Fitting the model

Cost function and log-loss

Logistic Regression Model

$$P(y=1 \mid X) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n)}}$$

- X are the independent variables.
- Y is the target, which is class target.
- β are the parameters of the model.

Probabilities

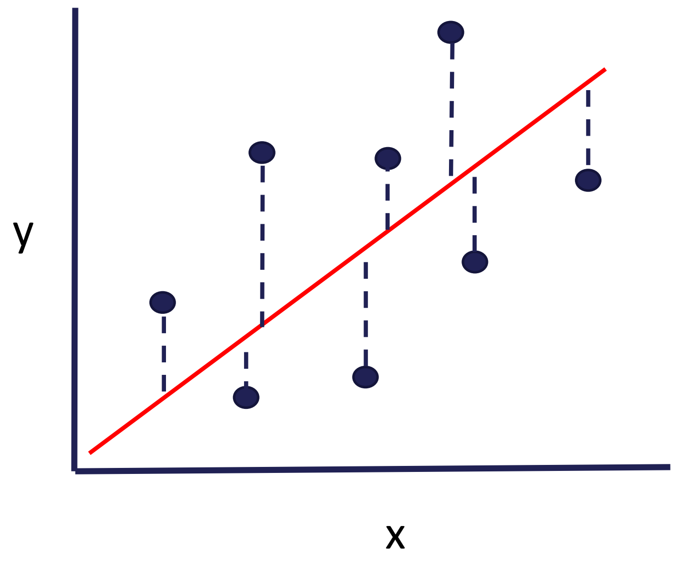
$$P(y=1 \mid X) = \frac{1}{1+e^{-(\beta_0+\beta_1 x_1+\beta_2 x_2+\ldots+\beta_n x_n)}}$$

- The sum of probabilities must equal 1.
- If we know P(y=1) ➔ we know P(y=0) = 1 – P(y=1)

Linear regression: residuals

$$\beta = \min \sum (y - predictions)^2$$

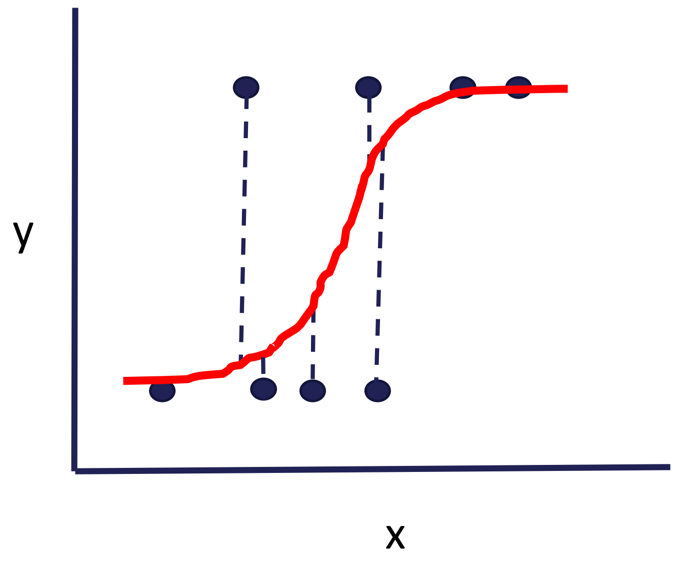
OLS ➔ Find the coefficients that minimize the squared difference of the target and the predictions.



Residuals – not useful

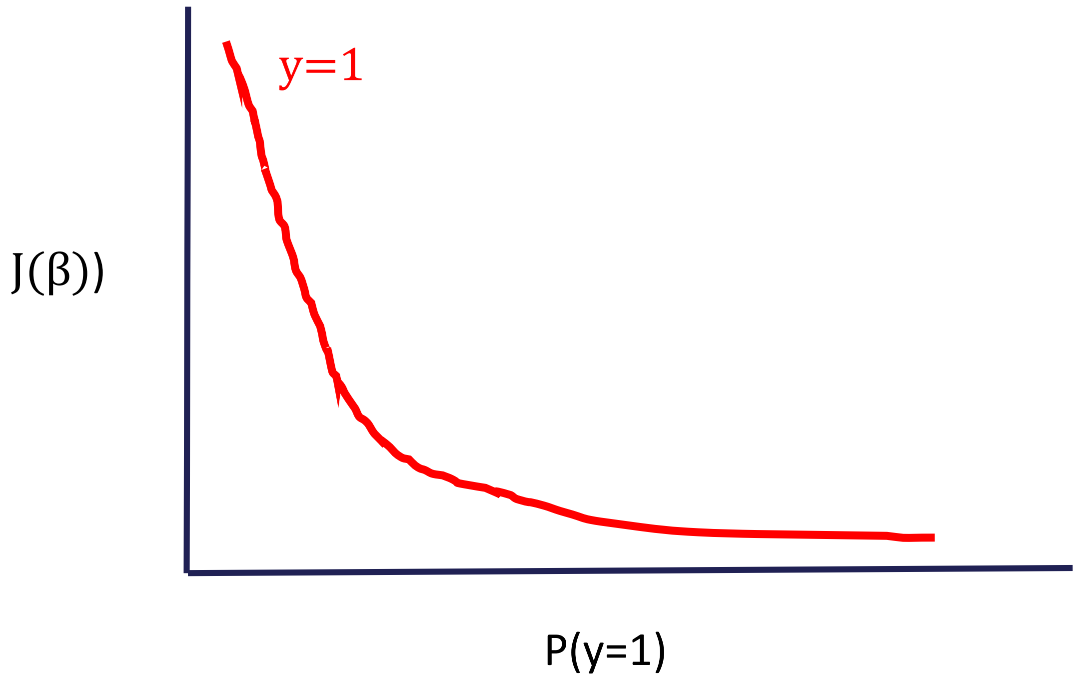
$$\beta = \min \sum (y - predictions)^2$$

Non-convex function ➔ not guaranteed to find the minimum.



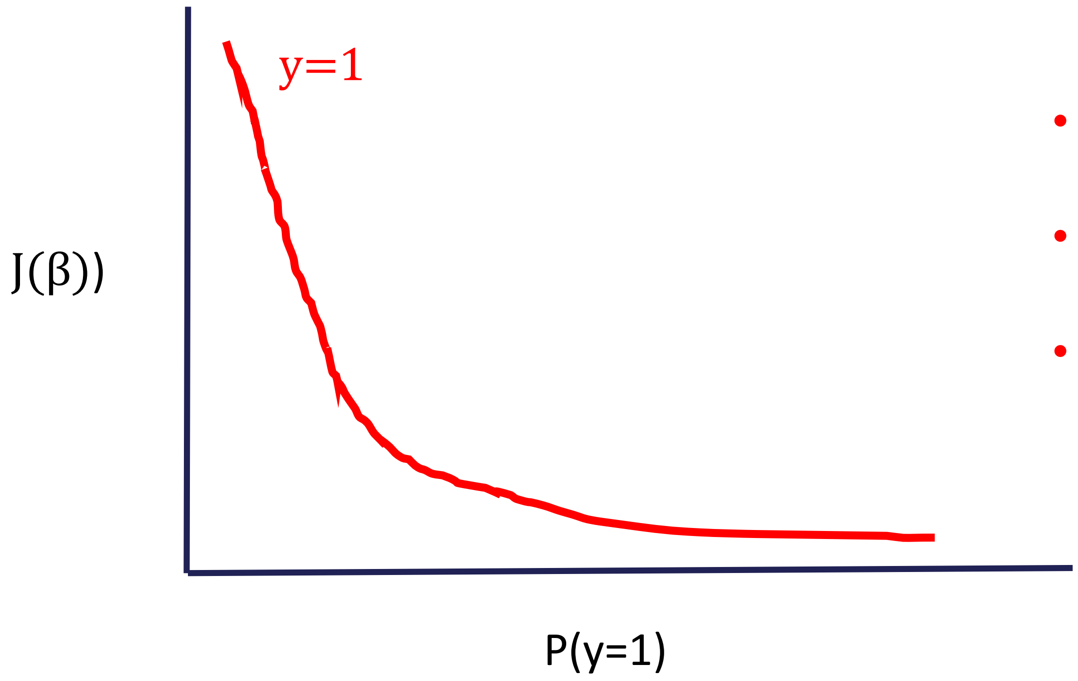
Cost

$$\text{Cost(model, y)} = \begin{cases} -\log(P(y=1)) & \text{if } y=1 \\ & \end{cases}$$



Cost

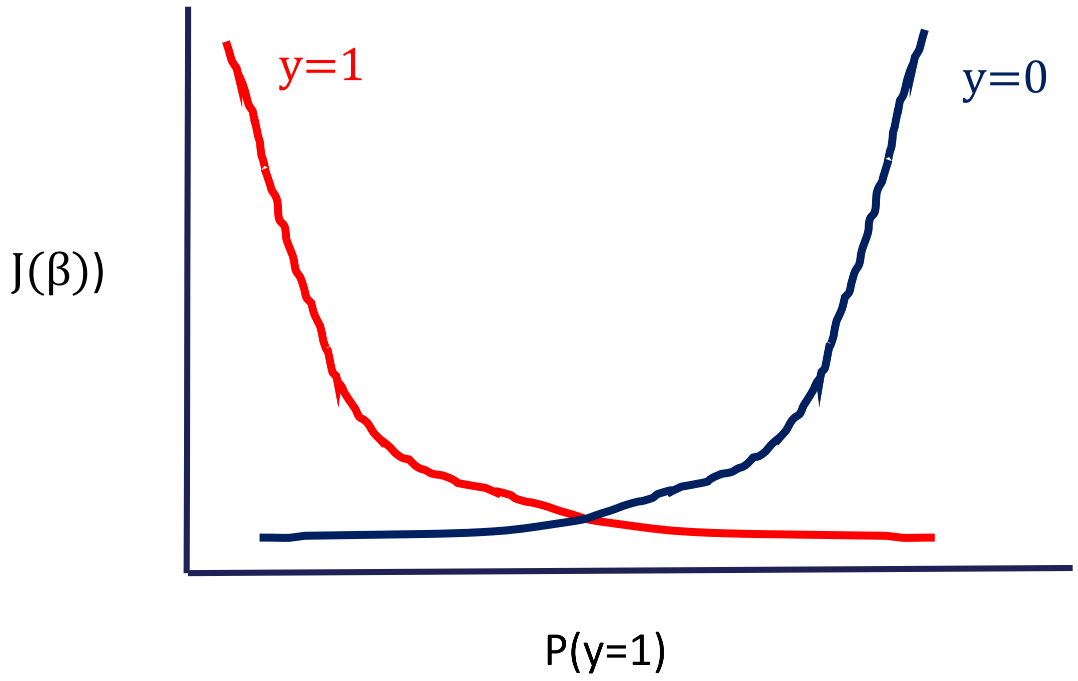
$$\text{Cost(model, y)} = \begin{cases} -\log(P(y=1)) & \text{if } y=1 \\ \end{cases}$$



- If P(y=1) = 0.3 ➔ -log(0.3) = 1.2
- If P(y=1) = 0.5 ➔ -log(0.5) = 0.69
- If P(y=1) = 0.8 ➔ -log(0.8) = 0.22

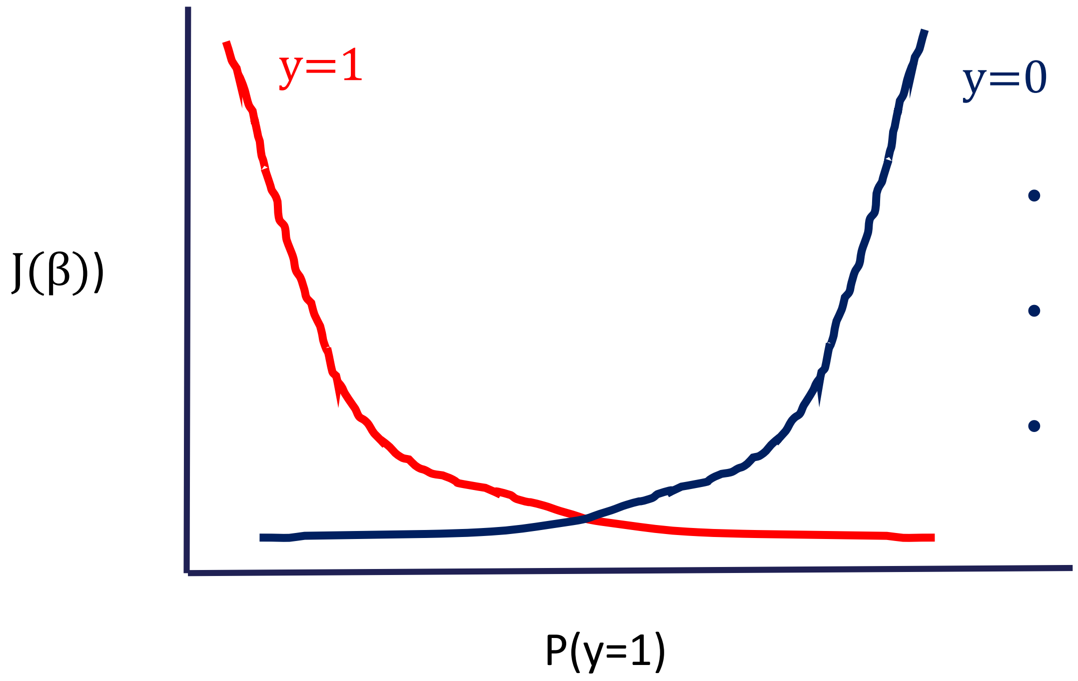
Cost

$$\text{Cost(model, y)} = \begin{cases} -\log(P(y=1)) & \text{if } y=1 \\ -\log(1-P(y=1)) & \text{if } y=0 \end{cases}$$



Cost

$$\text{Cost(model, y)} = \begin{cases} -\log(P(y=1)) & \text{if } y=1 \\ -\log(1-P(y=1)) & \text{if } y=0 \end{cases}$$



- If P(y=1) = 0.7 ➔ -log(1-0.7) = 1.2
- If P(y=1) = 0.4 ➔ -log(1-0.4) = 0.51
- If P(y=1) = 0.2 ➔ -log(1-0.2) = 0.22

Cost

$$\text{Cost(model, y)} = \begin{cases} -\log(P(y=1)) & \text{if } y=1 \\ -\log(1-P(y=1)) & \text{if } y=0 \end{cases}$$

$$\text{Cost(model, y)} = \sum -y \log(P(y=1)) - (1-y)\log(1-P(y=1))$$

Log-loss

$$\text{Cost(model, y)} = \begin{cases} -\log(P(y=1)) & \text{if } y=1 \\ -\log(1-P(y=1)) & \text{if } y=0 \end{cases}$$

$$\text{Cost(model, y)} = \sum -y \log(P(y=1)) - (1-y)\log(1-P(y=1))$$

$$J(\beta) = -\frac{1}{m}\sum y \log(P(y=1)) + (1-y)\log(1-P(y=1))$$

THANK YOU

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