Exponential growth

- $N(t) = \text{lambda}^t * N_0$
- Pop sizes at time t and 0 (N(t) and No)
- Lambda is per capita rate of increase

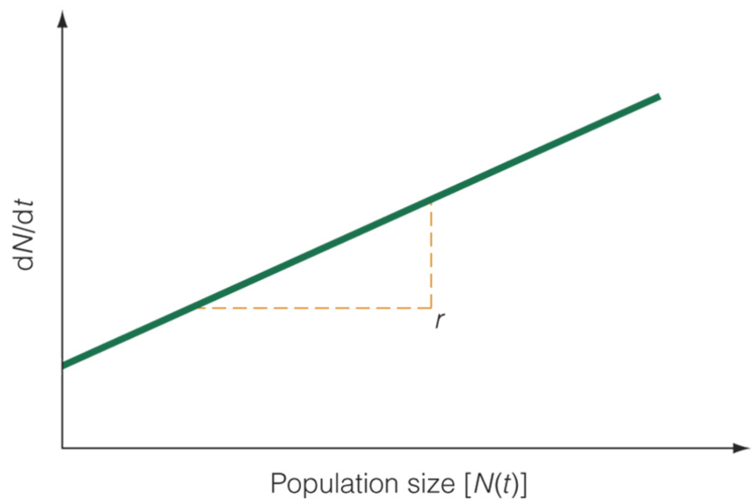
- $N(t)=N_0e^{rt}$
- where r is the exponential growth parameter
- N_0 is the starting population
- t is the time elapsed
- $r=0$ if the population is constant, $r>0$ if population is increasing, $r<0$ if the population is decreasing.

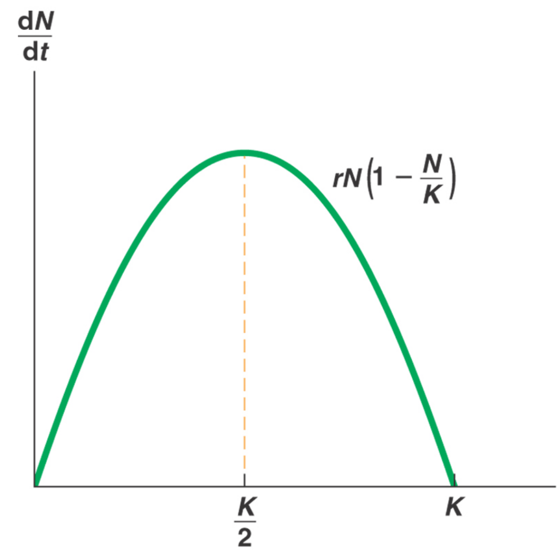
Logistic growth

- $dN/dT=r_{max}N(K-N)/K$
- $dN/dT=r_{max}*(1-(N/K)) * N$

$$\frac{dN}{dt} = rN \qquad \frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

From exponential growth to logistic growth





$$\frac{dN}{dt} = rN$$

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

Leslie Matrix (after Patrick Leslie, 1948)

discrete, age-structured model of population growth

$$\underbrace{\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}_{t+1}}_{\text{column matrix, } t+1} = \underbrace{\begin{pmatrix} b_1 & b_2 & b_3 \\ s_1 & 0 & 0 \\ 0 & s_2 & 0 \end{pmatrix}}_{\text{transition matrix}} \underbrace{\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}_t}_{\text{column matrix, } t}$$

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Transition (Leslie) matrix

Age Next Year	Age This Year		
	1	2	3
1	b_1	b_2	b_3
2	s_1	0	0
3	0	s_2	0

here, 3 ages = 3 rows & 3 columns (3 x 3 matrix)

Example calculation, Leslie matrix

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}_{t+1} = \begin{pmatrix} 0 & 2 & 3 \\ 0.2 & 0 & 0 \\ 0 & 0.3 & 0 \end{pmatrix} \begin{pmatrix} 100 \\ 50 \\ 20 \end{pmatrix}_t$$

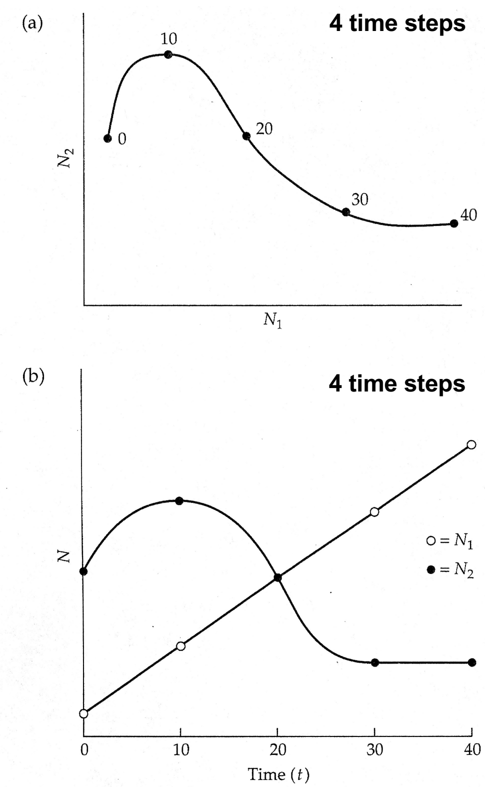
Lotka-Volterra

- $dN_1/dt = r_1N_1[(K_1-N_1-\alpha N_2)/K_1]$

- $dN_2/dt = r_2N_2[(K_2-N_2-\beta N_1)/K_2]$

State-space graphs help to track population trajectories (and assess stability) predicted by models

Mapping state-space trajectories onto single population trajectories



From Gotelli (2001)

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Lotka-Volterra Model

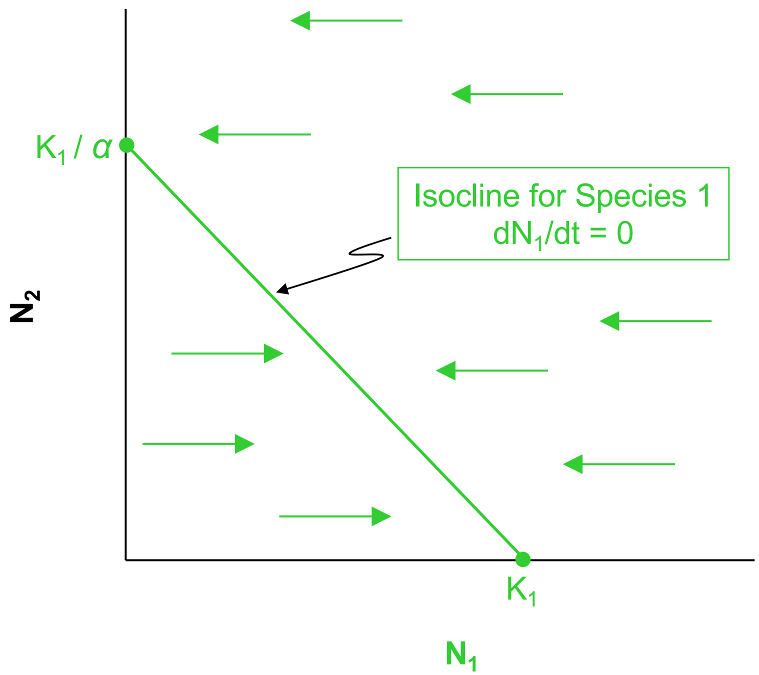
Remember that equilibrium solutions require $dN/dt = 0$

Species 1: $N_1 = K_1 - \alpha N_2$

Therefore:

When $N_2 = 0$, $N_1 = K_1$

When $N_1 = 0$, $N_2 = K_1/\alpha$



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