Final Project Report

Derivative Securities - MGT6081

Submitted by:

Akinhanmi, Samuel

Aparicio, Valentina

Hales, Morgan

Sarkari, Zenobia

Date:

05/03/2023

Table of Contents

1. Introduction

Investing in the stock market has always been a popular way of generating wealth. However, it is a challenging task due to the market's high volatility, so investors need to consider a wide variety of factors that can impact their investments. Moreover, to reduce risk and maximize returns, investors create well-diversified portfolios of stocks or ETFs.

In this project, we will select our favorite stocks and ETFs to create a portfolio that generates the highest return for the least variance. Once we have created our portfolio, we will simulate its performance into the future using various models such as Geometric Brownian Motion (GBM), the Merton model, the Constant Elasticity of Variance model (CEV), and the Heston model. Next, we will adjust our portfolio weights again to obtain the best return over all the models. Finally, we will calculate the Sharpe ratio and improve it by adding options. The Sharpe ratio measures the excess return of a portfolio relative to the risk-free rate per unit of volatility. Thus, incorporating options into our portfolio can improve its risk-adjusted return.

Finally, we will analyze which models are more relevant for the anticipated future. As the stock market is subject to various macroeconomic factors, it is crucial to identify which models better account for these factors accurately. By doing so, we can make informed investment decisions to optimize our portfolio and achieve our financial goals.

2. Data

In order to perform the quantitative analysis described above, historical data for our list of favorite stocks were extracted from Yahoo Finance using Python programming. This data is collected for a one-year period between March 1, 2022, and March 31, 2023, for the following stocks:

- **OXY:** Occidental Petroleum Corporation
- **META:** Facebook
- **MARA:** Marathon Digital Holdings
- **DAL:** Delta Air Lines
- **MPC:** Marathon Petroleum Corporation
- **VLO:** Valero Energy Corporation
- **XOM:** Exxon Mobil Corporation
- **BBIO:** BridgeBio Pharma Inc
- **AUPH:** Aurinia Pharmaceuticals Inc
- **QQQ:** Invesco QQQ Trust Series 1

After extracting the closing price after adjustments for all applicable splits and dividend distributions, normalized daily returns are computed and stored in a separate data frame and CSV file.

3. Initial Portfolio Optimization

To begin optimization, we started with an equally weighted portfolio for which we calculated return, variance, and standard deviation (risk). We then proceeded to build 20,000 portfolios using random weights, which we normalized so that they added up to one. We calculated their returns, variances, and standard deviations for each of these portfolios. As a result, we built an efficient frontier representing the set of optimal portfolios that offer the highest expected return for a given

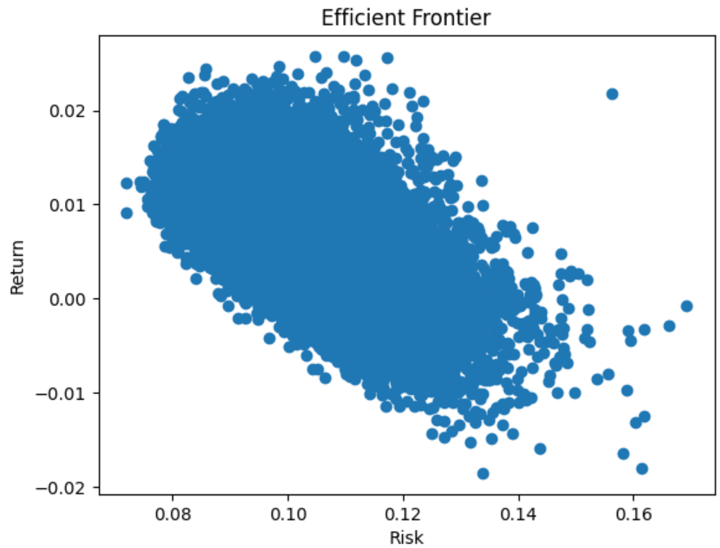

Figure 1: Efficient Frontier

level of risk. In other words, the frontier graphically displays the relationship between risk and return for a portfolio of assets – see our resulting efficient frontier in Figure 1.

Based on the 20,000 portfolios, we found the portfolio with the least variance and calculated its volatility, returns, weights, initial value, and final value – see Figure 2.

```
Least Variance Portfolio Volatility: 0.07325445188155455
Least Variance Portfolio Return: 0.013239209286214053
Least Variance Portfolio Weights: OXY_weight     0.093610
META_weight    0.138631
MARA_weight    0.000427
DAL_weight     0.029937
MPC_weight     0.214999
VLO_weight     0.001836
XOM_weight     0.223833
BBIO_weight    0.012339
AUPH_weight    0.048193
QQQ_weight     0.236195
Name: 14127, dtype: float64
Portfolio Initial Value: 0.018222176876413206
Portfolio Final Value: 0.004251709393868408
```

Figure 2: Least Variance Portfolio Metrics

We also found the portfolio with the highest Sharpe ratio. The Sharpe ratio helps investors assess the return on investment of a portfolio relative to its risk. A high Sharpe ratio indicates that a portfolio's returns are higher for the amount of risk taken, while a low Sharpe ratio indicates that the portfolio is not generating sufficient returns relative to the risk taken. Generally, a Sharpe ratio of greater than one is considered good, while a Sharpe ratio of less than one is considered poor. Moreover, the Sharpe Ratio is equal to *(Portfolio Return - Risk-Free Rate) / Portfolio Standard Deviation*. The resulting portfolio metrics can be found in Figure 3.

```
Highest Sharpe Portfolio Volatility: 0.11132682137730607
Highest Sharpe Portfolio Return: 0.029310162342087442
Highest Sharpe Portfolio Weights: OXY_weight     0.048142
META_weight    0.071292
MARA_weight    0.007270
DAL_weight     0.013165
MPC_weight     0.243142
VLO_weight     0.223050
XOM_weight     0.056481
BBIO_weight    0.289504
AUPH_weight    0.032584
QQQ_weight     0.015370
Name: 16866, dtype: float64
Portfolio Initial Value: 0.014672980629847628
Portfolio Final Value: -0.006175026320801706
```

Figure 3: Highest Sharpe Portfolio Metrics

4. Models

Geometric Brownian Motion (GBM)

Geometric Brownian Motion, also called GBM, is a model often used in finance to simulate stock prices that follow Brownian motion (BM) with drift. BM is a random walk in which the next movement is independent of the previous movement. The drift term represents the growth rate of the stock's price over time, while the diffusion term is the volatility multiplied by the change in Brownian motion, which represents random and unpredictable circumstances that affect the asset price. These terms combine into the following equation:

$$dS = \mu S dt + \sigma S dW_t$$

where drift is shown as *mu*, and diffusion is *sigma*.

In the context of this project, we utilized GBM to simulate forward the price of our ten favorite assets. We used a self-written function to price the stocks and Python's matplotlib library to plot the paths of each asset one-year into the future. These plots are shown below in Figure 4 – note that these plots are not "close enough" to show the usual erratic dynamics of GBM paths.

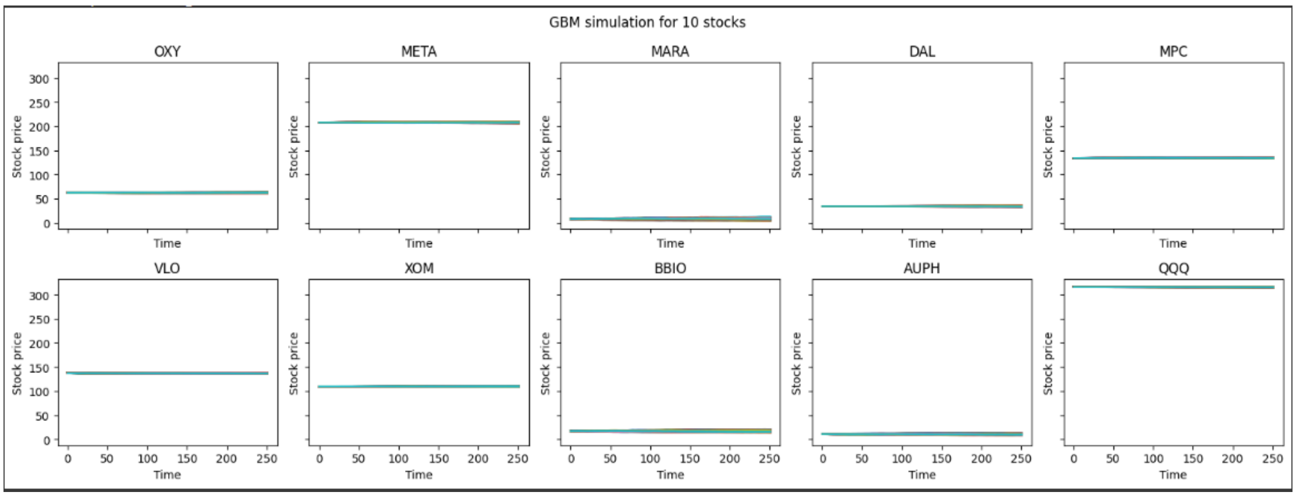

Figure 4: GBM Simulations for 10 Assets

The average price for each of the assets, given by GBM, is summarized in the following table:

OXY	META	MARA	DAL	MPC	VLO	XOM	BBIO	AUPH	QQQ
62.34	207.85	7.92	34.10	133.94	137.02	109.50	17.10	10.97	315.68

Constant Elasticity of Variance (CEV)

An extension of Geometric Brownian Motion is the Constant Elasticity of Variance (CEV) model, which is also used in options pricing. The Black-Scholes model assumes that the volatility of an asset is constant over time, whereas the CEV model allows the volatility to change over time. This change is known as the “elasticity of variance.” Thus, the CEV model follows Geometric Brownian Motion with stochastic volatility. This is useful to price assets that tend to have high degrees of volatility. This ability to account for volatility allows the CEV model to show the impact of the market on option prices. The asset price movement is modeled as follows:

$$\mathrm{d}s_t/s_t = \mu \mathrm{d}t + k s_t^{\gamma} \mathrm{d}w_t$$

Where *w* is the Brownian motion, mu is the drift term, and *y* is the elasticity parameter. The rest of the equation represents the variance of the asset price.

Similar to our GBM application, we used another hand-written function for the CEV model and matplotlib to plot the paths of the assets. The plots are shown below in Figure 5.

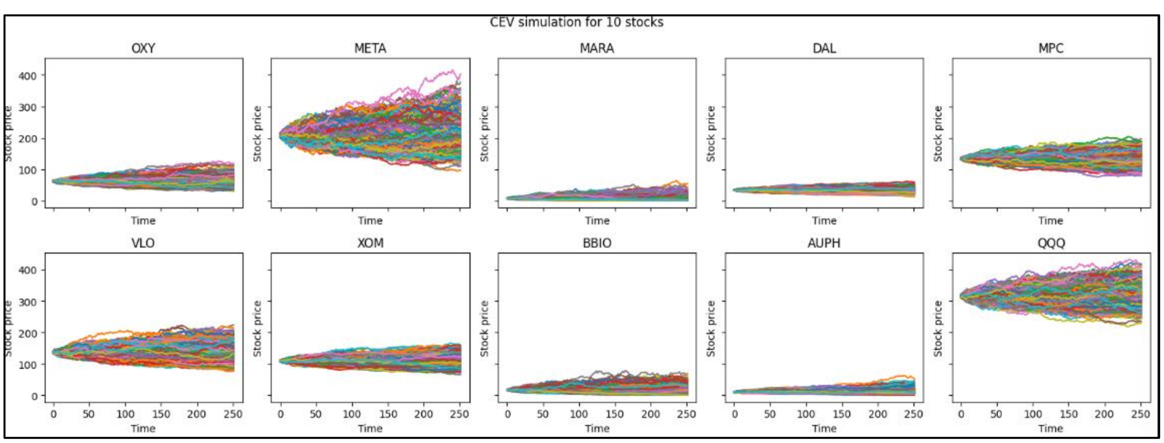

Figure 5: CEV Simulations for 10 Assets

The average price for each of the assets, given by CEV, are summarized in the following table:

OXY	META	MARA	DAL	MPC	VLO	XOM	BBIO	AUPH	QQQ
62.91	208.24	7.92	33.81	133.18	137.15	109.88	17.14	11.07	316.13

Merton Model

The Merton Model is similar to the CEV model because it assumes the asset follows Geometric Brownian Motion with constant drift and stochastic volatility. The difference is that the Merton model also assumes that the "company" has debt. Merton argues that the equity holders have a call option on the company's assets, with a strike price K. If the assets are valued at greater than K, the equity holders are paid the difference. If the assets are less than K, the holders get nothing. Bondholders are said to own a zero-coupon bond, and when the assets are worth more than K, they receive K. Otherwise, the bondholders only receive the asset value. In this case, bondholders are short a put option.

The Merton Model is an improvement on the previous models, accounting for the debt of the company issuing the asset, which allows for more accurate estimations of the option price.

The Merton Model follows the following equation, which values the equity of the assets, taking the value of debt into account:

$$E = V_t \cdot N(d_1) - K \cdot e^{-r \cdot \Delta T} \cdot N(d_2)$$

$$d_1 = \frac{ln\frac{V_t}{K} + (r + \frac{\sigma_v^2}{2}) \cdot \Delta T}{\sigma_v \cdot \sqrt{\Delta T}}$$

$$d_2 = d_1 - \sigma_v \cdot \sqrt{\Delta T}$$

The drawback of this formula is that the value and volatility of the assets are not observable, only the equity and its volatility. However, the following equation can be used to iteratively estimate the value of assets:

$$E \cdot \sigma_E = V \cdot \sigma_V \cdot N(d_1)$$

For our simulation purposes, we followed the same modeling procedure as before by writing a custom function to plot jump paths on matplotlib. The resulting plots are shown in Figure 6 below.

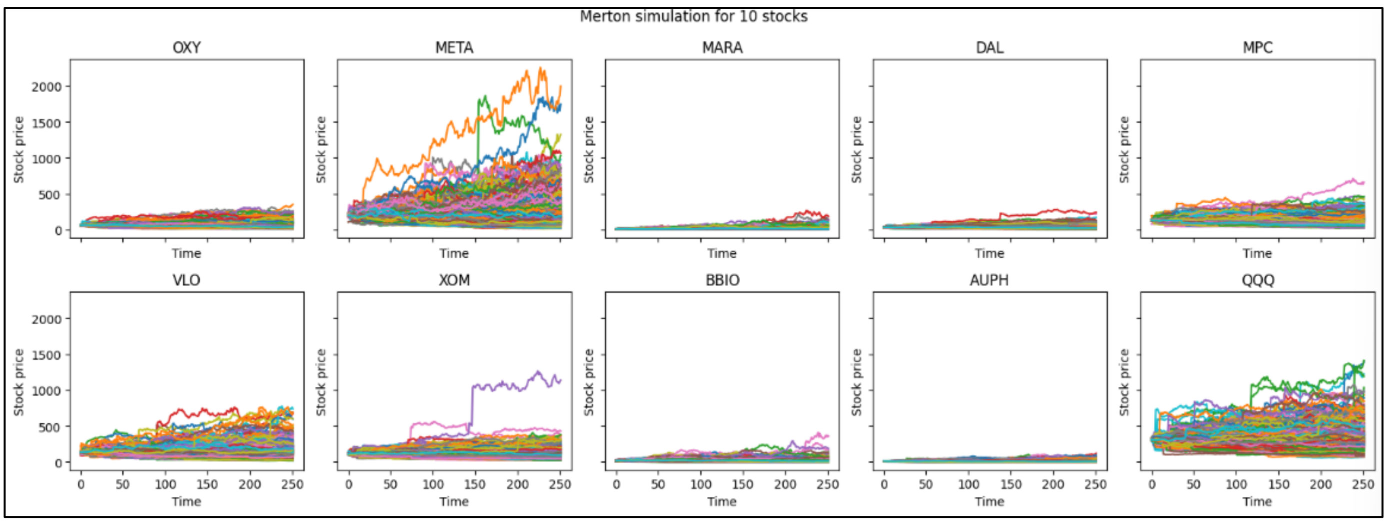

Figure 6: Merton Simulations for 10 Assets

The average price for each of the assets, given by the Merton model, is summarized in the following table:

OXY	**META**	**MARA**	**DAL**	**MPC**	**VLO**	**XOM**	**BBIO**	**AUPH**	**QQQ**
50.37	199.70	14.40	32.42	84.12	194.43	121.32	18.52	10.32	456.54

Heston Model

The Heston model is an extension of the previously mentioned models. When the Black-Scholes model assumes constant volatility, the Heston Model has two Brownian Motions. The first motion is the underlying asset price, while the second represents its volatility. The two are negatively correlated, with a correlation parameter to show the relationship between the two stochastic processes. These processes are shown in the formulas below:

$$dS_t = rS_t dt + \sqrt{V_t} S_t dW_{1t}$$
$$dV_t = k(\theta - V_t)dt + \sigma \sqrt{V_t} dW_{2t}$$

$$W_t^S * W_t^v = \rho dt$$

Once again, we used the same procedure as the other models and plotted the price simulations of the assets. The resulting plots are shown in Figure 7 below.

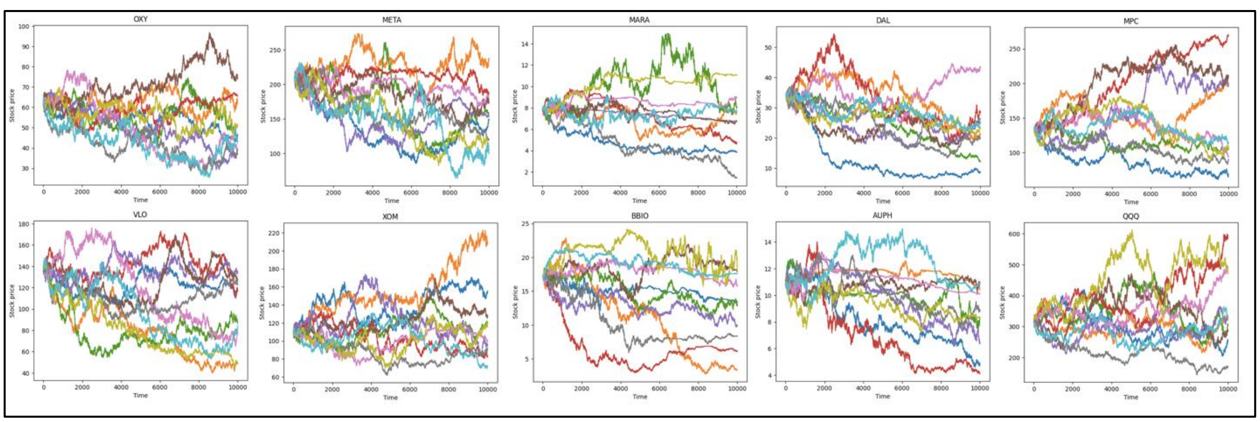

Figure 7: Heston Simulations for 10 Assets

The average price for each of the assets, given by the Heston model, is summarized in the following table:

OXY	META	MARA	DAL	MPC	VLO	XOM	BBIO	AUPH	QQQ
50.21	156.94	6.70	23.46	146.18	96.58	116.52	12.35	8.31	354.33

5. Final Portfolio Optimization

After applying various models such as Geometric Brownian Motion (GBM), Constant Elasticity of Variance (CEV), Merton, and Heston, we calculated the final average prices for each of the assets in our portfolio – that is, we took the average of the 10,000 ending asset prices of each stock. To optimize our portfolio further, we repeated the steps in the initial portfolio optimization using these average prices to calculate future returns, future variances, and future volatilities. We then calculated another efficient frontier of 20,000 portfolios – see Figure 8 – and then identified the portfolio with the highest Sharpe ratio – see highest Sharpe portfolio metrics below in Figure 9.

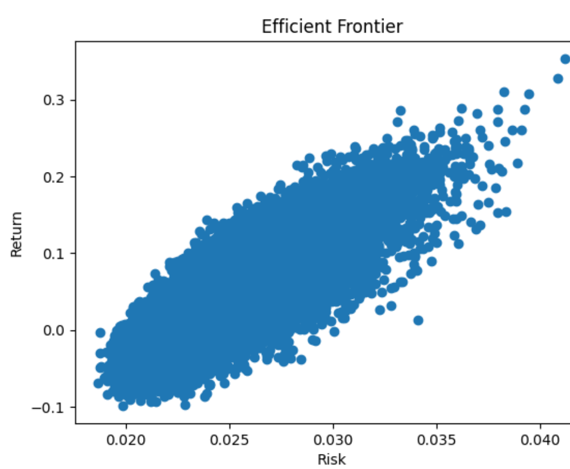

Figure 8: Efficient Frontier from 2nd Optimization

```
Highest Sharpe Portfolio Volatility: 0.0856640625690249
Highest Sharpe Portfolio Return: 0.024429200378842886
Highest Sharpe Portfolio Weights: OXY_weight     0.127280
META_weight    0.078654
MARA_weight    0.003367
DAL_weight     0.021549
MPC_weight     0.283760
VLO_weight     0.283437
XOM_weight     0.065495
BBIO_weight    0.052886
AUPH_weight    0.052655
QQQ_weight     0.030917
Name: 18552, dtype: float64
0.16843936589180994
```

Figure 9: Highest Sharpe Portfolio Metrics

As our final step, we added suitable options to improve the portfolio's Sharpe ratio. To do so, we priced ten European call options using the Black-Scholes model and added them to our portfolio. These options were priced using the final stock price averages mentioned earlier, which served as our strike prices. As a result, after conducting the portfolio optimization again, we obtained another 20,000 portfolios using a total of 20 assets – 10 stocks/ETFs and 10 options on these stocks/ETFs. We then found the highest Sharpe portfolio, which showed an improved return-for-risk metric of 0.178230.

6. Conclusion

To conclude, our analysis of the portfolio optimization process using various models, such as Geometric Brownian Motion, CEV, Merton, and Heston, has shown the potential benefits of utilizing these models to obtain the highest Sharpe ratio portfolio. By simulating the future one-year performance of our portfolio through these models, we identified the best combination of assets that can provide the best risk-adjusted return.

Overall, the Heston and Merton models are more commonly used in industry for predicting stock values into the future. As previously mentioned, the Heston model accounts for changes in the volatility of the underlying by using the stochastic volatility approach, while the Merton model accounts for the possibility of default by the underlying firm. Although the CEV model also uses stochastic volatility, it is less commonly used in the industry compared to the Heston model. Additionally, both the Heston and Merton models are extensions of the Geometric Brownian Motion model, making them more pertaining to the future than the plain GBM model.

It is important to mention that investors should carefully consider their investment objectives and risk tolerance before choosing a model for portfolio optimization. Each model can offer different insights into a portfolio's future performance, and understanding each model's underlying assumptions and limitations is crucial in making informed investment decisions.

By implementing these techniques, investors can build a well-diversified, optimized portfolio with maximized returns. However, it is crucial to regularly monitor and adjust the portfolio weights as market conditions and investment objectives evolve. Through continuous monitoring and analysis, investors can optimize their portfolios to achieve long-term financial goals.