$$E = \frac{J(J+1)\hbar^2}{2I} \tag{1}$$

$$I = \frac{m_1 \cdot m_2}{m_1 + m_2} r^2 \tag{2}$$

$$E = \hbar w(n + \frac{1}{2}) \tag{3}$$

$$\nu = \frac{\omega}{2\pi} = \sqrt{\frac{k}{\mu}} \tag{4}$$

$$E_{ges} = \left(n + \frac{1}{2}\right) h\nu + \frac{\hbar^2 J(J+1)}{2I} \tag{5}$$

$$P = \frac{N \cdot V_{\text{Atom}}}{V_{\text{Elementarzelle}}} \tag{6}$$

$$2d_{hkl} \sin\theta = \lambda \tag{7}$$

$$v_{ph} = \frac{\omega}{q} \tag{8}$$

$$v_g = \frac{\partial w}{\partial q} \tag{9}$$

$$\boldsymbol{G} = \frac{2\pi}{a} \tag{10}$$

Dispersionsrelation lineare Kette:

$$\omega = 2\sqrt{\frac{C_1}{M}} \left|\sin\left(\frac{qa}{2}\right)\right| \tag{11}$$

Lösungsansatz:

$$u_{s+n} = Ue^{-i[\omega t - q(s+n)a]} \tag{12}$$

Modifizierte Streubedingung:

$$\begin{aligned} \hbar\omega &= \hbar\omega_0 + \hbar\omega_{\boldsymbol{q}} \\ \hbar\boldsymbol{k} &= \hbar\boldsymbol{k}_0 \pm \hbar\boldsymbol{q} + \hbar\boldsymbol{G} \end{aligned} \tag{13}$$

Zustandsdichte im reziproken Raum:

$$\rho_q^{(n)} = \frac{L^n}{(2\pi)^n} \tag{14}$$

Debye-Näherung $\omega = vq$:

$$\mathcal{D}(\omega)d\omega = \rho_q \int_{\omega}^{\omega+dw} d^3q = \rho_q d\omega \int_{\omega=const} \frac{dS_\omega}{v_g} = \rho_q d\omega \frac{4\pi q^2}{v_g} = \frac{V}{2\pi^2}\frac{\omega^2}{v^3}d\omega \tag{15}$$

$$U = \int_0^{\omega_D} \hbar\omega\mathcal{D}(\omega)\langle n(\omega,T)\rangle d\omega \tag{16}$$

$$\begin{aligned} k_B\Theta &= \hbar\omega_D \\ x &= \frac{\hbar\omega}{k_BT} \\ x_D &= \frac{\hbar\omega_D}{k_BT} = \frac{\Theta}{T} \end{aligned} \tag{17}$$

Immer noch Debye-Näherung:

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = 9Nk_B\left(\frac{T}{\Theta}\right)^3 \int_0^{x_D} \frac{x^4e^x}{(e^x-1)^2}dx \tag{18}$$

Wärmetransport durch Gitterstöße:

$$\boldsymbol{j} = -\Lambda\nabla T \tag{19}$$

$$\Lambda = \frac{1}{3}Cvl \tag{20}$$

Bei tiefen Temperaturen:

$$\Lambda \propto T^3 d \tag{21}$$

$$\psi(\boldsymbol{r}) = \frac{1}{\sqrt{V}}e^{i\boldsymbol{kr}} \tag{22}$$

$$E = \frac{\hbar^2 k^2}{2m} \tag{23}$$

$$k_i = \frac{2\pi}{L} m_i \tag{24}$$

$$\rho_k = \frac{2V}{(2\pi^3)} \tag{25}$$

$$\mathcal{D}(E) = \frac{2V}{(2\pi^3)\hbar} \frac{4\pi k^2}{v_g} \overset{v_g = \frac{\hbar k}{m}}{=} \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \sqrt{E} \tag{26}$$

Zweidimensionale Zustandsdichte pro Volumen:

$$D^{(2)}(E) = \frac{\rho_k^{(2)}}{A\hbar} \frac{2\pi k}{v_g} = \frac{m}{\pi\hbar^2} \tag{27}$$

$$f(E) = \frac{1}{e^{\frac{(E-\mu)}{k_B T}} + 1} \tag{28}$$

$$n = \frac{N}{V} = \int_0^\infty D(E) f(E, T=0) dE = \int_0^{E_F} D(E) dE = \frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \frac{2E_F^{\frac{3}{2}}}{3} \tag{29}$$

$$\begin{aligned} E_F &= \frac{\hbar^2}{2m}(3\pi^2 n)^{\frac{2}{3}} && \\ k_F &= (3\pi^2 n)^{\frac{1}{3}} && \text{Fermi-Wellenvektor} \\ v_F &= \frac{\hbar}{m}(3\pi^2 n)^{\frac{1}{3}} && \text{Fermi-Geschwindigkeit} \\ T_F &= \frac{E_F}{k_B} && \text{Fermi-Temperatur} \end{aligned} \tag{30}$$

$$u_0 = \int_0^\infty E D(E) f(E, T=0) dE = \int_0^{E_F} E D(E) dE = \frac{3n}{5} E_F = \frac{3n}{5} k_B T_F \tag{31}$$

$$\delta u(T) = u(T) - u_0 = n k_B T \cdot \frac{T}{T_F} \tag{32}$$

$$c_V^{el} = \left(\frac{\partial u}{\partial T}\right)_V \approx \frac{2 n k_B T}{T_F} \approx \gamma T \tag{33}$$

$$c_V^{ges} = \gamma T + \begin{cases} 3n_A k_B & \text{für } T > \Theta \\ \beta T^3 & \text{für } T \ll \Theta \end{cases} \tag{34}$$

Dispersionsrelation quasi-freie Elektronen:

$$E_{\boldsymbol{K}} = \frac{\hbar^2 k^2}{2m} = E_{\boldsymbol{k}+\boldsymbol{G}} = \frac{\hbar^2}{2m}|\boldsymbol{k}+\boldsymbol{G}|^2 \tag{35}$$

$$\frac{d\boldsymbol{v}}{dt} = \frac{1}{\hbar}\frac{d}{dt}\left(\frac{\partial E(\boldsymbol{k})}{\partial \boldsymbol{k}}\right) = \frac{1}{\hbar}\frac{\partial^2 E(\boldsymbol{k})}{\partial \boldsymbol{k}\partial \boldsymbol{k}}\frac{\partial \boldsymbol{k}}{dt} = \frac{1}{\hbar^2}\frac{\partial^2 E(\boldsymbol{k})}{\partial \boldsymbol{k}\partial \boldsymbol{k}}\boldsymbol{F} \tag{36}$$

$$\left(\frac{1}{m^*}\right)_{ij} = \frac{1}{\hbar^2}\frac{\partial^2 E(\boldsymbol{k})}{\partial k_i \partial k_j} \tag{37}$$

Bloch-Oszillationen:

$$|\boldsymbol{v}| = \frac{e\mathcal{E}}{\hbar} \tag{38}$$

$$T_B = \frac{\frac{2\pi}{a}}{\frac{e\mathcal{E}}{\hbar}} = \frac{h}{ae\mathcal{E}} \tag{39}$$

Drude Modell: Bewegung Elektronen $\Longleftrightarrow$ Kinetische Gastheorie

$$m\frac{d\boldsymbol{v}}{dt} = -e\mathcal{E} - m\frac{\boldsymbol{v}_d}{\tau} \tag{40}$$

$$\boldsymbol{v}_d = -\frac{e\tau}{m}\boldsymbol{\mathcal{E}} = -\mu\boldsymbol{\mathcal{E}} \tag{41}$$

$$\boldsymbol{j} = -en\boldsymbol{v}_d = \frac{ne^2\tau}{m}\boldsymbol{\mathcal{E}} = ne\mu\boldsymbol{\mathcal{E}} \tag{42}$$

$$\sigma = \frac{j}{\mathcal{E}} = \frac{ne^2\tau}{m} = ne\mu \tag{43}$$

$$l = v_F \tau \tag{44}$$

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} = \rho_D + \rho_G = \frac{m}{ne^2\tau_D} + \frac{m}{ne^2\tau_G(T)} \tag{45}$$

$$\frac{\rho(300K)}{\rho(4.2K)} \tag{46}$$

$$B_C(T) = B_C(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right] \tag{47}$$

$$\sigma = e(n\mu_n + p\mu_p) \tag{48}$$

$$\begin{aligned} n &= \int_{E_L}^{\infty} D_L(E) f(E,T) dE \\ p &= \int_{-\infty}^{E_V} D_V(E)[1 - f(E,T)] dE \end{aligned} \tag{49}$$

$$\begin{aligned} D_L(E) &= \frac{1}{2\pi^2}\left(\frac{2m_n^*}{\hbar^2}\right)^{\frac{3}{2}} \sqrt{E - E_L} \\ D_V(E) &= \frac{1}{2\pi^2}\left(\frac{2m_p^*}{\hbar^2}\right)^{\frac{3}{2}} \sqrt{E_V - E} \end{aligned} \tag{50}$$

$$n = \int D_L(E) f(E) dE = \frac{1}{2\pi^2}\left(\frac{2m_n^*}{\hbar^2}\right)^{\frac{3}{2}} e^{\frac{E_F}{k_B T}} \int_{E_L}^{\infty} \sqrt{E - E_L} e^{\frac{E}{k_B T}} \tag{51}$$

Massenwirkung:

$$\begin{aligned} n =& \mathcal{N}_L e^{-\frac{(E_L - E_F)}{k_B T}} \\ p =& \mathcal{N}_V e^{\frac{(E_V - E_F)}{k_B T}} \end{aligned} \tag{52}$$

Für intrinsische Halbleiter:

$$n \cdot p = \mathcal{N}_L \mathcal{N}_V e^{-\frac{E_g}{k_B T}} \tag{53}$$

$$n_i = p_i = \sqrt{\mathcal{N}_L \mathcal{N}_V} e^{-\frac{E_g}{2 k_B T}} \tag{54}$$

$$E_F = \frac{E_L + E_V}{2} + \frac{k_B T}{2} \ln\left(\frac{\mathcal{N}_V}{\mathcal{N}_L}\right) = \frac{E_L + E_V}{2} + \frac{3}{4} k_B T \ln\left(\frac{m_p^*}{m_n^*}\right) \tag{55}$$

Dotierte Halbleiter: