Mancala Game Theory

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September 2022

1 Introduction

Game Theory is a branch of mathematics that focuses on the analysis of strategies for situations in which competitors make decisions that are interdependent. It is useful for making decisions in cases where two or more decision makers have conflicting interests. It seeks to find the optimal gaming strategy to win a game.

Within the branch of Game Theory, we will be taking a look at a zero-sum two-person game. A zero-sum two-person game is a game that consists of two players, called the *row* player and the *column* player. The row player must select 1 of their p strategies and the column player must select 1 of their q strategies concurrently. If the row player selects their mth strategy and the column player selects their nth strategy, then the row player receives reward a_{mn} and the column player loses the amount a_{mn}. Thus, the row player's reward a_{mn} comes from the column player [3]. For games like this, the sum of the rewards to the players is zero, meaning that both the row and column player's have conflicting interests. Hence, zero-sum games are situations where one person's gain is the other person's loss.

2 Rules & Strategies

2.1 The Game and Rules

Mancala is a game that involves two players in which one player's advantage is the other player's disadvantage. In game theory, this is called a zero-sum two-person game. The Mancala game board consists of twelve pits and two wells, or 'Mancala' pits. Each player has six small pits and one Mancala. Depending on the game, each pit can start with anywhere from one to four marbles. Figure 1 is a standard Mancala game board.

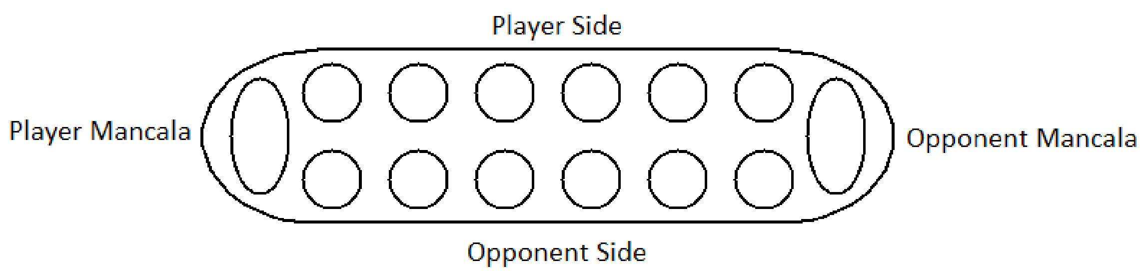

Figure 1: A standard Mancala game board. The pits are the small circles, and the Mancalas are the ovals labeled accordingly.

There are a couple different ways to play Mancala: avalanche and capture. In this paper, we will focus on the capture method. Starting on a player's turn, they choose a pit to take the marbles from, and deposit a single marble into each pit counter-clockwise in order on that player's side. Capture Mancala has these standard rules [2]:

1. If you pass your own Mancala, deposit one marble in it.
2. If you pass your opponent's Mancala, skip it.
3. If the last marble you drop is in your own Mancala, you get another turn.
4. If the last marble you drop is in an empty pit on your side, you capture that marble and any marbles in your opponents opposite adjacent pit.

5. Always place all captured marbles into your Mancala.

6. Without loss of generality, if Player's side no longer has marbles after their turn ends, the remaining marbles in play are immediately placed in Opponent's Mancala.

While a standard Mancala game normally has four marbles in each pit (a total of 48 marbles), for the purposes of game play and potential strategies, we will look at a Mancala game that only has one marble in each pit (a total of 12 marbles). Thus, this section will analyze the game play with only one marble in each pit. Later, we will discuss the extensions of Mancala game play when more marbles are incorporated.

3 Optimal Strategy (12-Marble Game)

3.1 Using a Decision Tree

In a Mancala game only using 12 marbles, there is one opening move that each player will normally make: move the first pit, and then the second in order to capture a marble. This results in each player having three marbles in their own Mancala, and a single Marble in pits 3 through 5 as shown in Figure 2. From this new starting point, the question then becomes which pit will result in the best chances of Player winning, and thus our three strategies are found.

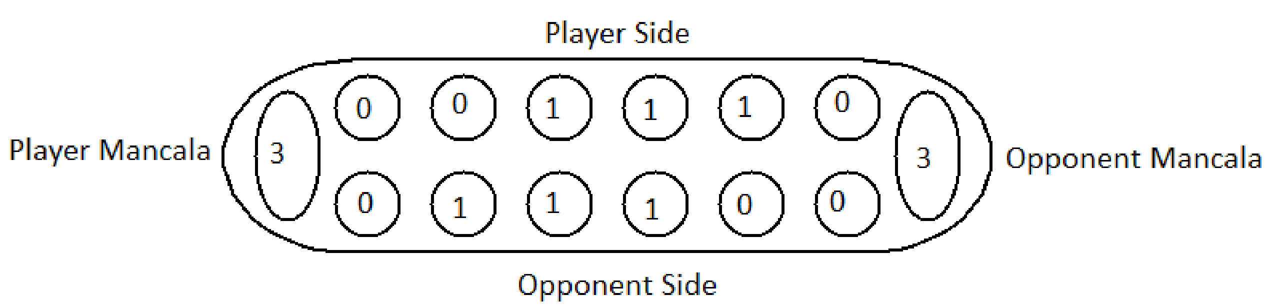

Figure 2: The new starting game. This is assuming both Player and Opponent make the same opening moves for moving pit 1, then pit 2.

Assuming Player starts first, the game tree in Figure 3 will be Player's options and the resulting game board for each option. These are the strategies that Player can choose from. Strategy 1 will be the event where Player chooses to move pit 3, Strategy 2 will be the choice to move pit 4, and Strategy 3 is the choice to move pit 5.

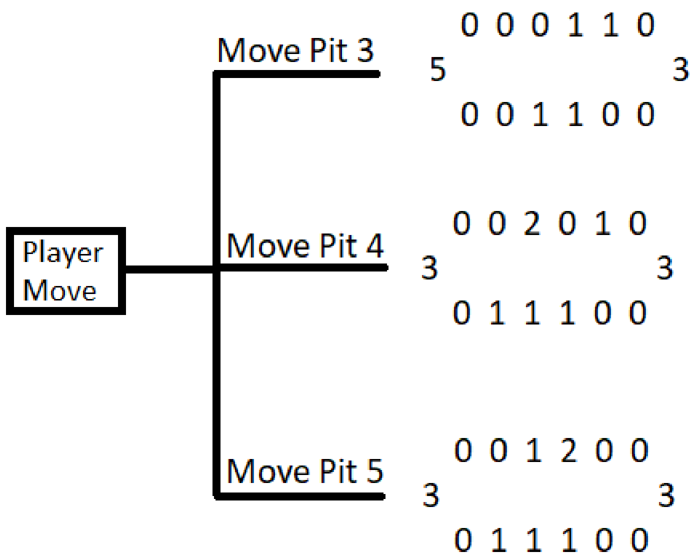

Figure 3: Player's options for their first round. The resulting Mancala game board is to the right of each option.

3.1.1 Strategy 1

Strategy 1 is the event where Player chooses to move pit 3. When Player makes this choice, Opponent is left with only two options. Figure 4 is a Game tree of the possible moves for Opponent to make, and the resulting outcomes of each choice. Opponent can either move pit 3 or pit 4, but no matter which pit Opponent chooses to move, Player will always win the game. This means with Strategy 1, there is a 100% chance that Player will win the game.

There is no need for a calculation for Strategy 1, but for clarity assume there is an equal chance that Opponent with make the choice to move either pit 3 or pit 4. This means with probability $p = 0.5$ Opponent makes their choices. Now, assigning a value of 1 to a favorable outcome for Player (1 for winning, 0 for losing, and 0 for tying), the chances of Player winning ($P(W)$) can be found:

$$P(W) = 0.5(1) + 0.5(1) = 1$$

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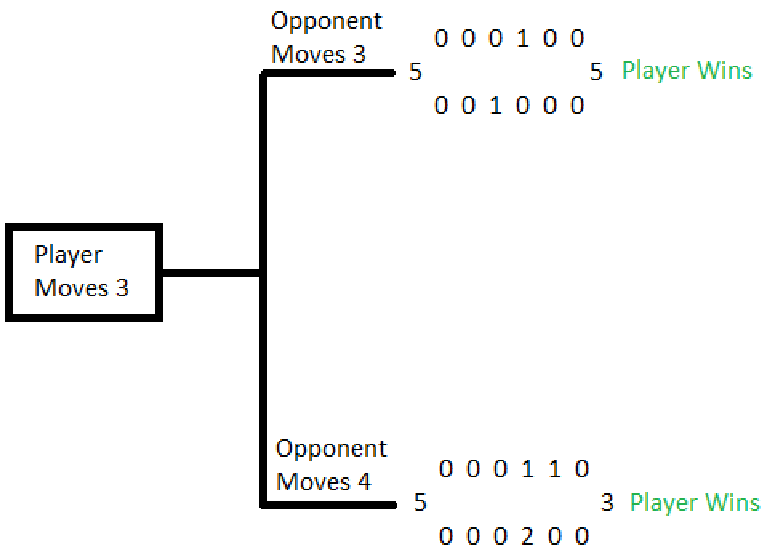

Figure 4: Opponent's options for their first round assuming Player chose to use Strategy 1. At the end of each branch is the resulting game board.

3.1.2 Strategy 2

Now, if Player chooses to move pit 4, Opponent has a few options. Figure 5 outlines the options and the resulting probabilities of Player winning. The game tree is truncated after four rounds of the game for legibility; the game tree continues for up to 30 branches. At the end of each branch, the chances of Player winning is given. These totals were found by using the same process outlined in Section 3.1.1.

To find the chances of Player winning, it is easier to look at each of Opponent's first options' branches individually in Figure 5. If Opponent chooses to move pit 3, there is a 0% chance of Player winning ($\frac{1}{3}(0) = 0$). If Opponent chooses to move pit 4, there is a 29.33%

chance of Player winning $(\frac{1}{3}((\frac{1}{2}(0) + \frac{1}{2}(\frac{1}{2}(\frac{1}{3})(0.459 + 0.582 + 0.5)) + (\frac{1}{2}(\frac{1}{3})(0.25 + 0.3333 + 0.214))) = 0.2933)$. Lastly, if Opponent chooses to move pit 5, there is a 16.67% chance of Player winning $(\frac{1}{3}(\frac{1}{2}(0) + frac12(1)) = 0.1667)$. Summing up these chances give Player a 46% chance of winning.

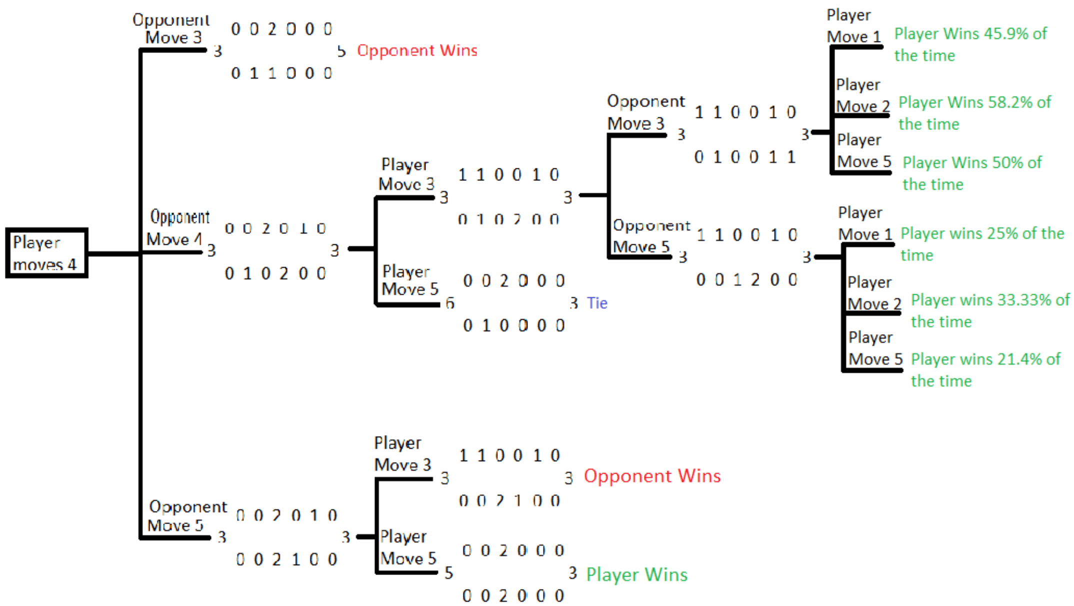

Figure 5: A game tree for all the possible outcomes for strategy 2. The tree is truncated for legibility. At the end of each branch is the chances of Player winning if they continue with that move.

3.1.3 Strategy 3

If player chooses to move pit 5, Opponent has three options. Figure 6 outlines the options and the resulting chances of Player winning. Using the same process outlined in sections 3.1.1 and 3.1.2, Player has a 25% chance of winning if Opponent moves pit 3 $(\frac{1}{3}(\frac{1}{2}(\frac{1}{2}(1)+\frac{1}{2}(0))+\frac{1}{2}(1)) = 0.25)$. Player then has a 33.33% chance of winning if Opponent moves pit 4, and a 0% chance of winning if Opponent moves pit 5. Summing up these chances gives Player a 58.33% chance of winning.

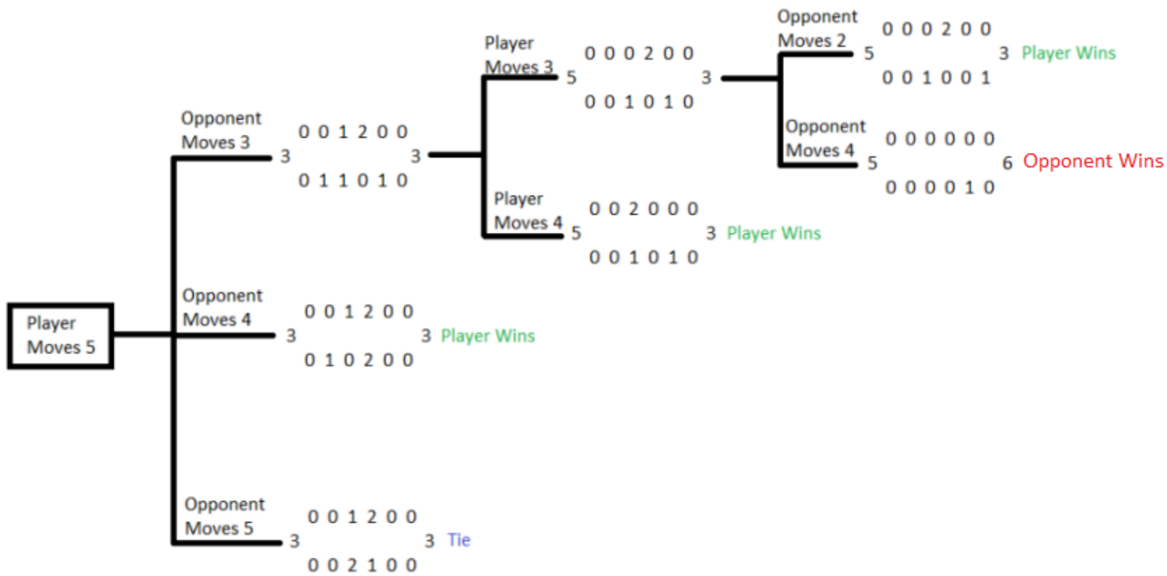

Figure 6: A game tree for all possible outcomes for strategy 3.

3.2 Results

The results of the three different strategies are as follows:

1. Strategy 1: Player has a 100% chance of winning.

2. Strategy 2: Player has a 46% chance of winning.

3. Strategy 3: Player has a 58.33% chance of winning.

Clearly, the optimal gaming strategy is for Player to move pit 3. This forces Opponent one of two options, and both result in Player gaining and maintaining the lead.

4 24-Marble Games

While the superior strategy for a 12-marble game was to move the marble in our first pit and gain an extra move, we will consider a different play strategy for the more complex 24-marble game.

It is important to note, however, that these decision trees become very convoluted. For this analysis, we will consider each player's best moves (to maximize the marbles in their mancala, either by multiple turns or captures) rather than showing all potential moves.

4.1 Strategy 1:

In this strategy, Player will move the two marbles from their first pocket; this will maximize their potential for multiple moves on their next turn. Regardless how Opponent moves (considering the two best moves from Opponent, see Figure 7), Player will gain six marbles from multiple turns and a capture (see Figure 8), and Opponent will follow with moving marbles from pit 3.

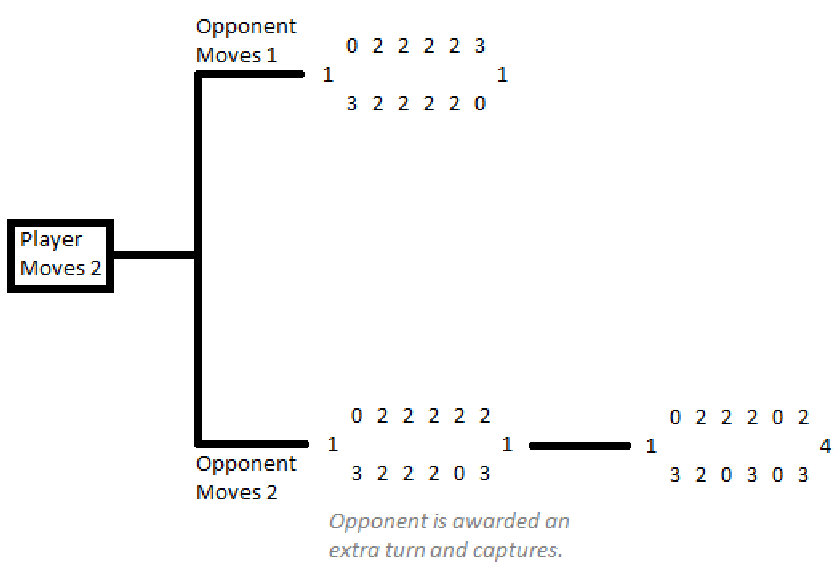

Figure 7: Opponent either mirrors Player or gains an extra turn.

While this move optimizes the marbles in Opponent's mancala, it is a misplay; Player should be able to maintain some marbles in their pits while Opponent moves their marbles to Player's side. Remember rule 6: any remaining marbles in play after one side is cleared goes to their mancala. Regardless of how Opponent moves their marbles, Player 1 will win by a slight margin.

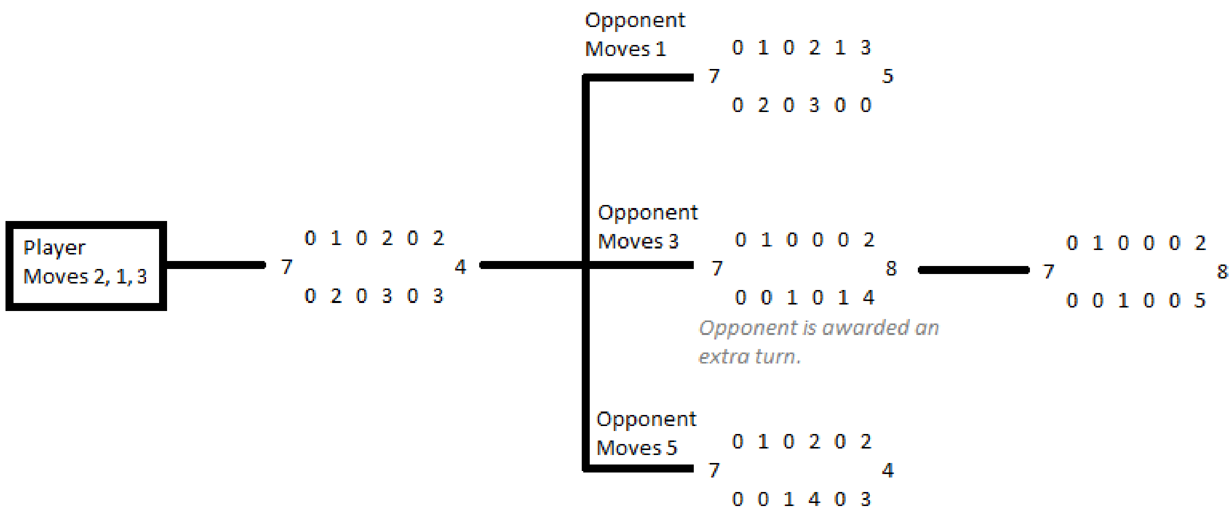

Figure 8: Opponent should move marbles from pit 3 to maximize their marbles in their mancala.

4.2 Strategy 2:

Now, consider the optimal strategy for the 12-marble game: replay then capture. After Player moves the marbles from pit 2 then pit 4 (to capture), Opponent should mirror Player's moves as they did in the 12-marble game [1]. See Figure 9.

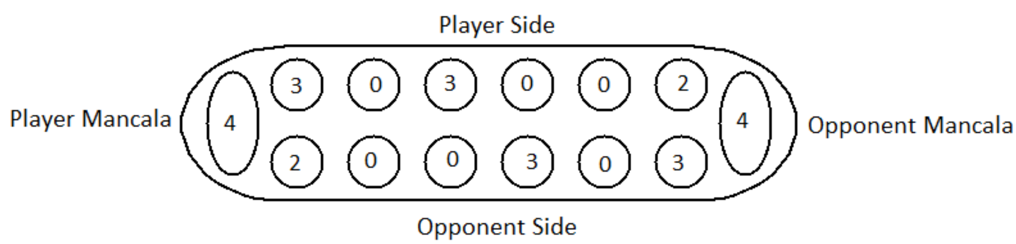

Figure 9: 24-marble, strategy 2 resulting game board.

It is clear that Player should move the marbles from pit 3 for the extra turn then capture once more. Regardless of Opponent's move, Player should stall the best they can and try not to move any marbles to Opponent's side.

There are two main plays that can continue the game: Opponent moves marbles from pit 1 or 6. See Figure 10 for when Opponent moves from pit 1 and see Figure 11.

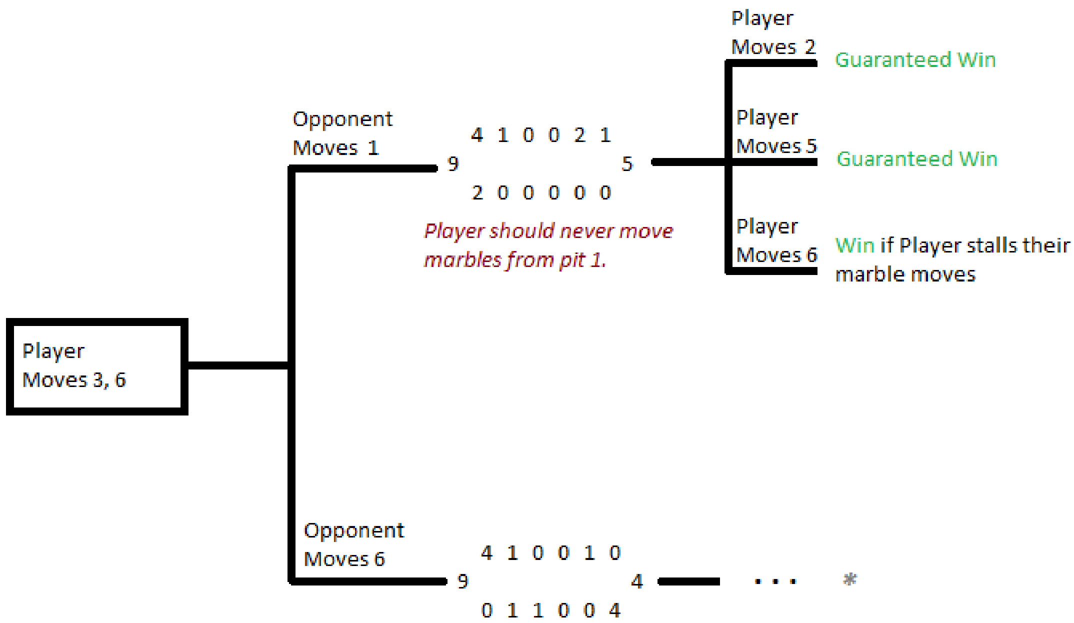

Figure 10: Continued sequence when Opponent moves from pit 1

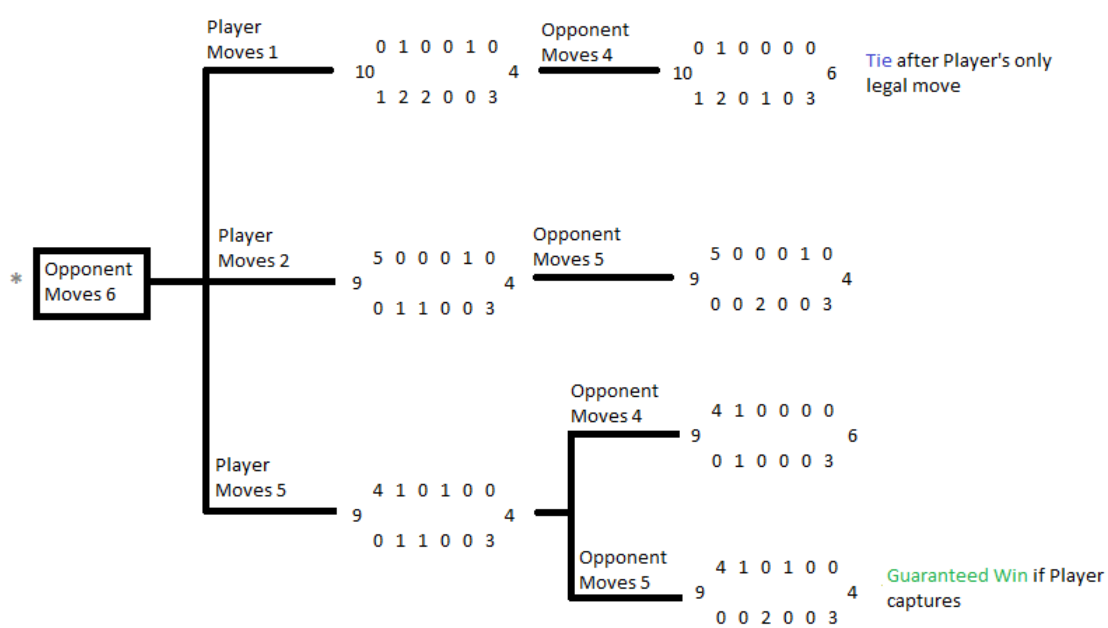

Figure 11: Continued sequence when Opponent moves from pit 6

It's somewhat easy to see that, dependent on Player's move, this will greatly influence the remainder of the game. If we continue the trees focusing on Player's "best" move (to maximize marbles in their mancala), i.e. moving from pit 1, it will result in a tie – which ultimately is not the best strategy.

It is more difficult, however, to determine the superior/optimal strategy as the trees become exponentially large for one-marble increases per pit. There are strong sequences of moves but it is nearly impossible to narrow it down to a guaranteed winning sequence. It was easier for this analysis since we forced intermediate goals throughout the game: to maximize replays/marbles in Player's mancala.

Thus, the stronger strategy is option 2, but wins are not guaranteed as they are in the 12-marble case.

5 48-Marble Games

Consider the standard 48-marble mancala game. Once the game has progressed above 24-marble level, the moves that Player chooses become more reliant on the choices that Opponent makes. Thus, it becomes increasingly difficult to plan future moves. Additionally, we should consider that many of the pits will be filled with a lot of marbles, making it difficult to maximize the amount of marbles in you mancala.

As with the 24-marble game, it is important to note that these decision trees become very convoluted. For this analysis, we will consider each player's best moves to maximize the marbles in their mancala, either by multiple turns or captures, rather than showing all potential moves with a decision tree.

When comparing a 48-marble game to the 12-marble and 24-marble games, we will consider a different playing strategy for this even more complex game.

5.1 Optimal Strategy (48-marble game)

We will assume that Player makes the first move of the game. The following table represents all the options Player one can choose:

Pit 1	Optimizes the possibility to have multiple turns during the next round.
Pit 2	Gives Opponent the option for two extra turns.
Pit 3	Gives Opponent the option for an extra turn with no significant advantage to themselves.
Pit 4	Earns an extra turn, in which they can follow up with another strong move.
Pit 5	Does not provide any significant advantage to themself.
Pit 6	Does not provide any significant advantage themself.

Based off the options Player can make, they should either choose to move pit 1 or pit 4.

5.2 Strategy 1:

If Player chooses to move pit 1, they will end up giving up the opportunity for an extra turn in the first round. However, this maximizes the possibility of multiple extra turns in the next round. Once the second round starts, Player will end up with an extra move no matter which pit Opponent chooses to play.

5.3 Strategy 2:

If Player chooses to move pit 4, they will gain an extra turn. In order to remain in the advantage, Player should chose pit 1 on their extra turn. Any other pit would result in an advantage for Opponent, thus a disadvantage for Player. Choosing to move pit 1 will then force Opponent to be defensive, causing Player to gain the advantage.

5.4 Results and Tips

Although it may appear that Player should move pit 1 because it seems like the most advantageous move, Player should actually move pit 4 for their opening move. This leads to a better chance of winning the game.

Along with the opening move, there are some additional strategies that Player should follow during the game rounds. If at all possible, Player should have at most one pit that has more than 5 marbles. It is better to take Opponents marbles than it is to drop one or two in their own mancala. Also, it is preferable to earn as many extra turns as possible in a single round. This maximizes the number of marbles Player will have in their mancala, which is accomplished by keeping pit 1 open.

References

[1] Kelly Beffert. It's your move: Optimal playing strategies for mancala. 2004.

[2] Official Game Rules. Mancala rules. Accessed: October 15, 2022.

[3] Wayne L. Winston. *Operations Research Applications and Algorithms.* Thomson Brooks/Cole, fourth edition, 2003. Accessed: October 15, 2022.