-x)^2}{2}\right) = 12x(1-x)^2$$

$$\boxed{g(x) = 12x(1-x)^2 \quad \text{FOR } 0 \le x \le 1}$$

(C) FIND THE PROBABILITY THAT THE WEIGHT OF THE TOFFEES IN A BOX IS LESS THAN 1/8 OF A KILOGRAM IF IT IS KNOWN THAT CREAMS CONSTITUTE 3/4 OF THE WEIGHT.

CONDITIONAL DISTRIBUTION $\boxed{P(Y < 1/8 \mid X = 3/4) = 1/4}$

$$P(0 < y < 1/8 \mid x = 3/4) = \int_0^{1/8} f(y \mid 3/4)\,dy = \int_0^{1/8} \frac{f(3/4, y)}{g(3/4)}\,dy = \int_0^{1/8} \frac{24 \cdot 3/4\, y}{12 \cdot 3/4\,(1 - 3/4)^2}\,dy$$

$$= \int_0^{1/8} \frac{18y}{9 \cdot (1/4)^2}\,dy = \left[34\left(\frac{y^2}{2}\right)\right]\Bigg|_0^{1/8}$$

$$= 32 \cdot (1/8)^2/2 = 1/4$$

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(3.43) LET X DENOTE THE REACTION TIME, IN SECONDS TO A CERTAIN STIMULUS & Y DENOTE THE TEMPERATURE °F AT WHICH A CERTAIN REACTION STARTS TO TAKE PLACE. SUPPOSE THAT TWO RANDOM VARIABLES X & Y HAVE JOINT DENSITY, FIND

$$f(x,y) = \begin{cases} 4xy, & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{ELSEWHERE} \end{cases}$$

(A) $P(0 \le X \le 1/2 \ \&\ 1/4 \le Y \le 1/2)$

$$\int_0^{1/2}\int_{1/4}^{1/2} f(x,y)\,dy\,dx = \int_0^{1/2}\int_{1/4}^{1/2} 4xy\,dy\,dx = \int_0^{1/2} 4x\int_{1/4}^{1/2} y\,dy\,dx = \int_0^{1/2} x\left[\frac{4y^2}{2}\right]\Bigg|_{1/4}^{1/2} dx$$

$$= \int_0^{1/2} x\left[\frac{4(1/2)^2}{2} - \frac{4(1/4)^2}{2}\right] dx$$

$$= \int_0^{1/2} x\left[\frac{1}{4} - \frac{1}{16}\right] dx$$

$$= \int_0^{1/2} x\left[\frac{4}{16} - \frac{1}{16}\right] dx$$

$$= \int_0^{1/2} \frac{3}{16} x\,dx$$

$$= \frac{3}{16}\int_0^{1/2} x\,dx$$

$$= \frac{3}{16}\left[\frac{x^2}{2}\right]\Bigg|_0^{1/2}$$

$$= \frac{3}{16}\left(\frac{(1/2)^2}{2} - \frac{0^2}{2}\right)$$

$$= \frac{3}{64}$$

$$\boxed{P(0 \le X \le 1/2 \ \&\ 1/4 \le Y \le 1/2) = \frac{3}{64}}$$

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(3 45) LET X DENOTE THE DIAMETER OF AN ARMORED ELECTRIC CABLE.
BOTH X & Y ARE SCALED SO THAT THEY RANGE BETWEEN 0 & 1
SUPPOSE THAT X & Y HAVE THE JOINT DENSITY FUNCTION

$$f(x,y) = \begin{cases} 1/y, & 0 < x < y < 1 \\ 0, & \text{ELSEWHERE} \end{cases}$$

FIND $P(x+y > 1/2) \Rightarrow P(x + y > 1/2)$

$$\Rightarrow \quad -y \quad -y$$

$$\Rightarrow P(x > 1/2 - y)$$

WE OBSERVE THAT FOR $0 \le y \le 1/4$ WE HAVE $1/2 - y \ge y$
SO THE FIRST INTEGRAL IS EQUAL TO ZERO $\int_0^1$
THIS IS BECAUSE $y \in [0, 1/4]$ & $x + y > 1/2$
THUS, $x > 1/2 - y > 1/2 - 1/4 = 1/4 \ge y$
HOWEVER THE JOINT DENSITY FUNCTION IS 0
WHEREVER $x \ge y$ SO THE INTEGRAL WOULD BE ZERO.

$$P(x+y > 1/2) = \int_{1/4}^{1/2}\int_{1/2-y}^{y} f(x,y)\,dx\,dy + \int_{1/2}^{1}\int_{0}^{y} f(x,y)\,dx\,dy$$

$$= \int_{1/4}^{1/2}\int_{1/2-y}^{y} \frac{1}{y}\,dx\,dy + \int_{1/2}^{1}\int_{0}^{y} \frac{1}{y}\,dx\,dy$$

$$= \int_{1/4}^{1/2} \frac{x}{y}\Big|_{1/2-y}^{y}\,dy + \int_{1/2}^{1}\left[\frac{x}{y}\right]\Big|_0^y\,dy$$

$$= \int_{1/4}^{1/2}\left[\frac{y}{y} - \frac{1/2 - y}{y}\right]dy + \int_{1/2}^{1}\left[\frac{y}{y} - \frac{0}{y}\right]dy$$

$$= \int_{1/4}^{1/2}\left[\frac{y}{y} - \frac{1}{2y} - \frac{y}{y}\right]dy + \int_{1/2}^{1} 1\,dy$$

$$= \int_{1/4}^{1/2} -\frac{1}{2y}\,dy + [y]\Big|_{1/2}^{1}$$

$$= \left[\frac{-\ln(y)}{2}\right]\Big|_{1/4}^{1/2} + (1/2)$$

$$= \frac{-\ln(1/2)}{2} + \frac{\ln(1/4)}{2} + 1/2$$

$$= \frac{-1}{2}\left(\ln(1/4) - \ln(1/2)\right) + \frac{1}{2}$$

$$= \frac{-1}{2}\left(\ln\left(\frac{1/2}{1/4}\right)\right) + \frac{1}{2}$$

$$\boxed{P(x+y > 1/2) = 1 - \frac{\ln(2)}{2}}$$

H ~~5, 7, 13, 14, 17, 33, 36, 38, 41, 43, 45~~, 47, 56

(3.47) THE AMOUNT OF KEROSENE, IN THOUSANDS OF LITERS, IN A TANK AT THE BEGINNING OF ANY DAY IS A RANDOM AMOUNT Y FROM WHICH A RANDOM AMOUNT X IS SOLD DURING THAT DAY. SUPPOSE THAT THE TANK IS NO RESUPPLIED DURING THE DAY SO THAT $X \leq Y$, & ASSUME THAT THE JOINT DENSITY FUNCTION OF THESE VARIABLES IS,

$$f(x,y) = \begin{cases} 2, & 0 < x \leq y < 1 \\ 0, & \text{ELSEWHERE} \end{cases}$$

(A) DETERMINE IF X & Y ARE INDEPENDENT
(B) FIND $P(1/4 < X < 1/2 \mid Y = 3/4)$

IF $f(x|y)$ THEN $f(x|y) = g(x)$ & $f(x,y) = g(x)h(y)$ IS TRUE

ASSUME X & Y ARE INDEPENDENT YOU MUST FIRST FIND THEIR MARGINAL DENSITIES

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_x^1 2\,dy = 2y\Big|_x^1$$

$$g(x) = 2 - 2x$$

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^y 2\,dx = (2x)\Big|_0^y$$

$$h(y) = 2y$$

$$\boxed{\begin{gathered} f(x,y) = g(x)\,h(y) \\ 2 \neq (2-2x)(2y) \\ \text{THUS, } X \text{ \& } Y \text{ ARE NOT INDEPENDENT} \end{gathered}}$$

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SUN FEB 12 2022

H 5, 7, 13, 14, 17, 33, 36, 38, 41, 43, 45, 47, 56

(4.56) THE JOINT DENSITY FUNCTION OF THE RANDOM VARIABLE X & Y IS

$$f(x,y) = \begin{cases} 6x, & 0 < x < 1,\ 0 < y < 1 < 1-x \\ 0, & \text{ELSEWHERE} \end{cases}$$

(B) FIND $P(X > 0.3 \mid Y = 0.5)$

$$P(X > 0.3 \mid Y = 0.5) = P(0.3 < X < 1 - 0.5 \mid Y = 0.5)$$

$$= \int_x f(x \mid 0.5)\, dx$$

$$= \int_{x=0.3}^{0.5} \left[\frac{f(x, 0.5)}{h(x, 0.5)} \right] dx$$

$$= \int_{0.3}^{0.5} \frac{6x}{3(1-0.5)^2}\, dx$$

$$= \int_{0.3}^{0.5} 8x\, dx$$

$$= (4x^2) \Big|_{0.3}^{0.5}$$

$$= 4 \cdot (0.5^2) - 4 \cdot 0.3^2$$

$$\boxed{P(X > 0.3 \mid Y = 0.5) = 0.64}$$