



Interpreting the coefficients

Odds, $\log(\text{Odds})$ and Odds ratio.





Coefficients

- Positive coefficients (β) \rightarrow increase $P(y=1)$
- Negative coefficients (β) \rightarrow decrease $P(y=1)$

But the increase is not linear \rightarrow hard to interpret.



Odds

Odds: the probability of an event occurring divided by the probability of the event not occurring.

$$Odds = \frac{P(y=1)}{P(y=0)} = \frac{P(y=1)}{1 - P(y=1)}$$

Odds

$$Odds = \frac{P(y = 1)}{1 - P(y = 1)}$$

$$Odds = \frac{P(y=1)}{1 - P(y=1)} = \frac{0.5}{0.5} = 1$$

Odds of flipping a coin with $P(y=1) = 0.5$

$$Odds = \frac{P(y=1)}{1 - P(y=1)} = \frac{0.35}{0.65} = 0.55$$

Odds of survival in the titanic $P(y=1) = 0.35$

$$Odds = \frac{P(y=1)}{1 - P(y=1)} = \frac{0.8}{0.2} = 4$$

Odds of making a sale $P(y=1) = 0.8$

Odds

$$Odds = \frac{P(y = 1)}{1 - P(y = 1)}$$

$$Odds = \frac{P(y=1)}{1 - P(y=1)} = \frac{0.5}{0.5} = 1$$

Heads is equally likely as tails.

$$Odds = \frac{P(y=1)}{1 - P(y=1)} = \frac{0.35}{0.65} = 0.55$$

Surviving is ~half as likely as dying.

$$Odds = \frac{P(y=1)}{1 - P(y=1)} = \frac{0.8}{0.2} = 4$$

Making a sale is 4 times more likely than not.

Odds

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$$Odds = \frac{P(y=1)}{1 - P(y=1)} = \frac{0.8}{0.2} = 4$$

- Odds > 1 → P(y=1) is more likely
- Odds < 1 → P(y=0) is more likely

Log odds

$$Odds = \frac{P(y = 1)}{1 - P(y = 1)}$$

$$\log(Odds) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$$

Logistic regression is a linear model for the log odds.

Log odds

$$Odds = \frac{P(y=1)}{1 - P(y=1)} = \frac{\frac{1}{1+e^{-z}}}{1 - \frac{1}{1+e^{-z}}} = \frac{\frac{1}{1+e^{-z}}}{\frac{1+e^{-z}-1}{1+e^{-z}}} = \frac{\frac{1}{1+e^{-z}}}{\frac{e^{-z}}{1+e^{-z}}} = \frac{1}{1+e^{-z}} \times \frac{1+e^{-z}}{e^{-z}} = \frac{1}{e^{-z}}$$

$$\log(Odds) = \log\left(\frac{1}{e^{-z}}\right) = \log(1) - \log(e^{-z}) = z$$

$$\log(Odds) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$$

Log odds

$$\log(Odds) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$$

The coefficient (β) indicates the change in the **log(Odds)** per unit change of the features.

Log odds

$$\log(Odds) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$$

The coefficient (β) indicates the change in the $\log(Odds)$ per unit change of the features.

For categorical variables → the change when a category is present respect to the base category.

Log odds

$$\log(Odds) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$$

The coefficient (β) indicates the change in the $\log(Odds)$ per unit change of the features.

For categorical variables → the change when a category is present respect to the base category.

The intercept (β_0) is $P(y=1)$ when all variables are 0 (not used much).



Odds ratio

Proportional change in odds after a unit change in a predictor variable.

$$dOdds = e^{\beta_1}$$

Odds ratio

$$\log(Odds) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$$

$$\log(Odds)' = \beta_0 + \beta_1(x_1 + 1) + \beta_2 x_2 + \dots + \beta_n x_n$$

$$\frac{Odds'}{Odds} = \frac{e^{\beta_0 + \beta_1(x_1 + 1) + \beta_2 x_2 + \dots + \beta_n x_n}}{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n}}$$

$$\frac{Odds'}{Odds} = e^{(\beta_0 + \beta_1(x_1 + 1) + \beta_2 x_2 + \dots + \beta_n x_n) - (\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n)}$$

$$\frac{Odds'}{Odds} = e^{(\beta_1(x_1 + 1) - \beta_1 x_1)} = e^{(\beta_1 x_1 + \beta_1 - \beta_1 x_1)} = e^{(\beta_1)}$$



Odds ratio

Proportional change in odds after a unit change in a predictor variable.

$$dOdds = e^{\beta_1}$$

THANK YOU

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