Circles On A Lattice

Example

Example: Find the number of lattice points on the circumference of circles with radius $a = \sqrt{3}, b = \sqrt{46}$ and $c = \sqrt{27}$. **Solution:** The prime decomposition of $3 = 3^1$, since $3 \equiv 3 (\text{mod} 4)$ and the exponent 1 is odd, we can immediately conclude a circle with radius a has no lattice points.

The divisors of 90 are; $1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90$. We can check which numbers are congruent to 1 modulo 4 and 3 modulo 4 and use the formula **??** giving : $r_2(90) = 4(4-2) = 8$. Using the python code from the supplementary material[**?**] we have the coordinates of the lattice points also which are; $(3, 9), (3, -9), (-3, 9), (-3, -9), (9, 3), (9, -3), (-9, 3), (-9, -3)$.

For c we can write $27 = 3^3$ as $3 \equiv 3 (\text{mod} 4)$ and the exponent 3 is odd we can conclude this circle will have 0 lattice points