

ESI 4606: Analytics I - Foundations of Data Science

Homework 2

Due: September 21st (11:00AM), 2022

Problem 1 (2 points)

Consider randomly selecting a student at USF, and let A be the event that the selected student has a Visa card and B be the analogous event for MasterCard. Suppose that $\Pr(A) = 0.6$ and $\Pr(B) = 0.4$

- (a) Could it be the case that $\Pr(A \cap B) = 0.5$? Why or why not?
- (b) From now on, suppose that $\Pr(A \cap B) = 0.3$. What is the probability that the selected student has at least one of these two types of cards?
- (c) What is the probability that the selected student has neither type of card?
- (d) Calculate the probability that the selected student has exactly one of the two types of cards.

Problem 2 (2 points)

A customer service center has 5 telephone lines. Let X denote the number of lines in use at a specified time. The probability mass function (pmf), $f(x)$, for X , is shown in Table 1.

Table 1: Table summary of the probability mass function

x	0	1	2	3	4	5
$f(x)$	0.10	0.15	0.20	0.25	0.20	0.10

- (a) What is the corresponding cumulative distribution function (cdf), $F(x)$?
- (b) Let event $A = \{\text{at most three lines are in use}\}$, compute $\Pr(A)$.
- (c) Let event $B = \{\text{fewer than two lines are in use}\}$, compute $\Pr(B)$.
- (d) Compute the mean and the variance of X , i.e., $E(X)$ and $\text{Var}(X)$.

Problem 3 (1 point)

The lifetime of a certain brand of lightbulb has an exponential distribution with a mean of 800 hours. **Use R** to answer the following questions. [Hint: Using `help()` to learn R functions (e.g., `dexp`, `pexp`, `qexp`, `rexp`) related to the exponential distribution.]

- (a) Find the probability that a randomly selected lightbulb of this kind lasts 700 to 900 hours.
- (b) Find the probability that a randomly selected lightbulb of this kind lasts longer than 850 hours.
- (c) Find the 80th percentile of the lifetime of this kind of lightbulb.

Note: **To get full points, include R codes in the appendix sections**