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$$E = \frac{J(J+1)\hbar^2}{2I} \quad (1)$$

$$I = \frac{m_1 \cdot m_2}{m_1 + m_2} r^2 \quad (2)$$

$$E = \hbar \omega \left( n + \frac{1}{2} \right) \quad (3)$$

$$\nu = \frac{\omega}{2\pi} = \sqrt{\frac{k}{\mu}} \quad (4)$$

$$E_{ges} = \left( n + \frac{1}{2} \right) \hbar \nu + \frac{\hbar^2 J(J+1)}{2I} \quad (5)$$

$$P = \frac{N \cdot V_{\text{Atom}}}{V_{\text{Elementarzelle}}} \quad (6)$$

$$2d_{hkl} \sin \theta = \lambda \quad (7)$$

$$v_{ph} = \frac{\omega}{q} \quad (8)$$

$$v_g = \frac{\partial \omega}{\partial q} \quad (9)$$

$$\mathbf{G} = \frac{2\pi}{a} \quad (10)$$

Dispersionsrelation lineare Kette:

$$\omega = 2\sqrt{\frac{C_1}{M}} \left| \sin \left( \frac{qa}{2} \right) \right| \quad (11)$$

Lösungsansatz:

$$u_{s+n} = U e^{-i[\omega t - q(s+n)a]} \quad (12)$$

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Modifizierte Streubedingung:

$$\begin{aligned}\hbar\omega &= \hbar\omega_0 + \hbar\omega_{\mathbf{q}} \\ \hbar\mathbf{k} &= \hbar\mathbf{k}_0 \pm \hbar\mathbf{q} + \hbar\mathbf{G}\end{aligned}\tag{13}$$

Zustandsdichte im reziproken Raum:

$$\rho_q^{(n)} = \frac{L^n}{(2\pi)^n}\tag{14}$$

Debye-Näherung  $\omega = vq$ :

$$\mathcal{D}(\omega) = \frac{\partial N}{\partial k} \frac{\partial k}{\partial \omega} = 4\pi k^2 \frac{V}{(2\pi)^3} \frac{1}{v_{ph}} = \frac{V}{2\pi^2} \frac{\omega^2}{v^3}\tag{15}$$

$$U = \int_0^{\omega_D} \hbar\omega \mathcal{D}(\omega) \langle n(\omega, T) \rangle d\omega\tag{16}$$

$$\omega_D = \frac{v}{a} \sqrt[3]{6\pi^2}\tag{17}$$

$$\begin{aligned}k_B\Theta &= \hbar\omega_D \\ x &= \frac{\hbar\omega}{k_B T} \\ x_D &= \frac{\hbar\omega_D}{k_B T} = \frac{\Theta}{T}\end{aligned}\tag{18}$$

Immer noch Debye-Näherung:

$$C_V = \left( \frac{\partial U}{\partial T} \right)_V = 9nk_B \left( \frac{T}{\Theta} \right)^3 \int_0^{x_D} \frac{x^4 e^x}{(e^x - 1)^2} dx\tag{19}$$

Für tiefe Temperature:

$$C_V = 3nk_B\tag{20}$$

Für hohe Temperaturen:

$$C_V = \frac{12\pi^4}{5} nk_B \left( \frac{T}{\Theta} \right)\tag{21}$$

Wärmetransport durch Gitterstöße:

$$\mathbf{j} = -\Lambda \nabla T\tag{22}$$

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$$\Lambda = \frac{1}{3}Cvl \quad (23)$$

Bei tiefen Temperaturen:

$$\Lambda \propto T^3 d \quad (24)$$

$$\psi(\mathbf{r}) = \frac{1}{\sqrt{V}} e^{i\mathbf{k}\mathbf{r}} \quad (25)$$

$$E = \frac{\hbar^2 k^2}{2m} \quad (26)$$

$$k_i = \frac{2\pi}{L} m_i \quad (27)$$

Generell für Zustandsdichte:

$$\mathcal{D}(E) = \frac{\partial N}{\partial k} \frac{\partial k}{\partial E} \quad (28)$$

N ist jeweils Volumen mal Dichte:

$$\begin{aligned} N^{(3)} &= \frac{4}{3} \pi k^3 \cdot \frac{2V}{(2\pi)^3} \\ N^{(2)} &= \pi k^2 \cdot \frac{2A}{(2\pi)^2} \\ N^{(1)} &= 2k \cdot \frac{2L}{2\pi} \end{aligned} \quad (29)$$

$$\frac{\partial k}{\partial E} = \sqrt{\frac{m}{2\hbar^2}} \frac{1}{\sqrt{E}} = \frac{m}{\hbar^2 k} = \frac{1}{v_g \hbar} \quad (30)$$

$$f(E) = \frac{1}{e^{\frac{(E-\mu)}{k_B T}} + 1} \quad (31)$$

$$n = \frac{N}{V} = \int_0^\infty D(E) f(E, T=0) dE = \int_0^{E_F} D(E) dE = \frac{1}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{\frac{3}{2}} \frac{2E_F^{\frac{3}{2}}}{3} \quad (32)$$

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$$\begin{aligned}
E_F &= \frac{\hbar^2}{2m} (3\pi^2 n)^{\frac{2}{3}} \\
k_F &= (3\pi^2 n)^{\frac{1}{3}} && \text{Fermi-Wellenvektor} \\
v_F &= \frac{\hbar}{m} (3\pi^2 n)^{\frac{1}{3}} && \text{Fermi-Geschwindigkeit} \\
T_F &= \frac{E_F}{k_B} && \text{Fermi-Temperatur}
\end{aligned} \tag{33}$$

$$e^{i\mathbf{q}\mathbf{r}} = e^{i\mathbf{q}(x+L,y,z)} = e^{i\mathbf{q}\mathbf{r}} \cdot e^{iq_x L} \tag{34}$$

$$u_0 = \int_0^\infty ED(E)f(E, T=0)dE = \int_0^{E_F} ED(E)dE = \frac{3n}{5}E_F = \frac{3n}{5}k_B T_F \tag{35}$$

$$\delta u(T) = u(T) - u_0 = nk_B T \cdot \frac{T}{T_F} \tag{36}$$

$$c_V^{el} = \left( \frac{\partial u}{\partial T} \right)_V \approx \frac{\pi^2 n k_B T}{2T_F} \approx \gamma T \tag{37}$$

$$c_V^{ges} = \gamma T + \begin{cases} 3n_A k_B & \text{für } T > \Theta \\ \beta T^3 & \text{für } T \ll \Theta \end{cases} \tag{38}$$

Dispersionsrelation quasi-freie Elektronen:

$$E_{\mathbf{K}} = \frac{\hbar^2 k^2}{2m} = E_{\mathbf{k}+\mathbf{G}} = \frac{\hbar^2}{2m} |\mathbf{k} + \mathbf{G}|^2 \tag{39}$$

$$\frac{d\mathbf{v}}{dt} = \frac{1}{\hbar} \frac{d}{dt} \left( \frac{\partial E(\mathbf{k})}{\partial \mathbf{k}} \right) = \frac{1}{\hbar} \frac{\partial^2 E(\mathbf{k})}{\partial \mathbf{k} \partial \mathbf{k}} \frac{\partial \mathbf{k}}{dt} = \frac{1}{\hbar^2} \frac{\partial^2 E(\mathbf{k})}{\partial \mathbf{k} \partial \mathbf{k}} \mathbf{F} \tag{40}$$

$$\left( \frac{1}{m^*} \right)_{ij} = \frac{1}{\hbar^2} \frac{\partial^2 E(\mathbf{k})}{\partial k_i \partial k_j} \tag{41}$$

Bloch-Oszillationen:

$$|\mathbf{v}| = \frac{e\mathcal{E}}{\hbar} \tag{42}$$

$$T_B = \frac{\frac{2\pi}{a}}{\frac{e\mathcal{E}}{\hbar}} = \frac{\hbar}{ae\mathcal{E}} \tag{43}$$

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Drude Modell: Bewegung Elektronen  $\iff$  Kinetische Gastheorie

$$m \frac{d\mathbf{v}}{dt} = -e\mathcal{E} - m \frac{\mathbf{v}_d}{\tau} \quad (44)$$

$$\mathbf{v}_d = -\frac{e\tau}{m} \mathcal{E} = -\mu \mathcal{E} \quad (45)$$

$$\mathbf{j} = -en\mathbf{v}_d = \frac{ne^2\tau}{m} \mathcal{E} = ne\mu \mathcal{E} \quad (46)$$

$$\sigma = \frac{j}{\mathcal{E}} = \frac{ne^2\tau}{m} = ne\mu \quad (47)$$

$$l = v_F \tau \quad (48)$$

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} = \rho_D + \rho_G = \frac{m}{ne^2\tau_D} + \frac{m}{ne^2\tau_G(T)} \quad (49)$$

$$\frac{\rho(300K)}{\rho(4.2K)} \quad (50)$$

$$B_C(T) = B_C(0) \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right] \quad (51)$$

$$\sigma = e(n\mu_n + p\mu_p) \quad (52)$$

$$\begin{aligned} n &= \int_{E_L}^{\infty} D_L(E) f(E, T) dE \\ p &= \int_{-\infty}^{E_V} D_V(E) [1 - f(E, T)] dE \end{aligned} \quad (53)$$

$$\begin{aligned} D_L(E) &= \frac{1}{2\pi^2} \left( \frac{2m_n^*}{\hbar^2} \right)^{\frac{3}{2}} \sqrt{E - E_L} \\ D_V(E) &= \frac{1}{2\pi^2} \left( \frac{2m_p^*}{\hbar^2} \right)^{\frac{3}{2}} \sqrt{E_V - E} \end{aligned} \quad (54)$$

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$$n = \int D_L(E) f(E) dE = \frac{1}{2\pi^2} \left( \frac{2m_n^*}{\hbar^2} \right)^{\frac{3}{2}} e^{\frac{E_F}{k_B T}} \int_{E_L}^{\infty} \sqrt{E - E_L} e^{\frac{E}{k_B T}} dE \quad (55)$$

Massenwirkung:

$$\begin{aligned} n &= \mathcal{N}_L e^{-\frac{(E_L - E_F)}{k_B T}} = 2 \left( \frac{m_n^* k_B T}{2\pi \hbar^2} \right)^{\frac{3}{2}} e^{-\frac{(E_L - E_F)}{k_B T}} \\ p &= \mathcal{N}_V e^{\frac{(E_V - E_F)}{k_B T}} = 2 \left( \frac{m_p^* k_B T}{2\pi \hbar^2} \right)^{\frac{3}{2}} e^{\frac{(E_V - E_F)}{k_B T}} \end{aligned} \quad (56)$$

$$n \cdot p = \mathcal{N}_L \mathcal{N}_V e^{-\frac{E_g}{k_B T}} = \text{const} \quad (57)$$

Für intrinsische Halbleiter:

$$n_i = p_i = \sqrt{\mathcal{N}_L \mathcal{N}_V} e^{-\frac{E_g}{2k_B T}} \quad (58)$$

$$E_F = \frac{E_L + E_V}{2} + \frac{k_B T}{2} \ln\left(\frac{\mathcal{N}_V}{\mathcal{N}_L}\right) = \frac{E_L + E_V}{2} + \frac{3}{4} k_B T \ln\left(\frac{m_p^*}{m_n^*}\right) \quad (59)$$

Dotierte Halbleiter:

$$\begin{aligned} n_A &= n_A^0 + n_A^- \\ n_D &= n_D^0 + n_D^+ \end{aligned} \quad (60)$$

$$n + n_A^- = p + n_D^+ \quad (61)$$

$$\frac{n_A^0}{n_A} = \frac{1}{e^{\frac{E_D - E_F}{k_B T}} + 1} \quad (62)$$

$$\frac{n_D^0}{n_D} = \frac{1}{e^{\frac{E_F - E_A}{k_B T}} + 1} \quad (63)$$

Supraleitung:

$$B_C(T) = B_C(0) \left[ 1 - \left( \frac{T}{T_C} \right) \right] \quad (64)$$

$$I_C = \frac{B_C 2\pi r}{\mu_0} \quad (65)$$

$$\text{rot} \mathbf{B} = \mu_0 \mathbf{j} \quad (66)$$

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$$|\mathbf{B}_z(r)| = B_z(0)e^{-\frac{r}{\lambda_l}} \quad (67)$$

$$|\mathbf{j}| = \frac{B_z(0)}{\mu_0 \lambda_l} e^{-\frac{r}{\lambda_l}} \quad (68)$$