$$E = \frac{J(J+1)\hbar^2}{2I} \tag{1}$$

$$I = \frac{m_1 \cdot m_2}{m_1 + m_2} r^2 \tag{2}$$

$$E = \hbar w (n + \frac{1}{2}) \tag{3}$$

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} \tag{4}$$

$$E_{ges} = \left(n + \frac{1}{2}\right) h\nu + \frac{\hbar^2 J(J+1)}{2I} \tag{5}$$

P-Zweig:

$$\Delta E = \hbar\omega - \frac{J\hbar^2}{I} \tag{6}$$

R-Zweig:

$$\Delta E = \hbar\omega + \frac{(J+1)\hbar^2}{I} \tag{7}$$

$$P = \frac{N \cdot V_{\text{Atom}}}{V_{\text{Elementarzelle}}} \tag{8}$$

$$2 d_{hkl} \sin\theta = \lambda \tag{9}$$

$$v_{ph} = \frac{\omega}{q} \tag{10}$$

$$v_g = \frac{\partial w}{\partial q} \tag{11}$$

$$\boldsymbol{G} = \frac{2\pi}{a} \tag{12}$$

Dispersionsrelation lineare Kette:

$$\omega = 2\sqrt{\frac{C_1}{M}}\left|\sin\left(\frac{qa}{2}\right)\right| \tag{13}$$

Lösungsansatz:

$$u_{s+n} = Ue^{-i[\omega t - q(s+n)a]} \tag{14}$$

Modifizierte Streubedingung:

$$\begin{aligned} \hbar\omega &= \hbar\omega_0 + \hbar\omega_{\boldsymbol{q}} \\ \hbar\boldsymbol{k} &= \hbar\boldsymbol{k}_0 \pm \hbar\boldsymbol{q} + \hbar\boldsymbol{G} \end{aligned} \tag{15}$$

Zustandsdichte im reziproken Raum:

$$\rho_q^{(n)} = \frac{L^n}{(2\pi)^n} \tag{16}$$

Debye-Näherung $\omega = vq$:

$$\mathcal{D}(\omega) = \frac{\partial N}{\partial k}\frac{\partial k}{\partial \omega} = 4\pi k^2 \frac{V}{(2\pi)^3}\frac{1}{v_{ph}} = \frac{V}{2\pi^2}\frac{\omega^2}{v^3} \tag{17}$$

$$U = \int_0^{\omega_D} \hbar\omega\mathcal{D}(\omega)\langle n(\omega, T)\rangle d\omega \tag{18}$$

$$\omega_D = \frac{v}{a}\sqrt[3]{6\pi^2} \tag{19}$$

$$\begin{aligned} k_B\Theta &= \hbar\omega_D \\ x &= \frac{\hbar\omega}{k_B T} \\ x_D &= \frac{\hbar\omega_D}{k_B T} = \frac{\Theta}{T} \end{aligned} \tag{20}$$

Immer noch Debye-Näherung:

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = 9nk_B\left(\frac{T}{\Theta}\right)^3 \int_0^{x_D} \frac{x^4 e^x}{(e^x - 1)^2} dx \tag{21}$$

Für tiefe Temperature:

$$C_V = 3nk_B \tag{22}$$

Für hohe Temperaturen:

$$C_V = \frac{12\pi^4}{5} n k_B \left(\frac{T}{\Theta}\right) \tag{23}$$

Wärmetransport durch Gitterstöße:

$$\boldsymbol{j} = -\Lambda \nabla T \tag{24}$$

$$\Lambda = \frac{1}{3} C v l \tag{25}$$

Bei tiefen Temperaturen:

$$\Lambda \propto T^3 d \tag{26}$$

$$\psi(\boldsymbol{r}) = \frac{1}{\sqrt{V}} e^{i\boldsymbol{k}\boldsymbol{r}} \tag{27}$$

$$E = \frac{\hbar^2 k^2}{2m} \tag{28}$$

$$k_i = \frac{2\pi}{L} m_i \tag{29}$$

Generell für Zustandsdichte:

$$\mathcal{D}(E) = \frac{\partial N}{\partial k} \frac{\partial k}{\partial E} \tag{30}$$

N ist jeweils Volumen mal Dichte:

$$\begin{aligned} N^{(3)} &= \frac{4}{3}\pi k^3 \cdot \frac{2V}{(2\pi)^3} \\ N^{(2)} &= \pi k^2 \cdot \frac{2A}{(2\pi)^2} \\ N^{(1)} &= 2k \cdot \frac{2L}{2\pi} \end{aligned} \tag{31}$$

$$\frac{\partial k}{\partial E} = \sqrt{\frac{m}{2\hbar^2}} \frac{1}{\sqrt{E}} = \frac{m}{\hbar^2 k} = \frac{1}{v_g \hbar} \tag{32}$$

$$f(E) = \frac{1}{e^{\frac{(E-\mu)}{k_B T}} + 1} \tag{33}$$

$$n = \frac{N}{V} = \int_0^\infty D(E) f(E, T=0) dE = \int_0^{E_F} D(E) dE = \frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \frac{2E_F^{\frac{3}{2}}}{3} \tag{34}$$

$$\begin{aligned} E_F &= \frac{\hbar^2}{2m}(3\pi^2 n)^{\frac{2}{3}} & \\ k_F &= (3\pi^2 n)^{\frac{1}{3}} & \text{Fermi-Wellenvektor} \\ v_F &= \frac{\hbar}{m}(3\pi^2 n)^{\frac{1}{3}} & \text{Fermi-Geschwindigkeit} \\ T_F &= \frac{E_F}{k_B} & \text{Fermi-Temperatur} \end{aligned} \tag{35}$$

$$e^{i\boldsymbol{qr}} = e^{i\boldsymbol{q}(x+L,y,z)} = e^{i\boldsymbol{qr}} \cdot e^{iq_x L} \tag{36}$$

$$u_0 = \int_0^\infty E D(E) f(E, T=0) dE = \int_0^{E_F} E D(E) dE = \frac{3n}{5} E_F = \frac{3n}{5} k_B T_F \tag{37}$$

$$\delta u(T) = u(T) - u_0 = n k_B T \cdot \frac{T}{T_F} \tag{38}$$

$$c_V^{el} = \left(\frac{\partial u}{\partial T}\right)_V \approx \frac{\pi^2 n k_B T}{2 T_F} \approx \gamma T \tag{39}$$

$$c_V^{ges} = \gamma T + \begin{cases} 3 n_A k_B & \text{für } T > \Theta \\ \beta T^3 & \text{für } T \ll \Theta \end{cases} \tag{40}$$

Dispersionsrelation quasi-freie Elektronen:

$$E_{\boldsymbol{K}} = \frac{\hbar^2 k^2}{2m} = E_{\boldsymbol{k}+\boldsymbol{G}} = \frac{\hbar^2}{2m} |\boldsymbol{k} + \boldsymbol{G}|^2 \tag{41}$$

$$\frac{d\boldsymbol{v}}{dt} = \frac{1}{\hbar} \frac{d}{dt} \left(\frac{\partial E(\boldsymbol{k})}{\partial \boldsymbol{k}}\right) = \frac{1}{\hbar} \frac{\partial^2 E(\boldsymbol{k})}{\partial \boldsymbol{k} \partial \boldsymbol{k}} \frac{\partial \boldsymbol{k}}{dt} = \frac{1}{\hbar^2} \frac{\partial^2 E(\boldsymbol{k})}{\partial \boldsymbol{k} \partial \boldsymbol{k}} \boldsymbol{F} \tag{42}$$

$$\left(\frac{1}{m^*}\right)_{ij} = \frac{1}{\hbar^2} \frac{\partial^2 E(\boldsymbol{k})}{\partial k_i \partial k_j} \tag{43}$$

Bloch-Oszillationen:

$$|\boldsymbol{v}| = \frac{e\mathcal{E}}{\hbar} \tag{44}$$

$$T_B = \frac{\frac{2\pi}{a}}{\frac{e\mathcal{E}}{\hbar}} = \frac{h}{ae\mathcal{E}} \tag{45}$$

Drude Modell: Bewegung Elektronen $\iff$ Kinetische Gastheorie

$$m\frac{d\boldsymbol{v}}{dt} = -e\mathcal{E} - m\frac{\boldsymbol{v}_d}{\tau} \tag{46}$$

$$\boldsymbol{v}_d = -\frac{e\tau}{m}\boldsymbol{\mathcal{E}} = -\mu\boldsymbol{\mathcal{E}} \tag{47}$$

$$\boldsymbol{j} = -en\boldsymbol{v}_d = \frac{ne^2\tau}{m}\boldsymbol{\mathcal{E}} = ne\mu\boldsymbol{\mathcal{E}} \tag{48}$$

$$\sigma = \frac{j}{\mathcal{E}} = \frac{ne^2\tau}{m} = ne\mu \tag{49}$$

$$l = v_F\tau \tag{50}$$

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} = \rho_D + \rho_G = \frac{m}{ne^2\tau_D} + \frac{m}{ne^2\tau_G(T)} \tag{51}$$

$$\frac{\rho(300K)}{\rho(4.2K)} \tag{52}$$

$$B_C(T) = B_C(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right] \tag{53}$$

$$\sigma = e(n\mu_n + p\mu_p) \tag{54}$$

$$\begin{aligned} n &= \int_{E_L}^{\infty} D_L(E) f(E,T) dE \\ p &= \int_{-\infty}^{E_V} D_V(E)[1 - f(E,T)] dE \end{aligned} \tag{55}$$

$$\begin{aligned} D_L(E) &= \frac{1}{2\pi^2}\left(\frac{2m_n^*}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E - E_L} \\ D_V(E) &= \frac{1}{2\pi^2}\left(\frac{2m_p^*}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E_V - E} \end{aligned} \tag{56}$$

$$n = \int D_L(E) f(E) dE = \frac{1}{2\pi^2}\left(\frac{2m_n^*}{\hbar^2}\right)^{\frac{3}{2}} e^{\frac{E_F}{k_B T}} \int_{E_L}^{\infty} \sqrt{E - E_L} e^{\frac{E}{k_B T}} \tag{57}$$

Massenwirkung:

$$\begin{aligned} n =& \mathcal{N}_L e^{-\frac{(E_L - E_F)}{k_B T}} = 2\left(\frac{m_n^* k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} e^{-\frac{(E_L - E_F)}{k_B T}} \\ p =& \mathcal{N}_V e^{\frac{(E_V - E_F)}{k_B T}} = 2\left(\frac{m_p^* k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} e^{\frac{(E_V - E_F)}{k_B T}} i \end{aligned} \tag{58}$$

$$n \cdot p = \mathcal{N}_L \mathcal{N}_V e^{-\frac{E_g}{k_B T}} = const \tag{59}$$

Für intrinsische Halbleiter:

$$n_i = p_i = \sqrt{\mathcal{N}_L \mathcal{N}_V} e^{-\frac{E_g}{2k_B T}} \tag{60}$$

$$E_F = \frac{E_L + E_V}{2} + \frac{k_B T}{2} \ln(\frac{\mathcal{N}_V}{\mathcal{N}_L}) = \frac{E_L + E_V}{2} + \frac{3}{4} k_B T \ln(\frac{m_p^*}{m_n^*}) \tag{61}$$

Dotierte Halbleiter:

$$\begin{aligned} n_A = n_A^0 + n_A^- \\ n_D = n_D^0 + n_D^+ \end{aligned} \tag{62}$$

$$n + n_A^- = p + n_D^+ \tag{63}$$

$$\frac{n_A^0}{n_A} = \frac{1}{e^{\frac{E_D - E_F}{k_B T}} + 1} \tag{64}$$

$$\frac{n_D^0}{n_D} = \frac{1}{e^{\frac{E_F - E_A}{k_B T}} + (65)}$$

Supraleitung:

$$B_C(T) = B_C(0)\left[1 - \left(\frac{T}{T_C}\right)\right] \tag{66}$$

$$I_C = \frac{B_C 2\pi r}{\mu_0} \tag{67}$$

$$rot\boldsymbol{B} = \mu_0 \boldsymbol{j} \tag{68}$$

$$|\boldsymbol{B}_z(r)| = B_z(0)e^{-\frac{r}{\lambda_l}} \tag{69}$$

$$|\boldsymbol{j}| = \frac{B_z(0)}{\mu_0\lambda_l}e^{-\frac{r}{\lambda_l}} \tag{70}$$

$$\lambda_l = \sqrt{\frac{m_e}{\mu_0 n e^2}} \tag{71}$$